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# Some Statistical Analysis of Poultry Feeds Data

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## Abstract

In this study we presented generalized exponential power distribution as an alternative to normal distribution commonly used in the analysis of agricultural data. The distribution which is more robust in modeling because of the present of shape parameters, which regulates it tails. Some of its mathematical and statistical properties are examined. The application of the probability density function is demonstrated in fitting poultry feeds data. The goodness-of-fit test was carried to show that it is a better substitute to normal distribution in applications.

**Keywords:** exponential power distribution, normal distribution, goodness-of-fit, poultry feeds data, Pearson's  $\chi^2$  test, Kolmogorov-Smirnov test

## 1. Introduction

Cholesterol is a waxy substance that comes from two main sources: the liver and food intake. It has been noted that high level of cholesterol can block the arteries, decrease blood flow to other tissue in the body, thereby causing heart diseases [1]. Eggs have commonly been jettisoned owing to high cholesterol contents, therefore lowering consumption rate. But [1] further note that eggs are high quality source protein and other nutrients. It is also well known that eggs contain lecithin and phospholipids, necessary for the construction of brain cell membrane. In terms of feeding intellect, their value lies mainly in the quality of their proteins, they are actually rich in amino acids, essential in the production of the principal neurotransmitters. Hence, instead of a total boycott, it is better to reduce the risk associated with eggs consumption.

In order to alleviate the problem associated with consumption, an organic copper salt combination was used instead of the inorganic combinations currently used in preparation of poultry feeds. Ninety-six chickens were randomly selected and randomly divided into 2 groups of 48 each. Each group was subjected to the same general conditions and treatments, differing only in that while, the chickens in the first group were fed with inorganic copper salt, those in the second group were fed with organic copper salt. After 4 months the weight (gram) and cholesterol level (mg/egg) of the eggs yielded by the two groups were measured. The excerpt from the data used in this analysis is presented in the Appendix A courtesy of Federal University of Agriculture, Abeokuta.

Carrying out analysis of the data obtained from the laboratory study, it is a common assumption in literature to assume normal distribution in the analysis of agricultural data. This assumption is not always true especially in the current case. Therefore, we proposed to study a generalized form of normal distribution called

the exponential power distribution and its multivariate extension, which contains normal distribution and others in the literature as special cases in fitting poultry feeds data. This unifying exponential power distribution is characterized by a parameter  $\beta$  and a function  $h(\beta)$  which regulates the tail behaviour of the distribution, thus making it more flexible and suitable for modeling than the usual normal distribution, while retaining symmetry properties. Finally we fit the generalized exponential power distribution as well as the normal distribution to data on eggs produced by chicken on each of two different poultry feeds (inorganic and organic copper salt compositions) and show that the generalized exponential power distribution fit is considerably better. We then use the Kolmogorov-Smirnov two samples one-tailed test to show that there is an increase in egg weights and decrease in cholesterol level when the feed is organic.

## 2. Exponential power distribution

The exponential power distribution is a class of densities which includes the normal and allows thick tails, Thus making it more suitable in modeling when compared with the usual normal distribution. In fact, it is a natural generalization of the normal distribution and also used in applications by [2–8]. They also presented a multivariate version of the exponential power distribution and [5] used this distribution to model repeated measurements. We present this distribution with a proposition. Codes were written in R environment to estimate the parameters of both the univariate and the multivariate version of the distribution see [7, 9, 10].

**Proposition 2.1** *let  $X$  be a random variable then,*

$$f(x; \mu, \sigma, \beta) = \frac{\beta h(\beta)}{2\Gamma\left(\frac{1}{\beta}\right)\sigma} \exp \left\{ - \left[ \frac{h(\beta)|x - \mu|}{\sigma} \right]^\beta \right\} \quad (1)$$

is a probability density function (p.d.f.) with three parameters  $\beta > 0$ ,  $\sigma > 0$ ,  $\mu \in \mathfrak{R}$ . The tail region is regulated by the function  $h(\beta)$ , which is positive for all  $\beta > 0$ .

If a random variable  $X$  has the p.d.f (1) then its  $m$ th moments can be obtained from the relation

$$E(X^m) = \int_0^\infty \left( \left( \left[ -1^m \left( \sigma(2z)^{\frac{1}{2\beta}} - \mu \right)^m \right] + \left( \sigma(2z)^{\frac{1}{2\beta}} + \mu \right)^m \right) \left( \frac{z^{\frac{1}{2\beta}-1} \exp^{-z}}{2\Gamma\left(\frac{1}{2\beta}\right)} \right) \right) dz$$

In addition, its central moment estimates Agro [11–14] are:

$$E(X) = \mu; E|X - E(X)| = \frac{\sigma 2^{\frac{1}{2\beta}} \Gamma\left(\frac{1}{\beta}\right)}{\Gamma\left(\frac{1}{2\beta}\right)}; Var(X) = \frac{\sigma^2 2^{\frac{2}{2\beta}} \Gamma\left(\frac{3}{2\beta}\right)}{\Gamma\left(\frac{1}{2\beta}\right)}; \quad (2)$$

$$E(X - E(X))^3 = 0; E(X - E(X))^4 = \frac{\sigma^4 2^{\frac{4}{2\beta}} \Gamma\left(\frac{5}{2\beta}\right)}{\Gamma\left(\frac{1}{2\beta}\right)}; \quad (3)$$

and Kurtosis=

$$\frac{\Gamma\left(\frac{5}{2\beta}\right)\Gamma\left(\frac{1}{2\beta}\right)}{\Gamma^2\left(\frac{3}{2\beta}\right)}. \quad (4)$$

The results indicate that the sample mean  $\bar{X}$  is the estimate of the true mean  $\mu$  while the shape parameter can be numerically obtained from the estimate of the kurtosis. Substituting shape parameter estimate into  $Var(X)$  we estimate the scale parameter  $\sigma$ .

Also the log-likelihood function [14] for random samples  $x_1, x_2, \dots, x_n$  from (1) is:

$$LogL(\mu, \sigma, \beta) = n \ln \left( \frac{1}{\sigma \Gamma\left(1 + \frac{1}{2\beta}\right) 2^{1+\frac{1}{2\beta}}}\right) - \sum_{i=1}^{i=n} \frac{1}{2} \left| \frac{x_i - \mu}{\sigma} \right|^{2\beta} \quad (5)$$

The derivatives of (8) with respect to  $\mu$ ,  $\sigma$ , and  $\beta$  are.

$$\frac{\partial LogL}{\partial \mu} = \frac{\beta}{\sigma^{2\beta}} \left( \sum_{x_i \geq \mu} (x_i - \mu) - \sum_{x_i < \mu} (x_i - \mu) \right); \quad \frac{\partial LogL}{\partial \sigma} = -\frac{n}{\sigma} + \frac{\beta}{\sigma} \left| \frac{x - \mu}{\sigma} \right|^{2\beta}; \quad \text{and}$$

$$\frac{\partial LogL}{\partial \beta} = \frac{n}{2\beta^2} \left[ \Psi\left(1 + \frac{1}{2\beta}\right) + 1 \right] - \sum_{i=1}^{i=n} \left| \frac{x_i - \mu}{\sigma} \right|^{2\beta} \ln \left| \frac{x_i - \mu}{\sigma} \right|.$$

The expected fisher information matrix of EPD is

$$E\left(-\frac{\partial^2 LogL}{\partial \mu^2}\right) = \frac{n\beta(2\beta-1)2^{1-\frac{1}{\beta}}\Gamma\left(1-\frac{1}{2\beta}\right)}{\sigma^2\Gamma\frac{1}{2\beta}}; \quad E\left(-\frac{\partial^2 LogL}{\partial \sigma^2}\right) = \frac{2\beta n}{\sigma^2};$$

