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# Inventory Policies for Deteriorating Items with Maximum Lifetime under Downstream Partial Trade Credits to Credit-Risk Customers by Discounted Cash Flow Analysis

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## Abstract

Getting loans from banks are almost impossible after 2008 global financial crisis. As a result, about 80% of companies in the United Kingdom and the United States offer their products on various short terms, free-interest loans to customers. To compute the interest earned and charged during the credit period but not to the revenue and other costs which are considerably larger than the interest earned and charged, numerous researchers and academicians apply merely the discounted cash flow (DCF) analysis. In addition, some products deteriorate continuously and cannot sell after expiration date. However, a little number of researchers have considered the product lifetime expectancy into their models. In this chapter, a supplier-retailer-customer chain model is developed. The supplier provides an upstream full trade credit to the retailer, and the credit-risk customer gets a downstream partial trade credit from the retailer. The non-decreasing deterioration rate is 100% near particularly close to its expiration date. To compute all relevant costs, DCF analysis is applied. The retailer's optimal replenishment cycle time is not only exists but also unique that demonstrated in this proposal and that has been shown by the numerical examples.

**Keywords:** supply chain management, deterioration, expiration dates, trade credit, discount cash flow, credit-risk customer

## 1. Introduction

In traditional business transactions, it was implicitly assumed that the buyer must pay the procurement cost when products are received. However, in today's competitive markets, most companies offer buyers various credit terms to impel sales and hence reduce inventory. In the United Kingdom, "estimates suggest" that more than 80% of the business transactions are which made on credit, while about

80% of the United States' firms offer their products on trade credit, has been stipulated by Seifert et al. [1]. Conversely, trade credit decreases the inventory holding cost, therefore affecting order quantity of buyer.

In literature for inventory models with trade credit financing, Goyal [2] derived the retailer's optimal EOQ when the supplier provides a permissible delay in payment. Aggarwal and Jaggi [3] elongated the EOQ model for nondeteriorating items to deteriorating items. Jamal et al. [4] elaborate the model considering shortages. Teng [5] modified the previous models by using sales revenue to compute the interest earned from sales. Huang [6] then explored the problem to a supply chain system in an upstream and downstream trade credit environment. Liao [7] further generalized Huang's model with an unlimited replenishment rate with a limited replenishment rate for deteriorating items. Min et al. [8] proposed an EPQ model with both upstream and downstream trade credits when the demand is stock dependent. Teng et al. [9] extended the constant demand rate to product's life cycle dependent demand pattern. Under different credit terms, Chern et al. [10, 11] discussed Nash two-player equilibrium solutions between the supplier and the retailer. Chen et al. [12, 13] contributed the retailer's optimal model. Liao et al. [14] and Wu et al. [15] discussed optimal strategy for deteriorating items with capacity constraints.

The products like volatile liquids, blood banks, fruits, fashion merchandises, vegetables, and high-tech products deteriorate continuously due to evaporation, spoilage, and obsolescence, among other reasons. An exponentially decaying inventory model is built by Ghare and Schrader [16]. The constant deterioration rate is extended to Weibull failure rate by Covert and Philip [17]. Dave and Patel [18] proposed linear time functional demand. For shortages, Sachan [19] further generalized the EOQ model. The demand is log-concave with time dependent derived by Hariga [20]. Teng et al. [21] further expanded allowing shortages and continuous type demand pattern. Teng et al. [22] have permitted partial backlogging. For deteriorating products, Dye [23] investigated the effect of technology investment on refrigeration. No one of the above cited papers took the expiration date into consideration before Chen and Teng [24], Sarkar [25], Wang et al. [26], Wu et al. [27], and Sarkar et al. [28].

DCF is an important tool in inventory management. Researchers such as Hill and Pakkala [29], Chung and Liao [30], Dye et al. [31], Chang et al. [32], Mousavi et al. [33] work related to DCF analysis. Recently, Chen and Teng [34] and Duari and Chakraborti [35] applied the DCF analysis to obtain the optimal lot size and credit period in a supply chain with upstream and downstream trade credit financing.

A credit-worthy retailer generally gains a permissible delay on the entire purchasing quantity, in reality. However, a retailer often asks for credit-risk customers to cover a fraction of the purchasing cost at the time of placing an order and then provides a permissible delay (downstream credit). To reduce default risks with credit-risk customers, they use downstream partial trade credit as a strategy that has received relatively little attention by the researchers. Additionally, the majority of the recent studies consider merely the opportunity loss of trade credit. Most of the time, they ignore to take the opportunity loss of the other different costs in their study in order to take the effect of inflation and time value of money. For an exquisite and sharp analysis, the DCF analysis must be used on all relevant revenue and costs. Due to this fact, here a supplier-retailer-customer supply chain model is proposed. The supplier provides an upstream full credit period of  $S$  years to the retailer and the retailer gives to the customer a downstream partial credit period of  $R$  years. The deteriorating rate is constant or increasing and closer to 100% near expiration date. With time-dependent demand, the DCF analysis is applied to

study the effects of inflation and time value of money. Weibull non-decreasing deterioration rate is considered mostly as a special case of the proposed generalized deterioration rate. The retailer's objective function is formulated under different possible alternatives, in this study. We derived an algorithm to get optimal solution to each alternative using the existing theorem on concave functions. Finally, two numerical examples are solved in order to illustrate the problem.

The remaining of the paper is follows as: Section 2 defines notations and makes necessary assumptions. Section 3 gives the mathematical model; Section 4 derives the present value of the retailer's annual total profit under each alternative. Section 5 provides the required algorithm which simplifies the search for the optimal solution. Section 6 presents numerical study. Finally, the conclusions and the future research direction are provided in Section 7.

## 2. Notation and assumption

The following notation and assumptions are used in this model.

### 2.1 Notations

The following notations are used in this model:

$\alpha$	fraction of the purchasing cost must be paid at the time of placing an order, $0 < \alpha < 1$
C	cost per unit
D	demand
h	holding cost per unit
I	interest per
M	expiration time or maximum lifetime
OC	ordering cost per order
p	selling price per unit
R	downstream credit period in years by the retailer
S	upstream credit period in years by the supplier
T	time
Ic	interest charged by the retailer
Ie	interest earned by the supplier
T	replenishment cycle time

Functions:

$Q(t)$	order quantity level at time t
$\theta(t)$	deterioration rate at time t
$I(t)$	inventory level at time t
$D(t)$	demand rate, $D = (a + bt)$
PTP(T)	present value of annual total profit

For convenience, the asterisk symbol on a variable is denoted the optimal solution of the variable. For instance,  $T^*$  is the optimal solution of  $T$ .

