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# Strengths and Limitations of Traditional Theoretical Approaches to FRP Laminate Design against Failure

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## Abstract

The strength of Fiber Reinforced Plastic laminated structures is strongly dependent on the stacking sequence of the laminate, and consequently the fiber orientations of the individual laminae (also referred to as layers or plies). Classical Lamination Theory (CLT) is a theoretical tool providing the strain and stress distribution in a laminate based on its stacking sequence and material properties. On the other hand, first ply, and consequent ply failure can be approximated with interactive failure criteria, such as the Tsai-Hill and Tsai-Wu. Technological advances often require material alternatives to metallic structures, and FRPs constitute optimum solutions to such selections. However, these structures are no longer just plain laminates with unidirectional fibers in their laminae, they include geometric discontinuities allowing ease of assembly. Such discontinuities become stress concentration regions, which require extra attention upon design against failure. This chapter discusses the extent to which the traditional analysis of FRP failure, using CLT and interactive failure criteria is adequate in structures with discontinuities, and suggests extra analysis steps to be considered when designing against failure in the area of the discontinuity.

**Keywords:** fiber reinforced plastics, classical lamination theory (CLT), interactive failure criteria, linear fracture mechanics, stacking sequence, fiber orientations, first ply failure, unidirectional fibers

## 1. Introduction

Industries are constantly turning towards new material alternatives that can provide lighter structures of high strength and customizable stiffness to the needs of the destined application. A polymeric matrix with an appropriate reinforcement comprises composite material solutions for a wide range of industries from the aeronautics and automotive to the battery industry.

A special case of such composite materials is Fiber Reinforced Plastics (FRPs). These composite materials have an epoxy resin matrix and a fibrous high-strength reinforcing phase. As a result, they provide high strength and stiffness, while being much lighter than any metal. Additionally, FRPs are highly corrosion resistant [1]. In a majority of applications FRP layers are laminated into beam like structures.

Therefore, they offer the option of being tailored to the desired properties of the destined application. FRP laminates of unidirectional laminae (i.e. long fiber reinforcement) can be optimized to have stacking sequences that will provide an optimum strength and stiffness at low weight.

The heterogeneous and highly anisotropic nature of composites, and consequently FRPs, is due to the fact that composite materials are made of two or more constituents that are insoluble in each other. As a result, the anisotropic nature of FRPs should always be considered when evaluating or predicting their failure mechanisms.

Failure in composite materials is defined as the point when the component ceases to perform adequately for the application it is designed for. At that point, failure may be described as catastrophic or simply degradation of the material properties. Understanding the mechanisms that lead to any type of undesired failure is very important when designing a component against failure.

In laminated FRPs there are ways to predict when failure will first occur in the laminate. First ply failure, will not always mean catastrophic or not failure of the composite, however, it will denote when failure is first observed in the laminate, and at which specific ply. It is possible that the FRP laminate will still function properly, as the load will be carried by the remaining plies. As a result, the design and choice of stacking sequence specify the maximum acceptable load for an application, and in the case of cyclic loading applications, can even specify which maximum applied load will cause first ply failure [2–5].

This chapter is divided into two sections; the first section discusses the heterogeneous and anisotropic nature of composites, and how Classical Lamination Theory (CLT) is used to determine the state of stress in unidirectional FRP laminates. Furthermore, two interactive failure criteria, the Tsai-Hill and Tsai-Wu, are discussed as the criteria of predicting first ply failure in FRP laminates. The above failure criteria are useful in conjunction with experiments in determining first ply failure in unidirectional FRP laminates. In the case a laminate contains a geometric discontinuity, such as a hole or tapered edge, the unidirectionality of the fibers is interrupted at the region of the discontinuity. As a result, the above failure criteria cease to accurately predict failure at the discontinuity, which additionally becomes a stress concentration region. The second section of this chapter, discusses the inadequacy of the above methods in designing against failure in laminated FRP components with such geometric discontinuities, and suggests additional analysis combining the Tsai-Wu failure criterion with fracture mechanics to better evaluate and predict failure in such regions.