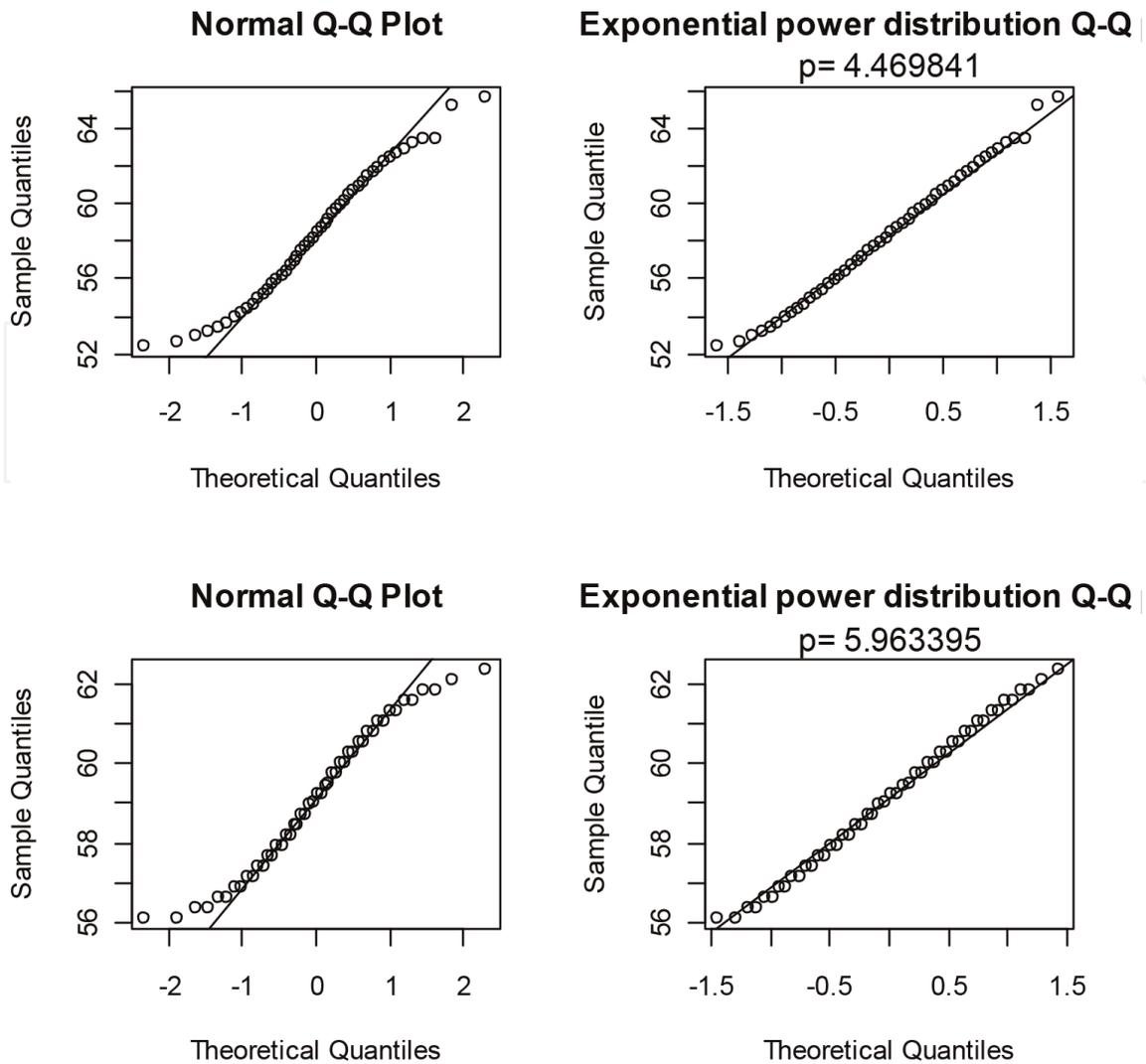
$$E\left(-\frac{\partial^2 LogL}{\partial \sigma \partial \beta}\right) = -\frac{1}{\sigma\beta} \left(1 + \Psi\left(1 + \frac{1}{2\beta}\right)\right) \ln 2; \quad \text{and}$$

$$E\left(-\frac{\partial^2 LogL}{\partial \beta^2}\right) = \frac{n}{\beta^3} \left(1 + \Psi\left(1 + \frac{1}{2\beta}\right) + \frac{\Psi'\left(1 + \frac{1}{2\beta}\right)}{2\beta}\right) + n \frac{(\ln 2)^2}{4\beta^3} \left(\Psi^2\left(1 + \frac{1}{2\beta}\right) + \Psi'\left(1 + \frac{1}{2\beta}\right)\right)$$

[10] developed codes in R programming environment to estimate these parameters from any given sample from (1), this also includes the parameter  $\beta$  which has no explicit solution.

### 3. Data analysis

Normality assumptions are common in data analysis, but a close look at the normal and generalized exponential power Q-Q plot in **Figures 1** and **2**, though by the normal distribution is a reasonable fit, the tails seem to be shorter than expected and hence the  $p$ -values resulting from the usual tests based on normal assumptions cannot be trusted. On the other hand, the generalized exponential power distribution proves to be a much better fit for egg weights as well as cholesterol level in both groups. We now carry out estimation of the parameters of the distribution using the method of moments and maximum likelihood estimate (MLE). These methods were preferred because they have many optimal properties in estimation: sufficiency; consistency; efficiency; and parametrization invariance which are rarely found in other approaches (For details on MLE see [4, 9, 10] and the estimated values from the observations namely the means, standard deviation (s.d.), the  $\beta$ , Akaike information Criterion (AIC) as well as Bayesian Information criterion (BIC) are given in **Table 1** with the corresponding log-likelihood ( $\log(A)$ ) estimates. Since the explicit expression cannot be obtained for  $\mu$  and  $\beta$  in the estimation of maximum likelihood in the Section 2 we employed a numerical approach using a written code in the R software programme. The statistical computing environment [10], was supplemented with the package called “normp” downloaded into R file from the site <http://cran.r-project.org/> and used to analyze the data. As earlier stated, though the normal distribution was a good fit to the sample data, but we have a better fit when we use the generalized exponential power distribution. This is evident when