## 2.2 Assumptions

The following assumptions are made to build the mathematical inventory model:

- a. In a supplier-retailer-customer supply chain system, the retailer obtains a full upstream credit period of  $S$  years from his/her supplier and in turn gives a partial downstream trade credit to his/her credit-risk customers who must cover  $\alpha$  portion of the purchasing cost at the time of placing an order and then get a credit period of  $R$  years on the outstanding quantity. For reliable customers, just set  $\alpha = 0$ , the retailer may provide a full trade credit.
- b. The retailer deposits the sales revenue into an interest-bearing account after time  $R$ , if  $S \geq R$ . When  $S \geq (T + R)$ , the retailer pays off the entire purchasing cost and collects all sales revenue at time  $S$ . Both, the credit payment sold by  $SR$  and the cash payment, are obtained from time  $0$  to  $S$  for  $S < T + R$ . To use the other activities and begin paying for the interest charges on the items sold, the retailer pays the supplier and retains the profit after  $(S - R)$ .
- c. The retailer gets cash payments from customers and immediately deposits those into an interest-bearing account until time  $S$ , if  $S \leq R$ . As to credit payments, the retailer must finance  $(1 - \alpha)c(a + bt)T$  at time  $S$  and then pay off the loan from time  $R$  to time  $T + R$ .
- d. A deteriorating item deteriorates continuously and cannot be sold after its maximum lifetime or expiration date. Therefore, its deterioration rate is percent near to its expiration date. As a result, it is assumed without loss of generality that the deterioration rate  $\theta(t)$  at time  $t$ ,  $0 \leq t \leq m$ , satisfies the following conditions:

$$0 \leq \theta(t) \leq 1, \theta'(t) \geq 0 \text{ and } \theta(m) = 1. \quad (1)$$

We assume the deterioration as

$$\theta(t) = \frac{1}{(1 + m - t)}, 0 \leq t \leq m \quad (2)$$

a special case of (1):

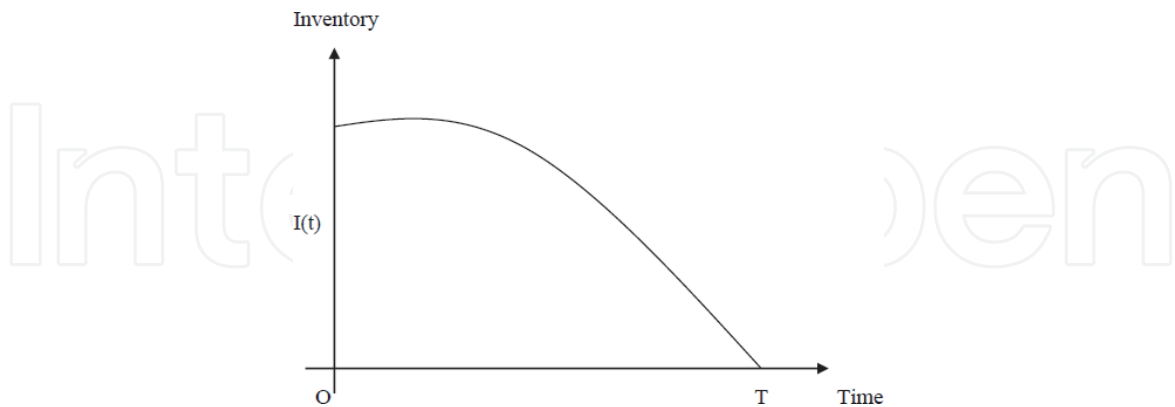
- e. It is assumed without loss of generality that both upstream and downstream credit periods  $R$  and  $S$  and the replenishment cycle time  $T$  are less than or equal to the expiration date  $m$ , since the deterioration rate reaches 100% after expiration date

$$R \leq m, S \leq m, \text{ and } T \leq m \quad (3)$$

- f. No shortages allowed.
- g. Replenishment rate is instantaneous.

### 3. Mathematical model

The inventory level is depleted by demand and deterioration, during the replenishment cycle  $[0, T]$ , and hence governed by the following differential equation (**Figure 1**):



**Figure 1.**  
Graphically shows the model.

$$I'(t) = -D - \theta(t)I(t), 0 \leq t \leq T, D = (a + bt) \quad (4)$$

With the boundary condition  $I(T) = 0$ . Note that the prime symbol on a variable is denoted the first order derivative with respect to the variable throughout the paper. Solving the differential Eq. (4), we get

$$\begin{aligned} I(t) &= e^{-\delta(t)} \int_t^T e^{\delta(u)} (a + bu) du, \text{ where } \delta(t) = \int_0^t \theta(u) du \\ &= \left(1 + m - \frac{t}{1+m}\right) (1+m)(b(t-T)) \\ &\quad + (a + b + bm)(\text{Log}[1 + m - t] - \text{Log}[1 + m - T]) \end{aligned} \quad (5)$$

#### 3.1 Sales revenue

The customers get a downstream credit period of  $R$  years from the retailer. Thus, the retailer receives the cash payment for the time  $0$  to  $T$ . Also the retailer receives the credit payment from  $R$  to  $T + R$ . Hence, the present value of sales revenue per cycle time  $T$  of the retailer is

$$SR = p \left[ \alpha \int_0^T (a + bt) e^{-it} dt + (1 - \alpha) \int_R^{T+R} (a + bt) e^{-it} dt \right] \quad (6)$$

$$= p \left[ \frac{e^{-i(R+T)} (b + ai + biR - e^{iT} (b + ai + biR) + biT) (-1 + \alpha) + (b + ai - e^{-iT} (b + ai + biT)) \alpha}{i^2} \right] \quad (7)$$



### 3.2 Different costs

#### 3.2.1 Ordering cost

At time 0, the retailer orders deteriorating items. Hence, the present value of the retailer's ordering cost per cycle time  $T$  is

$$OC \quad (8)$$

#### 3.2.2 Purchasing cost

Since the upstream trade credit is  $S$  years, the retailer must pay the supplier the whole purchasing cost  $cQ$  at time  $S$ . As a result, the present value of the retailer's purchasing cost per cycle time  $T$  is

$$\begin{aligned} PC &= ce^{-iS}Q = ce^{-iS}I(0) = c \int_0^T (a + bt)e^{\delta(t)-iS} dt \\ &= ce^{-iS}(1+m)^2(-bT + (a+b+bm)(\text{Log}[1+m] - \text{Log}[1+m-T])) \end{aligned} \quad (9)$$

