Chapter

Some Methods of Fuzzy Conditional Inference for Application to Fuzzy Control Systems

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Abstract

Zadeh proposed fuzzy logic with single membership function. Two Zadeh, Mamdani and TSK proposed fuzzy conditional inference. In many applications like fuzzy control systems, the consequent part may be derived from precedent part. Zadeh, Mamdani and TSK proposed different fuzzy conditional inferences for “if … then …” for approximate reasoning. The Zadeh and Mamdani fuzzy conditional inferences are know prior information for both precedent part and consequent part. The TSK fuzzy conditional inferences need not know prior information for consequent part but it is difficult to compute. In this chapter, fuzzy conditional inference is proposed for “if…then…” This fuzzy conditional inference need not know prior information of the consequent part. The fuzzy conditional inference is discussed using the single fuzzy membership function and twofold fuzzy membership functions. The fuzzy control system is given as an application.

Keywords: fuzzy logic, twofold fuzzy logic, fuzzy conditional inference, fuzzy control systems

1. Introduction

When information is incomplete, fuzzy logic is useful [10–26]. Many theories [1, 2] deal with incomplete information based on likelihood (probability), whereas fuzzy logic is based on belief. Zadeh defined fuzzy set with single membership function. Zadeh [3], Mamdani [4], TSK [2] and Reddy [5] are studied fuzzy conditional inferences. The fuzzy conditions are of the form “if <. Zadeh, Mamdani and TSK fuzzy conditional inference requires both precedent-part and consequent-part but 5fuzzy inferences don’t require consequent part. Precedent-part > then <consequent-part >.”

Zadeh [6] studied fuzzy logic with single membership function. The single membership function for the proposition “x is A” contains how much truth in the proposition. The fuzzy set with two membership functions will contain more information in terms of how much truth and false it has in the proposition. The fuzzy certainty factor is studied as difference on two membership functions “true” and “false” to eliminate conflict of evidence, and it becomes single membership function. The FCF is a fuzzy set with single fuzzy membership function of twofold fuzzy set.

The fuzzy control systems are considered in this chapter as application of single fuzzy membership function and twofold fuzzy set.

2. Fuzzy log with single membership function

Zadeh [6] has introduced a fuzzy set as a model to deal with imprecise, inconsistent and inexact information. The fuzzy set is a class of objects with a continuum of grades of membership.

The fuzzy set A of X is characterized as its membership function $A = \mu_A(x)$ and ranging values in the unit interval [0, 1]

$\mu_A(x): X \rightarrow [0, 1], x \in X$,

where X is the universe of discourse.

$A = \mu_A(x_1)/x1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n$,

where "+" is the union.

For instance, the fuzzy proposition "x is High"

$\text{High} = 0.2/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.2/x_5$

$\text{Not High} = 0.8/x_1 + 0.4/x_2 + 0.1/x_3 + 0.4/x_4 + 0.8/x_5$

For instance, the fuzziness of "Temperature is high" is 0.8

The graphical representation of young and not young is shown in **Figure 1**.

The fuzzy logic is defined as a combination of fuzzy sets using logical operators. Some of the logical operations are given below.

For example, A, B and C are fuzzy sets. The operations on fuzzy sets are given as:

Negation

If x is not A

$A' = 1 - \mu_A(x)/x$

Conjunction

x is A and y is B $\rightarrow$ (x, y) is A ΛB

$A\Lambda B = \min(\mu_A(x), \mu_B(y)\}(x,y)$

If x = y

x is A and y is B $\rightarrow$ (x, y) is A ΛB

$A\Lambda B = \min(\mu_A(x), \mu_B(y)\}/x$

For example

$A = 0.2/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.2/x_5$

$B = 0.4/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.1/x_5$

$A\Lambda B = 0.2/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.1/x_5$

The graphical representation is shown in **Figures 1** and **2**.

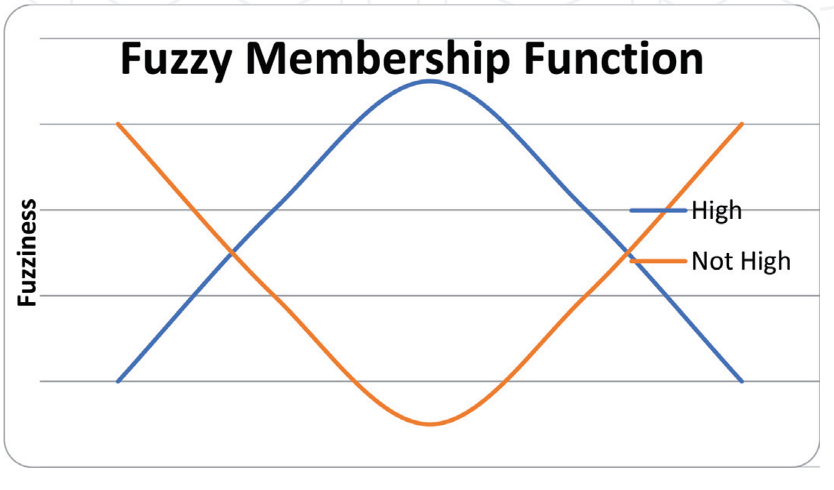

Figure 1.
Fuzzy membership function.

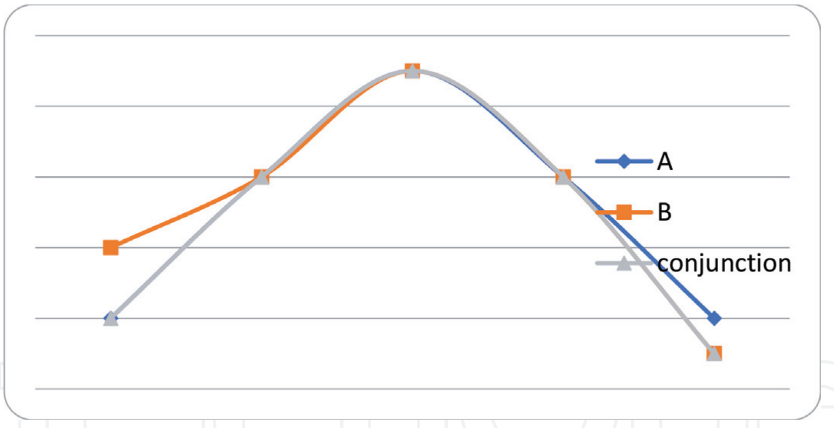

Figure 2.
Conjunction.

Disjunction

x is A and y is B→ (x, y) is A V B

A V B = max(μ_A(x), μ_B(y)}(x,y)

If x = y

x is A and y is B→ (x, y) is A V B

AVB = max(μ_A(x), μ_B(y)}/x

For instance,

A = 0.2/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.2/x_5

B = 0.4/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.1/x_5

AVB = 0.4/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.2/x_5

The graphical representation is shown in **Figure 3**.

Concentration

$\mu_{\text{very } A}(x) = \mu_A(x)^2$

Diffusion

$\mu_{\text{more or less } A}(x) = \mu_A(x)^{0.5}$

The graphical representation of concentration and diffusion is shown in **Figure 4**.