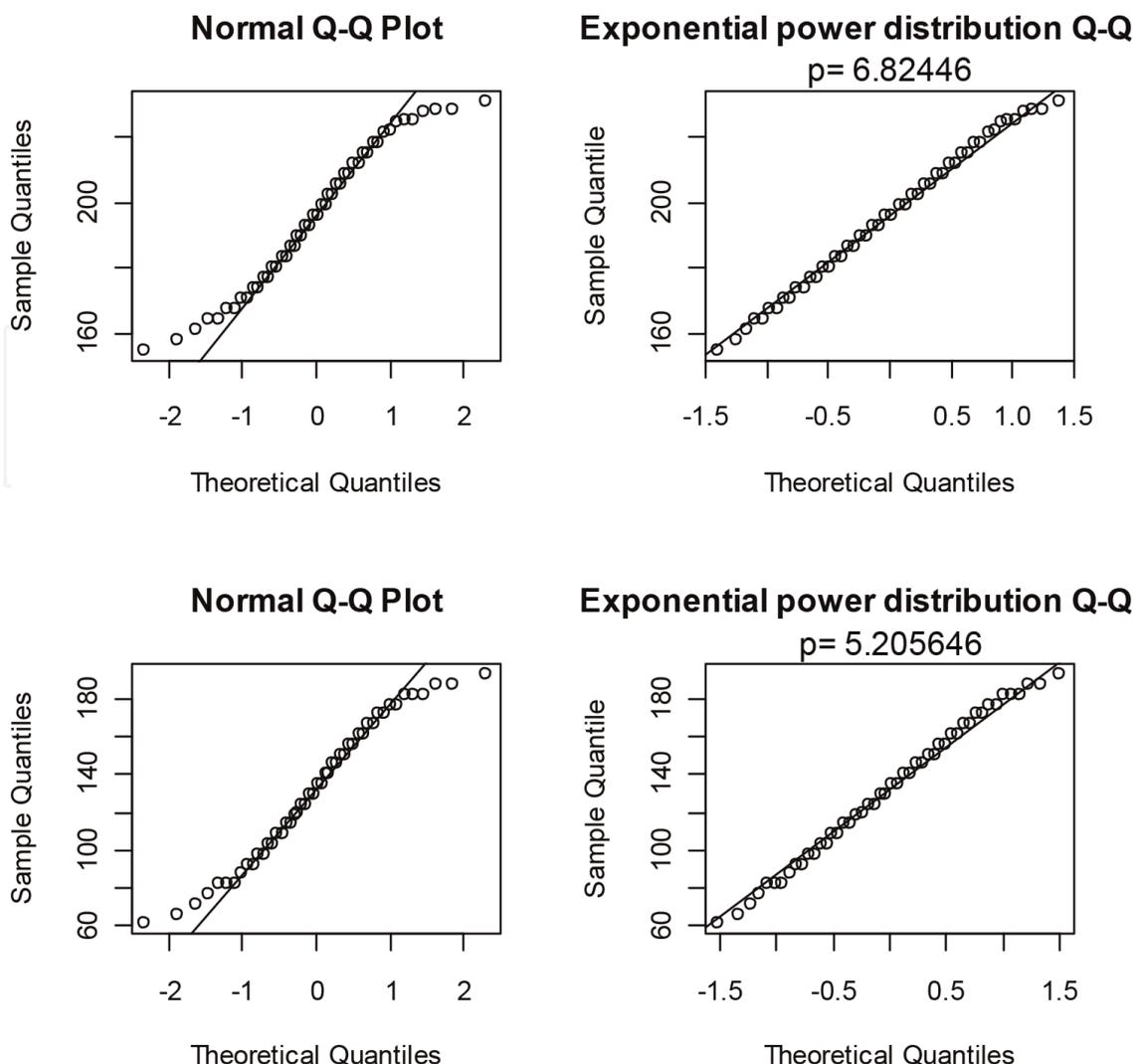


**Figure 1.** Top row are the Q-Q plots weight of the eggs for the inorganic copper salt, the first is the normal and the second is the Generalized Exponential distribution. The second row is the case of organic copper salt.

comparing the results from the log-likelihood functions, AIC and BIC given in **Table 1** below, as well as the plots in **Figures 1** and **2**.

### 3.1 Hypothesis testing

We have used a numerical algorithm written in the R software environment to find estimates of the parameters giving the best fit for the data in appendix B. These estimates are reported in **Table A1**. We used the Kolmogorov-Smirnov two-sample test [15] for large samples to decide whether there is significant difference in the weights and cholesterol levels of the two groups. Clearly, if we find that there is a significant difference, then feeds should be changed to organic copper salts. Note that we have used a one sided Kolmogorov-Smirnov two-sample test [15], that is, the test compares the cumulative frequency distributions of the two samples and decides if the observed  $D$  indicates that they were drawn from different populations and one of which is stochastically larger than the other. Let  $F_{n_1}(X)$  be the cumulative step function of the sample observations for the inorganic copper salt type and let  $F_{n_2}(Y)$  be the cumulative step function of the sample observations for organic copper salt type. We test the null hypothesis that the two samples have been drawn from the same population against the alternative hypothesis that the values of the population from which one the samples was drawn are stochastically larger than the values of the population from which the other sample was drawn. In other words, for the eggs weights the null and alternative hypotheses are of the form



**Figure 2.**  
 The Q-Q plots for the egg cholesterol content. The first row is the inorganic copper salt for normal and generalized exponential power distribution. The second row is the organic copper salt.

Density	Variables	Mean	s.d	Shape( $\omega$ )	$h(\omega)$	$\log(\ell)$	AIC	BIC
Normal	Weight (inorganic)	58.350	3.559	Nil	Nil	-129.055	262.11	273.5948
	Weight (organic)	59.10	1.822	Nil	Nil	-96.910	197.82	209.3048
GEP	Cholesterol (inorganic)	195.728	21.907	Nil	Nil	-216.282	436.564	448.0488
	Cholesterol (organic)	131.457	37.232	Nil	Nil	-241.739	487.478	498.9628
GEP	Weight (inorganic)	58.507	4.371	4.470	0.281	-126.028	258.056	275.2832
	Weight (organic)	59.118	2.306	5.963	0.168	-91.850	189.700	206.9272
	Cholesterol (inorganic)	194.481	28.359	6.825	0.147	-210.809	427.618	444.8452
	Cholesterol (organic)	129.720	46.327	5.206	0.192	-237.448	480.896	498.1232

**Table 1.**  
 Parameters estimation for eggs weights in Appendix B.

$H_0 : F_{n_1}(X) = F_{n_2}(Y)$  and  $H_A : F_{n_1}(X) < F_{n_2}(Y)$ ; also for the cholesterol levels while the null hypothesis remain the same, the alternative hypothesis is  $H_A : F_{n_2}(X) < F_{n_1}(Y)$ . We now define

$$D = \text{maximum}[F_{n_1}(X) - F_{n_2}(Y)]$$

The sampling distribution of  $D$  is assumed to be a generalized exponential power distribution. It has been shown by [16] that

$$\chi_2^2 = 4D^2 \frac{n}{2} \quad (6)$$

(for number of observations are the same)

1. for the egg weights  $D = 0.3125$  with corresponding  $P[\chi_2^2 > 9.21] < 0.01$ . Hence, we reject  $H_0$  and conclude that the weight of the eggs fed with inorganic copper salt is less than the organic type.
2. for the cholesterol level which is the most important, we have:  $D = 0.6875$  with  $P[\chi_2^2 > 45.375] < 0.0001$ . Hence we reject the null hypothesis and conclude that, using the organic copper salt type the cholesterol level significantly reduced.

#### 4. Exponential power distribution table

Given a set of data, one of the statistical issues is to see how well the data fit into postulated model. This technique necessitates the corresponding table of the probability distribution for the proposed model. The cumulative distribution for the exponential power distributions is not in explicit form, but some numerical approach was used to produce the abridge version of the table of the cumulative distribution for quick use for those who are not familiar with code written to solve such problem with different values of shape parameter  $\beta$  see Appendix C (**Tables A2–A6**). The table presented makes it workable to examine whether exponential power distribution is an appropriate model for any data set. Though we have the conventional testing method which is also discussed, one is Pearson's  $\chi^2$  test and the other one is Kolmogorov-Smirnov test. An example in poultry feeds data and a simulation example are included, we compare the fitting with the normal distribution. To illustrate the use of the table, the cumulative distribution function (cdf) for a standardized random variable having (1) with real  $\beta$  can be expressed has

$$P(X \leq x) = \int_{-\infty}^x \frac{1}{2\beta^{1/p} \left(1 + \frac{1}{\beta}\right)} \exp \left\{ -\frac{|x|^\beta}{\beta} \right\} \quad (7)$$

Thus, for each specified  $\beta$ , we can calculate the corresponding probability for each value of  $t$ . In the table, we present the corresponding probabilities for  $t$  ranging from 0.00 until  $P(X \leq x) \approx 1$  to 3 decimal places, with each increase in length by 0.01. We introduced Simpson rule in numerical computation coupled with R program developed by [9, 17]. We prefer Simpson's method compare to other methods because its guarantees the accuracy level of the table. The table is arranged as follows, if we wish to compute, say  $x = 0.15$ , the table in the appendix can used in the this way:

- $P(Y \leq 0.15) = 0.5910$ , when  $\beta = 0.5$
- $P(Y \leq 0.15) = 0.5695$ , when  $\beta = 1.0$
- $P(Y \leq 0.15) = 0.5583$ , when  $\beta = 3.6$

from the table we can see that the probability distribution of exponential power distribution depends on the shape parameter,  $\beta$ , and as  $\beta$  increases the cdf changed. For example, the  $P(Y \leq 3.0) = 0.9998$  remains the same at the accuracy of  $10^{-4}$  for

$\beta$  ranging from 2.40 to 10.00. Therefore, the tables were truncated at some points, when the resulting values of  $P(X \leq x)$  repeat the previous values for increase in shape parameter  $\beta$ . To check the accuracy of the table in the appendix, from our program we allowed  $\beta = 1$  which of course gave the values for the cdf of Laplace distribution otherwise known as double exponential. Also, when  $\beta = 2$  we have the values for the cdf of a random variable having a standard normal probability distribution function (not reproduce here, but available in many Statistical texts).

