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# Problems of Control Motion of Solar Sail Spacecraft in the Photogravitational Fields

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## Abstract

The problems of spacecrafts with a solar sail-controlled motion lead to the study of mathematical models for translational orbital motion and for the spaceship rotation about the mass center in photogravitational fields. There are opportunities to choose the optimal maneuvering conditions to realize orbital motion or to move to a given orbit point. The realization of the given optimal sail orientation about the sunlight flow allows to obtain motions in the vicinity of a possible relative equilibrium or stationary state. This realization also takes into account the stability change according to the process models with perturbations. For the motion control, we can change the properties, dimensions, or location of sail system elements. The spacecraft flights using light pressure are already a reality. Such space sailing ships may soon be used to fly to the big and small planets, for asteroids and comets meeting, to form special motion conditions in the vicinity of the Sun or the Earth. New technologies will bring visible benefits for solving complex problems, based on the direct use of practically unlimited source of solar energy.

**Keywords:** spaceflight, solar sail, control, stability

## 1. Introduction

### 1.1 Solar sail history

The principle of movement in space by solar sail is based on the light pressure effect on all the bodies, which experimentally are detected and measured by Russian scientist P.N. Lebedev in 1899 [1]. The development of the first engineering project of a space flight under a solar sail belongs to a Russian scientist and engineer Friedrich Zander (1887–1933).

In 1920, F.A. Zander and K.E. Tsiolkovsky (1857–1935) suggested that a very thin flat sheet, illuminated by sunlight, is able to achieve high speeds in space. As for the question of whether this property of photons can be used for space motion, they answered positively. The idea to use this effect for space flight was advanced by scientist and inventor F.A. Zander in 1924 [2]. He proposed the construction of solar sails and developed the foundation of the spacecraft motion theory. He was the first person to realize the potential of large specularly reflecting surfaces for space flight, proposed to build solar sails and developed the basis of the theory of motion of spacecraft. It can be considered the founding father of the innovative concept of

the solar sail, which was developed in two manuscripts but was not published until 1947. Flight spacecraft using light pressure energy is no longer a fantasy but the reality of the near future [3, 4]. The first attempt of the project implementation and deployment of a solar sail in space was made in 1993.

Over the years there has been appeared numerous studies of mathematical models of the motion and possible new versions of the form of solar sails [5, 6]. The technology of large-scale designs of sunlight reflectors is still in its initial state. The first practical development of flights with a solar sail began in the 1970s of the twentieth century. Of particular concern was the planned for 1986 flight of a spaceship on a solar sail to meet with Halley's comet. The first attempt to implement the project and deploy a solar sail in space was completed in 1993; a 20-m-diameter mirror sail was successfully deployed on the "Progress M-15" cargo ship as a result of the space experiment "Znamya-2."

If a real sail is at an angle to the flow, the force vector will be directed almost normal to the plane of the sail with a good reflection coefficient. The force of light pressure on the mirror at the same time would be almost twice as much on the black sail of equal area, which completely absorbs the radiation. If the solar sail is made of black material, its thrust is twice less than perfectly mirrored. In this case, the force is not directed along the normal to the surface and the direction of flow of sunlight. Light pressure forms a central photogravitational field, which operates when spacecraft sails moving in an interplanetary space. This will allow to select optimal control during maneuvering.

There are projects using the solar sail to put the spacecraft into geosynchronous orbit in the equatorial plane or to maintain motion in the orbit plane, which is parallel to the equatorial plane and has a nonzero latitude. These latitudinal orbits can create new systems for the deployment of satellite communication systems. One of the possible tasks is the creation of a cosmic solar screen located near the Lagrange point L1 of the Sun-Earth-spacecraft system, which can be useful for monitoring the global temperature of the Earth. Many promising projects for using the solar sail are published.

The solar radiation flux creates a force locally uniform pressure field on the surface of the spacecraft sails. If the surface has a symmetry, and the point of application of the resultant coincides with the center of mass of the spacecraft design, then any initial position relative to the light flux is a state of neutral equilibrium and peace. The action of other forces, even small in size, can produce disturbing moment, which can cause rotation about the center of mass. To damp or compensate the disturbance and to maintain the sail correct position with respect to the light flux it is necessary to use an additional control force. The special sails design allows to solve control and the spacecraft stability problems. Thrust vector and point spacecraft relative to the main body can be changed or if the value of the surface properties of the solar sail and arrangement of the elements may vary with respect to the device using the additional devices.