#### 3.2.3 Holding cost

The present value of the retailer's holding cost per cycle time  $T$  is

$$\begin{aligned} HC &= h \int_0^T e^{-it}I(t)dt = h \int_0^T \int_t^T (a + bt)e^{\delta(u)-\delta(t)-it} du dt \\ &= h \left( -\frac{b(1+m)^2(-1 + e^{-iT} + iT)}{i^2} + \frac{be^{-iT}(2 + iT + e^{iT}(-2 + iT))}{i^3} \right. \\ &\quad \left. + \frac{e^{-i(1+m)}(1+m)^2(a+b+bm) \left( \begin{aligned} &-\text{ExpIntegralEi}[i(1+m)] \\ &+\text{ExpIntegralEi}[i(1+m-T)] \\ &+e^{i(1+m)}\text{Log}\left[\frac{1+m}{1+m-T}\right] \end{aligned} \right)}{i} \right. \\ &\quad \left. - \frac{e^{-i(1+m)}(a+b+bm)(e^{i(1+m)} - e^{i(1+m-T)} - (1+i+im)\text{ExpIntegralEi}[i(1+m)] \right. \\ &\quad \left. + (1+i+im)\text{ExpIntegralEi}[i(1+m-T)] + e^{i(1+m)}\text{Log}\left[\frac{1+m}{1+m-T}\right])}{i^2} \right) \end{aligned} \quad (10)$$

According to the values of  $R$  and  $S$ , there are two potential cases:

Case I:  $R \leq S$ .

Case II:  $R \geq S$ .

Both cases are discussed separately.

### 3.2.4 Case I: when $R \leq S$

Based on the values of  $S$ ,  $T$ , and  $T + R$ , three sub-cases can occur:

i.  $S \leq T$ .

ii.  $T \leq S \leq T + R$ .

iii.  $T + R \leq S$ .

Notice that for both cases (i)  $S \leq T$  and (ii)  $T \leq S \leq T + R$ , the case  $S = T$  is applicable.

Similarly, the condition  $S = T + R$  is applicable for cases (ii)  $T \leq S \leq T + R$  and (iii)  $T + R \leq S$ .

The interest earned and the interest charged for the above three cases are investigated accordingly.

#### 3.2.4.1 Sub-case 1a: $S \leq T$

In this sub-case, the retailer gets revenue and receives interest from the two possible sources: (a) the cash payment for the time 0 to  $S$  and (b) the credit payment for the time  $R$  to  $S$ . So, the present value of the interest earned per cycle is

$$\begin{aligned}
 IE &= pI_e \left[ \alpha \int_0^S t(a + bt)e^{-it} dt + (1 - \alpha) \int_R^S (t - R)(a + bt)e^{-it} dt \right] \\
 &= pi_e \left[ \frac{e^{-i(R+S)} \left( e^{iS} (ai + b(2 + iR)) + e^{iR} \left( \frac{ai(-1 + i(R - S))}{+b(-2 + i(R + iRS - S(2 + iS)))} \right) \right) (1 - \alpha)}{i^3} \right. \\
 &\quad \left. + \frac{(2b + ai + e^{-iS}(-ai(1 + iS) - b(2 + iS(2 + iS))))\alpha}{i^3} \right] \quad (11)
 \end{aligned}$$

The retailer provides his customers a credit period of  $R$  years and gets customers' credit payments from time  $R$  through  $(T + R)$ , on the other hand. The retailer obtains  $\alpha c(a + bt)S$  dollars from cash payment and  $(1 - \alpha)c(a + bt)(S - R)$  dollars from credit payment, at time  $S$ , and therefore pays his/her supplier  $[\alpha c(a + bt)S + (1 - \alpha)c(a + bt)(S - R)]$  dollars. Consequently, the retailer must finance all items sold after time  $S$  for the cash payment and  $(S - R)$  for the credit payment at an interest charged  $I_c$  per dollar per year. So, the present amount of the interest charged per cycle is given by

$$IC = cI_c \left[ \alpha \int_S^T (T - t)(a + bt)e^{-it} dt + (1 - \alpha) \int_S^{T+R} (T + R - t)(a + bt)e^{-it} dt \right]$$



$$= ci_c \left[ \frac{\left( e^{-i(R+T)}(ai + b(2 + i(R + T))) + (1 - \alpha)e^{-iS} \left( \begin{matrix} ai(-1 + i(R - S + T)) \\ + b \left( \begin{matrix} -2 + i(R - 2S + T) \\ + i^2 S(R - S + T) \end{matrix} \end{matrix} \right) \right) \right)}{i^3} \right. \\ \left. + \frac{\left( \begin{matrix} e^{-iT}(ai + b(2 + iT)) \\ + e^{-iS} \left( \begin{matrix} -2 + i(T + S(-2 + i(-S + T))) \end{matrix} \right) \end{matrix} \right) \alpha}{i^3} \right] \quad (12)$$

## 4. Total profit of the model

### 4.1 Profit of the first sub-case of the model

As a result, the present value of the retailer's annual total profit by using (7)–(12) is

$$\begin{aligned} \text{PTP}_1(T) &= \frac{1}{T}(SR - PC - HC - OC - IC + IE) \\ &= \frac{1}{T} \left\{ p \left[ \alpha \int_0^T (a + bt)e^{-it} dt + (1 - \alpha) \int_R^{T+R} (a + bt)e^{-it} dt \right] \right. \\ &\quad - c \int_0^T (a + bt)e^{\delta(t)-iS} dt - h \int_0^T \int_t^T (a + bt)e^{\delta(u)-\delta(t)-it} du dt - O \\ &\quad - cI_c \left[ \alpha \int_S^T (T - t)(a + bt)e^{-it} dt + (1 - \alpha) \int_S^{T+R} (T + R - t)(a + bt)e^{-it} dt \right] \\ &\quad \left. + pI_e \left[ \alpha \int_0^S t(a + bt)e^{-it} dt + (1 - \alpha) \int_R^S (t - R)(a + bt)e^{-it} dt \right] \right\} \quad (13) \end{aligned}$$