## 5. Goodness-of-fit tests for the exponential power distribution

In this section, we present two procedures for goodness-of-fit test for the exponential power distribution. One is *Pearson's*  $\chi^2$  test and the other one is Kolmogorov-Smirnov test. These are two well-known tests in the literature to examine how well a set of data fits into a postulated model provided that the probability distribution of the postulated random variable is available.

### 5.1 $\chi^2$ procedure for exponential power distribution

Given a set of data  $X_1, \dots, X_n$ . To carry out Pearson's  $\chi^2$  test to ascertain if the data is well fit into exponential power distribution  $EP(p_0)$ , we proceed as follows:

- Partition the sample space into  $K$  disjoint intervals;
- Find the probability  $\beta_k$  that an outcome falls in the  $K$ th interval under the assumption that the underlying population has an  $EP(\beta_0)$  distribution.  $\beta_k$  can be found using the table in the appendix, then  $E_k = n\beta_k$  is the expected number of outcomes that falls in the  $K$ th interval in  $n$  repetitions of the experiment;
- The  $\chi^2$  test statistic with degree of freedoms  $K - 1$  is then defined as

$$\chi^2 = \sum_{i=1}^K \frac{(N_i - E_i)^2}{E_i} \quad (8)$$

where  $N_i$  is the number of outcomes that fall in the  $i$ th interval and  $E_i$  is the expected number in the  $i$ th interval. The selection of  $K$  follows the general rule in the application of Pearson's  $\chi^2$  test.

If the  $\chi^2$  value calculated from (8) is too large compared with the endpoint of  $\chi_{k-1}^2$  at certain significance level, say 0.05 (commonly used) but on some occasion 0.01, it implies that the differences between the expected and the observed values are too large, then the assumption of exponential power with  $p_0$  must be rejected. Other value of  $\beta$  or even other models may need to be considered. If the calculated  $\chi^2$  value is small, it implies that the data set fits well into the model. Therefore the model can be accepted at the significance level specified.

From this test procedure, conspicuously it is convenient to have the tables for practical purpose. For example, we can always compare the value of  $\chi^2$  to see whether normal distribution or exponential power is a better fit for the data.

### 5.2 Kolmogorov test procedure on the exponential power distribution

Suppose we have a random sample  $X_1, \dots, X_n$  from a population with distribution function  $F(x)$ , we desire to see if a postulated exponential power distribution (with

specified  $\beta_0$ ) can be used to fit the underlying population of the data. The null hypothesis can be stated as follows.

$$H_0 : F(x) = G_0(x) \text{ for all } x.$$

against the alternative.

$$H_1 : F(x) \neq G_0(x) \text{ for at least one } x.$$

where  $G_0(x)$  denotes the cdf of  $EP(\beta_0)$

$$D(F_n(x), G_\beta(x)) = \sup_z |F_n(x) - G_\beta(x)| \quad (9)$$

where

$$F_n(x) = \begin{cases} 0, & x < X_{(1)} \\ \frac{i}{n}, & X_{(i)} \leq x < X_{(i+1)}, k = 1, \dots, n - 1; \\ 1, & x \geq X_{(n)}. \end{cases}$$

where  $X_{(1)}, \dots, X_{(n)}$  in the expression of  $F_n(x)$  are the ordered statistics of  $X_1, \dots, X_n$ .  $G_\beta(x)$  at each sample points of  $X_i$  can be found from the exponential power distribution table. In this case, the Kolmogorov-Smirnov test statistic  $D(.,.)$  is the maximum distance between empirical distribution function and postulated distribution function at the sample points. At significant level of  $\alpha$ , the test endpoint  $d_\alpha$  for test statistic  $D$  can be found from [15, 16]. The rule is that if the calculated  $D$  is larger than  $d_\alpha$  the postulated exponential power distribution function is too far away from the observed distribution function. Thus  $H_0$  is rejected at  $\alpha$  level of significance, otherwise,  $H_0$  is accepted at the same significance level. To carry out this test, it is critical to find the  $F_n(x)$ 's for the postulated exponential power distribution. The table provide in this paper makes it possible for the implementation of this test.

**Example 1:** (Approximation of the exponential power distribution by the normal distribution). Normal distribution has been well known to be the limiting distribution for so many distribution in the literature. In this section with explore to what value of the parameter  $p$  will normal give an acceptable approximation to data having exponential power distribution with parameter  $p_i$ . This will also examine the closeness between exponential power and normal distributions, using the Kolmogorov-Smirnov test of normality distance. Let  $X \sim N(0, 1)$  and  $F(x)$  be the cdf, also let  $Y \sim EP(\beta)$  and  $G_\beta(y)$  be the cdf. The Kolmogorov distance between  $F(x)$  and  $G_\beta(y)$  is defined as

$$D(F, G_\beta) = \sup_z |F - G_\beta| \quad (10)$$

The values of  $D(F, G_p)$  can be obtained from the tables in the appendix. The values of  $D(F, G_p)$  from some selected  $p = 1.6 - 4.4$ . These are shown in the table below.

we observed from **Table 2**, that as  $p$  increases  $D(F, G_p)$  also increases, this implies that approximation by normal distribution becomes poorer with large

$p$	1.6	1.8	2.2	2.4	2.6	2.8	3.0
$D(F, G_p)$	0.0146	0.0072	0.0065	0.0180	0.0197	0.0226	0.0268
$p$	3.2	3.4	3.6	3.8	4.0	4.2	4.4
$D(F, G_p)$	0.0311	0.0348	0.0384	0.0415	0.0447	0.0478	0.0504

**Table 2.**  
Kolmogorov distance between  $F$  and  $G_p$ .

Intervals	$n$	$p_i$	$EP(4.4)$	normal
$(-\infty, -1.75]$	3	0.0033	3.3	40.1
$(-1.75, -1.25]$	55	0.0540	54.0	65.5
$(-1.25, -0.75]$	149	0.1524	152.4	121.0
$(-0.75, -0.25]$	193	0.1924	192.4	174.7
$(-0.25, 0.25]$	197	0.1958	195.8	197.4
$(0.25, 0.75]$	196	0.1924	192.4	174.7
$(0.75, 1.25]$	151	0.1524	152.4	121.0
$(1.25, 1.75]$	52	0.0540	54.0	65.5
$(1.75, \infty]$	4	0.0033	3.3	40.1

**Table 3.**  
 Pearson's  $\chi^2$  test.

estimated  $p$  from experimental samples. Large  $D(F, G_p)$  is noticeable in all  $p$ 's when  $t = 1.3$ . Therefore, normal assumption in such case of large  $p$  value may lead to error in conclusion. It should be noted that the significance of  $D(F, G_p)$  also depends on the sample size.

**Example 2:** (Simulation from exponential power distribution). **Table 3** shows a simulation of 1000 samples from exponential power distribution with  $p = 4.4$ , where  $n$  is the observed frequency in the  $i$ th interval,  $p_i((a, b]) = P(EPD(4.4)) - P(EPD(4.4) \leq a)$ . and  $Np_i$  and normal are the expected frequency in the  $i$ th interval for  $EPD(4.4)$  and normal distribution, respectively. We obtained  $\chi^2$  value of 0.4207 for  $EP(4.20)$  with degree of freedom 9, thus  $EP(4.20)$  is accepted as expected. However, the goodness-of-fit for  $N(0, 1)$  gives an observed  $\chi^2$  value of 832.559, which results in the rejection of  $N(0, 1)$  model for the same data set. See **Table 3** for detail report.

## 6. The Kullback-Leibler information

The Kullback-Leibler (K-L) information function [14] can be used to discriminate between two distributions  $F_\theta(x)$  and  $F(x)$  of  $F_\theta(x); \theta \in \Theta$ . It is defined as

$$I(\theta, \phi) = E_\theta \left\{ \ln \frac{f(X; \theta)}{f(X; \phi)} \right\}; \theta, \phi \in \Theta \quad (11)$$

The family of  $F$  is assumed to be regular.