## 2. Predicting failure in unidirectional laminated FRPs

### 2.1 Laminate stress distribution and classical lamination theory

The combination of the matrix and reinforcing phases (i.e. fibers in FRPs), which remain insoluble in each other, offer the composite material its anisotropic and heterogeneous nature, while at the same time a combination of the properties of both constituents. The volume percent of the total material occupied by the individual constituents (matrix and fibers) regulate the properties of the composite. As a result, the properties of FRPs may be tailored to the needs of an application by selecting the appropriate volume percent of fibers and matrix.

The anisotropic nature of the FRPs is mainly due to their reinforcing phase, the fibers, as the matrix phase is assumed to be homogenous and isotropic. The reinforcing phase may take the form of long unidirectional fibers, woven fibers,

short, or chopped fibers that are scattered in the matrix. The theories and criteria considered in this chapter only concern long unidirectional fibers.

Laminated FRPs are structures composed of two or more FRP layers. These layers are also referred to as plies or laminae. Each lamina has its reinforcing phase oriented in a specified way and the fibers occupy a given volumetric fraction of the lamina. As a result, the properties of the laminate are determined by each lamina. The laminae are stacked together to create the laminate. The order of stacking is very important as the different orientations of the fibers in the individual laminae, as well as their volume percentage, affect the mechanical properties of the whole laminate. The selection of the order of laminae stacking is called the stacking sequence of the laminate.

Although it is often the case to view a FRP laminate as a homogeneous structure of isotropic bulk mechanical properties, this approach should only be followed for macroscopic analysis, when geometry and loading conditions are investigated rather than the specific effect of the material properties. When investigating the strength, stiffness, and designing against failure, the anisotropic and heterogeneous nature of the FRP laminate should be considered. In such cases, the analysis is in the lamina level or even a microscopic level of the individual constituents of the composite: the matrix, reinforcement, and their interface. The discussion that follows concerns the lamina level.

The FRP mechanical properties, although affected by the mechanical properties of its constituents, differ greatly from them and depend additionally on the volume fraction that each phase occupies. Rules of mixtures is the set of equations that calculate the elastic properties of a composite material taking into account the individual properties of its constituents and their volume fractions. The Young's moduli ( $E_i$ ), shear moduli ( $G_{ij}$ ), and Poisson's ratios ( $\nu_{ij}$ ) are determined using Rules of Mixtures and the Halpin-Tsai equations. There exist therefore, three Young's moduli, one for each material direction (Eqs. (1) and (2)) and four shear moduli (Eq. (3)). To calculate Poisson's ratios the bulk moduli ( $K$ ) are used (Eqs. (5)–(9)) [6].

$$E_1 = (1 - f)E_m + fE_f \quad (1)$$

$$E_2 = E_3 = E_m \frac{(1 + \xi\eta f)}{(1 - \eta f)} \quad (2)$$

$$G_{12} = G_{21} = G_{13} = G_{31} = \frac{G_m(1 + \xi\eta f)}{(1 - \eta f)} \quad (3)$$

where the subscripts m and f, refer to matrix and fiber, respectively, and 1,2,3, to the directionality of the material. The constant  $f$  is the volume fraction of fibers in the composite such that  $0 \leq f \leq 1$ ,  $\xi \approx 1$ , and

$$\eta = \frac{\left(\frac{E}{E_m} - 1\right)}{\left(\frac{E_f}{E_m} + \xi\right)} \text{ or } \eta = \frac{\left(\frac{G}{G_m} - 1\right)}{\left(\frac{G_f}{G_m} + \xi\right)} \quad (4)$$

