Chapter

Optical Sensor for Nonlinear and Quantum Optical Effects

Antônio Carlos Amaro de Faria

Abstract

In this chapter, the main foundations for the conception, design, and the project of optical sensors that explore the effects of nonlinear and quantum optics are presented. These sensors have a variety of applications from the design of waveguides with self-selection of propagation modes to signal processing and quantum computing. The chapter seeks to present formal aspects of applied modern optics in a detailed, sequential, and concise manner.

Keywords: optical sensor, nonlinear optics, quantum optics, optical fiber, optical signal processing

1. Introduction

Classical electrodynamics is the basis for the analysis and formulation of electromagnetic waves. From the equations of Maxwell, it is possible to obtain the equation of the movement of the electric and magnetic fields whose solution describes the propagation of the electromagnetic wave. This formal treatment was originally developed by Maxwell [1], who verified that the electromagnetic wave propagated with the speed of light provided that the optics could be described from the electromagnetism. The medium through which the electromagnetic wave propagates responds in various ways to the electromagnetic field. This response depends on how the atoms and molecules are arranged spatially composing the constituent medium and how the interaction or scattering of the electromagnetic wave through the medium will occur. In other words, the way the medium responds to the electromagnetic excitation is contained in the middle polarization due to the propagation of the electromagnetic wave. It is in this context that some recent analyses have discovered some solutions from the nonlinear response of the medium to the propagation of the electromagnetic wave which may lead to an approach of some quantum effects from a nonlinear treatment of electromagnetism in the material medium.

The propagation of optical pulses through waveguides such as optical fibers can give rise to nonlinear optical effects and quantum effects. The appropriate modeling of these effects can be used for the development of sensors to the optical fiber whose resolution can be regulated properly. In addition, the method allows the selection of propagation modes by selecting the desired modes by knowing the band gap of the waveguide or the photonic crystal.

The development of sensors to the optical fiber is based on the propagation of optical pulses through waveguides like optical fibers and photonic crystals. The propagation of the pulses through waveguides can generate nonlinear and quantum effects as Raman and Brillouin effects. We have analytically modeled these effects from the Maxwell equations on dielectric media describing the propagation of these optical pulses by developing a model that can be implemented computationally for the processing and propagation of these optical signals. The optical sensor can select the modes of propagation of the optical beams through the natural conduction band of the photonic crystal since the function of our model, the so-called optical potential, describes the optical light scattering through the crystal [2].

2. Modeling the optical lattice

The modeling of an optical system is very important for the design and development of many applications of optics in electronics, photonics, integrated optics, and an array of devices based on the light. Nonlinear effects from the interaction of light with matter in waveguides and photonic crystals may be suitable for a variety of optical applications [3–5]. Quantum effects such as the Raman effect and the Brillouin effect can be conveniently dealt with by exploring the nonlinear aspects of optical wave propagation in waveguides. Effects, e.g., self-focusing and low dispersion, of a guided beam can be applied and exploited in several technologies that use waveguides and sensors to the optical fiber [6, 7]. Consider an optical field given by

$$E\left(\vec{r},t\right) = A\left(\vec{r}\right)\exp\left(i\beta_0 z\right). \tag{1}$$

In Eq. (1), β_0 is the propagation constant, and beam one propagates along the z direction and self-focuses along the transverse directions x and y. The function $A[r(x,y,z)]$ represents the evolution of the beam envelope. The nonlinearity refers to nonlinear medium polarization that is the change of the refraction index of the medium that is responsible by the self-focusing of the pulse.

Considering the Maxwell equations and the medium polarization, one can obtain the generalized nonlinear Schrödinger (NLS) equation that describes soliton solutions [8]:

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \pm F\left(|u|^2 u\right) = 0. \tag{2}$$

In Eq. (2), u is proportional to electric field. The term $F(|u|^2 u)$ represents a generalized term of nonlinear optical effects. The solutions of Eq. (2) can represent and describe the optical beam profile one propagating through an optical lattice and an optical fiber. It describes also quantum noise as Raman and Brillouin scattering in the optical system.