**Proposition:**  $I(\theta, \phi) \geq 0$  if and only if,  $f(X; \theta) = f(X; \phi)$  with probability one.

**Proof:** Recall that  $\ln x$  is the concave function of  $x$  and by Jensen's inequality  $\ln(E(Y)) \geq E(\ln Y)$  for every non-negative random variable  $Y$ , having a finite expectation. Accordingly,  $-I(\theta, \phi) = E_\theta \left\{ -\ln \frac{f(X; \theta)}{f(X; \phi)} \right\} = \int \ln \frac{f(X; \theta)}{f(X; \phi)} f(x; \theta) dx$

$\leq \ln \int f(x; \theta) dx = 0$  if both sides of the above equation is multiply by  $-1$  we have that  $I(\theta; \phi) \geq 0$ . Also, if  $P_\theta[f(X; \theta) = f(x; \phi)] = 1$  then  $I(\theta; \phi) = 0$ . *Q.E.D.*

It is worth noting that if  $X_1, \dots, X_n$  are identical and independent random variables then the  $K_L$  information function say  $I(\theta; \phi)$  is additive, that is,

$$I_n(\theta; \phi) = E_\theta \left\{ \ln \frac{f(X; \theta)}{f(X; \phi)} \right\} = E_\theta \left\{ \sum_{i=1}^n \ln \frac{f(X_i; \theta)}{f(X_i; \phi)} \right\} = nI(\theta; \phi).$$

**Example 1:** let  $F$  be the class of all normal distribution  $\{N(\mu, \sigma^2), \mu \in R, \sigma > 0\}$ . let  $\theta_1 = (\mu_1, \sigma_1)$  and  $\theta_2 = (\mu_2, \sigma_2)$ .

The likelihood ratio is

$$\frac{f(x; \theta_1)}{f(x; \theta_2)} = \frac{\sigma_1}{\sigma_2} \exp \left\{ -\frac{1}{2} \left[ \left( \frac{x - \mu_1}{\sigma_1} \right)^2 - \left( \frac{x - \mu_2}{\sigma_2} \right)^2 \right] \right\} \quad (12)$$

**Example 3:** (Applications to poultry feeds data) Now consider the data in Appendix B, where cholesterol level  $x_i$  of 48 eggs of chicken fed with organic copper salt are measured in  $mg/egg$ , where 5.20 is the estimated  $p$  value for exponential power distribution and 131.457 and 37.232 are the population mean and standard deviation, respectively. Also for Normal we have 59.10 and 1.822 as the estimated mean and standard deviation, respectively. The ordered data set  $x_i$  are given in **Table 4**,  $z_i$  and  $t_i$  is the standardized values for  $x_i$  for  $EP(5.20)$  and normal, respectively.  $Z_i = P(EP(5.20) \leq z_i)$  and  $T_i = P(N(0, 1) \leq t_i)$  is the normal counterpart. We define  $D_{EP}$  as the  $\max(|Z_i - i/n|, |Z_i - (i - 1)/n|)$  for  $EP(5.20)$  and  $D_N$  as  $\max(|T_i - i/n|, |T_i - (i - 1)/n|)$  for normal distribution. From **Table 4**, using Kolmogorov-Smirnov test, we find the corresponding  $|D| = 0.061833$  for  $EP(5.20)$  and  $|D| = 0.77742$  for normal distribution. One can easily see that the fit of exponential power cdf is uniformly better than that of the standard normal cdf in this example. All these have been made possible using the table in the appendix. Details are provided in **Table 4**.

$x_i$	$z_i$	$t_i$	$Z_i$	$ D_{EP} $	$T_i$	$ D_N $
60.73	-1.489196365	-1.899629351	0.0115	0.0115	0.5294	0.5294
66.03	-1.374792238	-1.757278685	0.0254	0.01627	0.5392	0.518367
71.33	-1.260388111	-1.614928019	0.0452	0.0173	0.5537	0.512033
76.63	-1.145983983	-1.472577353	0.0713	0.01203	0.5708	0.5083
81.86	-1.033090854	-1.33210679	0.1065	0.023167	0.5918	0.508467
81.93	-1.031579856	-1.330226687	0.1065	0.0185	0.5918	0.487633
81.93	-1.031579856	-1.330226687	0.1065	0.03933	0.5918	0.4668
87.16	-0.918686727	-1.189756124	0.1429	0.02377	0.617	0.471167
92.46	-0.8042826	-1.047405458	0.1829	0.016233	0.6469	0.480233
92.52	-0.802987459	-1.045793941	0.1864	0.02193	0.96492	0.77742
97.76	-0.689878473	-0.905054792	0.2284	0.020067	0.6814	0.473067
97.82	-0.688583332	-0.903443275	0.2284	0.0216	0.6841	0.454933
103.06	-0.575474345	-0.762704125	0.2707	0.0207	0.7236	0.4736
103.11	-0.574395061	-0.761361195	0.2747	0.01697	0.7236	0.452767
108.36	-0.461070218	-0.620353459	0.3182	0.026533	0.7676	0.475933
108.41	-0.459990934	-0.619010529	0.3182	0.01513	0.7709	0.4584
113.66	-0.346666091	-0.478002793	0.3613	0.027967	0.8156	0.482267
113.7	-0.345802664	-0.476928449	0.3613	0.0137	0.8156	0.461433
118.96	-0.232261964	-0.335652127	0.4088	0.0338	0.8669	0.4919
119	-0.231398536	-0.334577783	0.4088	0.012967	0.8707	0.474867
124.26	-0.117857837	-0.193301461	0.4563	0.039633	0.9247	0.508033

$x_i$	$z_i$	$t_i$	$Z_i$	$ D_{EP} $	$T_i$	$ D_N $
124.3	-0.116994409	-0.192227116	0.4524	0.0149	0.9247	0.4872
129.56	-0.003453709	-0.050950795	0.4998	0.041467	0.9801	0.521767
129.6	-0.002590282	-0.04987645	0.4996	0.020433	0.9801	0.500933
134.86	0.110950418	0.091399871	0.5437	0.0437	0.0359	0.48493
134.89	0.111597988	0.09220563	0.5437	0.022867	0.0359	0.50577
140.16	0.225354545	0.233750537	0.5912	0.049533	0.091	0.4715
140.19	0.226002115	0.234556296	0.5912	0.0287	0.091	0.49233
145.46	0.339758672	0.376101203	0.6347	0.051367	0.148	0.45617
145.48	0.340190386	0.376638376	0.6347	0.030533	0.148	0.477
150.76	0.454162799	0.518451869	0.6778	0.0528	0.1985	0.44733
150.78	0.454594513	0.518989042	0.6778	0.031967	0.1985	0.46817
161.06	0.568566926	0.660802535	0.7253	0.058633	0.2454	0.4421
161.08	0.56899864	0.661339708	0.7253	0.0378	0.2454	0.46293
166.36	0.682971054	0.803153202	0.7681	0.059767	0.2881	0.44107
161.37	0.68318691	0.803421788	0.7681	0.038933	0.2881	0.4619
166.66	0.797375181	0.945503868	0.8136	0.0636	0.3289	0.44193
166.67	0.797591038	0.945772454	0.8136	0.042767	0.3289	0.46277
171.96	0.911779308	1.087854534	0.8535	0.061833	0.3621	0.4504
171.97	0.911995165	1.08812312	0.8535	0.041	0.3621	0.47123
177.26	1.026183435	1.2302052	0.8935	0.060167	0.3907	0.46347
177.26	1.026183435	1.2302052	0.8935	0.039333	0.3907	0.4843
182.56	1.140587562	1.372555866	0.9259	0.0509	0.4147	0.48113
182.56	1.140587562	1.372555866	0.9259	0.030067	0.4147	0.50197
182.56	1.140587562	1.372555866	0.9259	0.0116	0.4147	0.5228
187.86	1.25499169	1.514906532	0.9528	0.0153	0.4345	0.52383
187.86	1.25499169	1.514906532	0.9528	0.02637	0.4345	0.54467
193.16	1.369395817	1.657257198	0.9746	0.0254	0.4515	0.5485

**Table 4.**  
*Kolmogorov goodness-of-fit test.*

## 7. Concluding remarks

We have proposed a generalized exponential power distribution, and studied some of its mathematical and statistical properties. We fitted this distribution to data arising from an experiment concerning the cholesterol level and weight of eggs. The Q-Q plots clearly show that the generalized exponential power distribution fits the data better than the usual normal distribution. Finally, hypotheses tests show that consumption of eggs from chicken fed with organic copper salt should not be boycotted for the fear of high cholesterol level. Therefore we recommend that the constituents of poultry feeds should change from the inorganic to organic combinations.

