Chapter

BH M87: Beyond the Gates of Hell

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Abstract

The supermassive black hole located in the galaxy M87 (BH M87) is four times larger than our solar system. If it is spherically symmetric, then a capsule free falling from a distance of 1 light year would cross BH M87's event horizon within some tens of years. Continuing that journey, any unfortunate astronomer traveling within the capsule would remain alive for a few further tens of hours; if the capsule were equipped with a powerful engine and could slow down, their lifetime inside the horizon beyond "the gates of Hell" would be slightly extended. How is this so? What are the other properties of the interior of BH M87? Maintaining the assumption of spherical symmetry of the exterior of BH M87, we briefly discuss some simple but intriguing properties of its interior, a region that turns out to be highly anisotropic, both expanding and contracting at the same time.

Keywords: Schwarzschild and Reissner-Nordström space-times, supermassive black hole M87, isotropic exterior, event horizon, anisotropic interior

1. Introduction

On April 10, 2019, the first ever image of a black hole was displayed. Due to the extensive efforts of very many teams of astronomers, working in parallel a picture of a supermassive black hole, of 6.5 billion solar masses, located in the galaxy M87, at a distance of 55 Mly, belonging to the Virgo supercluster, was produced. The size of that object, despite being four times the size of the solar system, is nonetheless still too small to be pictured by a single telescope, so worldwide cooperation through the Event Horizon Telescope project led to a synthesized Earth size-like device and the final vision (see **Figure 1**) [1].

The visible presence of such a supermassive black hole puts old questions in a new light. Traveling toward such an object, reaching its "edge"—the event horizon—crossing it, and entering the interior, what would be the experience of such a traveler, an unfortunate astronaut, who would be unable to share his/her views and experiences with colleagues who remain at the starting point in a "Mother Station?"

We will describe some particular features of such a trip focusing on the bizarre properties of the interior of the black holes.

In general there are four possible kinds of black holes (see, e.g., [2–4]). Isotropic, i.e., spherically symmetric and static, BHs are of the Schwarzschild type; rotating—hence axially symmetric—BHs are called Kerr BHs; both of these types could also be charged; then they are referred to as Reissner-Nordström and Kerr-Newman, respectively. The outer edge of the BHs, an event horizon, acts as a semitransparent membrane that might be crossed once and in one direction only. Apart from the Schwarzschild BH, the other three types of BH also possess an inner horizon referred to as a Cauchy horizon.

Figure 1.
First ever image of the black hole in galaxy Messier 87, here denoted as BH M87 (55 Mly from the Earth), April 10, 2019.

In our considerations we will limit our discussion to the case of spherically symmetric, static BHs: the Schwarzschild (S) and Reissner-Nordström (RN) types. In these two cases, the space-time metric tensor is diagonal in spherical polar coordinates t, r, θ, φ and is described by the line element:

$$ds^2 = g_{tt}c^2dt^2 - g_{tt}^{-1}dr^2 - r^2d\Omega^2 \tag{1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ is a unit sphere element. The $\{tt\}$ element of the metric tensor takes the following form:

$$g_{tt}^{(i)} = \begin{cases} 1 - \dfrac{2GM}{c^2r} & i = S \\ 1 - \dfrac{2GM}{c^2r} + \dfrac{Q^2}{r^2} & i = RN \end{cases} \tag{2}$$

where M is the mass and Q is the charge of the BH. Hereafter we will use the notation $c = G = 1$. The zero value of $g_{tt}^{(S)}$ determines the location of the event horizon or gravitational radius, r_g:

$$r_g = 2M \tag{3}$$

There are two zeros of $g_{tt}^{(RN)}$,

$$r_\pm = M \pm \sqrt{M^2 - Q^2} \tag{4}$$

determining an outer, r_+, event horizon and an inner, r_- Cauchy horizon.

2. A capsule radially falling toward a black hole horizon

Consider the case of a test object, a capsule radially freely falling in a spherically symmetric and static space-time (1). We shall assume that capsule A (for Alice, see below) starts from rest at some initial position located at a Mother Station (MS) fixed at radial position r_{MS}. We will describe this radial infall answering some simple questions:

1. How long does it take, measured by an observer, termed A for Alice, within capsule A to reach the event horizon?

2. How much time does such a trip take from the point of view of another observer, termed static observer (SO) located at the Mother Station?

3. How does the speed of A change during this journey?

4. How can we verify these predictions?

Before doing this we will introduce some useful definitions. Firstly, every observer O whose history in the space-time is described by a world line, $\{x^{\mu}_{O}(\tau)\}$ such that

$$d\tau^2 = g_{\alpha\beta} dx^{\alpha} dx^{\beta} \tag{5}$$

is specified by a **unit velocity four-vector** $\left\{u^{\mu}_{O}(\tau) = \frac{dx^{\mu}}{d\tau}\right\}$,

$$u^2 \equiv g_{\alpha\beta} u^{\alpha} u^{\beta} = 1. \tag{6}$$

Light rays $\{x^{\mu}(\sigma)\}$ belong to light cones, and they are specified by a **null wave vector** $\left\{k^{\mu}(\sigma) = \frac{dx^{\mu}}{d\sigma}\right\}$,

$$k^2 \equiv g_{\alpha\beta} k^{\alpha} k^{\beta} = 0, \tag{7}$$

where σ is an affine parameter of the null geodesic. Due to the symmetry properties of the static and isotropic character of the S and RN space-times, there are two conservation laws: energy and angular momentum are conserved quantities (see, e.g., [5]). Energy conservation means that the t-component of the covariant velocity u_t/wave k_t vector is conserved:

$$u_t = g_{t\beta} u^{\beta} \equiv g_{tt} u^t = \varepsilon, \tag{8}$$

$$k_t = g_{t\beta} k^{\beta} \equiv g_{tt} k^t = \omega. \tag{9}$$

Conservation of the angular momentum results in the planar motion of both time-like geodesics (8) and light-like geodesics (9). Without loss of generality, one can consider then equatorial planar motion, $\theta = \frac{\pi}{2}$, where the corresponding velocity/wave vector component vanishes:

$$u^{\theta} = 0, \tag{10}$$

$$k^{\theta} = 0. \tag{11}$$

The value of the angular momentum is conserved, i.e.,

$$u_{\varphi} = g_{\varphi\varphi} u^{\varphi} \equiv L, \tag{12}$$

$$k_{\varphi} = g_{\varphi\varphi} k^{\varphi} \equiv l. \tag{13}$$

Therefore, geodesics determined by three nonvanishing components of the tangent vector, which is the velocity vector for the time-like world lines, Eq. (6), and the wave vector for the light-like world lines, Eq. (7), may be found from the two conservation laws and the normalization condition:

$$u^r = \pm\sqrt{\varepsilon^2 - g_{tt}\left(1 + \frac{L^2}{r^2}\right)}, \tag{14}$$

$$k^r = \pm\sqrt{\omega^2 - g_{tt}\frac{l^2}{r^2}}. \tag{15}$$

A special class of non-geodesic trajectories represents static observers (SO), whose position is fixed $(r_0, \theta_0, \varphi_0)$. Their only nonvanishing component of the velocity vector is a temporal one u^t_{SO}. It is determined by the normalization condition (Eq. (6)):

$$u^t_{SO} = \frac{1}{\sqrt{g_{tt}(r_0)}}. \tag{16}$$

Hence one can describe the trajectory of A, which is radially infalling, and the Mother Station (MS), which is static at r_0, by using their velocity vectors u_A and u_{MS}:

$$u_A = \left(u^t_A, u^r_A, 0, 0\right) = \left(\frac{\varepsilon}{g_{tt}}, -\sqrt{\varepsilon^2 - g_{tt}}, 0, 0\right), \tag{17}$$

$$u_{MS} = \left(u^t_{MS}, 0, 0, 0\right) = \left(\frac{1}{\sqrt{g_{tt}(r_{MS})}}, 0, 0, 0\right), \tag{18}$$

If A starts from the location of the Mother Station, being initially at rest, then

$$\varepsilon = \sqrt{g_{tt}(r_{MS})} \tag{19}$$

(see also below).

During the infall of A, one can measure its speed at some intermediate point r_1 (between r_{MS} and the event horizon) by arranging at r_1 an observer O determined by velocity vector u_O who measures an infinitesimal "distance of A" covered within an infinitesimal "time period." This results in a speed for A as measured by O, $v_A(O)$ expressed in terms of a *scalar product* $u_A u_O$:

$$u_A u_O = g_{\alpha\beta} u^\alpha_A u^\beta_O = \frac{1}{\sqrt{1 - v_A^2}}. \tag{20}$$

If O is a static observer located at r_1, then

$$v_A = \frac{\sqrt{\varepsilon^2 - g_{tt}(r_1)}}{\varepsilon} \tag{21}$$

as one can verify by using Eqs. (20) and (16).

2.1 How long does it take to Alice to reach the event horizon?

Now we can answer the questions concerning the duration associated with the infall of A. Applying the equations for the nonvanishing components of its velocity vector (Eq. (17))

$$\frac{dt}{d\tau} = \frac{\varepsilon}{g_{tt}}, \tag{22}$$

$$\frac{dr}{d\tau} = -\sqrt{\varepsilon^2 - g_{tt}}, \tag{23}$$

one obtains the equations for the *coordinate time t* and for the *proper time τ*:

$$t + C = -\int \frac{\varepsilon}{g_{tt}\sqrt{\varepsilon^2 - g_{tt}}} dr, \tag{24}$$

$$\tau + C' = -\int \frac{1}{\sqrt{\varepsilon^2 - g_{tt}}} dr. \tag{25}$$

The proper time is actually the time measured by Alice (A) traveling within the capsule. Hence the trip from MS to the event horizon of the BH is completed by Alice within the period:

$$\tau(r_{MS}; r_g) = -\int_{r_{MS}}^{r_g} \frac{1}{\sqrt{\varepsilon^2 - g_{tt}}} dr. \tag{26}$$

Specific examples of the free fall for both Schwarzschild and Reissner-Nordström space-times will be presented later. The important fact is that expression (26) leads to a finite value of the time $\tau(r_{MS}; r_g)$ recorded by Alice.

On the other hand, the coordinate time corresponding to the trip from MS to the event horizon is infinite (see also [6]):

$$t(r_{MS}; r) = -\int_{r_{MS}}^{r} \frac{\varepsilon}{g_{tt}\sqrt{\varepsilon^2 - g_{tt}}} dr \xrightarrow{r \to r_g} \infty \tag{27}$$

(see, however, below, Section 5). Coordinate time is associated with the time recorded by an observer(s) belonging to "our" part of the universe. It means that the perception of observers located outside the event horizon of a black hole is such that Alice would never complete her trip toward the horizon. In other words she could never reach the horizon in a finite time period.

This process of the asymptotic approach to the BH horizon as perceived by MS observers can be illustrated in a way presented in the following subsection.

2.2 Communication between the capsule and the Mother Station

Let us consider an exchange of electromagnetic signals, light rays between two observers: Alice, traveling within the capsule and Bob located at the Mother Station. Such signals are represented by radial rays (9) and (15) where $l = 0$ and

$$k = \left(\frac{\omega}{g_{tt}}, \pm\omega, 0, 0\right). \tag{28}$$

The frequency of the signal recorded by an arbitrary observer O, ω_O, is given by the projection of the appropriate wave vector, k, on the unit time-like vector of O, i.e., on the four-velocity vector u_O. It is a scalar product $k \cdot u_O$, and

$$\omega_O = k \cdot u_O \equiv g_{\alpha\beta} k^\alpha u_O^\beta. \tag{29}$$

Hence, Alice sends back signals that are recorded by Bob (at MS), and the frequency ratio of the recorded, ω_B^r vs. emitted, ω_A^e signals, found from Eqs. (28), (29), (9), (15), (17) and (18) is (see also [7]):

$$\frac{\omega_B^r}{\omega_A^e} = \frac{g_{\alpha\beta}k^\alpha u_B^\beta}{g_{\alpha\beta}k^\alpha u_A^\beta} = \frac{\omega}{\sqrt{g_{tt}(r_{MS})}}\left[\frac{\omega\varepsilon}{g_{tt}(r_A)}\left(1+\frac{\sqrt{\varepsilon^2 - g_{tt}(r_A)}}{\varepsilon}\right)\right]^{-1} = \frac{g_{tt}(r_A)}{g_{tt}(r_{MS})}\frac{1}{1+v_A} \equiv 1 - v_A. \tag{30}$$

One can see that the signals are found to be **redshifted**: the frequency of the recorded signals is lower than the frequency of the emitted signals. But in this case it turns out to be of a special form: it may be referred to as a **critical redshift** as it tends to zero as A approaches the horizon. Indeed, the speed v_A of the capsule, once measured by the static observer, tends to the speed of light in a vacuum, $v_A \to 1$ (e.g., [6]) as the capsule approaches the horizon, $g_{tt}(r_g) = 0$ (see Eq. (21)). And it is the manifestation of the fact that from Bob's perspective, the capsule approaches event horizon asymptotically and will never reach the horizon (see however Section 5!): the frequency of the signals incoming from the capsule gradually decreases and eventually goes beyond the lower limits (however small!) of the sensitivity of recording devices.