Note the basic problems [6, 7] of engineering and realization of flights of the spacecraft with the solar sail:

- Creation of an effective reflective polymer film for sails.
- Packing of sails in special containers for delivery to space.
- Take into account the restrictions on the total weight of the spacecraft with a sail at launch.
- Special tools for deploying sails of a large area in the working position.

- The formation of special frame elements for control and support of the sail.
- Providing required initial orientation of the elements of the sail.
- Motion control and stability of the given position in flight.

Only by resolving all problems can we talk about space travel and maneuvering in reality. It should provide a sufficiently sophisticated control sail itself, as desired by changing its size, shape, and position relative to the main body. We can use the sail elements that can change the reflection coefficient of the surface of a given program. Successful construction has been recognized as the slit-like sails helicopter rotor, each blade which is rolled out from the container and can be rotated radially relative to the axis of fixing at a predetermined angle. In some projects, the spacecraft with the sail offers spinning relative to the main axis for the stability of the sail shape under the action of light pressure and disturbing forces.

The most successful may be the design of the sail system, which provides the installation of the desired orientation and control over its preservation. After creating and placing such mirror elements with certain proportions in orbit, we obtain orientation stability relative to the Sun for coplanar trajectories of the transition to a new orbit or preservation of a given final orbit. More complex options and models make it possible to program sequential control of the orbital or rotational motions of a spacecraft with a solar sail.

The efficiency of using solar sails is primarily associated with the angle of their orientation relative to the beam of rays. In a complex system of mirror surfaces, the beam path can be adjusted in such a way that the direction of the incident and reflected light beam is independent, creating new opportunities for a given direction of the thrust vector.

Only having solved all the problems, we can talk about space travel. This can be ensured by a difficult choice of controlling the elements of the sailing system, which will allow you to change the size or shape and position relative to the main body and the flow of sunlight. We can use sail elements that can change the reflection coefficient of a part of the surface according to a given program.

Of interest is the unique possibility of functioning in special zones in the vicinity of the Sun, even near the solar corona, where the sail can simultaneously play the role of a reliable heat-resistant screen that protects the main instrument compartment from overheating. This design will be indispensable for studying solar space and observing sunspots from close range. The disclosure of the sail creates a force in the direction that compensates for the force of gravity and, therefore, will change the parameters of a possible orbit.

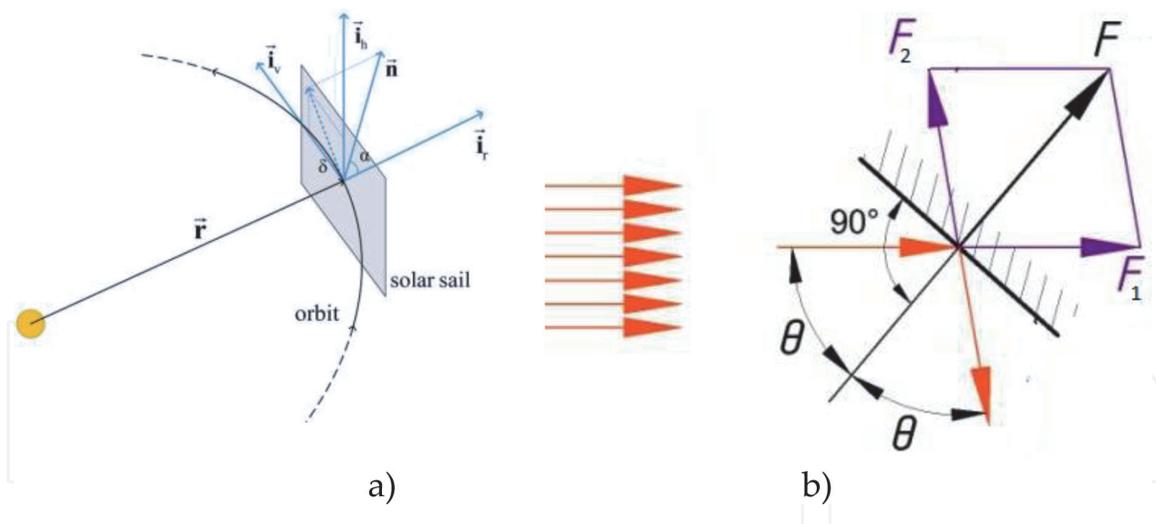
Many options relate to near-Earth space maneuvers. The use of the solar sail is possible to put the spacecraft into a given orbit and to support further stable operation of satellite systems without additional fuel consumption.