#### 4.1.1 Sub-case 1b: $T \leq S \leq T + R$

The retailer accumulates revenue and obtains interest from two sources: (a) the cash payment starting from time 0 to S and (b) the credit payment starting from time R to S. So, the present value of the interest earned per cycle is

$$IE = pI_e \left[ \alpha \left( \int_0^T t(a + bt)e^{-it} dt + \int_T^S T(a + bt)e^{-it} dt \right) + (1 - \alpha) \int_R^S (t - R)(a + bt)e^{-it} dt \right]$$

$$= pI_e \left[ \frac{e^{-i(R+S)} \left( e^{iS} (ai + b(2 + iR)) + e^{iR} \left( ai(-1 + i(R - S)) + b(-2 + i(R + iRS - S(2 + iS))) \right) \right) (1 - \alpha)}{i^3} \right. \\ \left. + \left( \frac{e^{-i(S+T)} (-e^{iT} (b + ai + biS)T + e^{iS}T(b + ai + biT))}{i^2} \right) \right. \\ \left. + \frac{2b + ai + e^{-iT} (-ai(1 + iT) - b(2 + iT(2 + iT)))}{i^3} \right) \alpha \quad (14)$$

By time  $T (\leq S)$ , the retailer receives all cash payments so that there is no interest charged for the cash payment. However, the retailer must pay up all items sold during the interval  $[S-R, T]$ . Therefore, the annual interest charged is

$$IC = (1 - \alpha) \int_S^{T+R} (T + R - t)(a + bt)e^{-it} dt \\ = \frac{(e^{-i(R+T)} (ai + b(2 + i(R + T))) + e^{-iS} (ai(-1 + i(R - S + T)) + b(-2 + i(R - 2S + T) + i^2S(R - S + T))))(1 - \alpha)}{i^3} \quad (15)$$

#### 4.2 Profit of the second sub-case of the model

From (7)–(10), (14), and (15), it is known that the present value of the retailer's annual total relevant profit is

$$PTP_2(T) = \frac{1}{T} (SR - PC - HC - OC - IC + IE) \\ = \frac{1}{T} \left\{ p \left[ \alpha \int_0^T (a + bt)e^{-it} dt + (1 - \alpha) \int_R^{T+R} (a + bt)e^{-it} dt \right] - c \int_0^T (a + bt)e^{\delta(t) - iS} dt \right. \\ \left. - \left[ h \int_0^T \int_t^T (a + bt)e^{\delta(u) - \delta(t) - it} du dt - OC - (1 - \alpha) \int_S^{T+R} (T + R - t)(a + bt)e^{-it} dt \right] \right. \\ \left. + pI_e \left[ \alpha \left( \int_0^T t(a + bt)e^{-it} dt + \int_T^S T(a + bt)e^{-it} dt \right) + (1 - \alpha) \int_R^S (t - R)(a + bt)e^{-it} dt \right] \right\} \quad (16)$$

##### 4.2.1 Sub-case 1c: $T + R \leq S$

The retailer receives all the cash and credit payments before the supplier's upstream credit period  $S$  and no interest charged. However, the present value of the interest earned per cycle is given by

$$\begin{aligned}
 IE &= pI_e \left[ \alpha \left( \int_0^T t(a + bt)e^{-it} dt + \int_T^S Te^{-it} dt \right) \right. \\
 &\quad \left. + (1 - \alpha) \left( \int_{T+R}^S T(a + bt)e^{-it} dt + \int_R^{T+R} (t - R)(a + bt)e^{-it} dt \right) \right] \\
 &= pi_e \left[ \left( -\frac{e^{-i(R+T)}(ai(1 - e^{iT} + iT) + b(2 - e^{iT}(2 + iR) + i^2T(R + T) + i(R + 2T)))}{i^3} \right. \right. \\
 &\quad \left. \left. + \frac{e^{-i(R+S+T)}(-e^{i(R+T)}(b + ai + biS)T + e^{iS}T(ai + b(1 + i(R + T))))}{i^2} \right) (1 - \alpha) \right. \\
 &\quad \left. + \left( \frac{e^{-i(S+T)}(-e^{iT}(b + ai + biS)T + e^{iS}T(b + ai + biT))}{i^2} \right. \right. \\
 &\quad \left. \left. + \frac{2b + ai + e^{-iT}(-ai(1 + iT) - b(2 + iT(2 + iT)))}{i^3} \right) \alpha \right]
 \end{aligned} \tag{17}$$

#### 4.3 Profit of the third sub-case of the model

So the present value of the retailer's annual total profit is

$$\begin{aligned}
 PTP_3(T) &= \frac{1}{T} (SR - PC - HC - OC + IE) \\
 &= \frac{1}{T} \left\{ p \left[ \alpha \int_0^T (a + bt)e^{-it} dt + (1 - \alpha) \int_R^{T+R} (a + bt)e^{-it} dt \right] \right. \\
 &\quad \left. - c \int_0^T (a + bt)e^{\delta(t) - iS} dt - h \int_0^T \int_t^T (a + bt)e^{\delta(u) - \delta(t) - it} du dt - OC \right. \\
 &\quad \left. + pI_e \left[ \alpha \left( \int_0^T t(a + bt)e^{-it} dt + \int_T^S Te^{-it} dt \right) \right. \right. \\
 &\quad \left. \left. + (1 - \alpha) \left( \int_{T+R}^S T(a + bt)e^{-it} dt + \int_R^{T+R} (t - R)(a + bt)e^{-it} dt \right) \right] \right\}
 \end{aligned} \tag{18}$$

Combining (13), (16), and (18), the present value of the retailer's annual total profit is given as

$$\begin{aligned}
 PTP(T) &= PTP_1(T) \text{ if } S \leq T \\
 PTP(T) &= PTP_2(T) \text{ if } S - R \leq T \leq S \\
 PTP(T) &= PTP_3(T) \text{ if } T \leq S - R
 \end{aligned} \tag{19}$$

It is clear from (13), (16), and (18) that

$$PTP_1(S) = PTP_2(S), \text{ and } PTP_2(S - R) = PTP_3(S - R) \tag{20}$$

This implies that  $PTP(T)$  is continuous in  $T \geq 0$ .

#### 4.3.1 Case II: when $R \geq S$

The following sub-cases may occur based on values of  $S$  and  $T$ :  $S \leq T$ , and  $S \geq T$ .