Summarizing the findings of this section, one would like to point out some of intuitive and counterintuitive conclusions. Obviously the speed of the capsule freely falling toward the BH horizon increases as measured by static observers placed above the event horizon. Quite non-obvious is that this value tends to the speed of light as it approaches the horizon. And what is even more important is that this outcome is independent of the initial conditions: wherever the capsule starts from, the rest of the value of its speed asymptotically approaches the value of the speed of light. Moreover, there are no static observers residing on the horizon, so one cannot claim that a test object reaches the speed of light when crossing the BH horizon (see also Refs. [8, 9]). Accompanying this highly nonclassical behavior of the free fall speed is the duration of this trip toward the horizon—it turns out to be infinite for an observer located beyond the event horizon (see also Section 5). Nevertheless the trip is completed within a well-defined time period for a traveler, Alice, who is confined within the capsule. This may be regarded as a most dramatic illustration of time dilation where both kinematic and gravitational time dilations are combined. It is confirmed by the generalized Doppler frequency shift: signals emitted by Alice and recorded by Bob at MS are critically redshifted.

3. Approaching and crossing the event horizon

When Alice, confined to the capsule, approaches the event horizon and then if the BH is massive enough—greater than millions of solar masses—then tidal forces are not particularly large (see, e.g., [9]), and it is believed that she would not even notice the instant of crossing the horizon. But the further consequences would be quite dramatic: one may cross the event horizon only once and only in one direction toward the BH. One may ask the general question: in such a situation of free fall, would it be possible to identify the presence of the horizon?

On the one hand, there is an obvious outcome arising from the equivalence principle: in a freely falling frame, one cannot determine an external gravitational field. But this refers to possible experiments performed within a freely falling frame. It has recently been shown [7] that by using an appropriate communication channel Alice could identify the presence of the event horizon quite precisely, in order to stop the capsule, if it is equipped with a powerful enough engine, or to determine the instant of crossing the horizon. Indeed, by recording the electromagnetic signals coming from Bob (placed at MS), with $k^r = +\omega$ (see Eq. (28)), Alice finds the following frequency ratio of recorded ω_A^r and emitted ω_B^e signals:

$$\frac{\omega_A^r}{\omega_B^e} = \frac{1}{1 + v_A}. \tag{31}$$

This ratio tends to $\frac{1}{2}$ as the capsule approaches the horizon (see Eq. (21)). And this is the way to identify the presence of the horizon in general and to identify crossing instant in particular: the redshift of signals coming from MS equals ½.

One of the specific features of the event horizon relates to the singular character of time dilation described above for the trip toward the horizon: nobody residing in "our" part of universe could record the instant when the capsule (or any other test particle) reaches the edge of an (arbitrary) BH. This results in an effect referred to as "image collision" [10, 11] (also termed touching ghosts). If capsule A is followed by another capsule C (carrying Cindy), which started its free fall later than A, how would Cindy perceive capsule A crossing the horizon? This problem could be formulated in the following way. Let Alice release a signal "I'm crossing the Black Hole horizon!" at the particular instant (known perfectly well to her from the method described above) just as she passes the horizon. It does not need to be the message—it could be a specific, encoded light ray signal. How would such a signal be recorded by Cindy? One can answer this question in various ways, for instance by illustrating this using Kruskal-Szekeres coordinates (see Ref. [11]) or invoking an analytical description within a different singularity-free coordinate frame. But one also can give a reverse argument! Cindy must record Alice's signal only when she, herself, crosses the horizon. Otherwise, recording this signal before reaching the horizon, Cindy would be able to share this message with other residents in our part of the universe; she could even stop her capsule. But this would contradict the above paradigm, namely, that one cannot record in our part of the universe the event horizon crossing instant by capsule A (or any other test particle).

4. The interior of black holes: there is no black hole inside a black hole

There are two singularities in the expression for the line element (1). One is defined as the horizon of a BH—a horizon of a BH (1) is determined from the zero value of g_{tt} or as a singularity of $g_{rr} = g_{tt}^{-1}$. It is well known (see e.g., [12]) that there are different coordinate systems, other than that used in (1), that are free of this singular characteristic at the horizon. These include Gullstrand-Painleve, Kruskal-Szekeres, Eddington-Finkelstein coordinates, and many others [9, 12]. The other type of singularity corresponds to $r = 0$ and cannot be removed or avoided by applying a different system of coordinates. One uses then the phrase "coordinate singularity" to refer to the former type as a "horizon singularity as opposed to the "true singularity" representing the latter one. By applying an appropriate frame of reference, we no longer deal with singular behavior at the event horizon.

4.1 Cylindrical-like shape

Hence, the interior of a BH could be described within such a singularity-free frame of reference. It has been shown, however [13], that the interior of a Schwarzschild BH may also be described in the terms of above-the-horizon coordinates, t, r, θ, φ (see Eq. (1)). There are two important consequences of such an approach. The first is the singular character at the horizon, $r = r_g$. The second is even more important: inside the horizon one has to accept the interchange of the roles of coordinates t and r. The former takes on the role of a spatial coordinate, and the latter has to be regarded as a temporal coordinate. This means that within the BH's horizon, $g_{tt} < 0$ and r can only decrease, and $dr < 0$ representing the passage of time. This interchange leads to a new interpretation of the conservation law associated with the t-invariance in this case. Outside the horizon it is interpreted as energy conservation (Eqs. (8) and (9)); inside the horizon it is manifested as momentum, t-component, and conservation. One arrives then at the first rather counterintuitive property of a BH.

The interior of spherically symmetric black holes described by Eq. (1) turns out to be a **cylindrical-like shape**, homogeneous along its axis with spheres at the two ends.

Other counterintuitive properties are associated with the dynamical character of the interior. Indeed, inside the horizon of BHs $r < r_g$, where r plays the role of a temporal coordinate, one can see in expression (1) that all of the metric tensor elements are r, "*time*" dependent. Therefore, it is a dynamical space-time, or in other words, it may be regarded as a *cosmology*. What are the properties of such a cosmology, for instance compared to our homogenous and isotropic, expanding universe?

