We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900
Open access books available

185,000
International authors and editors

200M
Downloads

Our authors are among the

154
Countries delivered to

TOP 1%
most cited scientists

12.2%
Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com



Chapter

Convection Currents in Nanofluids under Small Temperature Gradient

Jyoti Sharma and Urvashi Gupta

Abstract

Nanobiotechnology has huge number of applications in medical science thereby improving health care practices. Keeping in mind the applications of nanoparticles and the convection patterns in biological fields, behaviour of nanofluids is explored for small temperature difference in the layer. The flow of nanofluids is usually described by system of differential equations. A mathematical model for the system based on conservation laws of mass, momentum and energy is formed. To get the insight of the problem, complex equations are simplified wherever needed to get interesting results without violating the necessary physics. The influence of physical properties such as density and conductivity of metallic/non-metallic nanoparticles is examined on the onset of convection currents in the fluid layer.

Keywords: nanofluids, natural convection, conservation equations, metallic and non-metallic nanoparticles

1. Introduction

In 1959, the celebrated physicist and Nobel laureate Richard Feynman presented an idea of nanotechnology in his talk "There is a plenty of room at the bottom-An invitation to enter a new field of physics" by emphasizing on the fact that the laws of physics allow us to arrange the atoms the way we want. Almost a century ago, Maxwell [1] initiated working on this issue theoretically and unveiled that the particles of size of micrometer and millimeter, if used in traditional fluids can resolve the motive in a more efficient manner. Yet they had few drawbacks like clogging, erosion in micro channel and settling down which were curbed with the evolution of better substitute; nanosized particles (called as nanoparticles). The suspension of nanoparticles in the regular fluids comprised the nanofluids [2]. Nanofluids have also shown many interesting properties, and the distinctive features (refer **Table 1**) resulting in unprecedented potential for many applications particularly in biological, medical and biomedical applications.

The catalytic role of nanoparticles in intensifying the thermal conductivity of nanofluids is analyzed by many researchers: Masuda et al. [3], Eastman et al. [4], Das et al. [5] and others. In 2006, Buongiorno [6] pioneered the formulation of conservation equations of nanofluids by incorporating the impacts of diffusion due to Brownian motion and thermophoresis of nanoparticles. He made an observation that the velocity of nanoparticles can be perceived as a sum of base fluid and

Properties	Microparticles	Nanoparticles
Stability	Not stable	Stable
Surface/Volume ratio	One	One thousand times more than that of microparticles
Conductivity	Less	High
Clogging	More	Negligible
Erosion	Yes	No
Nanoscale phenomenon	No	Yes

Table 1.
Comparison of particles.

relative (slip) velocities. To prosecute his research, he considered seven slip mechanisms; inactivity, magnus effect, Brownian motion, diffusiophoresis, thermophoresis, gravitational settling and fluid drainage. Throughout his investigation, he agreed that from all these seven techniques, Brownian diffusion and thermophoresis have a significant part in the absence of turbulent effects. Choi et al. [7] found that carbon nanotubes provide highest thermal conductivity enhancement of nanofluids. There are ample number of evaluations on thermal conductivity of nanofluids [8–11] in which they discussed and analyzed the theoretical as well as experimental results. Heat transfer in nanofluids because of convection has been examined and contemplated by Das and Choi [12], Ding et al. [13] and Das et al. [14].

The ballistic character of heat transfer within nanoparticles has been studied by Chen [15]. Abnormal increase in viscosity is generally observed in relation to the base fluid. The presence of nanoparticles has found to enhance thermal conductivity [4, 7, 16–19]. At very low nanoparticle volume fractions ($<0.1\%$), a heat transfer enhancement up to 40% has been reported [8] and this percentage is found to enhance with temperature [5] and concentration of nanoparticles [16]. The results of Choi et al. [7] established the unexpected non-linear character of measured thermal conductivity with nanotube loadings at low concentration while all theoretical studies concluded a linear relationship. Also, it was discovered that thermal conductivity strongly depends on temperature [5] and particle size [20]. Pak and Cho [21] in their study also reported the heat transfer data for turbulent flow of nanofluids having nanoparticles as aluminum and titanium in circular tubes. They found that Nusselt number is up to 30% more than that of base fluid. Nowadays nanofluids are also used in drug delivery systems [22] and advanced nuclear systems [23] due to enriched thermal properties. The nanofluid technology is still in its early stage and various researchers are using nanofluids as a tool to solve technological riddles of the modern society. **Figure 1** establishes big impact of small particles in view of the diverse applications of nanofluids in fields of industrial, residential, biomedical and transportation.