##### 4.3.1.1 Sub-case 2a: $S \leq T$

As  $R \geq S$ , there is no interest earned from the credit payment. However, the present value of the annual interest earned from the cash payment is

$$IE = \alpha p I_e \int_0^S (a + bt) t e^{-it} dt$$

$$= \frac{p(2b + ai + e^{-iS}(-ai(1 + iS) - b(2 + iS(2 + iS))))\alpha i_e}{i^3} \quad (21)$$

At time  $S$ , the retailer should finance  $(1-\alpha)c(a + bt)T$  for the credit payment and  $\alpha c(a + bt)(T-S)$  for the cash payment, respectively. Then, the retailer renders the loan for the cash payment at time  $T$  and pays off the loan for the credit payment at  $t = T + R$ . So, the present amount of the interest charged per cycle is

$$IC = cI_c \left[ \alpha \left( \int_S^T (T-t)(a + bt)e^{-it} dt + (1-\alpha) \left\{ \int_S^R T(a + bt)e^{-it} dt + \int_R^{T+R} (T+R-t)(a + bt)e^{-it} dt \right\} \right) \right]$$

$$= ci_c \left[ \left( \frac{e^{-i(R+S)}(-e^{iS}(b + ai + biR)T + e^{iR}(b + ai + biS)T)}{i^2} + \frac{e^{-i(R+T)}(ai + b(2 + i(R+T)) + e^{iT}(ai(-1 + iT) + b(-2 + i(T+R(-1 + iT))))))}{i^3} \right) (1-\alpha) \right. \\ \left. + \frac{(e^{-iT}(ai + b(2 + iT)) + e^{-iS}(ai(-1 + i(-S+T)) + b(-2 + i(T+S(-2 + i(-S+T))))))\alpha}{i^3} \right]. \quad (22)$$

#### 4.4 Profit of the first sub-case of the second case of the model

Consequently, the present value of the retailer's annual total profit is

$$PTP_4(T) = \frac{1}{T}(SR - PC - HC - OC - IC + IE)$$

$$= \frac{1}{T} \left\{ p \left[ \alpha \int_0^T (a + bt)e^{-it} dt + (1-\alpha) \int_R^{T+R} (a + bt)e^{-it} dt \right] - c \int_0^T (a + bt)e^{\delta(t)-iS} dt - h \int_0^T \int_t^T (a + bt)e^{\delta(u)-\delta(t)-it} du dt - OC \right\}$$

$$\begin{aligned}
 & -cI_c \left[ \alpha \left( \int_S^T (T-t)(a+bt)e^{-it} dt + (1-\alpha) \left\{ \int_S^R T(a+bt)e^{-it} dt \right. \right. \right. \\
 & \left. \left. \left. + \int_R^{T+R} (T+R-t)(a+bt)e^{-it} dt \right\} \right) \right] + \alpha p I_e \int_0^S (a+bt)te^{-it} dt
 \end{aligned} \quad (23)$$

We then discuss the last sub-case in which  $R \geq S \geq T$ .

#### 4.4.1 Sub-case 2b: $S \geq T$

Similarly, the present value of the interest earned from the cash payment per cycle is

$$\begin{aligned}
 IE &= pI_e \left[ \alpha \left\{ \int_0^T t(a+bt)e^{-it} dt + \int_T^S T(a+bt)e^{-it} dt \right\} \right] \\
 &= p \left( \frac{e^{-i(S+T)} (-e^{iT}(b+ai+biS)T + e^{iS}T(b+ai+biT))}{i^2} \right. \\
 &\quad \left. + \frac{2b+ai+e^{-iT}(-ai(1+iT)-b(2+iT(2+iT)))}{i^3} \right) \alpha i_e
 \end{aligned} \quad (24)$$

For the cash payment, there is no interest to charge. On the other hand, the retailer must finance  $(1-\alpha)c(a+bt)T$  for the credit payment at time  $S$  and start paying off the loan from time  $R$  to  $T+R$ .

Hence, the present amount of the interest charged per cycle is

$$\begin{aligned}
 IC &= (1-\alpha)cI_c \left[ \left\{ \int_S^R T(a+bt)e^{-it} dt + \int_R^{T+R} (T+R-t)(a+bt)e^{-it} dt \right\} \right] \\
 &= ci_c \left[ \left( \frac{e^{-i(R+S)} (-e^{iS}(b+ai+biR)T + e^{iR}(b+ai+biS)T)}{i^2} \right. \right. \\
 &\quad \left. \left. + \frac{e^{-i(R+T)} (ai+b(2+i(R+T)) + e^{iT}(ai(-1+iT)+b(-2+i(T+R(-1+iT))))}{i^3} \right) (1-\alpha) \right. \\
 &\quad \left. + \frac{(e^{-iT}(ai+b(2+iT)) + e^{-iS}(ai(-1+i(-S+T)) + b(-2+i(T+S(-2+i(-S+T))))}{i^3} \right) \alpha \right]
 \end{aligned} \quad (25)$$

#### 4.5 Profit of the second sub-case of the second case of the model

Consequently, the present value of the retailer's annual total profit is

$$PTP_5(T) = \frac{1}{T} (SR - PC - HC - OC - IC + IE)$$

$$\begin{aligned}
 &= \frac{1}{T} \left\{ p \left[ \alpha \int_0^T (a + bt)e^{-it} dt + (1 - \alpha) \int_R^{T+R} (a + bt)e^{-it} dt \right] \right. \\
 &\quad - c \int_0^T (a + bt)e^{\delta(t)-iS} dt - h \int_0^T \int_t^T (a + bt)e^{\delta(u)-\delta(t)-it} du dt - O \\
 &\quad \left. - (1 - \alpha)cI_c \left\{ \int_S^R T(a + bt)e^{-it} dt + \int_R^{T+R} (T + R - t)(a + bt)e^{-it} dt \right\} \right] \\
 &\quad \left. + pI_e \left[ \alpha \left\{ \int_0^T t(a + bt)e^{-it} dt + \int_T^S T(a + bt)e^{-it} dt \right\} \right] \right\}
 \end{aligned} \tag{26}$$

Combining (23) and (26), we know that the present value of the retailer's annual total relevant profit is

$$\begin{aligned}
 PTP(T) &= PTP_4(T), \text{ if } S \leq T \\
 PTP(T) &= PTP_5(T), \text{ if } S > T
 \end{aligned} \tag{27}$$

It is clear that  $PTP(T)$  is continuous in  $T$  and has the following properties:

$$PTP_4(S) = PTP_5(S) \tag{28}$$

## 5. Algorithm

We developed the following algorithm to find the optimal solution of the problem. The algorithm is as follows:

**Step 1:** Input all the parameters values.

**Step 2:** Assimilate the values of  $R$  and  $S$ . If  $R \leq S$  then go to Step 3, otherwise go to Step 5.

**Step 3:** Compute all  $PTP_j(T_j^*)$ , for  $j = 1, 2$ , and  $3$ .

**Step 3.1:** Find the unique root  $T_1$  in (30). If  $S \leq T_1$  we set  $T_1^* = T_1$  else, we set  $T_1^* = S$ . Calculate  $PTP_1(T_1^*)$  by (13).