One can start with an extension of the case considered above of capsule A crossing the horizon and continuing its trip within the bounds of the horizon. As already mentioned we may apply the coordinates used outside the horizon remembering the important interchange of the roles of coordinates t and r. Hence, inside the horizon the velocity vector is still given by expression (17) where u_A^t and u_A^r refer now to spatial and temporal components, respectively. Alice, confined within the capsule, and being inside the horizon of the BH (and being aware of this!, see Section 3), still receives the electromagnetic signals released at the fixed location of MS by Bob. These are described by formula (28). Therefore, inside the horizon the frequency ratio

$$\frac{\omega_A^r}{\omega_C^e} = \frac{1}{1 + \frac{\sqrt{\varepsilon^2 - g_{tt}(r_A)}}{\varepsilon}} \xrightarrow{r \to 0} 0 \tag{32}$$

decreases further below the horizon's value of ½ and tends to zero at the final singularity, $-g_{tt}(r) \xrightarrow{r \to 0} \infty$.

This description may be deceptive when interpreted through the automatic application of formulae (31) naively leading to the (wrong!) conclusion that the speed of A, $v_A = \frac{\sqrt{\varepsilon^2 - g_{tt}(r_1)}}{\varepsilon} \xrightarrow{r \to 0} \infty$. What is wrong with such an extension of the former interpretation?

One can ask for the speed of capsule A within the horizon measured in a way similar to the one applied outside the horizon. In order to do this, we need to introduce an analogue of a static observer, called an r-observer, ro (see below). This is one whose only velocity component is a "temporal" one, i.e.,

$$u_{ro} = \left(0, u_{ro}^r, 0, 0\right) = \left(0, -\sqrt{-g_{tt}(r)}, 0, 0\right). \tag{33}$$

The speed $\tilde{v}_A$ of capsule A within the horizon measured by ro (see Eq. (33)), by definition, is given as (see also Eq. (20))

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$$u_A \cdot u_{ro} = \frac{1}{\sqrt{1 - \tilde{v}_A^2}} \tag{34}$$

which turns out to be:

$$\tilde{v}_A = \frac{\varepsilon}{\sqrt{\varepsilon^2 - g_{tt}(r_1)}}. \tag{35}$$

Hence inside the horizon, the speed of the capsule that has already crossed over the edge and entered that region is given by the expression that is formally inverse to the corresponding one above the horizon (c.f. Eqs. (18) and (35)). This outcome might be surprising only at first sight. Indeed, as the meaning of speed is the ratio of an (elementary) "distance"/(elementary) "time" and the numerator and denominator have already reversed their roles, then the ratio known as "speed" is expressed (formally) as the inverse of the one outside the horizon. That is why the speed outside the horizon, v_A Eq. (18), and the speed inside the horizon, $\tilde{v}_A$ Eq. (35), are expressed as mutually inverse quantities.

Another interesting feature of the speed inside the horizon $\tilde{v}_A$ (35) is that its value decreases from the asymptotic value 1 at the horizon to zero at the final singularity, $r = 0$. For different values of $\varepsilon = g_{tt}(r_{MS})$, i.e., different initial positions of the capsule, the speed changes differently (see **Figure 2**), but the asymptotic values at the horizon and at the ultimate singularity remain fixed.

The capsule's speed is plotted along the vertical axis (velocity) as a function of r for $M = 1$ and $r_g = 2$, and the horizontal axis represents distance for different initial conditions: the red line represents a fall from infinity, $\varepsilon = 1$.

This discussion throws new light on a BH's interior: the velocity of a freely falling test particle, which grows as it falls outside horizon, appears to decrease within the horizon (see also [7]).

To illustrate the behavior of the interior further, let us consider two r-observers placed along the axis of homogeneity t, exchanging electromagnetic signals. The frequency shift would in this case be a significant source of information about the dynamics of the BH's interior.

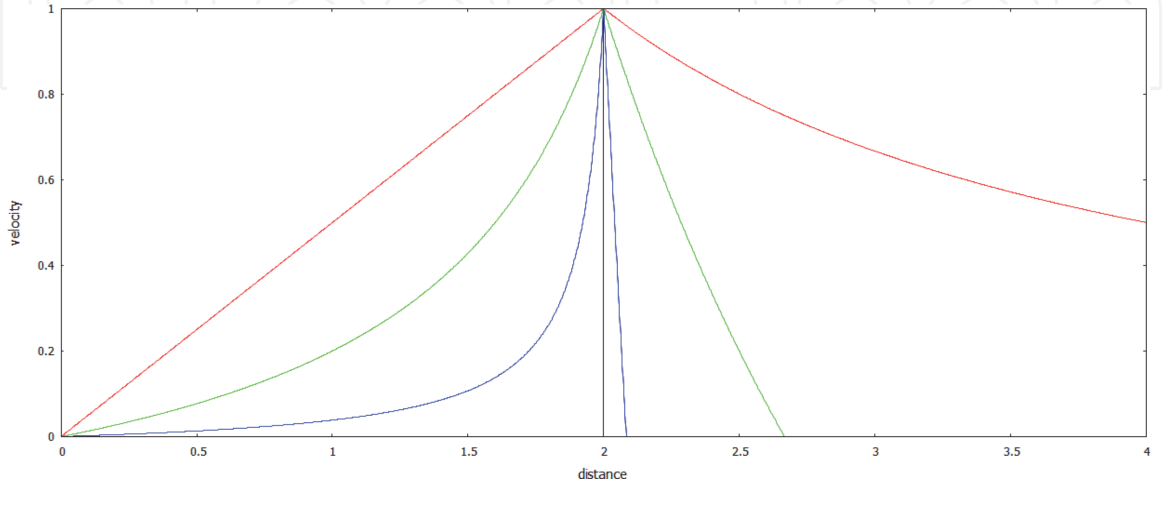

Figure 2.
The case of Schwarzschild space-time.

4.2 Expansion - exchange of electromagnetic signals along the t-axis

Let us consider then the exchange of signals between two observers located on the t-axis: Diana (D) receives signals sent by George (G), $r_D < r_G < r_g$. The frequency ratio is in this case expressed as follows:

$$\frac{\omega_D^r}{\omega_G^e} = \frac{\sqrt{-g_{tt}(r_G)}}{\sqrt{-g_{tt}(r_D)}} \tag{36}$$

It leads to distinct conclusions in S and RN space-times.

In the case of a Schwarzschild BH, $-g_{tt}(r) = \frac{2}{r} - 1$ is a monotonic function of r, and Eq. (36)

$$\frac{\omega_d^r}{\omega_G^e} < 1 \tag{37}$$

describes a Doppler-like *redshift* (see **Figure 3**). Hence, regarding this as a "cosmology," Eq. (36) represents a "cosmological redshift" due to expansion (along the t-axis!; see below).