These days, nanoparticles are used in almost every biomedical application. Recent usage of nanotechnology in medicine and cancer therapy has attracted a lot of interest in thermal properties of nanofluid such as blood with nanoparticles suspension. Researchers have made the efforts to construct a mathematical model that shows the physical system or phenomenon nearly exact behaviour [24, 25]. Motivated by their work, we also intended to form an analytical model for the analysis of the convection currents in a horizontal nanofluid layer which is in accordance with the physical laws. Consequently, the onset of convection currents in the nanofluid layer is investigated mathematically with the help of partial differential equations. To begin with, equations are non-dimensionalized to get Rayleigh

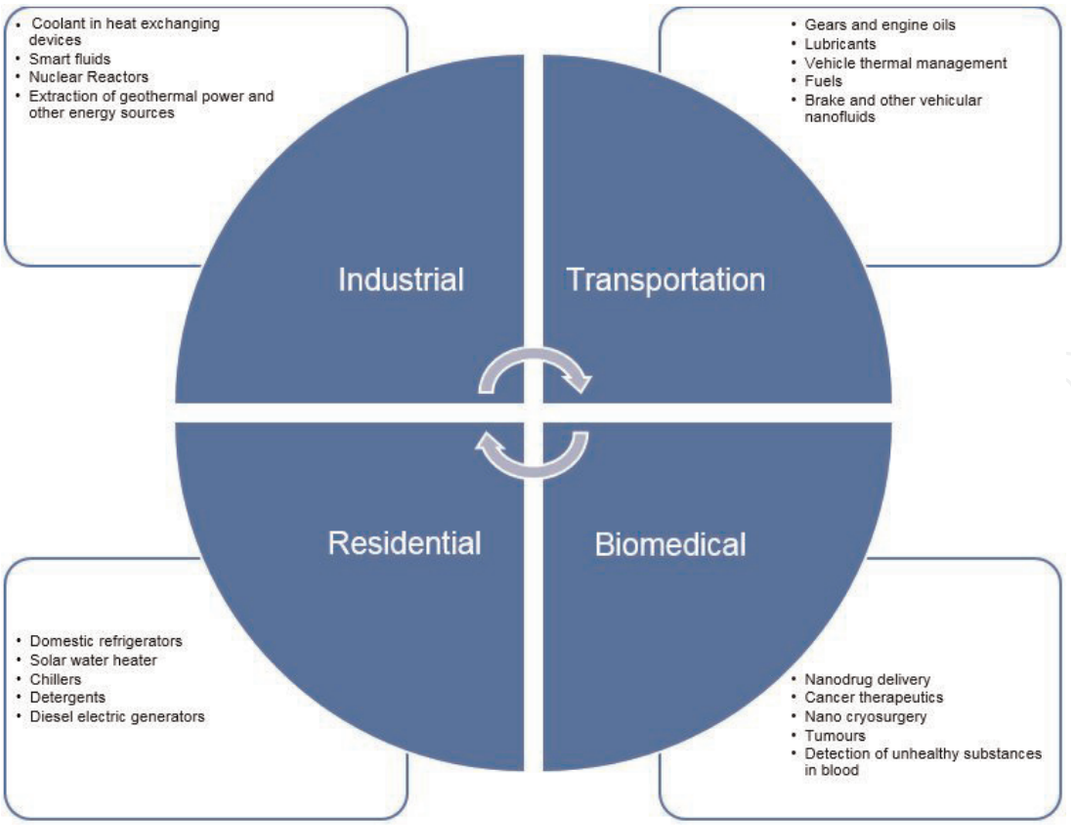


Figure 1.
Applications of nanofluids.

number in the system. Then small disturbances are added to the initial flow and new set of equations are obtained. Further PDE's are converted into ordinary differential equations using normal modes and expression for Rayleigh number is obtained. It is found that density and conductivity of nanoparticles are important parameters in deciding the stability of the system.

2. Instability of fluids under small temperature gradient: Rayleigh Bênard convection

The convective motions occur in a fluid layer heated underside in which a small temperature gradient is maintained across its boundaries. The maintained temperature across the boundaries must surpass a certain value before the instability can manifest itself. This Phenomenon was discovered by Bénard [26] in 1900. In most of his experiments, he found that if a fluid layer is heated underside, the layer at the bottom expands due to higher temperature. This makes the fluid density lighter at the bottom than that on the top making the system top heavy. Here viscosity and thermal diffusivity tend to oppose the convective motions but with the application of higher temperature gradient across the fluid layer, the thermal convection process gets initiated showing the pattern of cellular motions (called Bénard convection). Bénard [27] performed an experiment with metallic plate with a thin non-volatile liquid layer of 1 mm depth maintained under constant temperature.

Keeping the upper layer of fluid exposed to free air, he observed that the fluid layer was decomposed into number of cells (showing cellular motion) called Bénard cells. Thus in the standard Bénard problem, density difference due to variation in

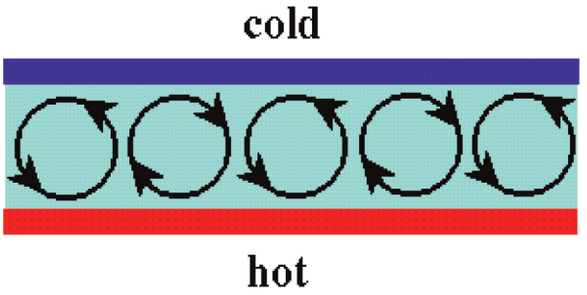


Figure 2.
A view of Rayleigh-Bénard convection.