Implication

Zadeh [6], Mamdani [7] and Reddy [5] fuzzy conditional inferences are considered for fuzzy control systems.

If x_1 is A_1 and x_2 is A_2 and … and x_n is A_n, then y is B

The presidency part may contain any number of “and/or”

Zadeh [6] fuzzy inference is given as:

If x_1 is A_1 and x_2 is A_2 and … and x_n is A_n, then y is B

= min(1, 1 − (A_1, A_2,…, A_n) + B)

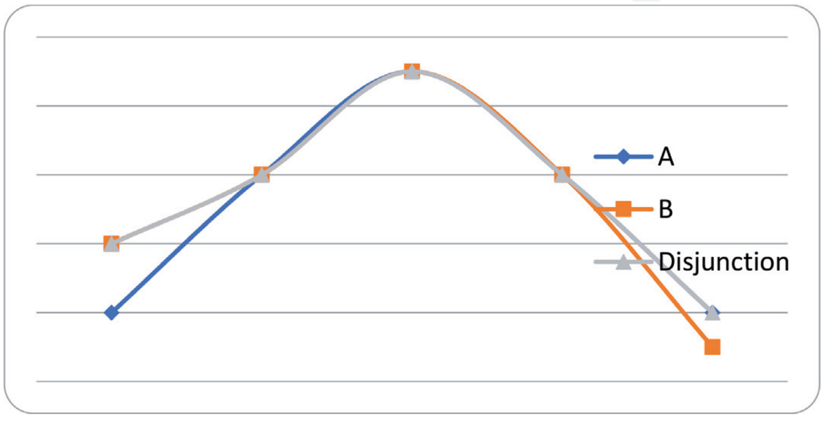

Figure 3.
Disjunction.

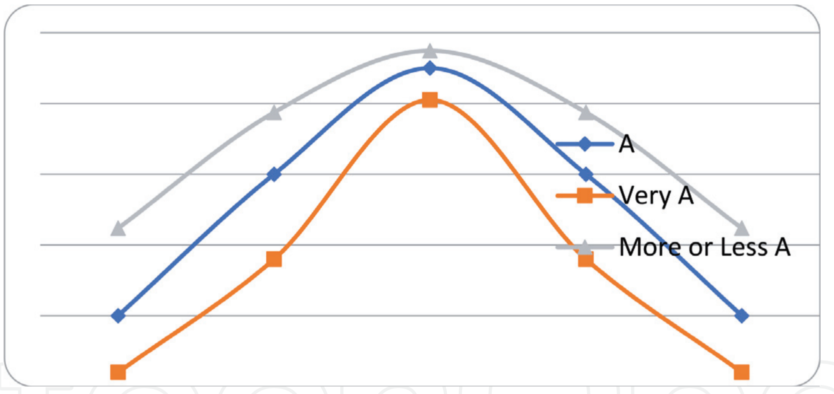

Figure 4.
Fuzzy quantifiers.

Mamdani [4] fuzzy inference is given as:
If x_1 is A_1 and x_2 is A_2 and … and x_n is A_n, then y is B
$= \min(A_1, A_2,\ldots, A_n, B)$
Zadeh and Mamdani fuzzy inference has prior information of A and B. The relation between A and B is known. Then, B is derived from A.
Reddy [2] inference is given by:
If x_1 is A_1 and x_2 is A_2 and … and x_n is A_n, then y is B
$= \min(A_1, A_2,\ldots,A_n)$
Consider the fuzzy rule:
If x_1 is A_1 and x_2 is A_2, then x is B
For instance,
$A_1 = 0.2/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.2/x_5$
$A_2 = 0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.3/x_5$
$B = 0.1/x_1 + 0.4/x_2 + 0.6/x_3 + 0.4/x_4 + 0.1/x_5$
The graphical representation of A1, A2 and B is shown in **Figure 5**.
The graphical representation of fuzzy inference is shown in **Figure 6**.
Composition
If some relation between R and A1 than B1 is to infer from R
B1 = A1 o R, where R = A➔B
Zadeh fuzzy inference is given by:
$B1 = A1 \text{ o } R = \min\{\mu_A(x), \mu_R(x)\}$
$= \min\{\mu_A(x), \min(1,1-\mu_{A1}(x) + \mu_B(x))\}$
Mamdani fuzzy inference is given by:
$= \min\{\mu_{A1}(x),\mu_A(x) + \mu_B(x)\}$
If there is some relation R between A and B, then Reddy fuzzy inference is given by:
$= \mu_{A1}(x)$

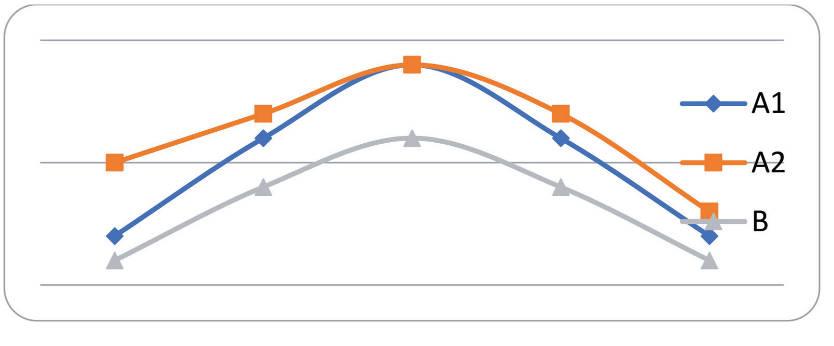

Figure 5.
Fuzzy sets.

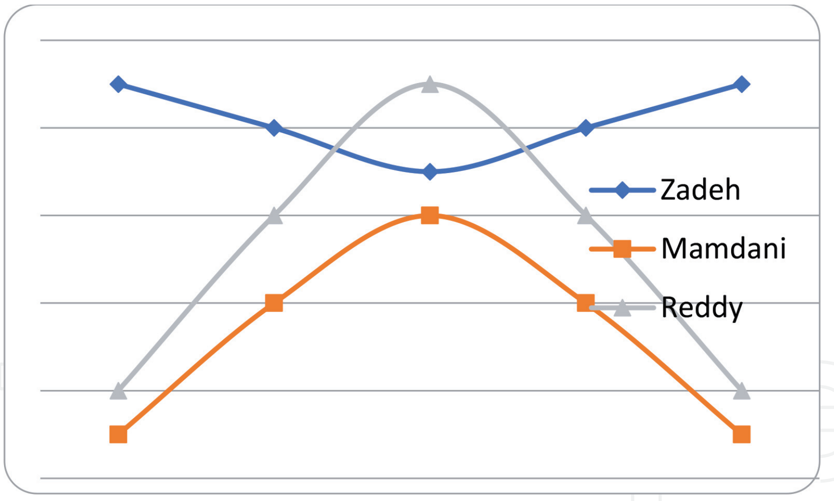

Figure 6.
Fuzzy conditional inference.