In the case of a Reissner-Nordström BH, $-g_{tt}(r) = \frac{2}{r} - 1 - \frac{Q^2}{r^2}$ is a non-monotonic function of r, and Eq. (36) leads to:

a Doppler *redshift*,

$$\frac{\omega_D^r}{\omega_G^s} < 1, \tag{38}$$

for $r_m < r_D < r_G$, and a Doppler *blueshift*

$$\frac{\omega_D^r}{\omega_G^s} > 1 \tag{39}$$

for $r_D < r_G < r_m = Q^2$. This is illustrated in **Figure 4**, the ratio (36) in the RN case, M = 1, $Q = 0.6$, for fixed $r_G = 1.6$.

In this case Eq. (36) represents "cosmological redshift" due to expansion, followed by "cosmological blueshift" due to contraction (along homogeneity t-axis).

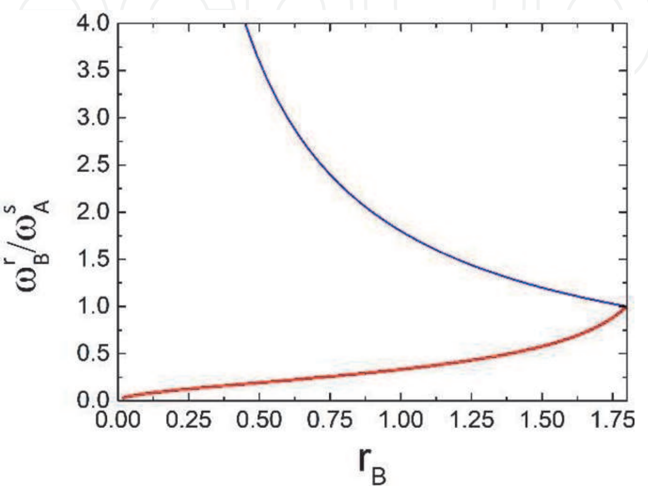

Figure 3.
The frequency shift inside the horizon of a Schwarzschild BH: signals propagating along the axis of homogeneity (t), Eq. (36) are redshifted (red), and signals propagating perpendicularly to the t-axis (57) are blueshifted (blue); (M = 1), $r_A = 1.75$.

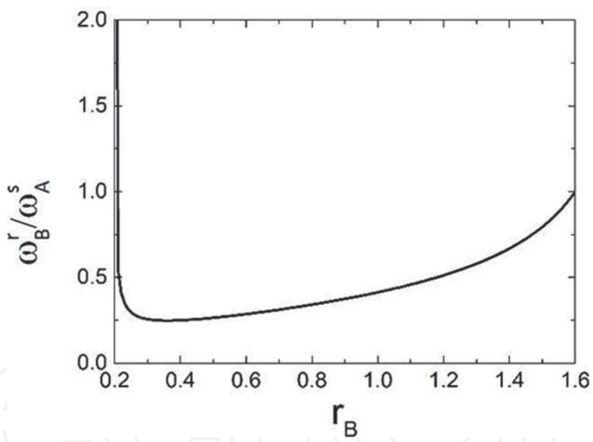

Figure 4.
Frequency shift inside the horizon of a RN BH (M = 1, Q = 0.6), for signals propagating along the t-axis $r_- = 0.2 < r_D < r_G = 1.6$: initial redshift Eq. (38) is followed by the final blueshift Eq. (39) (due to expansion followed by contraction).

4.3 Contraction – exchange of electromagnetic signals perpendicular to the *t*-axis

One may ask what happens if the exchanged signals travel perpendicularly to the t-direction? This means that the t-component of the position of Diana and George is the same. Assuming that the trajectory of the signal, the light ray, is confined within an equatorial plane, then it travels between φ_G and φ_D where D and G are placed at $\left(t_D = t_G, \frac{\pi}{2}, \varphi_D\right)$, $\left(t_G, \frac{\pi}{2}, \varphi_G\right)$. The signal is emitted at instant r_G and then recorded at instant r_D, so one will find (see [14]) both for S and RN BHs:

$$\frac{\omega^r_D}{\omega^s_G} = \frac{r_G}{r_D} > 1 \tag{40}$$

A Doppler blueshift is found in both S and RN space-times. This represents a contraction of this cosmology in a hyperplane perpendicular to the direction of homogeneity.

Therefore the cylindrically shaped interior of spherically symmetric, static (outside the horizon!), Schwarzschild and Reissner-Nordström black holes reveals anisotropic dynamics: they turn out to expand along the cylindrical axis of homogeneity and contract perpendicularly to this axis. In the case of Reissner-Nordström black holes, M = 1, the expansion stops at some instant, $r_m = Q^2$, and then contraction follows. It should be pointed out that the contraction perpendicular to the t-axis may simply be observed due to the form of the line element (1) inside the horizon, where the coefficient of its angular part, $r^2 d\Omega^2$, is an ever-decreasing coordinate r.

5. Traveling toward BH M87

The black hole in galaxy Messier 87, BH M87, is located at a distance of 55 Mly from the solar system. Its mass is estimated at 6.5 billion solar masses and its size, given as

$$R_{M87} \equiv \frac{2GM_{BHM87}}{c^2}, \tag{41}$$

appears to be 20 billion kilometers, four times the size of the solar system itself. It is probably rotating, so it cannot be regarded as spherically symmetric.

In our discussion we will assume, however, that this supermassive black hole, whose image was issued for the first time in history on April 10, 2019 (it looks like the gate to Hell) [1], is spherically symmetric and static—this implies that it is of Schwarzschild or Reissner-Nordström type. Having in mind our discussion above, we will try to indicate the specific features of such a trip, being of course absolutely fatal (as we will argue) once "the Gate" of the horizon of BH M87 has been crossed.

5.1 Free fall toward BH M87

Let us consider a spaceship starting its free fall from a Mother Station located at r_{MS} applying a coordinate system given by (1). We will consider various cases corresponding to different values of r_{MS}:

a. MS located at the Earth—$r_{MS} = 55\,Mly = 5.5 \cdot 10^{23} m$.

b. MS located within M 87—$r_{MS} = 1\,000\,ly = 10^{19} m$.

c. MS located (very) close to BH M87—$r_{MS} = 1\,ly = 10^{16} m$.

Our aim is to describe the trip itself and its perception by two specific observers: an astronaut, Archibald (A), located within a spaceship and a static observer, Barbara (B), located at MS. We will assume that A and B communicate simply by the exchange of electromagnetic signals and radial light rays of fixed frequency, characterized by a wave vector (28).