temperature across the upper and lower boundaries of the fluid becomes the main reason for the occurrence of instability. **Figure 2** shows the schematic representation of Rayleigh-Bénard convection. Rayleigh [28] was the first person who gave an analytical treatment of the problem related to identifying the conditions responsible for breakdown of basic state. As a subsequent work carried out by Rayleigh and Bénard, thermal instability of fluids is known as Rayleigh Bénard convection. The condition for convective motions (depends on temperature gradient) can be represented in dimensionless form by the critical Rayleigh number. He figured out the condition for the instability of free surfaces by showing that the instability would occur on a large temperature gradient $\beta = -dT/dz$ in such a way that the Rayleigh number; $R_A = \alpha\beta \mathbf{g} d^4/\kappa\nu$, exceeds a certain critical value; where acceleration due to gravity is represented by g, coefficient of thermal expansion by α, the depth of the layer by d, thermal diffusivity by κ and kinematic viscosity is given by ν. For the stabilizing viscous force, R_A parameter gives the force of destabilizing buoyancy. Chandra [29] found discrepancy between the theoretical and experimental work for the convective motions in fluids when heated underside. He explained it by conducting an experiment on the layer in air and observed that instability of the fluid layer was dependent on its depth. A simplification in the partial differential equations describing the flow of compressible fluid is done by Spiegel and Veronis [30] by assuming very small depth of the layer as compared to the height. The basic equations of a fluid layer in porous medium (when heated underside) were formulated and derived by Joseph [31] by using Boussinesq approximation. The problem of thermal convection of a fluid layer has been put forward by Chandrasekhar [32] by considering the implications of various aspects of hydrodynamics and hydromagnetics. He depicted the result that addition of rotation and magnetic field increases the stability of the system. Kim et al. [33] considered the same problem of thermal convection for nanofluids. They showed that convective motion directly depends on the two physical properties (heat capacity and density) of nanoparticles and adversely depends on the conductivity of nanoparticles. Buongiorno [6] was the first scientist who formulated the conservation equations of nanofluids by assimilating the effects of diffusion due to Brownian motion and thermophoresis of nanoparticles. During his analysis, he concluded that Brownian and thermophoretic diffusion play a significant role in the absence of turbulent effects as compared to other seven mechanisms. Hwang et al. [34] treated this problem analytically and put forth the result of thermal instability of water based nanofluid with alumina nanoparticles in a rectangular container which is heated from below. They found that stability of the base fluid is enhanced by adding alumina nanoparticles and further it is enhanced by increasing the volume fraction of nanoparticles, the average temperature of the nanofluids and by decreasing the size of nanoparticles. They observed the decrease in heat transfer coefficient of nanofluids with the increase of the size of nanoparticles and decrease in the

temperature of nanofluids. Tzou [35, 36] investigated the onset of convective instability of nanofluids using Buongiorno's model analytically and established that nanofluids exhibit much lower stability than regular fluids. The results depict the inverse relationship of density and heat capacity of nanoparticles with their thermal conductivity and the shape factor. The results also include that the heat transfer coefficient of nanofluid is enhanced relative to volume fraction of nanoparticles. Nield and Kuznetsov [37] reconsidered the instability problem for nanofluids to get the expression for Rayleigh number and found conditions for the existence of oscillatory convection. It was established that the buoyancy coupled with the conservation of nanoparticles lead to higher instability of nanofluids. Alloui et al. [38] considered the shallow cavity to study Rayleigh Bénard instability for nanofluids. They concluded that rate of heat transfer in nanofluids depends on the strength of convection and volume fraction of nanoparticles while the presence of nanoparticles increase the stability of the system. Thermal instability for a horizontal nanofluid layer was considered by Yadav et al. [39]. They found an expression of thermal Rayleigh number and observed that the temperature gradient delays the convective motions while volumetric fraction of nanoparticles and the ratio of the density of nanoparticles to that of base fluid have destabilizing impact on the layer. The joint behaviour of nano-effects (Brownian motion and thermophoresis) creates destabilizing effect and can reduce the values of critical Rayleigh number as compared to that of regular fluids.

3. Conservation equations for a nanofluid layer

We start this section with the description of Boussinesq approximation which is used to write the conservation equations of nanofluids in simplified form.

As is the case of regular fluid [32], equations of nanofluids are difficult to solve because of their non-linear character. Therefore some mathematical approximations are to be used to simplify the basic equations without violating the physical laws. The contribution of Boussinesq [40] in the solution of thermal instability problems is in the form of approximations which is after his name. This approximation has been used by a many researchers for solving different problems of fluids. Boussinesq suggested that inertial effects of density variations can be neglected as compared to its gravitational effects as such situations exist in the domain of meteorology and oceanography. So, density is assumed to be constant everywhere in the equations of motion except in the term with external force. Therefore, we change $\rho_0[1 + \alpha(T_0 - T)]$ by ρ_0 everywhere in the equations of motion except the term representing the external body force.

Anoop et al. [41] explained various experimental techniques using which nanoparticles can be suspended in the base fluid and that suspension remain stable for several weeks. Buongiorno [6] adopted the formalism of Bird et al. [42] and Chandrasekhar [32] to write conservation equations for nanofluids by considering nanoscale effects; Brownian diffusion and thermophoresis. A model for convective transport in regular fluids was reformulated for nanofluids to accommodate these nanoscale effects as follows.

The random motion of nanoparticles is called Brownian motion and results into the continuous collisions with the base fluid molecules. The Brownian diffusion coefficient due to Brownian motion is given by

$$D_B = \frac{k_B T}{3\pi\mu d_p}, \tag{1}$$

where d_p is the nanoparticle's diameter, k_B is the Boltzmann's constant and μ is the viscosity of the fluid. The nanoparticles mass flux due to Brownian diffusion, $\boldsymbol{j_{p,B}}$ is given as

$$\boldsymbol{j_{p,B}} = -\rho_p D_B \nabla \phi, \tag{2}$$

where ϕ is the nanoparticle volume fraction and ρ_p is the nanoparticle mass density.