## 1.2 Forces and their moments

The resulting light flux pressure force  $\vec{F}_i$  is proportional to the surface  $S_i$  area of the sail elements with a parameter (1), which is determined by a reflection coefficient  $q_i$  and inversely proportional to the square of the distance  $r$  from the Sun (**Figure 1a**).

It also depends on the direction force (**Figure 1b**) of the vector normal  $\vec{n}$  to the surface of the respective element  $b(\vartheta_i)$  relative to the radial direction with angle  $\vartheta$ .

Stability may be ensured by the torques of pressure forces about the center of mass, which can modify the value at change the settings. The main vector of the



**Figure 1.**  
The formation of the resultant force of the light flux acting on the surface of the sail.

forces and the sum of the moments of all the forces  $\vec{F}_i$  acting on the sail with a relative position  $\vec{\rho}_i$

$$\vec{F} = \sum_i \vec{F}_i = \sum_i q_i S_i \frac{b(\vartheta_i)}{r^2} \vec{n}(\vartheta_i), \quad \vec{M} = \sum_i \vec{\rho}_i \times \vec{F}_i(\vartheta_i). \quad (1)$$

These values determine the motion of the spacecraft center of mass and rotation relative to the orbital system that accompanies motion in a central field.

The main vector of the moment of acting forces relative to the center of mass may differ from zero for light pressure forces if the elements have different areas or angles of mutual arrangement. This determines the ability to return in the right direction in the event of a change in orientation by random interference or the use of new elements of the basic layout of the spacecraft for orientation in the right direction [8, 9].

## 2. Equations of motion

### 2.1 Different types of equations

The equations of motion with allowance for disturbances can be represented in different forms, based on models of the problem of two or three bodies using convenient coordinate systems and basic parameters. Heliocentric flight to planets, asteroids, or to the Sun can be considered, to a first approximation, the motion in a photogravitational field as a two-body problem under the action of the additional light pressure of the rays on the sail surface for a fixed angle of the normal position, taking into account the influence of additional disturbances.

Motion in a photogravitational field can be considered as a two-body problem or a central force field without taking into account the influence of other forces, when the action of additional light pressure from the rays reduces the influence of gravitational interaction. This change is especially noticeable for the case of a large sail surface.

When creating orbits near the Earth or to place a spacecraft at the libration points of the Sun-Earth-spacecraft system, it is necessary to use a more general model of the photogravitational restricted three-body problem, which takes into account the movement of two main bodies, as well as the direction of the

propagation of light radiation and direction from the spacecraft to the gravitational center: in this case it may not be the same.

Control using solar sails leads to complex problems and solving equations of mathematical models. There are basic versions of the equations of motion of the spacecraft in the central gravitational field, taking into account perturbations depending on the choice of the reference frame, absolute Cartesian, spherical and cylindrical coordinates, or Kepler's elements [5, 6, 10] for the orbit.

Changes in Cartesian coordinates  $x_i$  of the spacecraft center of mass in the absolute coordinate system based on the main operating force of gravity and the center of the field of light pressure at movement in three-dimensional case can be described by a second-order equation:

$$\frac{d^2x_i}{dt^2} + \frac{\mu}{r^3}x_i = \frac{\partial U}{\partial x_i} + F_i, \quad i = 1, 2, 3, \quad (2)$$

where we have used the notations  $x$ , the Cartesian coordinates;  $r$ , the module of the radius vector;  $\mu$ , the gravitational parameter;  $U$ , the force function of potential forces of the considered disturbances; and  $F_i$ , the nonpotential acceleration and control, including of the light pressure forces in projections on the axis coordinate system or jet forces on the active phases of orbit.