We will sequentially show how quantum effects from the interaction of an optical beam with the constituents of the waveguide or the crystal lattice through which it propagates can be described in the context of nonlinear electrodynamics. In other words, we will show an equivalence to quantum optics and nonlinear electrodynamics characterized by nonlinear polarization of the medium. We will demonstrate the equivalence between its properties and the properties of solutions from a dynamical action. This action can map optical systems, and the method is based on the variational principle whose solutions give the same from that of Eq. (2).

Considering the optical rays equation

$$\frac{d}{ds}\left(n\frac{d\vec{r}}{ds}\right) = \nabla n, \tag{3}$$

where n is the refraction index, s is a specific trajectory, and $\vec{r} = \vec{r}(s)$, the principle of least action may be written as

$$\delta \int nds = 0, \tag{4}$$

which is Fermat's principle for the paths of light rays. So the scalar wave equation in optical context can be written as

$$\nabla^2\phi - \frac{n^2}{c^2}\frac{d^2\phi}{dt^2} = 0, \tag{5}$$

that is satisfied by a plane wave solution

$$\phi = \phi_0 e^{i\left(\vec{k}.\vec{r} - \omega t\right)}. \tag{6}$$

The wave number k and the frequency ω are related by

$$k = \frac{2\pi}{\lambda} = \frac{n\omega}{c}. \tag{7}$$

So Eq. (6) can be written as

$$\phi = \phi_0 e^{i\vec{k}_0(nz - ct)}, \tag{8}$$

and it is assumed that $\vec{k}$ is in the z direction. The refractive index n can change very gradually in space, and a solution resembling the plane wave can be written as

$$\phi = e^{A\left(\vec{r}\right) + ik_0\left[L\left(\vec{r}\right) - ct\right]}, \tag{9}$$

and the quantities $A\left(\vec{r}\right)$ and $L\left(\vec{r}\right)$ are real functions.

Now we can establish a relation between this description of optical pulses, the NLS equation and the Schrödinger equation. This equivalence, in context of the classical level at least, between NLS equation solutions and the method that we propose in this work, can be constructed considering the relation

$$k_0(L - ct) = 2\pi\left(\frac{L}{\lambda_0} - \omega t\right). \tag{10}$$

In this sense, we are led to conclude that the energy E of the system and its frequency ω are proportional

$$E = h\omega. \tag{11}$$

The wave length and the frequency are related by

$$\lambda\omega = v, \tag{12}$$

and, now, considering that for a light ray one propagating in a certain medium with speed $v < c$, where c is the speed light in the vacuum

$$v = \frac{E}{p}, \tag{13}$$

and another simple relation

$$\lambda = \frac{v}{\omega}, \tag{14}$$

we come to an important relationship, using Eqs. (11) and (12):

$$\lambda = \frac{h}{p}. \tag{15}$$

So as an important consequence of this approach, we can write the wave Eq. (5) as

$$\nabla^2 \phi - \frac{1}{v^2}\frac{d^2\phi}{dt^2} = 0, \tag{16}$$

where v is the wave velocity in the medium of index n. This equation can yet be rewritten, considering a temporal dependence of kind $e^{-i\omega t}$ in the form

$$\nabla^2 \phi + \frac{4\pi^2}{\lambda^2}\phi = 0, \tag{17}$$

that is the time-independent wave equation. So it is perfectly acceptable that a certain optical field Ψ satisfies an equation of the same form as

$$\nabla^2 \psi + k^2 \psi = 0, \tag{18}$$

and doing the identifications

$$p = \sqrt{2m(E - V)}, \tag{19}$$

with p being the momentum and the terms inside the square root stands by kinetic energy or the difference between total energy E and the potential energy V of particle, we get the Schrödinger equation [9]

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0, \tag{20}$$

with m being the particle mass and h the Planck constant.