Thermophoresis is the phenomenon in which particles diffuse due to temperature gradient and the effect is similar to one of well-known effects of solute; Soret effect. The thermophoretic velocity is defined as.

$$\boldsymbol{V}_T = -\tilde{\beta}\frac{\mu}{\rho}\frac{\nabla T}{T} \text{ where } \tilde{\beta} = 0.26\frac{k}{2k + k_p}. \tag{3}$$

Here, ρ is the overall density of the nanofluid, k and k_p are the thermal conductivities of the fluid and the particle material, respectively. The negative sign in thermophoretic velocity represents movement of particles down the temperature gradient (from hot to cold). The nanoparticle mass flux due to thermophoresis, $j_{\boldsymbol{p,T}}$ is given as.

$$\boldsymbol{j_{p,T}} = -\rho_p \phi \boldsymbol{V}_T = -\rho_p D_T \frac{\nabla T}{T} \text{ with } D_T = \tilde{\beta}\frac{\mu}{\rho}\phi, \tag{4}$$

where D_T represents the thermophoretic diffusion coefficient.

The nanoparticles mass flux due to Brownian diffusion (Eq. (1)) and thermophoresis (Eq. (4)) are used to develop a two-component model for convective transport in nanofluids with the following assumptions:

- The nanofluid flow is incompressible.
- There are no chemical reactions in the fluid layer.
- The external forces are negligible.
- The mixture is dilute with nanoparticle volume fraction less than 1%.
- The viscous dissipation is negligible in the fluid.
- The radiative heat transfer is negligible.
- The nanoparticles and base fluid are locally in thermal equilibrium.

The seven equations based on basic conservation laws with the above mentioned assumptions are given as follows.

- Equation of state (one).
- Equation of continuity (one).
- Equation of nanoparticles (one).
- Equations of motion (three).
- Equation of energy (one).

3.1 Equation of state

Variables of state depend only upon the state of a system. The physical quantities: p; the pressure, T; the temperature and ρ; the density are the variables of state. We have three thermodynamic variables and a relation between them is given as.

$$F(p, \rho, T) = 0, \tag{5}$$

For substances with which we shall be principally concerned, the equation of state can be written as

$$\rho = \rho_0[1 + \alpha(T_0 - T)], \tag{6}$$

where T_0 is the temperature at which $\rho = \rho_0$.

3.2 Equation of continuity-conservation of mass

The equation of continuity for nanofluids is

$$\frac{\partial \rho}{\partial t} + u_j \frac{\partial \rho}{\partial x_j} = -\rho \frac{\partial u_j}{\partial x_j}, \tag{7}$$

where u_j is the jth component of nanofluid's velocity.
For an incompressible flow (using equation of state)

$$\frac{\partial \rho}{\partial t} + u_j \frac{\partial \rho}{\partial x_j} = 0, \tag{8}$$

so that the Eq. (7) reduces to

$$\frac{\partial u_j}{\partial x_j} = 0, \tag{9}$$

and in vector form continuity equation for nanofluid is expressed as

$$\nabla . \boldsymbol{v} = 0. \tag{10}$$

3.3 Equation of nanoparticles-conservation of mass

The conservation equation for nanoparticles in absence of chemical reactions is

$$\frac{\partial \phi}{\partial t} + \boldsymbol{v} . \nabla \phi = -\frac{1}{\rho_p} \nabla . \boldsymbol{j_p}, \tag{11}$$

where t is the time, ϕ is the nanoparticles volume fraction and $\boldsymbol{j}_p$ is the diffusion mass flux for nanoparticles and as external forces are negligible $\boldsymbol{j}_p$, the sum of two diffusion terms (Brownian diffusion and thermophoresis) using Eqs. (1) and (4) can be written as

$$\boldsymbol{j_p} = \boldsymbol{j_{p,B}} + \boldsymbol{j_{p,T}} = -\rho_p D_B \nabla \phi - \rho_p D_T \frac{\nabla T}{T}, \tag{12}$$

Combining Eqs. (11) and (12), nanoparticles conservation equation becomes

$$\frac{\partial \phi}{\partial t} + \boldsymbol{v}.\nabla\phi = \nabla.\left[D_B\nabla\phi + D_T\frac{\nabla T}{T}\right]. \tag{13}$$

Eq. (13) reveals that the nanoparticles move consistently with fluid (second term of left-hand side) and possess velocity relative to fluid (right-hand side) due to Brownian diffusion and thermophoresis.

3.4 Equations of motion-conservation of momentum

The equation of motion is derived from Newton's second law of motion which states that

$$\text{Rate of change of linear momentum} = \text{Total force}.$$

The momentum equation for nanofluid with negligible external forces is

$$\rho_0\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}.\nabla\boldsymbol{v}\right) = -\nabla p + \mu\nabla^2\boldsymbol{v} + \rho\boldsymbol{g}, \tag{14}$$

where ρ_0 is the nanofluid density at the reference temperature T_0 and the overall density of nanofluid; written as

$$\rho = \phi\rho_p + (1-\phi)\rho_f \cong \phi\rho_p + (1-\phi)\{\rho_0(1-\alpha(T-T_0))\} \tag{15}$$

Thus Eq. (14) becomes

$$\rho_0\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}.\nabla\boldsymbol{v}\right) = -\nabla p + \mu\nabla^2\boldsymbol{v} + \Big(\phi\rho_p + (1-\phi)\{\rho_0(1-\alpha(T-T_0))\}\Big)\boldsymbol{g}. \tag{16}$$

Note that in the absence of nanoparticles, Eq. (16) reduces to momentum equation for regular fluid.