3. Justification of Reddy and Mamdani fuzzy conditional inference

Justification of Reddy fuzzy conditional inference may be derived in the following:

Consider Reddy fuzzy conditional inference:

If x_1 is A_1 and x_2 is A_2 and … and x_n is A_n, then y is B = min{A_1, A_2,…, A_n}.

Consider TSK fuzzy conditional inference:

If x_1 is A_1 and x_2 is A_2 and … and x_n is A_n, then y is B = f(x_1, x_2,…, x_n).

The proposed method of fuzzy conditional inference may be defined by replacing x_1, x_2,…, x_n with A_1, A_2 and … and A_n

If x_1 is A_1 and/or A_2 and/or,…, and/or A_n, then y is B = f(A_1, A_2,…, A_n)

If x_1 is A_1 or A_2 and A_n, then y is B = f(A_1, A_2, A_3) = A_1 V A_2 Λ –Λ A_3

If x_1 is A_1 or A_2 and A_3, then y is B = f(A_1, A_2, A_3) = A_1 V A_2 Λ A_3

B = min(max($\mu_{A1}(x_1)$, $\mu_{A2}(x_2)$), $\mu_{A3}(x_3)$)

The fuzzy conditional inference is given by using Mamdani fuzzy inference

If x_1 is A_1 or A_2 and A_3, then y is B = min(A1 or A2 and A3, B)

If x_1 is A_1 or A_2 and A_3, then x is B = min(max($\mu_{A1}(x_1)$, $\mu_{A2}(x_2)$), $\mu_{A3}(x_3)$)

Thus, the Reddy fuzzy conditional inference is satisfied.

If x_1 is A_1 and x_2 is A_2 and … and x_n is A_n, then y is B = min{A_1, A_2,…,A_n}.

Justification of Mamdani fuzzy conditional inference may be derived in the following:

If some relation R between A and B is known, then Mamdani fuzzy conditional inference is given by:

If x is A, then y is B = A x B

Zadeh fuzzy conditional inference for "if … then … else …" is given by:

If x is A, then y is B else y is C = A x B v A′ x C

If x is A, then y is B else y is C = If x is A then y is B v If x is A′ then y is C = A x B v A′ x C

It is logically divided into:

If x is A, then y is B = A x B

If x is A′, then y is C = A′ x C

Thus, the Mamdani fuzzy conditional inference is satisfied.

If x is A, then y is B = A x B.

4. Fuzzy control systems using single fuzzy membership function

Zadeh introduced fuzzy algorithms. The fuzzy algorithm is a set of fuzzy statements. The fuzzy conditional statement is defined as fuzzy algorithm:

If x_i is $A1_i$ and x_i is $A2_i$ and… and x_i is An, then y is B_i

The consequent part may not be known in control systems

The fuzziness may be given for Reddy fuzzy inference as

If BZ is low (0.6)

and BE is normal (0.7)

then reduce fan speed

= min (0.6, 0.7)

= 0.6

The fuzzy set type-2 is a type of fuzzy set in which some additional degree of information is provided.

Definition: Given some universe of discourse X, a fuzzy set type-2 A of X is defined by its membership function $\mu_A(x)$ taking values on the unit interval [0,1], i.e., $\mu_{\tilde{A}}(x) \rightarrow [0,1]^{[0.1]}$

Suppose X is a finite set. The fuzzy set A of X may be represented as

$A = \mu_{\tilde{A}1}(x_1)/\tilde{A}_1 + \mu_{\tilde{A}2}(x_2)/\tilde{A}_2 + \ldots + \mu_{\tilde{A}n}(x_n)/\tilde{A}n$

Temperature = {0.4/low, 0.6/medium, 0.9/high}

John has "mild headache" with fuzziness 0.4

The fuzzy control system for boiler consists of a set of fuzzy rules [4].

If a set of conditions is satisfied, then the set of consequences is fired

The fuzzy control system is shown in **Figure 7**.

The fuzzy control system containing fuzzy variables are represented in decision **Table 1**.

The fuzzy control system of boiler is given in **Table 2**.

For instance,

If BZ is low

and BE is normal

then reduce fan speedFor instance, consider the fuzzy control system (**Table 3**).

The computation of proposed method (3.4) is given in **Table 4**.

Defuzzification

The centroid technique is used for defuzzification. It finds value representing the centre of gravity (COG) aggregated fuzzy generalized fuzzy set:

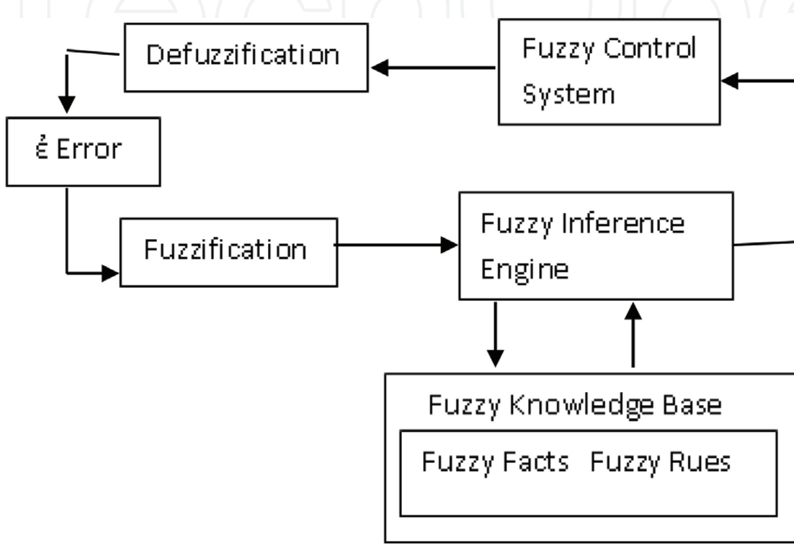

Figure 7.
Fuzzy control system.

A1	A2	...	An	B
A11	A12	…	A1n	B1
A21	A22	…	A2n	B2
⋮	⋮		⋮	⋮
Am1	Am2	…	Amn	Bmn

Table 1.
Fuzzy rules.

Condition	Burning zone (BZ) temperature	Back-end (BE) temperature	Action
AND	Drastically low	Low	Reduce Klin speed
AND	Drastically low	Low	Reduce fuel
AND	Slightly low	Low	Increase fan speed
AND	Low	High	Reduce fuel
AND	Low	Normal	Reduce fan speed

Table 2.
Boiler controller.

Condition	Burning zone (BZ) temperature	Back-end (BE) temperature	Action
AND	Drastically low (0.7)	Low (0.6)	Reduce Klin speed
AND	Drastically low (0.7)	Low (0.8)	Reduce fuel
AND	Slightly low (.8)	Low (.9)	Increase fan speed
AND	Low (0.7)	High (0.65)	Reduce fuel
AND	Low (0.6)	Normal (0.7)	Reduce fan speed

Table 3.
Boiler fuzzy controller.

Condition	Burning zone (BZ) temperature	Back-end (BE) temperature	Action
AND	Drastically low (0.7)	Low (0.6)	Reduce Klin speed (0.6)
AND	Drastically low (0.7)	Low (0.8)	Reduce fuel (0.7)
AND	Slightly low (.8)	Low (.9)	Increase fan speed (0.8)
AND	Low (0.7)	High (0.65)	Reduce fuel (0.65)
AND	Low (0.6)	Normal (0.7)	Reduce fan speed (0.6)

Table 4.
Fuzzy inference.