Firstly, free fall toward BH M87, the crossing of its horizon and eventually reaching the final outcome, will be considered within the two scenarios: fall from the rest (I) and fall with some nonzero initial speed simulating free fall from infinity (II).

5.1.1 How long does it take to reach position r_X?

The time to reach position r_X as measured by A is determined as follows (see Eq. (26)):

$$\tau(r_{MS}; r_X) = -\int_{r_{MS}}^{r_X} \frac{1}{\sqrt{\varepsilon^2 - g_{tt}}} dr = \begin{cases} -\int_{r_{MS}}^{r_X} \dfrac{1}{\sqrt{g_{tt}(r_{MS}) - g_{tt}}} dr \\ -\int_{r_{MS}}^{r_X} \dfrac{1}{\sqrt{1 - g_{tt}}} dr \end{cases} \tag{42}$$

for cases I and II, respectively. For Schwarzschild space-time, $Q = 0$, one finds in the scenarios (a)–(c) listed above the following results in cases I and II:

$$\tau_s(r_{MS}; r_X) = -\int_{r_{MS}}^{r_X} \frac{1}{\sqrt{\varepsilon^2 - g_{tt}}} dr = T_0 I_s(r_{MS}; r_X) \tag{43}$$

$$T_0 = \frac{R_g}{c} = 6.4 \cdot 10^4 s \tag{44}$$

$$I_s(r_2; r_1) = \begin{cases} -\int_{r_2}^{r_1} \dfrac{1}{\sqrt{g_{tt}(r_{MS}) - g_{tt}}} dr = I_{MS}((r_2; r_1)) = x_{MS}^{3/2} \left(arctgy + \dfrac{y}{1 + y^2} \right)_{y_2}^{y_1} \\ -\int_{r_2}^{r_1} \dfrac{1}{\sqrt{1 - g_{tt}}} dr = I_\infty(r_2; r_1) = \dfrac{2}{3} \left(x_2^{3/2} - x_1^{3/2} \right) \end{cases} \tag{45}$$

$$x = \frac{r}{R_g} \qquad y = \sqrt{x_{MS}}\sqrt{\frac{1}{x} - \frac{1}{x_{MS}}} \tag{46}$$

a. $r_X = \frac{r_{MS}}{2}$

$$\tau_s(r_{MS}; r_X) = \begin{cases} \tau_{MS} = T_0 x_{MS}^{3/2}\left(arctgy + \frac{y}{1+y^2}\right)_{y_2}^{y_1} \sim 10^{21}s \sim 3 \cdot 10^{13}y \\ \tau_\infty = T_0 \frac{2}{3}\left(x_2^{3/2} - x_1^{3/2}\right) \sim 10^{20}s \sim 3 \cdot 10^{12}y \end{cases} \tag{47}$$

$$\tau_s(r_{MS}; r_X) = \begin{cases} \sim 10^6 y \\ \sim 5 \cdot 10^5 y \end{cases} \tag{48}$$

$$\tau_s(r_{MS}; r_X) = \begin{cases} \tau_{MS} \sim 30y \\ \tau_\infty \sim 10y \end{cases} \tag{49}$$

b. $r_X = 1.1 r_g$

$$\tau(r_{MS}; r_X) = \begin{cases} \tau_{MS} \sim\sim 3 \cdot 10^{13} y \\ \tau_\infty \sim\sim 3 \cdot 10^{12} y \end{cases} \tag{50}$$

$$\tau(r_{MS}; r_X) = \begin{cases} \tau_{MS} \sim 10^6 y \\ \tau_\infty \sim 5 \cdot 10^5 y \end{cases} \tag{51}$$

$$\tau(r_{MS}; r_X) = \begin{cases} \tau_{MS} \sim 30y \\ \tau_\infty \sim 13.5y \end{cases} \tag{52}$$

Barbara may make her own measurements of the time representing the instants indicated above in different ways: recording signals coming from A, communicating with A about his perception of time, etc. One may prefer to use a compromise based on this variety of approaches, namely, indicating the coordinate instant t_X corresponding to $\tau(r_{MS}; r_X)$. Indeed one or other method of measuring the instant when reaching coordinate position r_X by A applied by B refers to t_X and is determined by (see also Eq. (42)):

$$t_X \equiv t(r_{MS}; r_X) = -\int_{r_{MS}}^{r_X} \frac{\varepsilon}{g_{tt}\sqrt{\varepsilon^2 - g_{tt}}} dr = \begin{cases} -\int_{r_{MS}}^{r_X} \frac{\sqrt{g_{tt}(r_{MS})}}{g_{tt}\sqrt{g_{tt}(r_{MS}) - g_{tt}}} dr \\ -\int_{r_{MS}}^{r_X} \frac{1}{g_{tt}\sqrt{1 - g_{tt}}} dr \end{cases} \tag{53}$$

for cases I and II, respectively. As indicated in former sections, the coordinate time period becomes singular (goes to infinity) as A approaches the horizon, independently of the initial conditions:

$$r_X \to r_g, t_X \to \infty. \tag{54}$$

In analogy with the above results for A, one finds for B the following outcomes:

$$t_{\infty X} \equiv t(r_{MS}; r_X) = -\int_{r_{MS}}^{r_X} \frac{1}{g_{tt}\sqrt{1 - g_{tt}}} dr = T_0 I_{\infty t} \tag{55}$$

$$
\begin{aligned}
I_{\infty t} &= \int_{x_{MS}}^{x_X} \frac{\sqrt{x}dx}{\left(1-\frac{1}{x}\right)} dr \\
&= \frac{2}{3}\left(x_{MS}^{3/2} - x_X^{3/2}\right) + 2(\sqrt{x_{MS}} - \sqrt{x_X}) + ln\left|\frac{x_{MS}-1}{x_{MS}+1}\right| - ln\left|\frac{x_X-1}{x_X+1}\right|
\end{aligned} \tag{56}
$$

$$
t_{\infty X} = T_0\left\{\frac{2}{3}\left(x_{MS}^{3/2} - x_X^{3/2}\right) + 2(\sqrt{x_{MS}} - \sqrt{x_X}) + ln\left|\frac{x_{MS}-1}{x_{MS}+1}\right| - ln\left|\frac{x_X-1}{x_X+1}\right|\right\} \tag{57}
$$

The dominant term in the coordinate time is the first one if the final position, r_X, is not too close to the horizon:

$$
t_{\infty X} \approx T_0\left\{\frac{2}{3}\left(x_2^{\frac{3}{2}} - x_1^{\frac{3}{2}}\right)\right\} = \tau_{\infty X} \tag{58}
$$