3. Brief introduction on optical solitons

3.1 Nonlinear Schrödinger equation

The equation that describes optical fields in a nonlinear medium is known as the nonlinear Schrödinger equation. In this section, we succinctly present the origin of the NLS equation for a CW beam propagating inside a nonlinear optical medium. From Maxwell equations in a nonlinear medium, one gets the wave equation for the electric field [7, 10]:

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 P}{\partial t^2} \tag{21}$$

with c being the speed of light in vacuum and ε_0 the vacuum permittivity. The total polarization $\vec{P}$ is

$$P(r,t) = P_L(r,t) + P_{NL}(r,t). \tag{22}$$

P_L is the linear part, and P_{NL} is the nonlinear part, given respectively by [8, 10]

$$P_L(r,t) = \varepsilon_0 \int \chi^1(t-t')E(r,t')dt' \tag{23}$$

and

$$P_{NL}(r,t) = \varepsilon_0 \int\int\int \chi^3(t-t_1, t-t_2, t-t_3) \times E(r,t_1)E(r,t_2)E(r,t_3)dt_1dt_2dt_3, \tag{24}$$

χ^1 and χ^3 are the first- and third-order susceptibility tensors. A general solution of Eq. (21) will be

$$E(r,t) = \frac{1}{2}\hat{x}\left[E(r,t)e^{-i\omega_0 t} + cc\right] \tag{25}$$

where $E(r,t) = A(r)e^{i\beta_0 Z}$ and $\beta_0 = k_0 n_0 \equiv \frac{2\pi n_0}{\lambda}$ are the propagation constants with the wavelength $\lambda = \frac{2\pi c}{\omega_0}$. The beam diffracts and self-focuses along the two transverse directions X and Y where X, Y, and Z are the spatial coordinates associated with r. The function A(X,Y,Z) is the evolution of the beam envelope; it would be a constant in the absence of nonlinear and diffractive effects. Nonlinear and diffractive effects and neglecting $\frac{d^2A}{dz^2}$ the beam envelope satisfy the following nonlinear parabolic equation:

$$2i\beta_0\frac{\partial A}{\partial Z} + \left(\frac{\partial^2 A}{\partial X^2} + \frac{\partial^2 A}{\partial Y^2}\right) + 2\beta_0 k_0 n_{nl}(I)A = 0. \tag{26}$$

Introducing the following variables

$$x = \frac{X}{\omega_0}, y = \frac{Y}{\omega_0}, z = \frac{Z}{L_d}, u = (k_0|n_2|L_d)^{1/2}A \tag{27}$$

where ω_0 is a transverse scaling parameter related to the input beam width and $L_d = \beta_0\omega_{0_2}$ is the diffraction length; Eq. (26) takes the form of a NLS equation:

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \pm |u|^2 u = 0. \tag{28}$$

Now one can consider the NLS equation in the form

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\left(\frac{\partial^2 u}{\partial x^2} + \sigma_d\frac{\partial^2 u}{\partial y^2}\right) \pm |u|^2 u = 0, \tag{29}$$

and the y independent form of it

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\frac{\partial^2 u}{\partial x^2}\pm|u|^2u=0. \tag{30}$$

Eq. (30) describes the spatial and temporal optical solitons as treated in Refs. [5,6, 11–15], respectively.

4. Nonlinear and quantum optical sensor principles

Nonlinear effects in optical fibers are common when, for example, increasing the power of the optical source. In this case, optical noise such as Raman effect and Kerr effect originate the interaction of optical fields with matter. These effects, depending on their technological application, may be undesirable. In our work, we discuss one set of solutions for optical fields whose nonlinear effects can be used to suppress certain propagation modes harmful to technological applications. On propagation of signals, for example, the Kerr effect can be a factor representing the loss of optical signal, and in this sense, using the solutions presented in this work, an optical network can be specially designed to suppress the distortion of the optical signal by the Kerr effect. The fiber or group of optical fibers can be designed so that the distortion of optical signals through nonlinear effects is eliminated. In another perspective, our method can be combined with space-division multiplexing (SDM) and nonlinear cancelation methods that offer the opportunity to reverse the effect of Kerr distortion [16]. The method developed in this work can be implemented to identify patterns of nonlinear modes which contribute to the distortion of the optical signal in the transmission system [17].

In this case, the propagation modes of the optical beam by the system will be conveniently selected and processed in the transmission link as shown in **Figure 1**. In this section, we will mathematically demonstrate how any optical signal can be transformed conveniently into appropriate optical pulse. In other words, any sign optical can be mapped to a field originally known.