$COG = \Sigma C_i \mu_{Ai}(x) / \Sigma C_i$
For instance,
Speed = $\{0.1/20 + 0.3/40 + 0.5/60 + 0.7/80 + 0.9/100\}$
$COG = (0.1{*}20 + 0.3{*}40 + 0.5{*}60 + 0.7{*}80 + 0.9{*}100)/$
$(0.1 + 0.3 + 0.5 + 0.7 + 0.9) = 73.6$

Condition	Burning zone (BZ) temperature	Back-end (BE) temperature	Action
AND/OR	Drastically low (0.7,0.1)	Low (0.8,0.1)	Reduce Klin speed (0.6,0.2)
AND/OR	Drastically low (0.8,0.1)	Low (0.9,0.1)	Reduce fuel (0.7,0.2)
AND/OR	Slightly low (1.0,0.2)	Low (1.0,0.1)	Increase fan speed (0.9,0.2)
AND/OR	Low (0.8,0.1)	High (0.9,0.2)	Reduce fuel (0.6,0.1)
AND/OR	Low (0.7,0.1)	Normal (0.8,0.2)	Reduce fan speed (0.5,0.1)

Table 5.
Twofold fuzziness.

5. Fuzzy logic with twofold fuzzy sets

Generalized fuzzy logic is studied for incomplete information [8, 9].

Given some universe of discourse X, the proposition "x is A" is defined as its twofold fuzzy set with membership function as

$\mu_A(x) = \{\mu_A{}^{True}(x), \mu_A{}^{False}(x)\}$

or

$A = \{\mu_A{}^{True}(x), \mu_A{}^{False}(x)\}$

where A is the seneralized fuzzy set and x Є X,

$0 < = \mu_A{}^{True}(x) < = 1$ and, $0 < = \mu_A{}^{False}(x) < = 1$

$A = \{\mu_A{}^{True}(x_1)/x_1 + \dots + \mu_A{}^{True}(x_n)/x_n,$

$\mu_A{}^{False}(x_1)/x_1 + \dots + \mu_A{}^{True}(x_n)/x_n, x_i$ Є X,

$\mu_A{}^{True}(x) + \mu_A{}^{False}(x) < 1,$

$\mu_A{}^{True}(x) + \mu_A{}^{False}(x) > 1$ and

$\mu_A{}^{True}(x) + \mu_A{}^{False}(x) = 1$

The conditions are interpreted as redundant, insufficient and sufficient, respectively.

For instance,

$A = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5,$

$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$

The graphical representation is shown in **Figure 8**.

The fuzzy logic is defined as a combination of fuzzy sets using logical operators. Some of the logical operations are given below.

Let A, B and C be the fuzzy sets. The operations on fuzzy sets are given below for twofold fuzzy sets.

Negation

$A' = \{1-\mu_A{}^{True}(x), 1-\mu_A{}^{False}(x)\}/x$

Disjunction

$AVB = \{\max(\mu_A{}^{True}(x), \mu_A{}^{True}(y)), \max(\mu_B{}^{False}(x), \mu_B{}^{False}(y))\}(x,y)$

Conjunction

$A\Lambda B = \{\min(\mu_A{}^{True}(x), \mu_A{}^{True}(y)), \min(\mu_B{}^{False}(x), \mu_B{}^{False}(y))\}/(x,y)$

Composition

$A \text{ o } R = \{\min_x (\mu_A{}^{True}(x), \mu_A{}^{True}(x)), \min_x(\mu_R{}^{False}(x), \mu_R{}^{False}(x))\}/y$

The fuzzy propositions may contain quantifiers like "very", "more or less". These fuzzy quantifiers may be eliminated as follows:

Concentration

"x is very A"

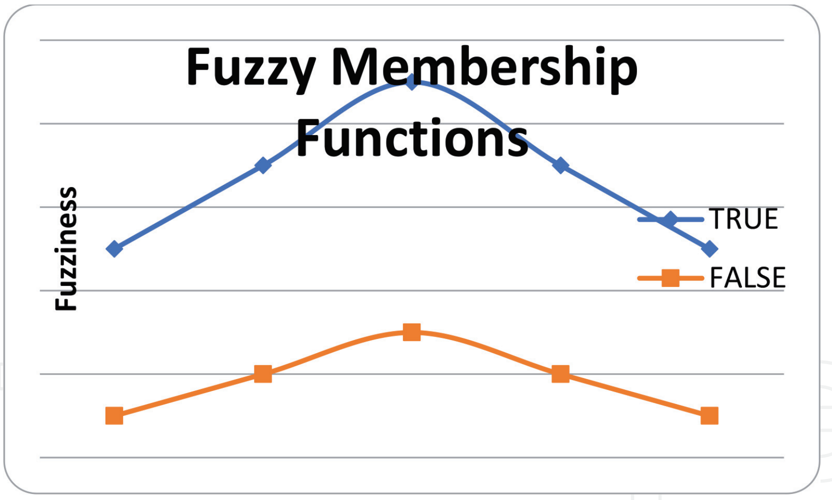

Figure 8.
Fuzzy membership function.

$\mu_{\text{very A}}(x) = \{\mu_A^{True}(x)^2, \mu_A^{False}(x)\mu_A(x)^2\}$

Diffusion

"x is more or less A"

$\mu_{\text{more or less A}}(x) = (\mu_A^{True}(x)^{1/2}, \mu_A^{False}(x)\mu_A(x)^{0.5}$

$A = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5,$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$

$B = \{0.4/x_1 + 0.6/x_2 + 0.8/x_3 + 0.6/x_4 + 0.4/x_5,$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$

$A' = \text{not } A = \{0.5/x_1 + 0.3/x_2 + 0.1/x_3 + 0.3/x_4 + 0.5/x_5,$
$0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.8/x_4 + 0.9/x_5\}$

$A \vee B = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5,$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$

$A \wedge B = \{0.4/x_1 + 0.6/x_2 + 0.8/x_3 + 0.6/x_4 + 0.4/x_5,$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$

$\text{Very } A = \{0.25/x_1 + 0.49/x_2 + 0.81/x_3 + 0.49/x_4 + 0.25/x_5,$
$0.01/x_1 + 0.04/x_2 + 0.09/x_3 + 0.04/x_4 + 0.01/x_5\}$

$\text{More or less } A = \{0.70/x_1 + 0.83/x_2 + 0.94/x_3 + 0.83/x_4 + 0.70/x_5,$
$0.31/x_1 + 0.44/x_2 + 0.54/x_3 + 0.44/x_4 + 0.31/x_5\}$

$A \rightarrow B = \{1/x_1 + 0.8/x_2 + /x_3 + 0.9/x_4 + 1/x_5,$
$1/x_1 + 1/x_2 + 1/x_3 + 0.8/x_4 + 1/x_5\}$

$A \text{ o } B = \{0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5,$
$0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5\}$

Implication

Consider the fuzzy condition "if x is A_1 and x is A_2 and .. and x is A_n, then y is B."