If the destination station, X gets close to the horizon, the duration of travel (45) becomes dominated by the last term which tends to infinity:

$$
t_{\infty X} \approx T_0\left\{-ln\left|\frac{x_1-1}{x_1+1}\right|\right\} \xrightarrow{x_1\to 1} \infty \tag{59}
$$

However, in practical terms, i.e., in all of the cases listed above

$$
t_{\infty X} \approx T_0\left\{\frac{2}{3}\left(x_2^{\frac{3}{2}} - x_1^{\frac{3}{2}}\right)\right\} = \tau_{\infty X}. \tag{60}
$$

The last term starts to dominate for

$$
-ln\left|\frac{x_1-1}{x_1+1}\right| = x_2^{\frac{3}{2}} \tag{61}
$$

i.e., it depends on the initial conditions. In case (c), the logarithmic term starts to dominate incrementally close to the horizon (on the horizon in fact, see below):

$$
r_X = R_g\left(1 + e^{-6000}\right) \tag{62}
$$

The meaning of this result is that the coordinate time is the corrected proper time, and the correction is moderate up to the vicinity of the horizon. In the close vicinity of the horizon, the singular term starts to dominate, and the coordinate time tends to infinity in this range. However, as shown above the "close vicinity of the horizon" means

$$
\Delta r_X = R_g e^{-6000} \tag{63}
$$

"effectively on the horizon!"

If the initial conditions are those described in (a) and (b), then that range is (formally) even smaller, i.e., it is a "stronger" zero.

Before entering the interior of the Schwarzschild BH, we will illustrate trip A via the Doppler shift.

5.1.2 Doppler shift

A and B are exchanging electromagnetic signals of fixed (emitter) frequency. Let us present the list of frequency ratios at various $r_X \equiv r_A$ as in the former subsection. Applying expressions (30) and (31), one finds:

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$$v_A^2 = \frac{\varepsilon^2 - g_{tt}(r_A)}{\varepsilon^2} = \begin{cases} \dfrac{g_{tt}(r_{MS}) - g_{tt}(r_A)}{g_{tt}(r_{MS})} = \dfrac{\dfrac{R_g}{r_A} - \dfrac{1}{x_{MS}}}{1 - \dfrac{R_g}{r_{MS}}} = \dfrac{1}{x_{MS}} \dfrac{\dfrac{r_{MS}}{r_A} - 1}{1 - \dfrac{1}{x_{MS}}} & I \\ \dfrac{R_g}{r_A} = \dfrac{1}{x_{MS}} \dfrac{r_{MS}}{r_A} & II \end{cases} \tag{64}$$

a. $r_A = \dfrac{r_{MS}}{2}$

- $v_A^2 = \begin{cases} \dfrac{1}{x_{MS}} \dfrac{\dfrac{r_{MS}}{r_A} - 1}{1 - \dfrac{1}{x_{MS}}} = \dfrac{1}{x_{MS}} \dfrac{1}{1 - \dfrac{1}{x_{MS}}} \approx \dfrac{1}{x_{MS}} & I \\ 2\dfrac{1}{x_{MS}} & II \end{cases}$
- $\dfrac{\omega_A^r}{\omega_B^e} = \dfrac{1}{1+v_A} \approx 1 - v_A = \begin{cases} 1 - \dfrac{1}{\sqrt{2.75}} \cdot 10^{-5} & I \\ 1 - 10^{-5} & II \end{cases}$
- $\dfrac{\omega_B^r}{\omega_A^e} = 1 - v_A$
- $\dfrac{\omega_A^r}{\omega_B^e} = \dfrac{1}{1+v_A} \approx 1 - v_A = \begin{cases} 1 - \dfrac{1}{7} \cdot 10^{-2} & I \\ 1 - \dfrac{\sqrt{2}}{7} \cdot 10^{-2} & II \end{cases}$
- $\dfrac{\omega_B^r}{\omega_A^e} = 1 - v_A$
- $\dfrac{\omega_A^r}{\omega_B^e} = \dfrac{1}{1+v_A} \approx 1 - v_A = \begin{cases} 1 - \dfrac{1}{22} & I \\ 1 - \dfrac{\sqrt{2}}{22} & II \end{cases}$
- $\dfrac{\omega_B^r}{\omega_A^e} = 1 - v_A$

b. $r_A = 1.1 R_{M87}$

- $v_A^2 = \begin{cases} \approx \dfrac{1}{1.1} & I \\ \dfrac{R_g}{r_A} = \dfrac{1}{1.1} & II \end{cases}$
- $\dfrac{\omega_A^r}{\omega_B^e} = \dfrac{1}{1+v_A} = 0.512$
- $\dfrac{\omega_B^r}{\omega_A^e} = 1 - v_A \approx 0.046$

and (c) the same as (a)

c. $r_A = 1.01 R_{M87}$

- $\dfrac{\omega_A^r}{\omega_B^e} = \dfrac{1}{1+v_A} = 0.5012$

- $\dfrac{\omega_B^r}{\omega_A^e} = 1 - v_A \approx 0.00496$

and (c) the same as (a)

d. $r_A = 1.001 R_{M87}$

- $v_A^2 = \dfrac{1}{1.001}$
- $\dfrac{\omega_A^r}{\omega_B^e} = \dfrac{1}{1 + v_A} = 0.50012$
- $\dfrac{\omega_B^r}{\omega_A^e} = 1 - v_A \approx 0.0004996.$

When A approaches the horizon of BH M87, $r_X \to R_{M87}$, the frequency of signals reaching B tends to zero

$$\frac{\omega_B^r}{\omega_A^e} \to 0 \tag{65}$$

and the signals themselves gradually disappear from the recording devices. Such a process becomes unboundedly extended in time. On the other hand, A receives the signals from B as redshifted toward a well-defined limit, and one finds in all cases (a–c)

$$\frac{\omega_A^r}{\omega_B^e}(r_X = R_{M87}) = 0.5 \tag{66}$$

as the indicator of the instant of crossing the horizon (see also Section 3).

5.2 Beyond "the gate of BH M87": how much time remains?

Archibald knows precisely the instant of his crossing of the horizon: whatever his starting point was, (a)–(c), he passes the horizon BH M87 when the frequency ratio hits ½. It is the irreversible instant in the whole trip: after this there is no way back. One may ask, however, the provocative question: why is there no way back?