$$K = \left[ \frac{f}{K_f} + \frac{(1 - f)}{K_m} \right]^{-1} \quad (5)$$

$$K_f = \frac{E_f}{3(1 - 2\nu_f)} \quad (6)$$

$$K_m = \frac{E_m}{3(1 - 2\nu_m)} \quad (7)$$

$$\nu_{12} = (1 - f)\nu_m + f\nu_f \quad (8)$$

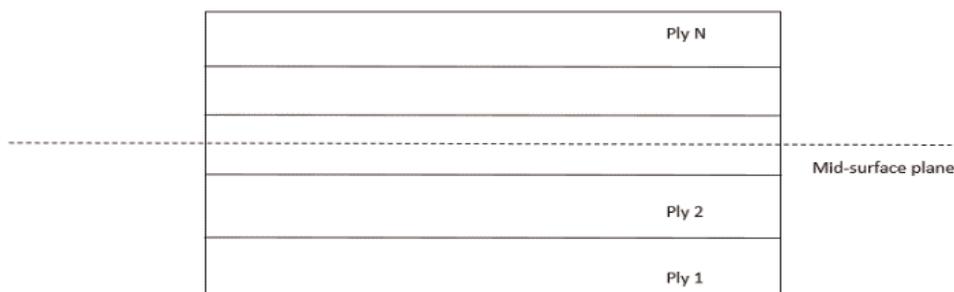
$$\nu_{23} = 1 - \nu_{21} - \frac{E_2}{3K} \quad (9)$$

To design against failure the FRP laminae should be built with an optimizable strength, and a desirable stiffness, while maintaining a low weight. The strength of the FRP laminate depends on that of the individual laminae, and can be optimized by choosing an appropriate stacking sequence. Therefore, although the above equations play an important role in determining the mechanical properties in different directions, it is important to start accounting for the orientation of the fibers in each lamina.

The constitutive relationships for FRPs use the generalized Hooke's Law (Eq. (10)). A total of 81 elastic constant would be required to fully characterize the FRP behavior. However, by assuming symmetric stresses and strains, the required elastic constants become 36. The lamina level contains two sets of axes that express the material direction; a set of local and a set of global axes. The local axes, also referred to as principal axes, have the longitudinal axis parallel to the longitudinal fibers. The global axes, are a reference frame of the laminate, where the horizontal, transverse, and normal directions coincide with the dimensional directions of the laminate. As a result, the longitudinal axis of the local reference system makes an angle with the global horizontal direction, thus allowing measuring the angle of the fiber orientation in each lamina. Consequently, each lamina has three mutually orthogonal axes of rotational symmetry, which further reduce the 36 elastic constant to 12. Only 9 of these constant are independent.

$$\sigma_{ij} = E_{ijkl}\epsilon_{kl} \quad (10)$$

As seen from Eq. (2) of the Rules of Mixtures, the properties of the lamina in directions 2 and 3, the directions normal to the longitudinal fibers, are the same. As a result, the plane 23 of the lamina is an isotropy plane. Therefore, the FRP lamina characterized as transversely isotropic, a special case of orthotropic materials, requires just 5 independent elastic constants to fully determine its behavior. Classical Lamination Theory (CLT) used with orthotropic continuous laminated composite materials builds a set of equations that lead to the development of constitutive relationships that determine the state of stress in each layer [7–9]. CLT accounts for both the lamina orientations and its position in the laminate, showing therefore, the significance of the stacking sequence to the strength and performance of the laminate. To determine the position of a lamina in the laminate a common starting reference point of lamina numbering is the bottom layer. This bottom lamina, becomes lamina 1. There also exists a fictitious plane dividing the laminate in two equal half portions, called the mid-surface plane. This plane serves as a position datum for the laminae (**Figure 1**).