## Appendix A

See Figures 1 and 2.

## Appendix B

See Table A1.

Inorganic copper salt		Organic copper salt	
Weight	Cholesterol	Weight	Cholesterol
52.67	164.23	56.08	56.08
53.17	167.42	56.34	56.34
53.67	170.6	56.61	56.61
54.17	173.78	56.87	56.87
54.67	176.96	57.13	57.13
55.17	180.14	57.39	57.39
55.67	183.32	57.65	57.65
56.17	186.51	57.92	57.92
56.67	189.69	58.18	58.18
57.17	192.87	58.44	58.44
57.67	196.05	58.7	58.7
58.17	199.24	58.96	58.96
58.67	202.42	59.23	59.23
59.17	205.6	59.45	59.45
59.67	208.78	59.75	59.75
60.17	211.96	60.01	60.01
60.67	215.14	60.27	60.27
61.17	218.33	60.54	60.54
61.67	221.52	60.8	60.8
62.17	224.69	61.06	61.06
62.67	224.85	61.32	61.32
63.17	227.88	61.58	61.58
63.43	228.03	61.85	61.85
65.67	231.06	62.34	62.34
65.15	228.01	62.11	62.11
63.43	224.83	61.85	61.85
62.93	221.65	61.58	61.58
62.43	218.46	61.32	61.32
61.93	215.28	61.06	61.06
61.43	212.1	60.8	60.8
60.93	208.92	60.54	60.54
60.43	205.74	60.27	60.27

Inorganic copper salt		Organic copper salt	
Weight	Cholesterol	Weight	Cholesterol
59.93	202.56	60.01	60.01
59.43	199.37	59.75	59.75
58.93	196.19	59.49	59.49
58.43	193.01	59.23	59.23
57.93	189.83	59	59
57.43	186.65	58.7	58.7
56.93	183.46	58.44	58.44
56.43	180.28	58.18	58.18
55.93	177.1	57.92	57.92
55.43	173.72	57.65	57.65
54.93	170.74	57.39	57.39
54.43	167.55	57.13	57.13
53.93	164.37	56.87	56.87

**Table A1.**  
 Observed data from inorganic and organic copper salt.

## Appendix C

See Tables A2–A6.

t	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5087	0.5166	0.5239	0.5308	0.5373	0.5436	0.5495	0.5554	0.5609
0.10	0.5663	0.5716	0.5767	0.5816	0.5864	0.5910	0.5956	0.6000	0.6044	0.6086
0.20	0.6127	0.6168	0.6207	0.6246	0.6284	0.6321	0.6358	0.6393	0.6428	0.6463
0.30	0.6497	0.6530	0.6564	0.6594	0.6626	0.6656	0.6687	0.6717	0.6746	0.6775
0.40	0.6804	0.6831	0.6859	0.6886	0.6913	0.6939	0.6965	0.6991	0.7016	0.7041
0.50	0.7065	0.7089	0.7113	0.7137	0.7160	0.7183	0.7205	0.7227	0.7249	0.7273
0.60	0.7293	0.7315	0.7334	0.7355	0.7375	0.7395	0.7415	0.7435	0.7454	0.7473
0.70	0.7492	0.7511	0.7529	0.7547	0.7565	0.7583	0.7601	0.7618	0.7635	0.7652
0.80	0.7669	0.7686	0.7702	0.7719	0.7735	0.7750	0.7766	0.7782	0.7797	0.7812
0.90	0.7827	0.7841	0.7857	0.7876	0.7886	0.7901	0.7915	0.7929	0.7941	0.7955
1.0	0.7970	0.7982	0.7997	0.8010	0.8023	0.8036	0.8049	0.8061	0.8074	0.8087
1.1	0.8099	0.8111	0.8115	0.8135	0.8147	0.8159	0.8170	0.8182	0.8193	0.8204
1.2	0.8216	0.8227	0.8238	0.8249	0.8260	0.8271	0.8281	0.8292	0.8302	0.8313
1.3	0.8323	0.8334	0.8343	0.8353	0.8363	0.8373	0.8383	0.8392	0.8402	0.8411
1.4	0.8421	0.8430	0.8439	0.8449	0.8458	0.8467	0.8476	0.8485	0.8493	0.8502
1.5	0.8511	0.8519	0.8528	0.8536	0.8545	0.8553	0.8561	0.8570	0.8578	0.8586
1.6	0.8594	0.8602	0.8610	0.8617	0.8625	0.8633	0.8641	0.8648	0.8657	0.8663
1.7	0.8670	0.8678	0.8685	0.8692	0.8700	0.8706	0.8714	0.8721	0.8728	0.8735
1.8	0.8741	0.8748	0.8755	0.8762	0.8768	0.8775	0.8781	0.8788	0.8794	0.8801

t	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.9	0.8809	0.8814	0.8819	0.8826	0.8832	0.8839	0.8844	0.8850	0.8857	0.8863
2.0	0.8869	0.8875	0.8880	0.8886	0.8892	0.8898	0.8902	0.8909	0.8915	0.8920
2.1	0.8926	0.8931	0.8937	0.8942	0.8948	0.8953	0.8958	0.8963	0.8973	0.8974