3.5 Equation of energy-conservation of energy

The thermal energy equation for nanofluid with the assumptions (i)–(v) is

$$(\rho c)\left[\frac{\partial T}{\partial t} + \boldsymbol{v}.\nabla T\right] = -\nabla.\boldsymbol{q} + h_p\nabla.\boldsymbol{j_p}, \tag{17}$$

where c and h_p are the specific heat of fluid (at constant pressure) and the specific enthalpy of nanoparticles, respectively and $\boldsymbol{q}$ is the energy flux, neglecting radiative heat transfer, the sum of heat fluxes due to conduction and nanoparticle diffusion, written as

$$\boldsymbol{q} = -k\nabla T + h_p\boldsymbol{j_p}, \tag{18}$$

Substituting Eq. (18) in Eq. (17), we get

$$(\rho c)\left[\frac{\partial T}{\partial t} + \boldsymbol{v}.\nabla T\right] = \nabla.(k\nabla T) - c_p\boldsymbol{j_p}.\nabla T. \tag{19}$$

with assumption of negligible external forces $\nabla h_p = c_p\nabla T$. Substituting Eq. (12) in Eq. (19); gives final form of thermal energy equation as

$$(\rho c)\left[\frac{\partial T}{\partial t}+\boldsymbol{v}.\nabla T\right]=k\nabla^2 T+(\rho c)_p\left[D_B\nabla\phi.\nabla T+D_T\frac{\nabla T.\nabla T}{T}\right]. \tag{20}$$

Note that if $\boldsymbol{j}_p$ is zero, Eq.(19) and hence Eq. (20) reduces to the familiar energy equation for regular fluid and therefore last two terms on right-hand side truly account for contributions of nanoparticle motion relative to fluid. Eq. (20) establishes that the transport of heat in nanofluids is possible by convection (second term on left-hand side), by conduction (first term on right-hand side), and also by virtue of nanoparticle diffusion (second and third terms on right-hand side).

Thus, Eqs. (10), (13), (16), (20) constitute the convective transport model for nanofluids which further can be solved for different parameters once the initial and boundary conditions are known. It is interesting to note that all the equations are strongly coupled meaning thereby that the one parameter depends on various other parameters.

Let us introduce non-dimensional variables to get the expression for thermal Rayleigh number as.

$$(x',y',z')=\frac{(x,y,z)}{d},t'=\frac{t\,\alpha_f}{d^2},\boldsymbol{v}'=\frac{\boldsymbol{v}\,d}{\alpha_f},p'=\frac{p\,d^2}{\mu\alpha_f},\phi'=\frac{\phi}{\phi_b},T'=\frac{T-T_0}{T_1-T_0}, \tag{21}$$

where $\alpha_f=\frac{k}{\rho C}$..

Using Eqs. (21), (10), (13), (16), (20) after dropping the dashes are

$$\nabla.\boldsymbol{v}=0, \tag{22}$$

$$\frac{\rho\alpha_f}{\mu}\left(\frac{\partial\boldsymbol{v}}{\partial t}+\boldsymbol{v}.\nabla\boldsymbol{v}\right)=-\nabla p+\nabla^2\boldsymbol{v}-\frac{\rho\boldsymbol{g}d^3}{\mu\alpha_f}\hat{k}+R_A T\hat{k}-\frac{\left(\rho_p-\rho\right)\phi_b\boldsymbol{g}\,d^3}{\mu\alpha_f}\phi\,\hat{k}, \tag{23}$$

$$\frac{\partial T}{\partial t}+\boldsymbol{v}.\nabla T=\nabla^2 T+\frac{(\rho C)_{\mathrm{P}}}{\rho C}\phi_b\frac{D_B}{\alpha_f}\nabla\phi.\nabla T+\frac{D_T(T_1-T_0)}{D_B T_0\phi_b}\frac{(\rho C)_{\mathrm{P}}}{\rho C}\phi_b\frac{D_B}{\alpha_f}\nabla T.\nabla T, \tag{24}$$

$$\frac{\partial\phi}{\partial t}+\boldsymbol{v}.\nabla\phi=\frac{D_B}{\alpha_f}\nabla^2\phi+\frac{D_T(T_1-T_0)}{D_B T_0\phi_b}\frac{D_B}{\alpha_f}\nabla^2 T, \tag{25}$$

where thermal Rayleigh number $R_A=\frac{\rho\,\boldsymbol{g}\,\beta_T d^3(T_1-T_0)}{\mu\alpha_f}$.