**Step 3.2:** Find the unique root  $T_2$  in (31). If  $T_2 \leq S - R$ ; we set  $T_2^* \leq S - R$ . If  $S - R \leq T_2 \leq S$ ; we set  $T_2^* = T_2$ . If  $T_2 \geq S$ ; we set  $T_2^* = S$ . Calculate  $PTP_2(T_2^*)$  by (16).

**Step 3.3:** Find the unique root  $T_3$  in (32). If  $T_3 \leq S - R$ ; we set  $T_3^* \leq T_3$ . Otherwise we set  $T_3^* = S - R$ . Calculate  $PTP_3(T_3^*)$  by (18).

**Step 4:** Find the maximum among  $PTP_j(T_j^*)$  for  $j = 1, 2$ , and  $3$ , set the optimal solution  $\{T^*, PTP(T^*)\}$  accordingly, and then stop.

**Step 5:** Compute  $PTP_j(T_j^*)$ , for  $j = 4$  and  $5$ .

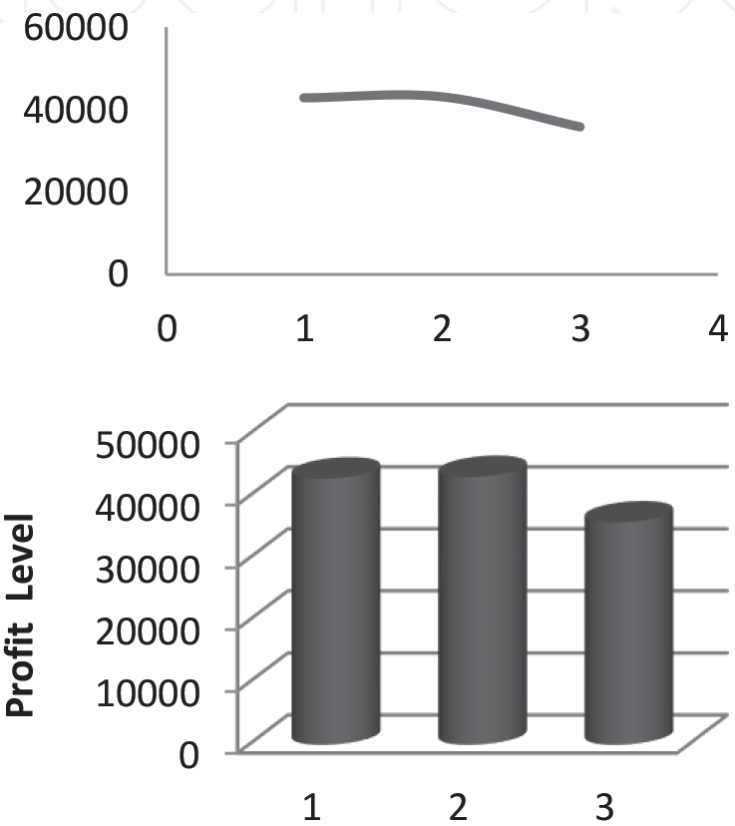
**Step 5.1:** Find the unique root  $T_4$  in (33). If  $S \leq T_4$  we set  $T_4^* = T_4$  else we set  $T_4^* = S$ . Calculate  $PTP_4(T_4^*)$  by (23).

**Step 5.2:** Find the unique root  $T_5$  in (34). If  $S \geq T_5$  we set  $T_5^* = T_5$  else we set  $T_5^* = S$ . Calculate  $PTP_5(T_5^*)$  by (26).

**Step 6:** Find the maximum among  $PTP_j(T_j^*)$  for  $j = 4$  and  $5$ , Set the optimal solution  $\{T^*, PTP(T^*)\}$  accordingly, and then stop.

6. Numerical examples

**Example 1:** Let us assume that  $\theta(t) = \frac{1}{(1+m-t)}$ ,  $m = 1$ ,  $\alpha = 0.20$ ,  $c = \$10$  per unit,  $a = 50$  units per year,  $b = 30$  units per year,  $h = \$15$  per unit per year  $= 0.4$ ,  $I_e = 0.4$ ,  $I_c = 0.5$ ,  $p = \$10$  per unit,  $R = 0.25$ , and  $T = 0.18$ . Optimal solutions when  $\theta(t) = \frac{1}{(1+m-t)}$  and  $S \geq R$  (**Figure 2**).



**Figure 2.**  
Graphically the results of sub-cases of the first case.

Case I	$Q_i^*$	$T_i^*$	$PTP_i^*$
1a(1)	21.91	0.18	42861.95
1b(2)	21.91	0.18	43119.31
1c(3)	21.91	0.18	35794.41

**Table 1.**  
The optimal solution to the three sub-cases of case I.

Case II	$Q_i^*$	$T_i^*$	$PTP_i^*$
2a(4)	22.21	0.18	30994.26
2b(5)	22.21	0.18	31892.34

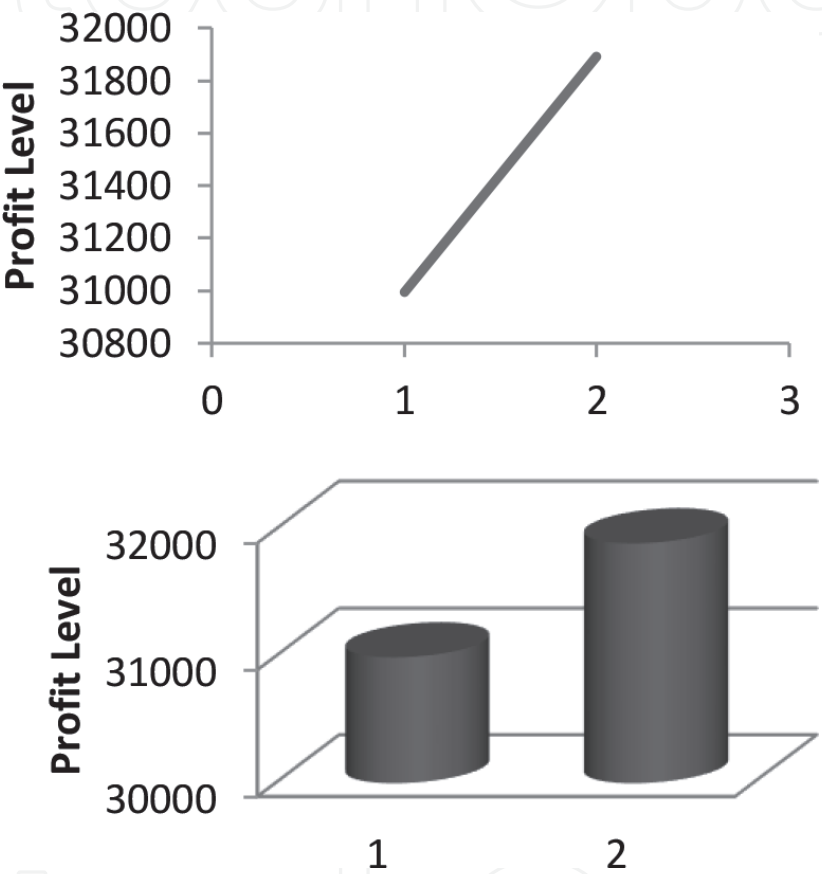
**Table 2.**  
The optimal solution to the two sub-cases of case II.