You can use the polar coordinates  $(r, \varphi)$  in the study of movement in the orbital plane:

$$\frac{d^2r}{dt^2} + \frac{\mu}{r^2} - r(\dot{\varphi})^2 = P_1, \quad \frac{d}{dt}(r^2\dot{\varphi}) = P_2, \quad (3)$$

where  $P_i$  ( $i = 1, 2$ ) is the radial and transversal components of the perturbing acceleration, which depend on the installation angle of the sail elements to implement the control law. The position relative to the flow of sunlight is taken into account at the corresponding pressure value far from the Sun.

The control algorithms  $u(t)$  are numerous and are determined through the parameters of the initial and final orbits or by the tasks of maneuvering the spacecraft in the process of movement. A fixed constant angle will determine the change in the parameters of the orbit. Without taking into account all perturbations and controls, we can use the classical solution of the two-body problem.

The movement in the central gravitational field has a solution, which in the absence of disturbing forces is determined by the initial values of the radius vector, velocity vector, and the gravitational parameter of the central body. They determine the constant Kepler elements  $k = (a, e, i, \Omega, \omega, M_0)$  which allow us to calculate the Cartesian coordinates  $x_i(t)$  and components  $v_i(t)$  of the velocity vector for the unperturbed motion at any time using the following formulas:

$$\begin{aligned} x_1 &= r(\cos u \cos \Omega - \sin u \sin \Omega \cos i), \\ x_2 &= r(\cos u \sin \Omega + \sin u \cos \Omega \cos i), \\ x_3 &= r \sin u \sin i, \\ v_1 &= \alpha(\cos u \cos \Omega - \sin u \sin \Omega \cos i) \\ &\quad - \beta(\sin u \cos \Omega + \cos u \sin \Omega \cos i), \\ v_2 &= \alpha(\cos u \sin \Omega + \sin u \cos \Omega \cos i) \\ &\quad - \beta(\sin u \sin \Omega + \cos u \cos \Omega \cos i), \\ v_3 &= \alpha \sin u \sin i + \beta \cos u \sin i. \end{aligned} \quad (4)$$

Notation used here

$$r = a(1 - e \cos E), \quad p = a(1 - e^2),$$

$$\alpha = \sqrt{\frac{\mu}{p}} er^{-1} \sin \vartheta, \quad \beta = \sqrt{\mu p} r^{-1},$$

and Kepler's equation

$$E - e \sin E = M_0 + n(t - t_0) = M. \quad (5)$$

Moving time between two points of the orbit can be determined from the above equation which is called the equation of Kepler (5).

The equations of motion while taking into account the perturbations can be represented as osculating elements  $k(t) = (a, e, i, \Omega, \omega, M_0)$ . The orbit elements for the perturbed motion of the spacecraft are functions of time  $k(t)$ . You can use Euler differential equations in which the functions on the right-hand sides of the equations are determined by the current values of the elements and the projections of the disturbing acceleration on the axis of the orbital coordinate system.

The contribution of radiation pressure is determined by the angle  $\phi$  of deviation, the normal vector  $\vec{n}$  from direction  $\vec{r}_0$  of flow. If we turn the flat mirror sail at an angle to the rays, the momentum transferred to the solar sail will be directed almost perpendicular to the reflective surface. Part of the momentum directed parallel to the sail, the photons will remain at home, so that the sail will get less than in the full disclosure of the rays. Turning the sail, we are able to control the direction of the thrust vector. However, for it to pay its value. If the vector of normal for flat sail is perpendicular to the flow of rays, the sail does not give any traction. The projections of the vector on the radial and transverse directions will be influenced by a change in the parameters of the orbit motion. The projection on the normal to the plane of the orbit will allow to change its inclination with respect to the initial position. Acceleration, which tells the stream of rays, also depends on the ratio of the area of the sail to the weight of the entire structure and the surface properties.