4. Initial and perturbed flow

At the initial state, it is assumed that nanoparticle volume fraction is constant and fluid layer is still while temperature and pressure vary in horizontal direction. We get initial solution of Eqs. (22)–(25) using the fact that thermal diffusivity is very large as compared to Brownian diffusion coefficient (refer Buongiorno [1]) as

$$v_i=0,\phi_i=1,T_i=1-z \tag{26}$$

Let us add perturbations to initial solution and write

$$(\boldsymbol{v},p,T,\phi)=\left(\boldsymbol{v}_i+\tilde{\boldsymbol{v}},p_i+\tilde{p},T_i+\tilde{T},\ \phi_i+\tilde{\phi}\right). \tag{27}$$

The Eq. (27) in Eqs. (22)–(25) give

$$\nabla.\tilde{\boldsymbol{v}} = 0, \tag{28}$$

$$\frac{\rho\alpha_f}{\mu}\frac{\partial\tilde{\boldsymbol{v}}}{\partial t} = -\nabla\tilde{p} + \nabla^2\tilde{\boldsymbol{v}} + R_A\tilde{T}\hat{k} - \frac{\left(\rho_p - \rho\right)\phi_b\boldsymbol{g}\,d^3}{\mu\alpha_f}\tilde{\phi}\hat{k}, \tag{29}$$

$$\frac{\partial\tilde{T}}{\partial t} - \tilde{u}_3 = \nabla^2\tilde{T} - \frac{(\rho C)_{\mathrm{P}}}{\rho C}\phi_b\frac{D_B}{\alpha_f}\frac{\partial\tilde{\phi}}{\partial z} - 2\frac{D_T(T_1 - T_0)}{D_B T_0\phi_b}\frac{(\rho C)_{\mathrm{P}}}{\rho C}\phi_b\frac{D_B}{\alpha_f}\frac{\partial\tilde{T}}{\partial z}, \tag{30}$$

$$\frac{\partial\tilde{\phi}}{\partial t} = \frac{D_B}{\alpha_f}\nabla^2\tilde{\phi} + \frac{D_T(T_1 - T_0)}{D_B T_0\phi_b}\frac{D_B}{\alpha_f}\nabla^2\tilde{T}. \tag{31}$$

Making use of the identity *curlcurl* $\equiv$ *graddiv* $-\nabla^2$ curlcurl = graddiv $-\nabla^2$ on Eq. (29) together with Eq. (28), we get

$$\frac{\rho\alpha_f}{\mu}\frac{\partial}{\partial t}\left(\nabla^2\tilde{u}_3\right) - \nabla^4\tilde{u}_3 = R_A\nabla_H^2\tilde{T} - \frac{\left(\rho_p - \rho\right)\phi_b\boldsymbol{g}\,d^3}{\mu\alpha_f}\nabla_H^2\tilde{\phi}, \tag{32}$$

where $\nabla_{\mathrm{H}}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

5. Method of normal modes

To change PDE's to ODE's, Eqs. (30)–(32) are solved using normal mode analysis and perturbed variables are written as

$$\left(\tilde{u}_3, \tilde{T}, \tilde{\phi}\right) = (W(z), \mathrm{T}(z), \Phi(z))\exp\left(ik_x x + ik_y y + st\right), \tag{33}$$

Thus above mentioned equations reduce to

$$\left(\left(D^2 - \alpha^2\right)^2 - \frac{s\rho\alpha_f}{\mu}\left(D^2 - \alpha^2\right)\right)W - R_A\alpha^2\,\mathrm{T} + \frac{\left(\rho_p - \rho\right)\phi_b\boldsymbol{g}\,d^3}{\mu\alpha_f}\alpha^2\,\Phi = 0, \tag{34}$$

$$W + \left(\left(D^2 - \alpha^2\right) - s - 2\frac{D_T(T_1 - T_0)}{D_B T_0\phi_b}\frac{(\rho C)_{\mathrm{P}}}{\rho C}\phi_b\frac{D_B}{\alpha_f}D\right)\mathrm{T} - \frac{(\rho C)_{\mathrm{P}}}{\rho C}\phi_b\frac{D_B}{\alpha_f}D\Phi = 0, \tag{35}$$

$$\left(\frac{D_B}{\alpha_f}\left(D^2 - \alpha^2\right) - s\right)\Phi + \frac{D_T(T_1 - T_0)}{D_B T_0\phi_b}\frac{D_B}{\alpha_f}\left(D^2 - \alpha^2\right)\mathrm{T} = 0, \tag{36}$$

where $D \equiv \frac{d}{dz}$, $\alpha = \left({k_x}^2 + {k_y}^2\right)^{1/2}$. Using one term Galerkin weighted residual method and free-free boundaries conditions

$$W = \quad D^2W = \quad \mathrm{T} \quad = 0 \text{ at } z = 0 \text{ and } z = 1. \tag{37}$$

We write

$$W = A\sin\pi z, \text{ and } T = B\sin\pi z, \tag{38}$$

using of the orthogonality to the functions; gives eigenvalue equation as

$$\left((J+s)\left(\frac{JD_B}{\alpha_f}+s\right)\left(J^2+\frac{\rho\alpha_f Js}{\mu}\right)-\alpha^2\left(\frac{JD_T\left(\rho_p-\rho\right)(T_1-T_0)\boldsymbol{g}\,d^3}{D_B\mu\,\alpha_f^2T_0}+R_A\left(\frac{JD_B}{\alpha_f}+s\right)\right)\right)$$
$$-\alpha^2J\left(\frac{JD_T\left(\rho_p-\rho\right)(T_1-T_0)\boldsymbol{g}\,d^3}{D_B\mu\,\alpha_f^2T_0}+R_A\left(\frac{JD_T\left(\rho_p-\rho\right)(T_1-T_0)\boldsymbol{g}\,d^3}{D_B\mu\,\alpha_f^2T_0}+s\right)\right)=0 \tag{39}$$

where $J=\pi^2+\alpha^2$.