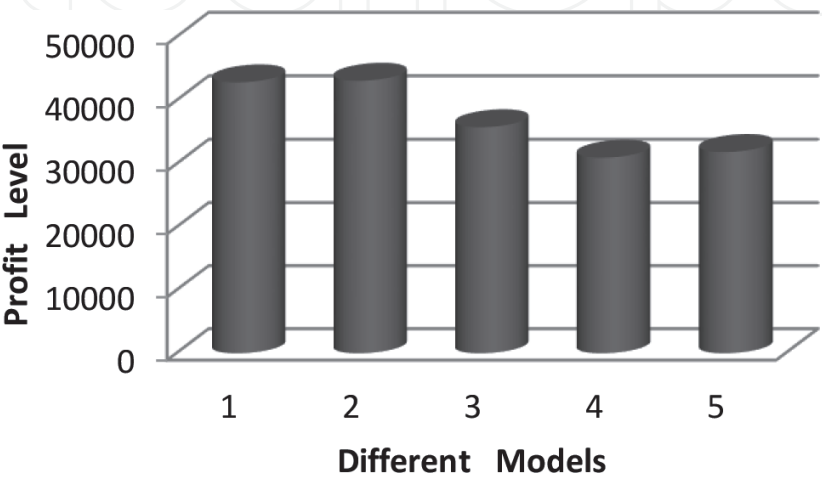
By using the proposed algorithm, we obtain the optimal solutions of each case shown in **Table 1**. As a result, for  $S = 0.74$ , then the optimal solution to the problem is  $Q^* = 21.91 \text{ units}$ ,  $T^* = 0.18, \text{ years} = 25 \text{ days}$ , and  $PTP^* = 43119.31$ .

**Example 2:** Using the same data as those in Example 1 except  $R = 0.45 \text{ years}$  again using the proposed algorithm, we obtain the optimal solutions for  $S = 0.18$  and  $0.44$ , respectively, each case shown in **Table 2**. Optimal solutions when  $\theta(t) = \frac{1}{(1+m-t)}$  and  $S \leq R$  (**Figure 3**).

As a result, at  $S = 0.44$ , the optimal solution to the problem is  $Q^* = 22.21 \text{ units}$ ,  $T^* = 0.18 \text{ yrs} = 45 \text{ days}$  and  $\text{Total Profit}^* = 31892.34$ . Considering all the sub-cases, we can conclude that the second sub-case of the first case is more



**Figure 3.**  
Graphically the results of sub-cases of the second case.



**Figure 4.**  
Graphical comparison of profit functions of both cases I and II.

profitable for the retailer. Consequently, the background and the conditions to take the best policy of that model are better than irrespective of all other cases (Figure 4).

## 7. Conclusions and future research

Usually, the retailer uses a downstream partial trade credit as a strategy to reduced fault risks with credit-risk customers. The deterioration rate is time dependent and near cent percent near expiration date. We have investigated an EOQ model for deteriorating items in a general framework that Goyal [2], Teng [5, 36], Huang [6], Teng and Goyal [37], Chen and Teng [24], Wu and Chan [38], and Wu et al. [15] did special cases. In addition, to reflect the effects of inflation and time value of money, a discounted cash flow analysis has been adopted to obtain the present value of the total profit for time-dependent demand. By applying the existing theoretical results in concave functions, we have demonstrated the proper algorithm to find the optimal solution for possible alternatives. Then we have used the two most commonly used deterioration rates to run several numerical simulations. With increase in the purchasing cost, the holding cost, or the interest rate reduces the order quantity, the cycle time and the annual total profit is sensitive. In contrast, along product maximum life-span elevates the order quantity, the cycle time, and annual total profit is also sensitive. In addition, the total profit is very sensitive to the selling price. Consequently, to prolong product life-span and increase profit, retailers must negotiate with suppliers for low purchase cost and invest in preservation technology (such as refrigeration).

If time approaches to the expiration date, it may be profitable to have a closeout sale at a markdown price. One may extend the model from zero-ending inventory to nonzero-ending inventory. Furthermore, a researcher might consider allowing for shortages, allow partial backlogging, and allow for failure, scrap, and rework. Finally, the proposed model with a single player can be extended to an integrated cooperative model for both the retailer and the customer.