Equations Hill-Clohessy-Wiltshire [11–15] which managed orbital motion of the moving coordinate system in the spatial case are

$$\begin{aligned} \ddot{x} + 2\omega\dot{y} &= u_x(t), \\ \ddot{y} - 2\omega\dot{x} - 3\omega^2y &= u_y(t), \\ \ddot{z} + \omega^2z &= u_z(t), \end{aligned} \quad (6)$$

The solution nonlinear equations (6) can be presented in the form of changes or deviations from the given movement of the reference point and then add a particular solution with the selected control function. The solution of a homogeneous system can be represented as the following (7) system of six equations:

$$\begin{aligned} x(t) &= \left(x_0 - 2\frac{y_0}{\omega}\right) - 3\omega t \left(2y_0 + \frac{x_0}{\omega}\right) + 2\left(3y_0 + \frac{2x_0}{\omega}\right) \sin \omega t + 2\frac{y_0}{\omega} \cos \omega t, \\ y(t) &= 2\left(2y_0 + \frac{x_0}{\omega}\right) - \left(3y_0 + \frac{2x_0}{\omega}\right) \cos \omega t + \frac{y_0}{\omega} \sin \omega t, \\ z(t) &= z_0 \cos \omega t + \frac{z_0}{\omega} \sin \omega t, \end{aligned} \quad (7)$$

$$\begin{aligned}\dot{x}(t) &= -3\omega\left(2y_0 + \frac{\dot{x}_0}{\omega}\right) + 2\omega\left(3y_0 + \frac{2\dot{x}_0}{\omega}\right) \cos \omega t - 2\dot{y}_0 \sin \omega t, \\ \dot{y}(t) &= \omega\left(3y_0 + \frac{2\dot{x}_0}{\omega}\right) \sin \omega t + \dot{y}_0 \cos \omega t, \\ \dot{z}(t) &= \dot{z}_0 \cos \omega t - z_0 \omega \sin \omega t,\end{aligned}$$

The unfolding of the sails on a circular heliocentric orbit will lead to the fact that the light pressure partially compensates the Sun's gravity. This is a reason to use a mathematical model of photogravitational force field [3, 7, 10].

The orbital elements for perturbed motion of the spacecraft are functions of time. We can use the differential equations of Euler, where the right-hand sides are determined by the current values of the elements  $k(t) = (a, e, i, \Omega, \omega, M_0)$  and projections of the perturbing acceleration  $P_i$  on the axes of the orbital coordinate system:

$$\begin{aligned}\frac{da}{dt} &= 2a^2(e \sin \vartheta P_1 + p r^{-1} P_2), \\ \frac{de}{dt} &= p(\sin \vartheta P_1 + \cos \vartheta P_2 + \cos E P_2), \\ \frac{di}{dt} &= r \cos \vartheta P_3, \quad \frac{d\Omega}{dt} = r \sin i \sin^{-1} i P_3, \\ \frac{d\omega}{dt} &= e^{-1}[(r + p) \sin \vartheta P_2 - p \cos \vartheta P_1] - \cos i \frac{d\Omega}{dt}, \\ \frac{dM_0}{dt} &= \sqrt{e^{-2} - 1}[(p \cos \vartheta - 2er)P_1 - (r + p) \sin \vartheta P_2].\end{aligned}\tag{8}$$

Here  $P_i$  ( $i = 1, 2, 3$ ) are the components of the disturbing acceleration in the projection on the axis of the orbital coordinate system. They depend on the installation angle of the sail elements for the implementation of the control law and determine a further change in the parameters of the orbit. The position relative to the stream of sunlight is taken into account with the corresponding value of light pressure far from the Sun.

The third and fourth equation of system shows that the plane orbit state is maintained if there is no projection of the disturbing forces in the normal to the plane. The behavior and properties of the solutions are analyzed in a dynamic system, which are simulating the controlled processes. The research methods of nonlinear continuous or discrete systems' quality of the movements, absolute or asymptotic stability, can be obtained [6, 12, 16].