Let us briefly discuss this point. During the radial fall toward BH M87, outside the horizon, $r > R_{M87}$, A can "see" both MS and BH M87, i.e., he can perceive the signals coming from B (located at MS) as well as the signals coming from regions located closer to BH M87 than his own current location. Radial light rays can obviously propagate along both increasing r and diminishing r. Upon crossing the horizon, the situation becomes quite different. The coordinate r changes its character—it becomes time-like, such that $dr < 0$. This means that the r coordinate only diminishes, reducing from R_{M87} to 0. Therefore, there is no way "back to the horizon" inside BH M87 because the horizon is "an instant in the past"—there is no way to "travel" to the past. It should be pointed out that this conclusion presented within this "pathological" (i.e., singular behavior at the horizon) system of coordinates remains valid as this also occurs in other, nonsingular coordinate systems.

Therefore, after crossing the horizon BH M87, Archibald no longer travels toward the center of black hole M87, but he travels along the t-axis of homogeneity until the final instant, $r = 0$.

How much time does this trip take? The answer is given by applying expression (42) to the interior of BH M87, $r < R_{M87} \equiv r_X, g_{tt} < 0$,

$$\tau(r_X; 0) = -\int_{r_X}^{0} \frac{\varepsilon}{\sqrt{\varepsilon^2 - g_{tt}}} dr = \begin{cases} -\int_{r_X}^{0} \frac{1}{\sqrt{g_{tt}(r_{MS}) - g_{tt}}} dr \\ -\int_{r_X}^{0} \frac{1}{\sqrt{1 - g_{tt}}} dr \end{cases} \tag{67}$$

and it depends on the scenario, i.e., the boundary conditions, I or II.

Hence, in the case of free fall from infinity (or its simulation), one finds:

$$\tau_\infty(r_X; 0) = 12\, hrs \tag{68}$$

In case I, a) – c) one obtains the same outcome:

Archibald, upon entering the interior of BH M87, would be left with only 12 hours in this fatal trip. Could this period be extended? Or what would be, if it were to exist, the maximal period, the maximal lifetime inside BH M87, hereafter termed lft BH M87?

As illustrated above lft BH M87 depends on the history (i.e., the initial conditions) of the trip, and it gets longer once MS gets closer to the horizon (much closer than 1 light year!). Actually as one finds from expression I (69), its maximal value corresponds to the case $g_{tt}(r_{MS}) = 0$. This cannot be achieved but it should be regarded as a limiting case. This limit represents the situation of Archibald's spaceship stopping just before reaching the horizon and then being released, maybe without Archibald who would prefer to avoid the particular experience of crossing the horizon. Then one finds the value of maximal lifetime of BH M87 as

$$\tau_{max}(r_X; 0) = -\int_{r_X}^{0} \frac{1}{\sqrt{-g_{tt}}} dr = 28.4 hrs. \tag{69}$$

This is then the maximal extension of time, the maximal lifetime within the black hole M87.

So despite the fact that BH M87 is an enormously large object, you do not have much time left once you have crossed its border.

5.2.1 Tidal forces at the gate and beyond

Discussing even a hypothetical trip to the interior of BH M87, one should take into account aspects of human frailty. One of them concerns the forces applied to the human body during this particular journey. There are tidal forces applied to the body of the astronaut, in this case Archibald. They turn out to be quite moderate on the horizon in the case of a supermassive black hole as is a well-known fact. So at the horizon, $r \cong R_{M87}$, the differential force acting along Archibald's body, leads to a pressure of the order of (see, e.g., [9]) 10^{-15} *atm*. This effect increases, however, and at some stage, when $r \cong \frac{1}{1000000} R_{M87}$, it leads to a limiting value of the pressure, some 10^2 *atm*. And for Archibald who decided to undergo the unique experience of crossing the horizon of BH M87, that would be the ultimate *end*.

5.3 RN scenario

What changes if BH M87 is electrified with a charge Q? Then BH M87 is of the RN kind; it is a little smaller, but its radius cannot never be smaller than half of the

Schwarzschild value (see Eq. (4)), $\frac{GM_{M87}}{c^2} \equiv M_{M87} = 10^{13}m$. Moreover, for particular values of the electric charge, the estimations of this section remain to be verified, leading to different final outcomes. However, the qualitative character of the results remains unchanged: the frequency ratio at the horizon hits ½ for A, and signals coming to B are critically redshifted; there is the most dramatic manifestation of time dilation illustrating the “image collision” or “touching ghosts” effect, and there is a significant difference between the interiors of these two kinds of BHs. If BH M87 is electrically charged, then it possesses an inner horizon, and the process of expansion along the homogeneity axis, the t-axis, would stop at the instant

$$r_{min} \equiv \frac{Q^2}{M_{M87}} > r_- \tag{70}$$

and then contraction would follow. That process of contraction would continue up to the instant

$$r = r_- \tag{71}$$

During contraction along the homogeneity axis, it becomes of infinite length apart from the final instant (86) when its length suddenly becomes zero – the system reaches its inner horizon. However, the physical character of the inner horizon remains a questionable point (see [12]).

6. Concluding remarks

Supermassive BH M87 is a very large object with a size of some 20 light hours. Located at a distance 55 Mly, it does not seem to be reachable from the Earth. However, looking at its image (the very first of a black hole), it might be of interest to consider and present some issues representing and characterizing this kind of object. As a supermassive black hole, it exerts a very strong gravitational pull (see also: “strong gravitational fields” [15–17]). To illustrate this one could consider free fall due to the gravitational attraction of BH M87. The trip from the Earth would last 10,000 times longer than the age of the universe. But a test object falling from a distance of 1 light year would reach the BH M87 event horizon within some 30 years. On the other hand, traveling with a constant speed of 300,000 km/h (at the moment the greatest speed achieved by an object produced by humans), one could cover a distance of 1 light year within 3600 years, 120 times longer than the period given above.

Assuming it is spherical, we have presented a variety of features related to the hypothetical trip toward and within BH M87, emphasizing the dynamics of its anisotropic interior.

Finally we would like to comment on a remark on the image of BH M87 made by an anonymous columnists who said:

“ ... it looks like the Gate to Hell”.

Considering a hypothetical trip toward BH M87, one finds that the anonymous columnist was wrong: looking at the image of BH M87, one has to remember that in fact it functions in a way much worse than the Gate to Hell. After crossing such an “invisible, so apparently gentle gate,” you are trapped: there is no way back and you are left with no more than 28 hours. By that time, your body would be stretched and compressed at the same time with no limits.

If BH M87 confines an electric charge, then it is possible that the process of stretching would be stopped, and contraction would follow. But this could hardly change your perspective: your lifetime within the horizon could never be substantially extended. And there is no way out.

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