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## References

- [1] Seifert D, Seifert RW, Protopappa-Sieke M. A review of trade credit literature: Opportunity for research in operations. *European Journal of Operational Research*. 2013;**231**(2): 245-256
- [2] Goyal SK. Economic order quantity under conditions of permissible delay in payments. *The Journal of the Operational Research Society*. 1985; **36**(4):335-338
- [3] Aggarwal SP, Jaggi CK. Ordering policies of deteriorating items under permissible delay in payments. *The Journal of the Operational Research Society*. 1995;**46**(5):658-662
- [4] Jamal AMM, Sarker BR, Wang S. An ordering policy for deteriorating items with allowable shortage and permissible delay in payment. *The Journal of the Operational Research Society*. 1997; **48**(8):826-833
- [5] Teng JT. On the economic order quantity under conditions of permissible delay in payments. *The Journal of the Operational Research Society*. 2002;**53**(8):915-918
- [6] Huang YF. Optimal retailer's ordering policies in the EOQ model under trade credit financing. *The Journal of the Operational Research Society*. 2003;**54**(9):1011-1015
- [7] Liao JJ. An EOQ model with non-instantaneous receipt and exponentially deteriorating items under two-level trade credit. *International Journal of Production Economics*. 2008;**113**(2): 852-861
- [8] Min J, Zhou YW, Zhao J. An inventory model for deteriorating items under stock-dependent demand and two-level trade credit. *Applied Mathematical Modelling*. 2010;**34**(11): 3273-3285
- [9] Teng JT, Min J, Pan Q. Economic order quantity model with trade credit financing for non-decreasing demand. *Omega*. 2012;**40**(3):328-335
- [10] Chern MS, Chan YL, Teng JT, Goyal SK. Nash equilibrium solution in a vendor-buyer supply chain model with permissible delay in payments. *Computers and Industrial Engineering*. 2014;**70**(1):116-123
- [11] Chern MS, Pan Q, Teng JT, Chan YL, Chen SC. Stackelberg solution in a vendor-buyer supply chain model with permissible delay in payments. *International Journal of Production Economics*. 2013;**144**(1): 397-404
- [12] Chen SC, Cárdenas-Barrón LE, Teng JT. Retailer's economic order quantity when the supplier offers conditionally permissible delay in payments link to order quantity. *International Journal of Production Economics*. 2014a;**155**:284-291
- [13] Chen SC, Teng JT, Skouri K. Economic production quantity models for deteriorating items with up-stream full trade credit and down-stream partial trade credits. *International Journal of Production Economics*. 2014b;**155**:302-309
- [14] Liao JJ, Huang KN, Ting PS. Optimal strategy of deteriorating items with capacity constraints under two-levels of trade credit policy. *Applied Mathematics and Computation*. 2014; **233**:647-658
- [15] Wu J, Al-khateeb FB, Teng J-T, Cárdenas-Barrón LE. Inventory models for deteriorating items with maximum lifetime under downstream partial trade credits to credit-risk customers by discounted cash-flow analysis. *International Journal of Production Economics*. 2016a;**171**:105-115

- [16] Ghare PM, Schrader GP. A model for an exponentially decaying inventory. *Journal of Industrial Engineering*. 1963;**14**(5):238-243
- [17] Covert RB, Philip GS. An EOQ model with Weibull distribution deterioration. *AIIE Transactions*. 1973; **5**(4):323-326
- [18] Dave U, Patel LK. (T, Si) policy inventory model for deteriorating items with time proportional demand. *The Journal of the Operational Research Society*. 1981;**32**(2):137-142
- [19] Sachan RS. On (T, Si) policy inventory model for deteriorating items with time proportional demand. *The Journal of the Operational Research Society*. 1984;**35**(11):1013-1019
- [20] Hariga MA. Optimal EOQ models for deteriorating items with time-varying demand. *The Journal of the Operational Research Society*. 1996; **47**(10):1228-1246
- [21] Teng JT, Chern MS, Yang HL, Wang YJ. Deterministic lot-size inventory models with shortages and deterioration for fluctuating demand. *Operations Research Letters*. 1999;**24** (1-2):65-72
- [22] Teng JT, Chang HJ, Dye CY, Hung CH. An optimal replenishment policy for deteriorating items with time-varying demand and partial backlogging. *Operations Research Letters*. 2002;**30**(6):387-393
- [23] Dye CY. The effect of preservation technology investment on a non-instantaneous deteriorating inventory model. *Omega*. 2013;**41**(5):872-880
- [24] Chen SC, Teng JT. Retailer's optimal ordering policy for deteriorating items with maximum life time under supplier's trade credit financing. *Applied Mathematical Modelling*. 2014; **38**(15-16):4049-4061
- [25] Sarkar B. An EOQ model with delay in payments and time varying deterioration rate. *Mathematical and Computer Modelling*. 2012;**55**(3-4): 367-377
- [26] Wang WC, Teng JT, Lou KR. Seller's optimal credit period and cycle time in a supply chain for deteriorating items with maximum life time. *European Journal of Operational Research*. 2014; **232**(2):315-321
- [27] Wu J, Ouyang LY, Cárdenas-Barrón LE, Goyal SK. Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing. *European Journal of Operational Research*. 2014; **237**(3):898-908
- [28] Sarkar B, Saren S, Cárdenas-Barrón LE. An inventory model with trade-credit policy and variable deterioration for fixed life time products. *Annals of Operations Research*. 2015;**229**(1):677-702
- [29] Hill RM, Pakkala TPM. A discounted cash flow approach to the base stock inventory model. *International Journal of Production Economics*. 2005;**93-94**:439-445
- [30] Chung KJ, Liao JJ. The optimal ordering policy in a DCF analysis for deteriorating items when trade credit depends on the order quantity. *International Journal of Production Economics*. 2006;**100**(1):116-130
- [31] Dye CY, Ouyang LH, Hsieh TP. Inventory and pricing strategies for deteriorating items with shortages: A discounted cash flow approach. *Computers and Industrial Engineering*. 2007;**52**(1):29-40
- [32] Chang CT, Ouyang LY, Teng JT, Cheng MC. Optimal ordering policies for deteriorating items using a discounted cash-flow analysis when a trade credit is linked to order quantity.

Computers and Industrial Engineering.  
2010;**59**(4):770-777

[33] Mousavi SM, Hajipour V, Niaki STA, Alikar N. Optimizing multi-item multi-period inventory control system with discounted cash flow and inflation: Two calibrated meta-heuristic algorithms. *Applied Mathematical Modelling*. 2013;**37**(4): 2241-2256

[34] Chen SC, Teng JT. Inventory and credit decisions for time-varying deteriorating items with up-stream and down-stream trade credit financing by discounted cash flow analysis. *European Journal of Operational Research*. 2015; **243**(2):566-575

[35] Duari NK, Chakraborti T. A cash flow-oriented EOQ model of deteriorating items with increasing demand rate and shortages under permissible delay in payment. *International Journal of Operational Research*. 2015;**24**(2):145-160

[36] Teng JT. Optimal ordering policies for a retailer who offers distinct trade credits to its good and bad credit customers. *International Journal of Production Economics*. 2009;**119**(2): 415-423

[37] Teng JT, Goyal SK. Optimal ordering policies for a retailer in a supply chain with up-stream and down-stream trade credits. *The Journal of the Operational Research Society*. 2007; **58**(9):1252-1255

[38] Wu J, Chan YL. Lot-sizing policies for deteriorating items with expiration dates and partial trade credit to credit-risk customers. *International Journal of Production Economics*. 2014;**155**: 292-301