## 2.2 Control of motion

The influence of solar pressure on the sail is determined by the angle of deviation of the normal vector to the surface from the direction of flow. If the plane of an ideal specular sail is at an angle to the rays, then the momentum transmitted to the solar sail will be directed almost perpendicular to the reflecting surface. By turning the elements of the sailing system, you can control the direction of the thrust vector. The arrangement of elements relative to the housing can be changed with the help of electric motors, supporting their work on the basis of solar batteries.

If the spacecraft with the folded solar sail is already delivered into orbit around the Earth or move around the Sun, the container of the sails will provide disclosure for the spacecraft new thrusters, providing a virtually unlimited supply of energy.

However, the sail has one major drawback: unlike jet engines, we cannot use its thrust in any direction with the same efficiency. It is necessary to orient the sail in a special way, to achieve the desired changes in the orbital parameters of outer space.

When you turn the sail so that the photons are reflected back relative to the direction of orbital motion, we get an additional force that gradually accelerates the spacecraft, which will move in a spinning spiral. When you turn the sail in a different direction, you get a decrease in speed or braking on the way to the sun.

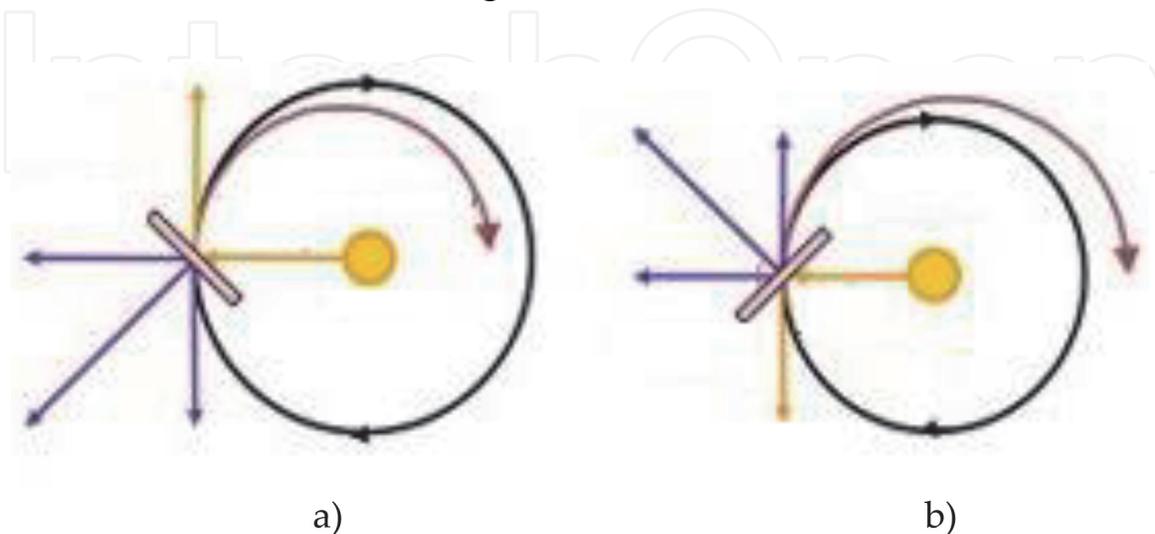
To change the inclination of the orbital plane of the spacecraft using sails, it is necessary to direct the reflected flow perpendicular to the initial plane. In addition, the elongated elliptical orbit can rotate sequentially, changing the longitude of the pericenter relative to the central body, so that over time, it approaches the orbit of an asteroid or comet for a possible encounter [17].

Even more interesting maneuvers can be performed using a solar sail in the near-Earth space, as the propagation direction of light emission in this case coincides with the direction of the center of gravity. In addition, throughout the year the Sun makes a complete revolution in the celestial sphere relative to the Earth, so be patient, you can wait for the right time of the year for optimum flexibility and translate using the spacecraft sails in the desired orbit.

We get the opportunity to control the movement by changing the direction of the thrust vector of the current light pressure when the sail plane is rotated. Vector projections of the force on the radial and transverse directions will affect the change in the basic parameters of the orbit (size, shape) in the process of movement.

The projection of the force to the normal to the orbit plane changes the inclination relative to the initial position of the orbit plane. Acceleration also depends on the ratio of the sail area to the mass of the entire structure and surface properties.