It is appropriate to point out that the mathematical approach is necessary to detail the practical implementation of the recognition and processing of optical signals that can reach the optical beam level in the waveguide or any optical network.

4.1 Optical systems

The discovery of non-Hermitian observables with real spectrum in optical systems may provide an important and useful relationship between the crystal structure of an optical system and parity-time (*PT*) symmetry [18]. Optical

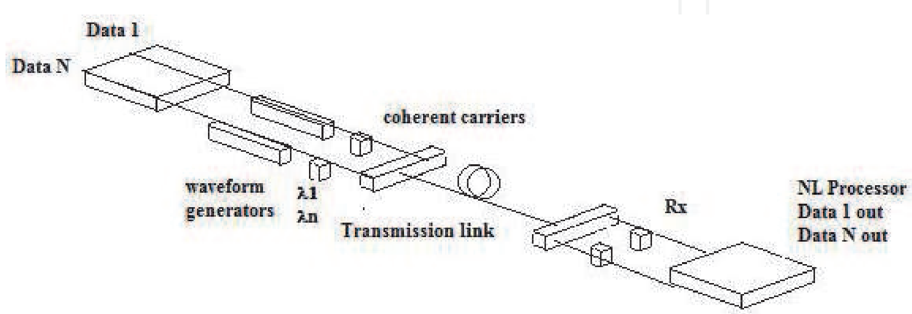

Figure 1.
Architecture of a transmission system. The nonlinear (NL) processor identifies **λI** *e* **λn** *modes and rectifies the optical distortions through the preprocessed patterns of non-Hermitian solitons diffracted by the optical network. Like any signal can be mapped, as will be presented in Section 4, the processor may recognize any optical pattern from the original signal, as a nonlinear distortion or optical soliton, and eliminate it, reverse it, or pass it on. In figure* **Rx** *is a signal identifier (basically an oscilloscope) with output to the Nonlinear Processor (NL Processor).*

potentials that have *PT* symmetry are potential that obey the following relation $V(x) = V^*(-x)$, where the superscript star denotes the conjugate complex. The symmetry PT may describe an optical system as a lattice structure or a waveguide. From the formal point of view, the light that propagates through a waveguide can be described by classical electrodynamics and the quantum mechanics, which seems to establish a deep relationship between them. However nonlinear optical phenomena lead to the same dynamic equations describing the propagation of quantum light in some materials media. We can explore the optical dynamics of a beam and waves propagating by an optical lattice which can be described by PT symmetric complex potentials. The optical beam propagation can be described by the Schröndinger-like equation:

$$i\frac{\partial\psi}{\partial z} + \frac{\partial^2\psi}{\partial x^2} + [V(x) + iW(x)]\psi + |\psi|^2\psi = 0. \tag{31}$$

Eq. (31) describes an optical beam one propagating in a self-focusing Kerr nonlinear PT symmetric potential as V(x) = V(−x) and W(−x) = − W(x). The solutions of Eq. (31) can be expressed as follows:

$$\psi(x, z) = \phi(x)\exp(i\lambda z), \tag{32}$$

where $\phi(x)$ is the nonlinear eigenmode and λ is the propagation constant. Then one obtains the following equation for the field $\phi(x)$:

$$\frac{\partial^2\phi}{\partial x^2} + [V(x) + iW(x)]\phi + |\phi|^2\phi = \lambda\phi. \tag{33}$$

In this point it is important to note that the optical system can be mapped by the band structure of the optical lattice or waveguide through the propagation constant λ. In fact the eigenvalue (λ) and the eigenmode $\phi(x)$ satisfy

$$\left[\frac{d^2}{dx^2} + F(x)\right]\phi = \lambda\phi, \tag{34}$$

whose solutions can be write in the form

$$\phi_n(k, x) = u_n(k, x)e^{ikx}, u_n(x + D, k) = u_n(x, k) \tag{35}$$

with $u_n(x,k)$ and satisfying

$$\frac{d^2u_n(x, k)}{dx^2} + 2ik\frac{du}{dx} + \left[F(x) - k^2\right]\phi = \lambda(k)\phi. \tag{36}$$

From Eq. (35) result important properties of optical systems described by symmetry *PT*, as the selection of propagation modes $\lambda(k)$.