6. Results and discussion

6.1 Stationary convection

For non-oscillatory motions $s=0$, this gives the expression for R_A from Eq. (39) as

$$R_A=\frac{J^3}{\alpha^2}-\frac{D_T\left(\rho_p-\rho\right)(T_1-T_0)\boldsymbol{g}\,d^3}{D_B\mu\,\alpha_fT_0}. \tag{40}$$

where $D_B=\frac{k_BT}{3\pi\mu d_p}$ and $D_T=\tilde{\beta}\frac{\mu}{\rho}\phi$ with $\tilde{\beta}=0.26\frac{k}{2k+k_p}$ as given by Nield and Kuznetsov [35].

Also

$$R_A=\frac{J^3}{\alpha^2}-\frac{\left(\rho_p-\rho\right)}{2k+k_p}A; \tag{41}$$

where A depends on the base fluid properties.

7. Discussions on analytical results using various metallic/non-metallic nanoparticles

Table 2 shows the ratios of density to conductivity of various metallic/non-metallic nanoparticles and density 997.1 and conductivity 0.613 of water is used.

It is observed that ratio of density to conductivity is accountable for hastening the onset of convection in the system. The ratio is more for non-metals than metals establishes the lesser stability of non-metallic nanoparticles than metals. Alumina is

Physical properties	Al	Cu	Ag	Fe	Al_2O_3	SiO_2	CuO	TiO_2
$\rho\,(\mathrm{Kg/m^3})$	2700	9000	10,500	7900	3970	2600	6510	4250
$\mathrm{k}\,(\mathrm{W/m}K)$	237	401	429	80	40	10.4	18	8.9
ρ_p/k_p	11.3	22.4	24.47	98.7	99.25	250	361.6	477.5

Table 2.
Ratios of density to conductivity of metallic and non-metallic nanoparticles.

most stable and titanium oxide is least stable among the nanoparticles under consideration. Density is found to be more influential than conductivity towards deciding the onset of convection in the layer.

8. Conclusions

Tremendous applications of nanofluids in pharmaceutical industry with respect to drug invention and cancer imaging motivated the scientists to study convection currents in fluid layer mathematically as well as experimentally. In the present work, the onset of instability in layer is studied under small temperature difference with the help of equations based on conservation laws. The expression of non-dimensional number Rayleigh number is found analytically which decides the instability of the system. Approximations are made whenever needed without violating the necessary physics to get the useful results. Analysis reveals that lesser the ratio of density to conductivity, higher is the stability of the layer. It is found that convection currents majorly depends on density and conductivity and precisely concluding density is more pronounced property than conductivity of nanoparticles. Metallic oxides make the system more stable than metallic nanoparticles in the fluid.

Author details

Jyoti Sharma[1*] and Urvashi Gupta[2]

1 U.I.E.T, Panjab University, Chandigarh, India

2 Dr. S. S. Bhatnagar University Institute of Chemical Engineering and Technology, Panjab University, Chandigarh, India

*Address all correspondence to: jyoti.maths@gmail.com

IntechOpen

© 2019 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. CC BY

References

[1] Maxwell JC. Electricity and Magnetism. Oxford, UK: Clarendon Press; 1873

[2] Choi S. Enhancing thermal conductivity of fluids with nanoparticles. In: Siginer DA, Wang HP, editors. Development and Applications of Non-Newtonian Flows. FED- 231/ MD-Vol. 66. New York; The American Society of Mechanical Engineers (ASME); 1995. pp. 99-105

[3] Masuda H, Ebata A, Teramae K, Hishinuma N. Alteration of thermal conductivity and viscosity of liquid by dispersing ultra fine particles. Netsu Bussei. 1993;**7**:227-233

[4] Eastman JA, Choi SUS, Li S, Yu W, Thompson LJ. Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles. Applied Physics Letters. 2001;**78**:718-720

[5] Das SK, Putra N, Thiesen P, Roetzel W. Temperature dependence of thermal conductivity enhancement for nanofluids. ASME Journal of Heat Transfer. 2003;**125**:567-574

[6] Buongiorno J. Convective transport in nanofluids. ASME Journal of Heat Transfer. 2006;**128**(3):240-250

[7] Choi SUS, Zhang ZG, Yu W, Lockwood FE, Grulke EA. Anomalous thermal conductivity enhancement in nanotube suspensions. Applied Physics Letters. 2001;**79**:2252-2254

[8] Wang X, Xu X, Choi SUS. Thermal conductivity of nanoparticle fluid mixture. Journal of Thermophysics and Heat Transfer. 1999;**13**(4):474-480

[9] Murshed SMS, Leong KC, Yang C. Enhanced thermal conductivity of TiO_2-water based nanofluids. International Journal of Thermal Sciences. 2005;**44**: 367-373

[10] Choi SUS. Nanofluids: From vision to reality through research. Journal of Heat Transfer. 2009;**131**(3):033106

[11] Wen D, Lin G, Vafaei S, Zhang K. Review of nanofluids for heat transfer applications. Particuology. 2009;**7**: 141-150

[12] Das SK, Choi SUS. A review of heat transfer in nanofluids. Advances in Heat Transfer. 2009;**41**:81-197