The presidency part may contain any number of "and"/"or."

Zadeh fuzzy conditional inference given as

$= \{\min(1, 1 - \min(\mu_{A1}^{True}(x), \mu_{A2}^{True}(x),\ldots, \mu_{An}^{True}(x)) + \mu_B^{True}(y)),$
$\min(1, 1 - \min(\mu_{A1}^{False}(x), \mu_{A2}^{TrueFalse}(x),\ldots, \mu_{An}^{False}(x)) + \mu_B^{False}(y))\}(x,y)$

Mamdani fuzzy conditional inference given as

$= \{\min(\mu_{A1}^{True}(x), \mu_{A2}^{True}(x),\ldots, \mu_{An}^{True}(x), \mu_B^{True}(y)), \min(\mu_{A1}^{False}(x),$
$\mu_{A2}^{TrueFalse}(x),\ldots, \mu_{An}^{False}(x),\mu_B^{False}(y))\}(x,y)$

Reddy [5] fuzzy conditional inference given by

$= \{\min(\mu_{A1}^{True}(x), \mu_{A2}^{True}(x),\ldots, \mu_{An}^{True}(x)), \min(\mu_{A1}^{False}(x), \mu_{A2}^{TrueFalse}(x),\ldots,$
$\mu_{An}^{False}(x))\}(x,y)$

Consider the fuzzy condition "if x is A_1 and x is A_2, then x is B"

The presidency part may contain any number of "and"/"or."

For instance,

$A1 = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5,$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$

$A2 = \{0.4/x_1 + 0.6/x_2 + 0.8/x_3 + 0.6/x_4 + 0.4/x_5,$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$

$B = \{0.5/x_1 + 0.7/x_2 + 1/x_3 + 0.7/x_4 + 0.5/x_5,$
$0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.4/x_5\}$

Zadeh fuzzy conditional inference given as

$=\{\min (1, 1-\min(\mu_{A1}{}^{True}(x), \mu_{A2}{}^{True}(x)) + \mu_B{}^{True}(x)), \min (1, 1-\min (\mu_{A1}{}^{False}(x), \mu_{A2}{}^{TrueFalse}(x)) + \mu_B{}^{False}(x))\}$

$= \{1/x_1 + 0.1/x_2 + 1/x_3 + 1/x_4 + 1/x_5,$
$1/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5\}$

Mamdani fuzzy conditional inference given as

$= \{\min(\mu_{A1}{}^{True}(x), \mu_{A2}{}^{True}(x),\ldots, \mu_{An}{}^{True}(x), \mu_B{}^{True}(x)), \min(\mu_{A1}{}^{False}(x), \mu_{A2}{}^{TrueFalse}(x),\ldots, \mu_{An}{}^{False}(x), \mu_B{}^{False}(x))\}$

$= \{0.4/x_1 + 0.6/x_2 + 0.8/x_3 + 0.6/x_4 + 0.4/x_5,$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$

Reddy fuzzy conditional inference given as

$=\{\min(\mu_{A1}{}^{True}(x), \mu_{A2}{}^{True}(x)), \min(\mu_{A1}{}^{False}(x), \mu_{A2}{}^{TrueFalse}(x))\}$

$= \{0.4/x_1 + 0.6/x_2 + 0.8/x_3 + 0.6/x_4 + 0.4/x_5,$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$

Composition

If some relation R between A and B is known and some value A1 than B1 is inferred from R,

B1 = A1 o R,

where R = A➔B

Zadeh fuzzy inference is given by

$B1 = A1 \text{ o } R == A1\text{o}\{\min (1, 1 - \mu_A{}^{True}(x) + \mu_B{}^{True}(x)), \min (1, 1 - \mu_A{}^{False}(x) + \mu_B{}^{False}(x))\}$

$= \min\{\mu_A(x), \min(1,1\text{-} \mu_{A1}(x) + \mu_B(x)\}$

Mamdani fuzzy inference is given by

$= A1\text{o}\{\min (\mu_A{}^{True}(x), \mu_B{}^{True}(x)), \min (\mu_A{}^{TrueFalse}(x), \mu_B{}^{False}(x)\}$

If some relation R between A and B is not known,

according to Reddy fuzzy inference,

$= \{\min (\mu_{A1}{}^{True}(x), \mu_A{}^{True}(x)), \min (\mu_{A1}{}^{TrueFalse}(x), \mu_A{}^{False}(x)\}$

The fuzzy set A of X is characterized as its membership function $A = \mu_A(x)$ and ranging values in the unit interval [0, 1]

$\mu_A(x)$: X ➔[0, 1], x Є X, where X is universe of discourse.

$A = \mu_A(x) = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n$, "+" is union

The generalized fuzzy certainty factor (GFCF) is defined as

$\mu_A{}^{GFCF}(x) = \mu_A{}^{True}(x) - \mu_A{}^{False}(x)$

The generalized fuzzy certainty factor becomes single fuzzy membership function.

$\mu_A{}^{GFCF(x)}$: X➔[0, 1], x Є X, where X is universe of discourse.

The generalized fuzzy certainty factor (GFCF) will compute the conflict of evidence in the uncertain information.

For example,

$A = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5,$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$

$\mu_A{}^{GFCF}(x) = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5 - 0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$

$= 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.4/x_5$

For instance, "x is high temperature" with fuzziness {0.8,0.2}

The GFCF is 0.6
The graphical representation of GFCF is shown in **Figure 9**.
For example, A and B are generalized fuzzy sets.
$A = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5 -$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$
$= 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.4/x_5$
$B = \{0.4/x_1 + 0.6/x_2 + 0.8/x_3 + 0.6/x_4 + 0.4/x_5 =$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$
$= 0.3/x_1 + 0.4x_2 + 0.5/x_3 + 0.4/x_4 + 0.3/x_5$
The operations on GFCF are given as follows:
Negation
$A' = 1 - \mu_A{}^{GFCF}(x)/x$
$= 0.6/x_1 + 0.5/x_2 + 0.4/x_3 + 0.5x_4 + 0.6/x_5$
The graphical representation is shown in **Figure 10**.
Conjunction
$A\Lambda B = \min(\mu_A(x), \mu_B(x)\}/x$
$A\Lambda B = 0.3/x_1 + 0.4/x_2 + 0.5/x_3 + 0.4/x_4 + 0.3/x_5$
The graphical representation is shown in **Figure 11**.
Disjunction
$AVB = \max(\mu_A(x), \mu_B(y)\}/x$
$AVB = .4/x_1 + 0.6/x_2 + 0.9/x_3 + 0.6/x_4 + 0.2/x_5$
The graphical representation is shown in **Figure 12**.
Concentration
$\mu_{vey\ A}{}^{GFCF}(x) = \mu_A{}^{GFCF}(x)^2$