4.2 Mapping optical systems

In order to show how the *PT* symmetric complex potentials can map optical lattice structures, consider Eq. (31) where

$$V(x) = V_0 \sec h^2(x), \quad W(x) = W_0 \sec h(x) \tanh(x), \tag{37}$$

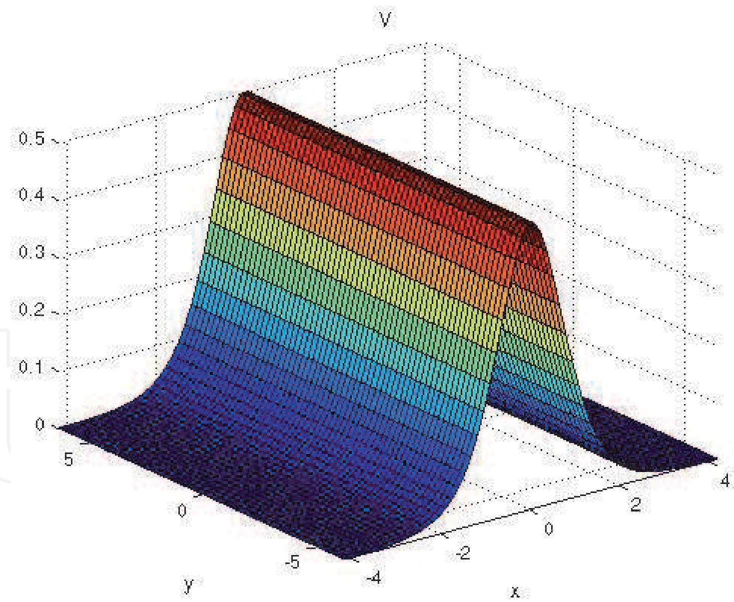

Figure 2.
Profile of potential V(x) *along the optical lattice.*

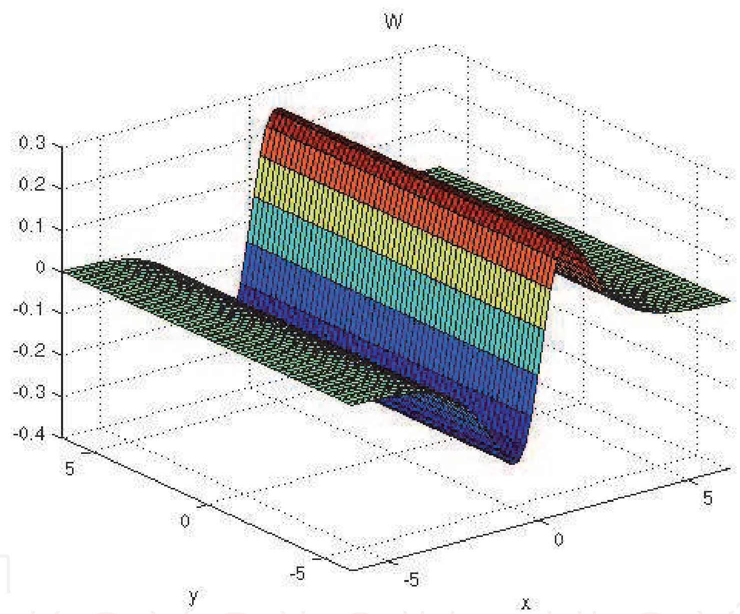

Figure 3.
Profile of potential W(x) *along the optical lattice.*

whose solution can be given by

$$\phi = \phi_0 \sec h(x) \exp\left\{i\mu \tan^{-1}[\sinh(x)]\right\}, \tag{38}$$

that represents a nonlinear mode with $\lambda = 1$, when $V_0 = 1$, $W_0 = 0.5$, and $\mu = W_0/3$. The profiles of the potentials $V(x)$ and $W(x)$ are shown in **Figures 2** and **3**.