Control algorithms  $u(t)$  are numerous. They are determined by the parameters of the orbit or the conditions and purpose of the maneuver. Motion in a gravitational field can be precisely determined if disturbing forces are absent. Then the orbit is determined by the initial values of the radius vector, velocity vector, and gravitational parameter of the central body. For a fixed position of the sail, we can consider a photogravitational field with a small perturbation, which determines the corrections to the parameters of the orbit, reducing the size of the orbit (**Figure 2a**) or increase the size of the orbit (**Figure 2b**).



**Figure 2.**

*Motion control is carried out by the position of the solar sail, when the vector of light pressure forces determines braking (a) or acceleration (b). In case (a) there will be a decrease in the size of the orbit; case (b) increases the size of the orbit.*

The presence of perturbations of a periodic or random nature can change the nature of the solutions of such systems. The properties of solutions of dynamic systems are determined by the selected feedback control function. The orbital stability of trajectories or the stability with respect to a part of variables [11–13] is also considered.

### **2.3 The stability of the orbital motion**

The system is called stable if it returns to the equilibrium or rest state after the termination of external influences that moved it out of this position. If after the termination of the external impact of the system does not return to a state of equilibrium, it is unsustainable. Stable equilibrium position becomes asymptotically stable with the addition of dissipative forces with complete dissipation.

Determining the motion of any mechanical system is often required to assess the stability and control of motion states. The strict definition of a stable equilibrium position and other solutions of dynamical systems were given in 1892 by the Russian scientist Lyapunov [6–9, 14, 18].

The movement or behavior of the solution of the dynamical system is called Lyapunov stable if small variations in the initial data from the reference phase variables selected for the study, solving a system of differential equations, lead to small deviations in the future. If the deviation over time tends to be zero, the reference solution is called asymptotically stable.

The system is called instable in case when even very small perturbation influence leads to large deviations or change the motion character, including the equilibrium position displacement, which is not stable if the velocity initial value is different from zero.

We also consider the stability of the orbital trajectory or stability of some of the variables [6–12]. In this case, it appears that the phase trajectory and its projection onto the corresponding subspace are close enough to the base path, although the representative points can be arbitrarily disperse, away from each other over time. Periodic solution of the system is not asymptotically stable. But if in such system all multipliers' modules but one are less than one, then according to Andronov-Witt theorem [14], the trajectory of the system periodic solution is asymptotically orbital stable.

Stability with respect to part of variables for partial differential equations involved Rumyantsev [19, 20], who published an article on the analogue of the theory of the second Lyapunov method for the stability problems for some of the variables [21]. He and his students developed methods for research on some of the variables of stability problems.

If the dynamic equations are written in the canonical form, and there are  $n$  first integrals, then the Arnold's theorem [12–14], all phase trajectories lie on the  $n$ -dimensional torus, and the motion are conditionally periodic system. This set is called the equilibrium or stationary state of motion of the system.

In the field of action of the geopotential excluding other perturbing forces exists stable equilibrium for body position while maintaining the orientation of the major axis of the ellipsoid of inertia in the direction of the center of gravity.

### **2.4 Control of orientation motion**

In the Cartesian coordinate system for the main body, the known parameters of the axial moments of inertia are taken into account. In the field of the geopotential force without other perturbing forces, there are stable equilibrium positions for the

body while maintaining the orientation of the main axes (x, y, z) of the inertia ellipsoid with the main moments towards the center of attraction, taking into account the angular velocity of the orbit.

If we consider only the first linear approximation, then the equation of oscillations of the spacecraft with a small perturbation or deviation from the equilibrium position has the form, which at the next step turns into the equation of harmonic oscillations with additional control functions.

Therefore, small oscillations of the mathematical pendulum will be isochronous. We get the orbital stability of motion or stability with respect to part of the variables. For small vibrations in the vicinity of the equilibrium position, the damping effect of additional gyroscopic devices or motors can be used. The action of disturbances can be compensated by a change in the size or reflective properties of the elements of the sailing ship, as well as their relative position. This creates additional moments that can be used for control and stability.