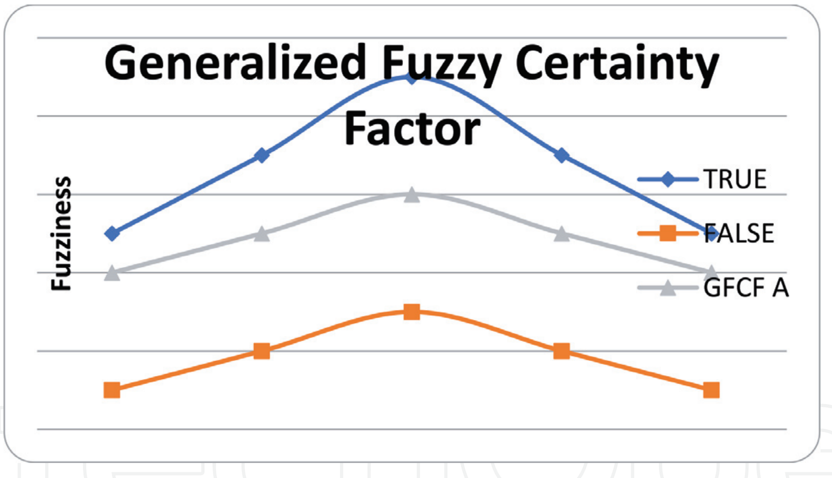

Figure 9.
Generalized fuzzy certainty factor.

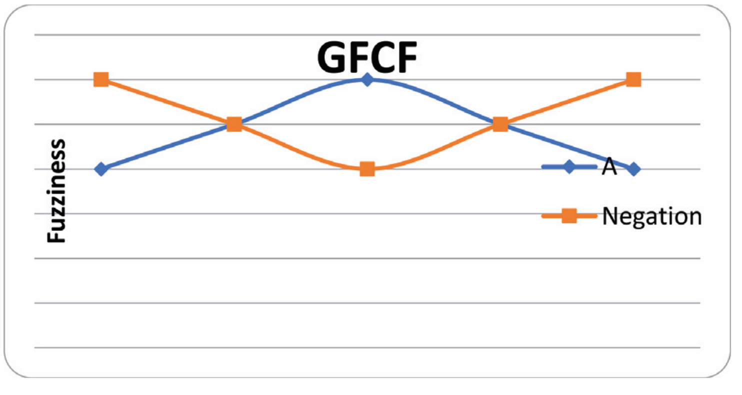

Figure 10.
Negation.

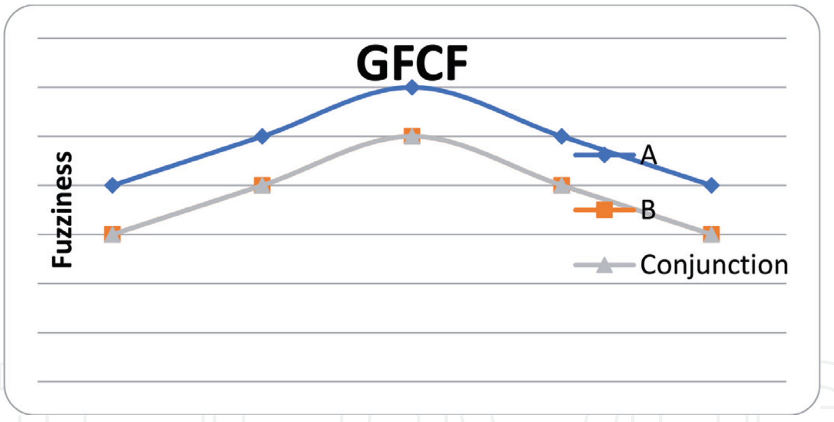

Figure 11.
Conjunction.

$= 0.16/x_1 + 0.25/x_2 + 0.36/x_3 + 0.25/x_4 + 0.16/x_5$

Diffusion

$\mu_{\text{more or less } A}{}^{GFCF}(x) = \mu_A{}^{GFCF}(x)^{0.5}$

$= 0.63/x_1 + 0.71/x_2 + 0.77/x_3 + 0.71/x_4 + 0.63/x_5$

The graphical representation of concentration and diffusion are shown in **Figure 13**.

Implication

Zadeh [9], Mamdani [7] and Reddy [5] fuzzy conditional inferences are considered keeping in view of fuzzy control systems.

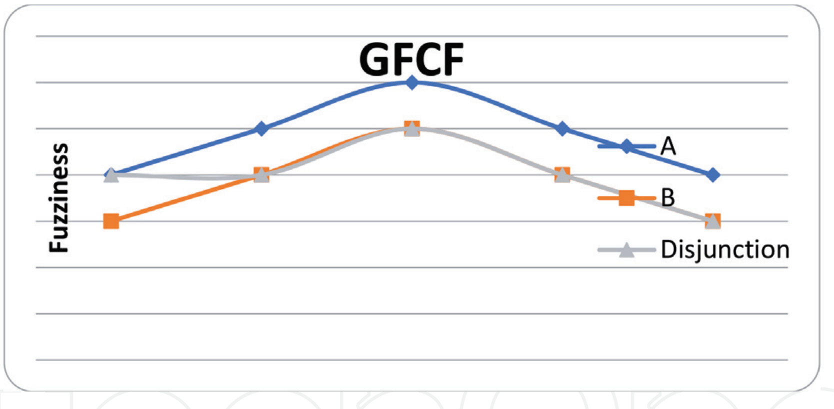

Figure 12.
Disjunction.

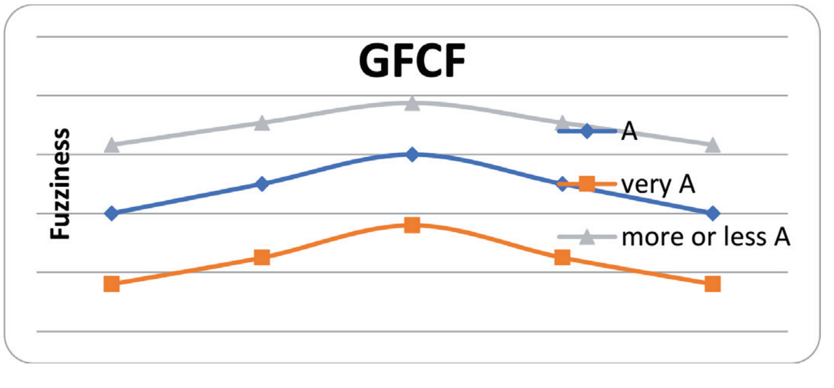

Figure 13.
Implication.