5. New non-Hermitian optical systems

The application of non-Hermitian optical systems in the modeling of the optical lattices and waveguides [18] can be performed by the variational method proposed by [19]. In this approach, a generalization of the NLS equation (29)

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\pm F\left(|u|^2 u\right)=0, \tag{39}$$

can be obtained by a properly Lagrangian density L, introducing the action as

$$S=\int_0^z dz\int\int_{-\infty}^{\infty} L(x,y,z)dxdy. \tag{40}$$

In this approach, a mechanism to describe nonlinear and non-Hermitian optical systems can be obtained starting from an adequate action given by Eq. (40) with a Lagrangian

$$L=\frac{1}{2}(\nabla\phi)^2-V(\phi), \tag{41}$$

where ϕ is proportional to the electric field as in Eq. (32) and $V(\phi)$ is the optical potential that model the optical lattice of the system. In this sense, Eq. (33) can be written as

$$\frac{1}{2}\frac{d^2\phi}{dx^2}=\frac{dV(\phi)}{d\phi}, \tag{42}$$

and solved as follows

$$\int\frac{d\phi}{\sqrt{2V(\phi)}}=\pm(x-x_0). \tag{43}$$

On the other hand, these same solutions emerge from the energy calculation of optical system, which can be written as

$$E=\frac{1}{2}\int_{-\infty}^{\infty} dx\left[\left(\frac{d\phi}{dx}-W_\phi\right)^2+2W_\phi\frac{d\phi}{dx}\right], \tag{44}$$

where $\frac{dW(\phi)}{d\phi}=W_\phi=\sqrt{2V\phi}$.

The configuration that minimizes the energy of system, given by Eq. (19), is obtained by the following differential equation:

$$\frac{d\phi}{dx}=W_\phi. \tag{45}$$

5.1 New models and sensors

Nonlinear and non-Hermitian optical systems can be modeled now by the method based on Lagrangian. The motion equations will describe the propagation of the optical field through of a generic optical lattice.

Consider a model that is based on nonlinear and non-Hermitian optical potential

$$W_\phi=\phi^2+2ia\phi-b^2, \tag{46}$$

where a and b are optical parameters. The optical field solution, using, is

$$\phi(x) = -ia + \alpha \tanh(\alpha x), \tag{47}$$

where $\alpha = \sqrt{b^2 - a^2}$, and the energy has real spectrum that can be calculated by

$$E = \frac{4}{3}\alpha^3. \tag{48}$$

Another nonlinear and non-Hermitian optical potential is given by

$$W_\phi = \cos(\phi) + ia\sin(\phi) + b \tag{49}$$

whose solution for the optical field is

$$\phi(x) = 2\arctan\left[\frac{-ia - 2\beta\tanh(\beta x)}{b-1}\right], \tag{50}$$

where $2\beta = \sqrt{1 - (a^2 + b^2)}$, whose field configuration is plotted in **Figure 4**.

It is important to emphasize that a new class of non-Hermitian optical systems can be generated by Eq. (42) and from Eq. (43) follows that

$$\int \frac{d\phi}{\sqrt{2V(\phi)}} = x - x_0 = \int \frac{d\phi}{\sqrt{2V(\phi)}}. \tag{51}$$

In general one can rewrite the above equation simply as $\phi = f(\varphi)$ then we can try to generate a new non-Hermitian model with real energy from non-Hermitian model given by Eq. (46). In this case, we can use:

$$f(\phi) = \sinh(\phi), \tag{52}$$

getting the non-Hermitian potential

$$W(\phi) = \sinh(\phi) + 2ia\ln\left[\cosh(\phi) - 2(1+b^2)\tanh^{-1}[\tanh(\phi)]\right], \tag{53}$$