[13] Ding Y, Alias H, Wen D, Williams RA. Heat transfer of aqueous suspension of carbon nanotubes (CNT nanofluids). International Journal of Heat and Mass Transfer. 2006;**49**: 240-250

[14] Das SK, Choi SUS, Yu W, Pradeep T. Nanofluids: Science and Technology. Hoboken, New York: Wiley; 2008

[15] Chen G. Ballistic-diffusive heat conduction equations. Physical Review Letters. 2001;**86**:2297-2300

[16] Syam Sundar L, Sharma KV. Thermal conductivity enhancement of nanoparticles in distilled water. International Journal of Nanoparticles. 2008;**1**(1):66-77

[17] Xuan Y, Li Q. Investigation on convective heat transfer and flow features of nanofluids. Journal of Heat Transfer. 2003;**125**:151-155

[18] Yu W, Choi SUS. The role of interfacial layers in the enhanced thermal of nanofluids: A renovated Maxwell model. Journal of Nanoparticle Research. 2003;**5**(1–2):167-171

[19] Vassallo P, Kumar R, Amico SD. Pool boiling heat transfer experiments in silica-water nano-fluids. International Journal of Heat and Mass Transfer. 2004;**47**:407-411

[20] Chon CH, Kihm KD. Thermal conductivity enhancement of nanofluids by Brownian motion. ASME Journal of Heat Transfer. 2005;**127**:810

[21] Pak BC, Cho Y. Hydrodynamic and heat transfer study of dispersed fluids with submicron metallic oxide particles. Journal of Experimental Heat Transfer. 1998;**11**(2):151-170

[22] Kleinstreuer C, Li J, Koo J. Microfluidics of nano-drug delivery. International Journal of Heat and Mass Transfer. 2008;**51**:5590-5597

[23] Buongiorno J, Hu W. Nanofluid coolants for advanced nuclear power plants. In: Proceedings of ICAPP'05, Seoul; 2005. Paper No. 5705

[24] Saedi Ardahaie S, Jafarian Amiri A, Amouei A, Hosseinzadeh K, Ganji DD. Investigating the effect of adding nanoparticles to the blood flow in presence of magnetic field in a porous blood arterial. Informatics in Medicine Unlocked. 2018;**10**:71-81. ISSN: 2352-9148

[25] Sayed HM, Aly EH, Vajravelu K. Influence of slip and convective boundary conditions on peristaltic transport of non-Newtonian nanofluids in an inclined asymmetric channel. Alexandria Engineering Journal. 2016; **55**(3):2209-2220. ISSN: 1110-0168

[26] Bénard H. Les Tourbillons cellulaires dans une nappe liquide. Revue Générale des Sciences Pures et Appliquées. 1900;**11**:1261-1271; 1309-1328

[27] Bénard H. Les tourbillions cellularies dans une nappe liquide transportant de la chaleur par convection en regime permanent. Annales de Chimie et de Physique. 1901;**23**:62-144

[28] Rayleigh L. On convective currents in a horizontal layer of fluid when the higher temperature is on the under side. Philosophical Magazine. 1916;**32**: 529-546

[29] Chandra K. Instability of fluids heated from below. Proceedings. Royal Society of London. 1938;**A164**:231-242

[30] Spiegal EA, Veronis G. On the Bousinessq approximation for a compressible fluid. The Astrophysical Journal. 1960;**131**:442

[31] Joseph DD. Stability of Fluid Motions. Vol. 1 & 2. Berlin: Springer-Verlag; 1976

[32] Chandrasekhar S. Hydrodynamic and Hydromagnetic Stability. New York: Dover Publication; 1981

[33] Kim J, Kang YT, Choi CK. Analysis of convective instability and heat transfer characteristics of nanofluids. Physics of Fluids. 2004;**16**:2395-2401

[34] Hwang KS, Lee JH, Jang SP. Buoyancy driven heat transfer of water based Al_2O_3 nanofluids in a rectangular cavity. International Journal of Heat and Mass Transfer. 2007;**50**:4003-4010

[35] Tzou DY. Instability of nanofluids in natural convection. ASME Journal of Heat Transfer. 2008;**130**:372-401

[36] Tzou DY. Thermal instability of nanofluids in natural convection. International Journal of Heat and Mass Transfer. 2008;**51**:2967-2979

[37] Nield DA, Kuznetsov AV. The onset of convection in a horizontal nanofluid layer of finite depth. European Journal of Mechanics - B/Fluids. 2010;**29**: 217-223

[38] Alloui Z, Vasseur P, Reggio M. Natural convection of nanofluids in a shallow cavity heated from below. International Journal of Thermal Sciences. 2010;**50**(3):385-393

[39] Yadav D, Agrawal GS, Bhargava R. Rayleigh Bénard convection in

nanofluid. International Journal of Applied Mathematics and Mechanics. 2011;7(2):61-76

[40] Boussinesq J. Théorie Analytiquede la Chaleur. Vol. 2. Paris: Gauthier-Villars; 1903. p. 172

[41] Anoop KB, Das SK, Sundararajan T. Effect of particle size on the convective heat transfer in nanofluid in the developing region. International Journal of Heat and Mass Transfer. 2009;**52**: 2189-2195

[42] Byron Bird R, Stewart WE, Lightfoot EN. Transport Phenomena. New York: Wiley; 1960