If x_1 is A_1 and x_2 is A_2 and … and x_n is A_n, then y is B
The presidency part may contain any number of “and”/“or.”
Zadeh fuzzy inference is given as follows:
If x_1 is A_1 and x_2 is A_2 and … and x_n is A_n, then y is B
$= \min(1, 1 - (A_1, A_2,\ldots,A_n) + B)$
Mamdani fuzzy inference is given as follows:
If x_1 is A_1 and x_2 is A_2 and … and x_n is A_n, then y is B
$= \min(A_1, A_2,\ldots, A_n, B)$
Reddy inference is given as follows:
If x_1 is A_1 and x_2 is A_2 and … and x_n is A_n, then y is B
$= \min(A_1, A_2,\ldots, A_n)$
Consider the fuzzy rule:
If x_1 is A_1 and x_2 is A_2, then x is B
For instance,
$A1 = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5 -$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$
$= 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.4/x_5$
$A2 = \{0.4/x_1 + 0.6/x_2 + 0.8/x_3 + 0.6/x_4 + 0.4/x_5 =$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$
$= 0.3/x_1 + 0.4x_2 + 0.5/x_3 + 0.4/x_4 + 0.3/x_5$
$B = \{0.5/x_1 + 0.7/x_2 + 1/x_3 + 0.7/x_4 + 0.5/x_5,$
$0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.4/x_5\}$
$= 0.1/x_1 + 0.2x_2 + 0.4/x_3 + 0.2/x_4 + 0.1/x_5$
The graphical representation of A1, A2 and B is shown in **Figure 14**.
Zadeh fuzzy inference is given as
$= \min(1, 1 - (A_1, A_2) + B)$
$= 0.8/x_1 + 0.8/x_2 + 0.9/x_3 + 0.8/x_4 + 0.8/x_5$
Mamdani fuzzy inference is given as
$\min(A_1, A_2,\ldots, A_n, B)$
$= 0.1/x_1 + 0.2/x_2 + 0.4/x_3 + 0.2/x_4 + 0.1/x_5$
Reddy fuzzy inference is given as
$\min(A_1, A_2,\ldots, A_n)$
$= 0.2/x_1 + 0.4/x_2 + 0.5/x_3 + 0.4/x_4 + 0.3/x_5$
The graphical representation of fuzzy inference is shown in **Figure 15**.
Composition
The GFCF is a single fuzzy membership function

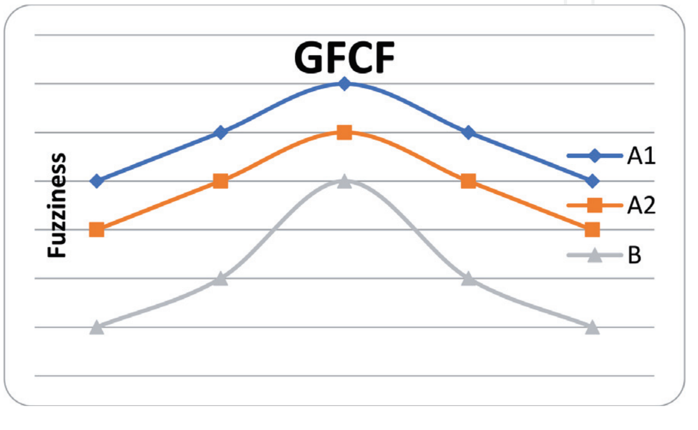

Figure 14.
GFCF for fuzzy rule.

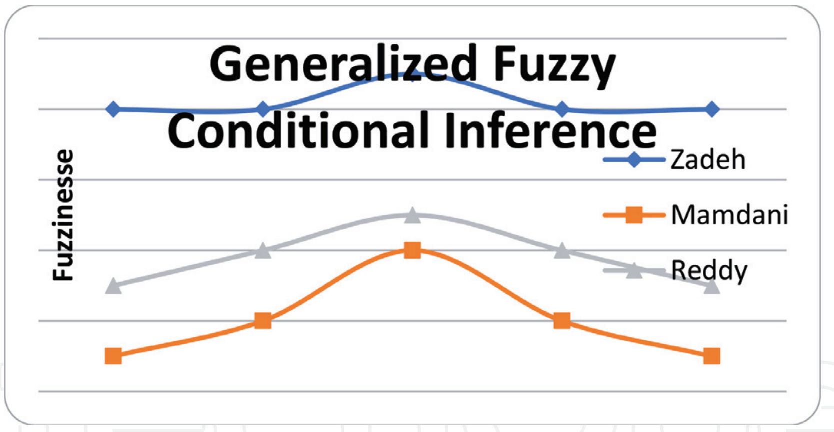

Figure 15.
Implication.

If some relation R between A1, then B1 is to infer from R:
B1 = A1 o R = $\min\{\mu_{A1}{}^{GFCF}(x), \mu_R{}^{GFCF}(x)\}/x$
Zadeh fuzzy inference is given by
B1 = A1 o R = $\min\{\mu_{A1}{}^{GFCF}(x), \mu_R{}^{GFCF}(x)\}$
$= \min\{\mu_{A1}{}^{GFCF}(x), \min(1, 1- \mu_{A1}{}^{GFCF}(x) + \mu_B{}^{GFCF}(x))\}$
Mamdani fuzzy inference is given by
$= \min\{\mu_{A1}{}^{GFCF}(x), \mu_{A1}{}^{GFCF}(x), \mu_B{}^{GFCF}(x)\}$
If there is some relation R between A and B, then Reddy fuzzy inference is given by
$= \mu_{A1}{}^{GFCF}(x)$
where A, B, A1, and B1 are the GFCF.
$A = \{0.5/x_1 + 0.7/x_2 + 0.9/x_3 + 0.7/x_4 + 0.5/x_5 -$
$0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$
$= 0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.4/x_5$
$B = \{0.5/x_1 + 0.7/x_2 + 1/x_3 + 0.7/x_4 + 0.5/x_5,$
$0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.4/x_5\}$
$= 0.1/x_1 + 0.2x_2 + 0.4/x_3 + 0.2/x_4 + 0.1/x_5$
A1 = more or less A
$= = 0.55/x_1 + 0.63/x_2 + 0.71/x_3 + 0.63/x_4 + 0.55/x_5$
The composition of Zadeh, Mamdani and Reddy fuzzy inference is shown in **Figure 16**.

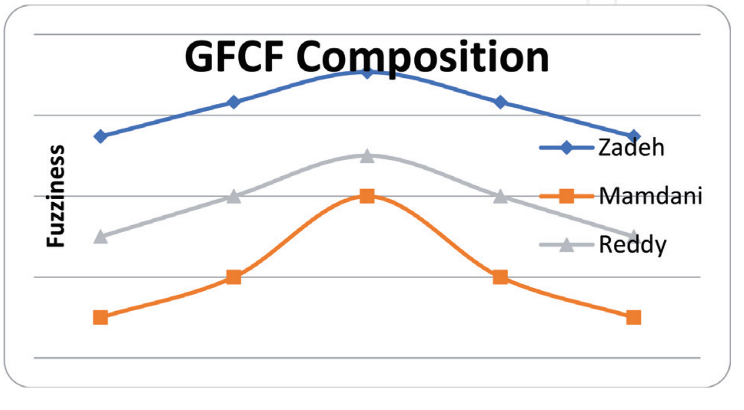

Figure 16.
Composition.

6. Fuzzy control systems using two fuzzy membership functions

Zadeh [6] introduced fuzzy algorithms. The fuzzy algorithm is a set of fuzzy statements. The fuzzy conditional statement is defined as follows:

If x_i is $A1_i$ and x_i is $A2_i$ and … and x_i is An, then y_i is B_i

The precedence part may contain and/or/not.

The fuzzy control system consist of a set of fuzzy rules.