In the case of possible oscillations of the satellite in the orbit plane while maintaining the orientation of the other major axis orthogonal to this plane, the law of change in kinetic momentum takes into account the effect of the Earth's gravitational field [9, 12, 16, 22]. This leads to the equation.

$$I_z \ddot{\vartheta} = M_z = 3\omega_0^2 (I_y - I_x) \sin \vartheta \cos \vartheta. \quad (9)$$

Let us denote  $\omega^2 = 3\omega_0^2 (I_y - I_x) (2I_z)^{-1}$ , where  $I_x, I_y, I_z$  are moments of inertia and  $\omega_0$  is the angular velocity motion on orbit and a new variable  $\varphi = 2\vartheta$ . Then we come to the ordinary equations of oscillations with perturbation. If we consider the first linear approximation only, the equation of oscillations of a spacecraft under a small perturbation or deviation from the equilibrium position has a form, which on the next step turns into the equation of harmonic oscillations with additional control function:

$$\ddot{\varphi} = -\omega^2 \sin \varphi + u(t, \varphi), \quad (10)$$

The period in linear approximation depends on the initial data. Therefore, small oscillations of mathematical pendulum

$$\ddot{\varphi} + \omega^2 \varphi = u(t, \varphi) \quad (11)$$

will be isochronous. We get the orbital stability of the movement or the stability with respect to the part of the variables. For the small oscillations in the neighborhood of the equilibrium position, it is possible to use the damping action of additional gyroscopic devices or engines [15, 17]. To damp the oscillations, a control is proposed in the form of piecewise constant functions

$$u(t, \varphi) = -u_{max} \text{sign}(\varphi) \quad (12)$$

of a relay type with a switching period, which is determined by the frequency  $\omega$ .

Euler's equation (8) of rotation of a rigid solid about a center of mass show that there are three options for steady motions in the form of stationary rotations about the three principal axes of the ellipsoid of inertia when the two components of the angular velocity are equal to zero and the third is a constant [7, 8].

The Euler equation in the general case [8, 22] determines the rotation of a body under the action of moments of force relative to the center of mass. In the case of the body rotation around the instantaneous axis we need the forces moment about the axis to turn the body or to stop rotation.

### 3. Conclusions

When a rigid body moves in the Earth's gravitational field, there are location options for stable orientation relative to the orbital system. Consideration of the effect of light pressure on a sailing spacecraft leads to the appearance of other possible provisions or stability conditions that can be used in the process of motion control.

The perturbations action can be compensated by the varying of size or reflective properties of spacecraft sail's elements, as well as their mutual arrangement. It creates additional torques, which can be used for a control and stability.

In the case where the main forces can be considered the gravitational interaction with the Sun and its light pressure, you can use the model photogravitational central field to interplanetary space flights to asteroids or other planets.

In the case of movement in orbits near the Earth, the directions of the main acting forces (gravity and light pressure) do not coincide. However, as a first approximation, it can be assumed that the luminous flux determines the force of a constant value, which is directed collinearly to a straight line passing through the two main bodies of the Sun-Earth-spacecraft system in a restricted circular three-body problem.

Then amendments to control the orientation by the changing in the angular position or shape of the sail can be taken into account.

The particular interest is the case of placing the spacecraft in the vicinity of the Euler-Lagrange libration points where a small disturbing force determine the motion character and stability. Optimal control theory leads to complicated formulations of the problem for the solving of additional equations of mathematical models that can use the Pontryagin maximum principle or Bellman equation [15, 18] for different cases and tasks. There are analytical and numerical methods of research and analysis of the basic properties of the equations that allow to obtain exact or approximate solution of set of the necessary conditions of the extremum of the quality functional [16, 17].

Solar sailing in space is a matter of the future. This will require sophisticated design solutions and space technology [7–12, 23–27]. A special spacecraft's control uses solar sail as a motioned forcement of the thruster units.

Then we take into account corrections for attitude control due to changes of the angular or rotation the sail geometry.

Thus, the nonlinear equations of motion will include a permanent disturbance, which can easily be taken into account. Of particular interest is the case of location the spacecraft in the vicinity of the libration points, where small perturbation forces will be determined by stability conditions.

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