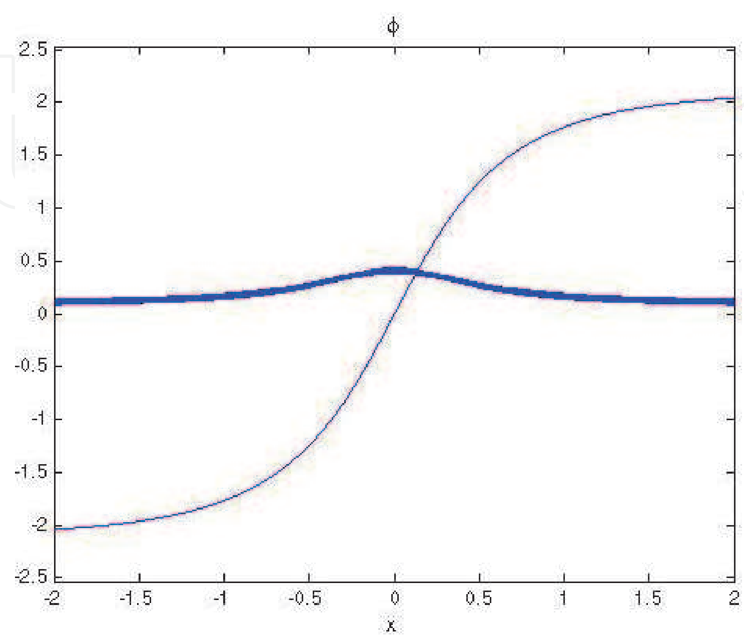

Figure 4.
The real (thin line) and imaginary (thick line) parts of optical field configurations of nonlinear and non-Hermitian optical potentials.

with the correspondent solution

$$\phi(x) = \sinh^{-1}[-ia + \alpha a \tanh(kax)]. \tag{54}$$

It is important to note that the optical field solution ϕ has a real component and an imaginary component. The real component represents the diffracted component of the optical field as shown in **Figure 4**. In our investigation, we have found that non-Hermitian optical systems can describe optical crystal structures and optical pulses propagating through such structures. We can observe that the modes of propagation, in Eq. (10), describe a particular conduction band of the optical lattice. In this sense, we can design a waveguide to act as a natural selector of certain wavelengths. In some photonic devices using sensors to optical fiber, for example, gyroscopes, optical noises from quantum scattering as the Raman effect is undesirable. So we can design a waveguide which suppresses the Raman propagation modes in the conduction band of the crystal lattice. Another important application of the modeling that we propose in this paper is that certain quantum noise from the interaction of light with matter can be treated very easily, as is the case for Raman scattering. In this case, the scattering propagation mode and its interaction with the crystal lattice can be modeled by the procedure that we have described. Thus we can simulate the propagation modes of an optical pulse by a crystalline network and mapping the crystalline structure whose information is present in the potential optical $V(x)$ and $W(x)$ of Eq. (12) by the symmetry parameters present in motion equations describing the propagation of an optical beam through an optical network. It is interesting to note that our procedure covers a wide range of nonlinear scatterings, which allow the numerical implementation of this method to recognize and model the patterns of quantum noise from the scattering of light with matter. Another interesting application of the method we have developed in soliton-based communications is that the crystal lattice can be designed such that the λ_k propagation modes can be used to adjust the bit size of the slot [14, 20]. This may be performed once for soliton train; it is possible to relate the soliton width T_0 with bit size B as

$$B = \frac{1}{T_B} = \frac{1}{2q_0 T_0}, \tag{55}$$

where T_B is the bit time slot and $2q_0 = T_B/T_0$ is the separation between neighboring soliton, as shown in **Figure 5**.

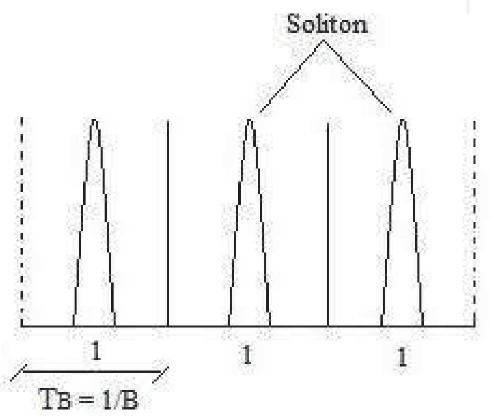

Figure 5.
Soliton bit stream. Each soliton occupying a bit slot can represent binary 1, and the absence of soliton can represent a binary 0.