If a set of conditions is satisfied, then a set of consequences is inferred.

The fuzzy set with twofold membership function will give more information than the single membership function.

The generalized fuzzy certainty factor (GFCF) is given as

$\mu_A{}^{GFCF}(x) = \{\mu_A{}^{True}(x) - \mu_A{}^{False}(x)\}$

For instance, "x has fever"

The GFCF for fever given as

$\mu_{Low}{}^{GFCF}(x) = \{\mu_{Low}{}^{True}(x) - \mu_{Low}{}^{False}(x)\}$

Consider the rule in fuzzy control system

If BZ is low

and BE is normal

then reduce fan speed

For instance, fuzziness may be given as follows:

If BZ is low (0.9,0.2)

and BE is normal (0.8,0.2)

then reduce fan speed (0.6, 0.3)

Fuzziness of GFCF may be given as follows:

If BZ is low (0.7)

and BE is normal (0.6)

then reduce fan speed (0.3)

For instance, consider the twofold fuzzy relational model of fuzzy control system.

The graphical representation of twofold fuzzy relational model is shown in **Figure 17**.

The graphical representation of fuzzy inference for condition part containing "AND" is shown in **Figure 18**.

Graphical representation of fuzzy inference for condition part containing "OR" is shown in **Figure 19**.

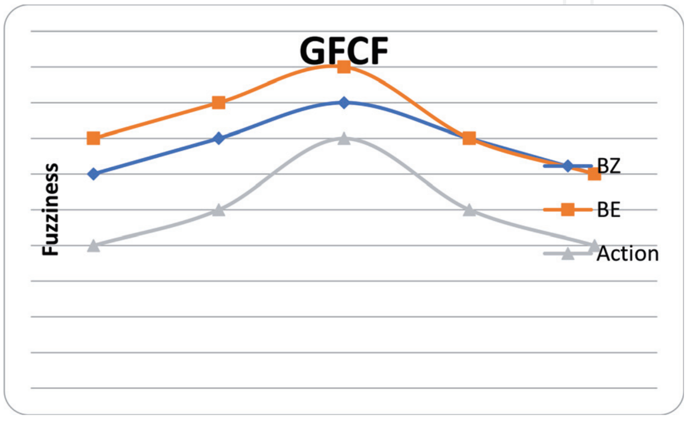

Figure 17.
GFCF for ***Table 5****.*

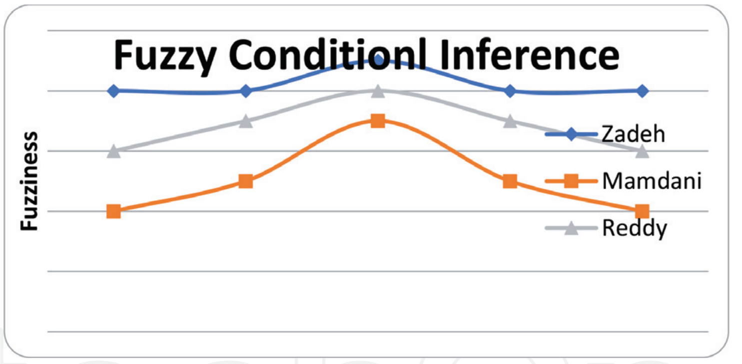

Figure 18.
Fuzzy conditional inference for "AND."

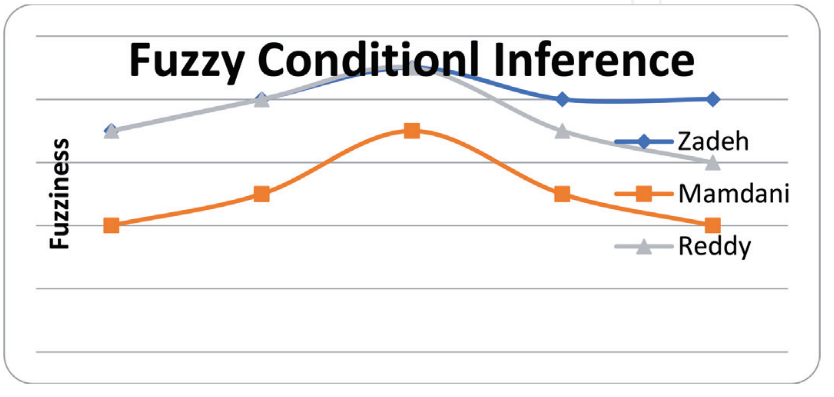

Figure 19.
Fuzzy conditional inference for "OR."

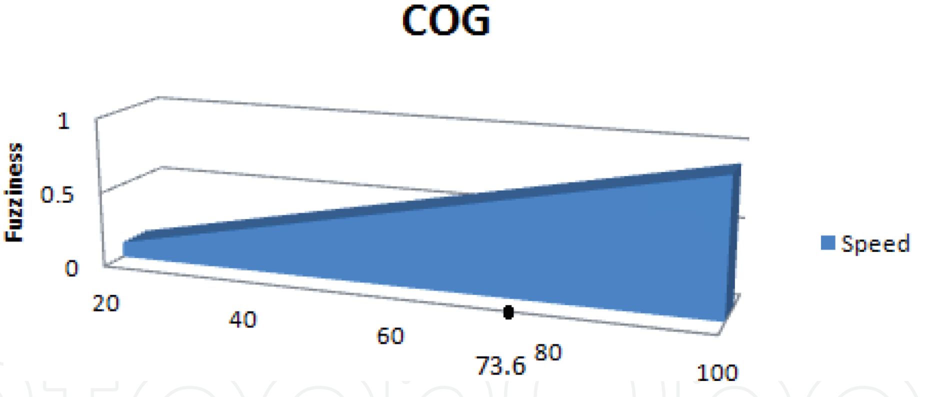

Figure 20.
Defuzzification.

Defuzzification

Usually, centroid technique is used for defuzzification. It finds value representing the centre of gravity (COG) aggregated fuzzy generalized fuzzy set.

$\mathrm{COG} = \Sigma\, C_i\, \mu_{Ai}{}^{GFCF}(x)/\ \Sigma\, C_i$

For instance,

Speed = {0.1/20 + 0.3/40 + 0.5/60 + 0.7/80 + 0.9/100}

COG = (0.1*20 + 0.3*40 + 0.5*60 + 0.7*80 + 0.9*100)/
(0.1 + 0.3 + 0.5 + 0.7 + 0.9) = 73.6

The defuzzification is shown in **Figure 20**.

7. Conclusion

The fuzzy set of two membership function will give more information than single fuzzy membership function for incomplete information. The fuzzy logic and fuzzy conditional inference based on single membership function and twofold fuzzy set are studied. The FCF is studied as difference between "True" and "False" membership functions to eliminate conflict of evidence and to make as single fuzzy membership function. FCF = [True-False] will correct truthiness of single membership function. The methods of Zadeh, Mamdani and Reddy fuzzy conditional inference studied for fuzzy control systems are given as application.

Conflict of interest

The author states that he has no conflict of interest and that he has permission to use parts of his previously published work from the original publisher.

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