**Table A2.**  
Cumulative distribution table for exponential power at  $p = 0.5$ .

t	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.2	0.8979	0.8984	0.8993	0.8994	0.8999	0.9004	0.9009	0.9014	0.9019	0.9021
2.3	0.9029	0.9034	0.9038	0.9043	0.9048	0.9052	0.9057	0.9062	0.9066	0.9071
2.4	0.9075	0.9080	0.9084	0.9089	0.9093	0.9097	0.9102	0.9106	0.9111	0.9115
2.5	0.9119	0.9123	0.9127	0.9132	0.9135	0.9140	0.9144	0.9149	0.9152	0.9156
2.6	0.9160	0.9163	0.9168	0.9172	0.9176	0.9180	0.9183	0.9188	0.9191	0.9195
2.7	0.9199	0.9202	0.9206	0.9210	0.9214	0.9217	0.9221	0.9224	0.9228	0.9231
2.8	0.9235	0.9238	0.9242	0.9245	0.9249	0.9252	0.9256	0.9259	0.9262	0.9260
2.9	0.9269	0.9272	0.9276	0.9279	0.9282	0.9285	0.9289	0.9292	0.9292	0.9297
3.0	0.9301	0.9303	0.9304	0.9311	0.9314	0.9317	0.9320	0.9323	0.9326	0.9329
3.1	0.9332	0.9335	0.9338	0.9340	0.9343	0.9346	0.9349	0.9352	0.9355	0.9358
3.2	0.9360	0.9363	0.9366	0.9369	0.9351	0.9374	0.9377	0.9380	0.9382	0.9384
3.3	0.9388	0.9390	0.9393	0.9396	0.9398	0.9401	0.9403	0.9405	0.9407	0.9411
3.4	0.9413	0.9416	0.9418	0.9421	0.9423	0.9426	0.9428	0.9430	0.9433	0.9435
3.5	0.9438	0.9440	0.9442	0.9445	0.9447	0.9449	0.9452	0.9454	0.9457	0.9458
3.6	0.9461	0.9463	0.9465	0.9467	0.9470	0.9472	0.9473	0.9476	0.9479	0.9481
3.7	0.9483	0.9485	0.9487	0.9488	0.9491	0.9494	0.9495	0.9497	0.9499	0.9501
3.8	0.9504	0.9506	0.9507	0.9510	0.9511	0.9514	0.9516	0.9517	0.9519	0.9521
3.9	0.9523	0.9527	0.9528	0.9529	0.9531	0.9533	0.9535	0.9537	0.9538	0.9540
4.0	0.9542	0.9544	0.9546	0.9541	0.9549	0.9551	0.9553	0.9555	0.9557	0.9559
4.1	0.9560	0.9562	0.9563	0.9566	0.9567	0.9569	0.9570	0.9572	0.9574	0.9575
4.2	0.9577	0.9579	0.9580	0.9582	0.9583	0.9585	0.9587	0.9588	0.9590	0.9592
4.3	0.9593	0.9595	0.9596	0.9598	0.9599	0.9601	0.9606	0.9603	0.9606	0.9607
4.4	0.9609	0.9610	0.9612	0.9613	0.9615	0.9616	0.9617	0.9619	0.9620	0.9622
4.5	0.9623	0.9625	0.9626	0.9627	0.9629	0.9630	0.9632	0.9633	0.9635	0.9636
4.6	0.9637	0.9640	0.9640	0.9641	0.9643	0.9644	0.9645	0.9647	0.9648	0.9649
4.7	0.9651	0.9652	0.9654	0.9655	0.9656	0.9657	0.9658	0.9660	0.9661	0.9662
4.8	0.9664	0.9664	0.9666	0.9667	0.9668	0.9670	0.9671	0.9672	0.9673	0.9675
4.9	0.9676	0.9677	0.9678	0.9679	0.9681	0.9681	0.9683	0.9684	0.9685	0.9686
5.0	0.9687	0.9689	0.9690	0.9690	0.9692	0.9693	0.9695	0.9695	0.9696	0.9698
5.1	0.9699	0.9700	0.9698	0.9702	0.9703	0.9704	0.9705	0.9706	0.9708	0.9708
5.2	0.9709	0.9709	0.9711	0.9710	0.9713	0.9714	0.9716	0.9716	0.9718	0.9719
5.3	0.9719	0.9721	0.9721	0.9721	0.9723	0.9724	0.9725	0.9726	0.9727	0.9728
5.4	0.9729	0.9730	0.9731	0.9732	0.9743	0.9734	0.9735	0.9736	0.9737	0.9738

<b>t</b>	<b>0.0</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
5.5	0.9739	0.9740	0.9741	0.9741	0.9743	0.9743	0.9744	0.9746	0.9746	0.9747
5.6	0.9748	0.9749	0.9749	0.9750	0.9751	0.9740	0.9753	0.9754	0.9755	0.9755
5.7	0.9756	0.9757	0.9758	0.9759	0.9760	0.9760	0.9761	0.9762	0.9763	0.9764
5.8	0.9765	0.9765	0.9766	0.9767	0.9768	0.9767	0.9769	0.9770	0.9770	0.9772
5.9	0.9773	0.9773	0.9774	0.9774	0.9776	0.9776	0.9777	0.9778	0.9779	0.9778
6.0	0.9780	0.9780	0.9782	0.9782	0.9783	0.9783	0.9785	0.9785	0.9786	0.9787
6.1	0.9787	0.9788	0.9789	0.9790	0.9792	0.9791	0.9792	0.9792	0.9793	0.9794
6.2	0.9794	0.9795	0.9792	0.9797	0.9797	0.9798	0.9799	0.9799	0.9800	0.9800
6.3	0.9801	0.9802	0.9802	0.9803	0.9804	0.9804	0.9805	0.9806	0.9806	0.9807
6.4	0.9808	0.9809	0.9809	0.9810	0.9810	0.9811	0.9812	0.9812	0.9813	0.9813
6.5	0.9814	0.9815	0.9815	0.9816	0.9816	0.9817	0.9817	0.9777	0.9818	0.9818
6.6	0.9820	0.9820	0.9821	0.9822	0.9822	0.9823	0.9823	0.9824	0.9825	0.9825
6.7	0.9826	0.9826	0.9827	0.9827	0.9828	0.9830	0.9829	0.9830	0.9814	0.9830
6.8	0.9831	0.9832	0.9832	0.9833	0.9833	0.9834	0.9835	0.9835	0.9836	0.9830
6.9	0.9837	0.9837	0.9837	0.9838	0.9839	0.9839	0.9831	0.9840	0.9838	0.9840
7.0	0.9842	0.9841	0.9839	0.9820	0.9835	0.9848	0.9844	0.9845	0.9846	0.9846
7.1	0.9847	0.9847	0.9848	0.9848	0.9849	0.9849	0.9849	0.9850	0.9850	0.9851
7.2	0.9851	0.9852	0.9852	0.9853	0.9853	0.9854	0.9854	0.9855	0.9855	0.9855
7.3	0.9856	0.9856	0.9857	0.9857	0.9858	0.9858	0.9859	0.9859	0.9859	0.9860
7.4	0.9860	0.9861	0.9861	0.9862	0.9862	0.9862	0.9863	0.9863	0.9863	0.9864

**Table A3.**  
 Cumulative distribution table for exponential power at  $p = 1.0$ .

<b>t</b>	<b>0.0</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
7.5	0.9865	0.9865	0.9865	0.9866	0.9866	0.9866	0.9867	0.9868	0.9868	0.9868
7.6	0.9869	0.9869	0.9869	0.9868	0.9870	0.9871	0.9871	0.9872	0.9872	0.9872
7.7	0.9873	0.9873	0.9873	0.9874	0.9873	0.9875	0.9871	0.9875	0.9876	0.9873
7.8	0.9876	0.9877	0.9877	0.9878	0.9878	0.9878	0.9879	0.9878	0.9879	0.9880
7.9	0.9880	0.9881	0.9881	0.9881	0.9882	0.9882	0.9883	0.9883	0.9883	0.9884
8.0	0.9884	0.9884	0.9884	0.9885	0.9885	0.9885	0.9886	0.9884	0.9887	0.9887
8.1	0.9887	0.9887	0.9888	0.9887	0.9888	0.9888	0.9889	0.9889	0.9888	0.9892
8.2	0.9890	0.9891	0.9891	0.9891	0.9892	0.9892	0.9892	0.9892	0.9893	0.9893
8.3	0.9895	0.9894	0.9894	0.9895	0.9895	0.9895	0.9896	0.9896	0.9896	0.9896
8.4	0.9897	0.9897	0.9897	0.9898	0.9898	0.9898	0.9899	0.9900	0.9899	0.9900
8.5	0.9900	0.9899	0.9900	0.9901	0.9901	0.9901	0.9901	0.9902	0.9902	0.9902
8.6	0.9903	0.9903	0.9903	0.9903	0.9904	0.9904	0.9904	0.9905	0.9905	0.9905
8.7	0.9905	0.9906	0.9906	0.9906	0.9906	0.9907	0.9907	0.9907	0.9908	0.9908
8.8	0.9908	0.9908	0.9909	0.9909	0.9910	0.9909	0.9910	0.9910	0.9912	0.9910
8.9	0.9911	0.9911	0.9911	0.9911	0.9912	0.9909	0.9912	0.9912	0.9913	0.9913