6. Conclusions

In this chapter, we describe how nonlinear and quantum optical effects can be applied to the development of optical sensors. From Maxwell's equations, one can obtain an equation that describes the nonlinear behavior of an optical pulse that propagates through an optical system such as a waveguide. The nonlinear behavior of an optical pulse can be understood here as optical noise, the propagation of an optical pulse through a general optical system, or even quantum optical effects that can be properly described by a nonlinear second-order equation known as generalized nonlinear Schrödinger equation as described in Section 2. This equation has simple solutions called optical solitons. In Sections 3 and 4, we show how the term optical potential $F(|u|^2u)$ from NLS, where u describes the electric field, can model several optical systems through which a nonlinear optical pulse can propagate, generating physical information that can be transmitted and processed electronically. In Section 5, we show that nonlinear optical systems, which are optical systems capable of transmitting such nonlinear optical pulses, can be modeled simply by a first-order equation from non-Hermitian optical systems, which are systems that present a type of symmetry in spatial directions and in time, that is, $x \to -x$ and $t \to -t$. We apply these results by describing how different optical systems can be equivalent or mapped together. In principle, a particular optical system, such as an approximately one-dimensional waveguide, can be designed by reproducing the same optical effects as another waveguide. In the example, we apply these results although the beam optical profile is different in the measurement instrument data output, an oscilloscope, for example, the optical system which in this case may be a waveguide, can be mapped through its *design function* $W(\phi)$. The electric field amplitudes $\phi(x)$ can be appropriately combined to generate optical pulses in various optical systems and can be transmitted and processed from photonics and integrated optics and applied to the development of various emerging technologies such as quantum computing and quantum information.

Author details

Antônio Carlos Amaro de Faria[1,2]

1 Institute for Advanced Studies—IAS, São José dos Campos, SP, Brazil

2 Federal Technological University of Parana, Parana, Brazil

*Address all correspondence to: atoni.carlos@gmail.com

References

[1] Maxwell JC. Electrodynamics. London: Cambridge Press; 1889

[2] Taylor JT. Optical Solitons-Theory and Experiment. New York: Cambridge University Press; 1992

[3] Shen Y. Principles of Nonlinear Optics. New York: Wiley; 1984

[4] Boyd RW. Nonlinear Optics. San Diego, CA: Academic Press; 1992

[5] Agrawal GP. Applications of Nonlinear Fiber Optics. San Diego, CA: Academic Press; 2001

[6] Haus HA, Wong WS. Solitons in optical communications. Reviews of Modern Physics. 1996;**68**:423

[7] Kivshar YS, Agrawal GP. Optical Solitons from Fibers to Photonic Crystals. San Diego, CA: Academic Press; 2003

[8] Agrawal GP. Nonlinear Fiber Optics. San Diego, CA: Academic Press; 2001

[9] Born M, Wolf E. Principle of Optics. New York: Cambridge University Press; 1999

[10] Ablowitz MJ, Clarkson PA. Solitons Nonlinear Evolution Equations and Inverse Scattering. New York: Cambridge University Press; 1991

[11] Luther-Davis B, Stegman GI. Spatial Solitons. New York: Springer; 2001

[12] Hasegawa A, Tappert F. Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. Applied Physics Letters. 1973;**23**:142

[13] Mollenauer LF et al. Experimental observation of picosecond pulse narrowing and solitons in optical fibers. Physical Review Letters. 1980;**45**:1095

[14] Hasegawa H, Kodama Y. Solitons in Optical Communications. New York: Oxford University Press; 1995

[15] Mollenauer LF et al. Optical Fiber Telecommunications III. San Diego, CA: Academic Press; 1997

[16] Temprana E et al. Two-fold transmission reach enhancement enabled by transmitter-side digital backpropagation and optical frequency comb-derived information carriers. Optics Express. 2015;**23**:20774

[17] Temprana E et al. Beating the nonlinear capacity limit. Optics & Photonics News. 2016;**30**:32

[18] Musslimani ZH et al. Optical solitons in pt periodic potentials. Physical Review Letters. 2008;**100**: 030402

[19] Whitham GB. Linear and Nonlinear Waves. New York: Wiley; 1974

[20] Mollenauer LF, Smith K. Demonstration of soliton transmission over more than 4000 km in fiber with loss periodically compensated by Raman gain. Optics Letters. 1988;**13**:675