<b>t</b>	<b>0.0</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
9.0	0.9913	0.9914	0.9914	0.9914	0.9914	0.9914	0.9915	0.9915	0.9915	0.9915
9.1	0.9916	0.9916	0.9916	0.9916	0.9917	0.9917	0.9917	0.9911	0.9918	0.9918
9.2	0.9918	0.9918	0.9918	0.9919	0.9919	0.9920	0.9919	0.9921	0.9920	0.9920
9.3	0.9920	0.9920	0.9921	0.9921	0.9922	0.9921	0.9922	0.9922	0.9922	0.9922
9.4	0.9922	0.9924	0.9923	0.9923	0.9923	0.9924	0.9924	0.9924	0.9924	0.9924
9.5	0.9925	0.9925	0.9925	0.9925	0.9925	0.9926	0.9926	0.9926	0.9926	0.9926
9.6	0.9927	0.9927	0.9927	0.9922	0.9908	0.9931	0.9928	0.9928	0.9928	0.9928
9.7	0.9929	0.9929	0.9929	0.9929	0.9930	0.9930	0.9930	0.9930	0.9930	0.9930
9.8	0.9930	0.9931	0.9931	0.9931	0.9931	0.9931	0.9932	0.9932	0.9932	0.9932
9.9	0.9932	0.9933	0.9933	0.9933	0.9933	0.9934	0.9934	0.9934	0.9934	0.9934

**Table A4.**  
Cumulative distribution table for exponential power at  $p = 1.0$ .

<b>t</b>	<b>0.0</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
0.0	0.5000	0.5050	0.5100	0.5150	0.5195	0.5245	0.5290	0.5340	0.5385	0.5430
0.10	0.5475	0.5520	0.5565	0.5610	0.5655	0.5695	0.5740	0.5780	0.5825	0.5865
0.20	0.5905	0.5945	0.5985	0.6025	0.6065	0.6105	0.6145	0.6185	0.6220	0.6260
0.30	0.6295	0.6335	0.6370	0.6405	0.6440	0.6475	0.6510	0.6545	0.6580	0.6615
0.40	0.6650	0.6680	0.6715	0.6745	0.6780	0.6810	0.6845	0.6875	0.6905	0.6935
0.50	0.6965	0.7000	0.7025	0.7055	0.7085	0.7115	0.7145	0.7170	0.7200	0.7230
0.60	0.7255	0.7285	0.7310	0.7335	0.7365	0.7390	0.7415	0.7440	0.7465	0.7490
0.70	0.7515	0.7540	0.7565	0.7590	0.7615	0.7640	0.7660	0.7685	0.7710	0.7730
0.80	0.7755	0.7775	0.7800	0.7820	0.7840	0.7860	0.7885	0.7905	0.7925	0.7945
0.90	0.7965	0.7985	0.8005	0.8025	0.8045	0.8065	0.8085	0.8105	0.8125	0.8140
1.0	0.8160	0.8180	0.8195	0.8215	0.8235	0.8250	0.8270	0.8285	0.8300	0.8320
1.1	0.8335	0.8350	0.8370	0.8385	0.8400	0.8415	0.8435	0.8450	0.8465	0.8480
1.2	0.8495	0.8510	0.8525	0.8540	0.8555	0.8565	0.8580	0.8595	0.8610	0.8625
1.3	0.8635	0.8650	0.8665	0.8680	0.8690	0.8705	0.8715	0.8730	0.8740	0.8755
1.4	0.8765	0.8780	0.8790	0.8805	0.8815	0.8825	0.8840	0.8850	0.8860	0.8875
1.5	0.8885	0.8895	0.8905	0.8915	0.8930	0.8940	0.8950	0.8960	0.8970	0.8980
1.6	0.8990	0.9000	0.9010	0.9020	0.9030	0.9040	0.9050	0.9060	0.9070	0.9075
1.7	0.9085	0.9095	0.9105	0.9115	0.9120	0.9130	0.9140	0.9150	0.9155	0.9165
1.8	0.9175	0.9180	0.9190	0.9200	0.9205	0.9215	0.9220	0.9230	0.9235	0.9245
1.9	0.9250	0.9260	0.9265	0.9275	0.9280	0.9290	0.9295	0.9305	0.9310	0.9315
2.0	0.9325	0.9330	0.9335	0.9345	0.9350	0.9355	0.9365	0.9370	0.9375	0.9380
2.1	0.9385	0.9395	0.9400	0.9405	0.9410	0.9420	0.9425	0.9430	0.9435	0.9440
2.2	0.9445	0.9450	0.9455	0.9460	0.9470	0.9475	0.9480	0.9485	0.9490	0.9495

**Table A5.**  
Cumulative distribution table for exponential power at  $p = 1.0$ .

t	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.3	0.9500	0.9505	0.9510	0.9515	0.9520	0.9525	0.9530	0.9535	0.9535	0.9540
2.4	0.9545	0.9550	0.9555	0.9560	0.9565	0.9570	0.9575	0.9575	0.9580	0.9585
2.5	0.9590	0.9595	0.9595	0.9600	0.9605	0.9610	0.9615	0.9615	0.9620	0.9625
2.6	0.9630	0.9630	0.9635	0.9640	0.9645	0.9645	0.9650	0.9655	0.9655	0.9660
2.7	0.9665	0.9665	0.9670	0.9675	0.9675	0.9680	0.9685	0.9685	0.9690	0.9695
2.8	0.9695	0.9700	0.9700	0.9705	0.9705	0.9710	0.9715	0.9715	0.9720	0.9720
2.9	0.9725	0.9730	0.9730	0.9735	0.9735	0.9740	0.9740	0.9745	0.9745	0.9750
3.0	0.9750	0.9755	0.9755	0.9760	0.9760	0.9765	0.9770	0.9770	0.9770	0.9770
3.1	0.9775	0.9775	0.9780	0.9785	0.9785	0.9785	0.9790	0.9790	0.9790	0.9795
3.2	0.9795	0.9800	0.9800	0.9800	0.9805	0.9805	0.9810	0.9810	0.9810	0.9815
3.3	0.9815	0.9815	0.9820	0.9820	0.9825	0.9825	0.9825	0.9830	0.9830	0.9830
3.4	0.9835	0.9835	0.9835	0.9840	0.9840	0.9840	0.9845	0.9845	0.9845	0.9850
3.5	0.9840	0.9850	0.9850	0.9855	0.9855	0.9855	0.9860	0.9860	0.9860	0.9860
3.6	0.9865	0.9865	0.9865	0.9865	0.9870	0.9870	0.9870	0.9875	0.9875	0.9875
3.7	0.9875	0.9880	0.9880	0.9880	0.9880	0.9880	0.9885	0.9885	0.9885	0.9885
3.8	0.9890	0.9890	0.9890	0.9890	0.9890	0.9895	0.9895	0.9895	0.9895	0.9900
3.9	0.9900	0.9900	0.9900	0.9900	0.9905	0.9905	0.9905	0.9905	0.9905	0.9910
4.0	0.9910	0.9910	0.9910	0.9910	0.9910	0.9915	0.9905	0.9915	0.9915	0.9915
4.1	0.9915	0.9920	0.9920	0.9920	0.9920	0.9920	0.9920	0.9925	0.9925	0.9925
4.2	0.9925	0.9925	0.9925	0.9925	0.9930	0.9930	0.9930	0.9930	0.9930	0.9930
4.3	0.9930	0.9935	0.9935	0.9935	0.9935	0.9935	0.9935	0.9935	0.9935	0.9940
4.4	0.9940	0.9940	0.9940	0.9940	0.9940	0.9940	0.9940	0.9945	0.9945	0.9945
4.5	0.9945	0.9945	0.9945	0.9945	0.9945	0.9945	0.9950	0.9950	0.9950	0.9950
4.6	0.9950	0.9950	0.9950	0.9950	0.9950	0.9950	0.9955	0.9955	0.9955	0.9955
4.7	0.9955	0.9955	0.9955	0.9955	0.9955	0.9955	0.9955	0.9960	0.9960	0.9960
4.8	0.9960	0.9960	0.9960	0.9960	0.9960	0.9960	0.9960	0.9960	0.9960	0.9960
4.9	0.9965	0.9965	0.9965	0.9965	0.9965	0.9965	0.9965	0.9965	0.9965	0.9965
5.0	0.9965	0.9965	0.9965	0.9965	0.9970	0.9970	0.9970	0.9970	0.9970	0.9970
5.1	0.9970	0.9970	0.9970	0.9970	0.9970	0.9970	0.9970	0.9970	0.9970	0.9970

**Table A6.**  
 Cumulative distribution table for exponential power at  $p = 1.0$ .

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