**Figure 1.**  
Stacking sequence and nomenclature.

CLT builds constitutive relationships using elastic properties, which can be determined by Rules of Mixtures and the Halpin-Tsai equations or experimental data, as well as thermal expansion properties at each material direction. Depending on the nature of the fibers and the destined application of the structure, hygroscopic coefficients may also be considered. In the case of a transversely isotropic material, only two sets of material properties are required: one set in direction 1 and one set in either direction 2 or 3. CLT first evaluates a stiffness matrix ( $Q$  and  $\bar{Q}$ ) for each lamina accounting for the elastic properties in the required directions and the orientation of the fibers. The overbar above  $Q$  denotes all off-axis laminae, i.e. those whose fibers make an angle with the global horizontal laminate direction, while the absence of a bar above  $Q$  refers to the stiffness matrix of on-axis laminae, i.e. those whose fibers are parallel to the global horizontal direction, having a  $0^\circ$  orientation. To distinguish between the different lamina stiffness matrices a subscript ( $k$ ) is used, denoting the  $k^{\text{th}}$  lamina in the laminate.

$$\bar{Q}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66} \end{bmatrix}$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta \\ \bar{Q}_{12} &= \bar{Q}_{21} = (Q_{11} + Q_{22} - 4Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} (\sin^4 \theta + \cos^4 \theta) \\ \bar{Q}_{22} &= Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta \\ \bar{Q}_{16} &= \bar{Q}_{61} = (Q_{11} - Q_{12} - 2Q_{66}) \sin \theta \cos^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \cos \theta \sin^3 \theta \\ \bar{Q}_{26} &= \bar{Q}_{62} = (Q_{11} - Q_{12} - 2Q_{66}) \cos \theta \sin^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin \theta \cos^3 \theta \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{66} (\sin^4 \theta + \cos^4 \theta) \end{aligned}$$

where

$$m = \cos \theta, \text{ and } n = \sin \theta \quad (11)$$

$$[Q]_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$$

where

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}} \\ Q_{12} &= Q_{21} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} \\ Q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}} \\ Q_{66} &= G_{12} \end{aligned}$$

As mentioned above, CLT focuses on each lamina individually. The constitutive equation of the  $k^{\text{th}}$  lamina (Eq. (12)) relates the stress distribution in the lamina to the lamina strain through the stiffness matrix. The strain distributions is a function

of the mid-surface strains ( $\varepsilon_{ij}^o$ ) and curvatures ( $\kappa_{ij}$ ), which are common to all laminae of the laminate and depend on loading conditions. The effect of thermal ( $\alpha_{ij}$ ) and hygral effects ( $\beta_{ij}$ ) is also included in the strain calculation as they are responsible for residual strains in the laminate that may be induced during the manufacturing and curing process or service life of the composite.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = [\bar{Q}_k] \left( \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k \Delta T - \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k \bar{M} \right) \quad (12)$$

The above relationship is the stress–strain relationship of the  $k^{\text{th}}$  lamina. In order to build relationships for the stress and strain distributions in the laminate, which can then be used to determine first ply failure and the strength of the whole laminate, the loading conditions of the laminate should be considered. Three matrices in CLT: the Extension Stiffness Matrix, ( $A_{ij}$ ), the Extension-Bending Coupling Matrix, ( $B_{ij}$ ), and the Bending Stiffness Matrix, ( $D_{ij}$ ), bring together the stiffness effects from each lamina, and consequently fiber orientation, accounting for the position ( $z$ ) of each lamina in the laminate (Eqs. (13)–(15)). These matrices account for the lamina thickness ( $t$ ) and calculate the stress distribution based on the different loading conditions applied.  $A_{ij}$  considers the tension-compression effects of longitudinal and transverse loading, matrix  $D_{ij}$  considers the effects of bending moments, while matrix  $B_{ij}$  couples the effects of both types of loading. A relationship calculating normal forces and moments includes the above matrices as well as mid-surface strains and curvatures (Eq. (16)<sup>1</sup>) [10].

$$[A_{ij}] = \sum_{k=1}^n [\bar{Q}_{ij}]_k t_k \quad (13)$$

$$[B_{ij}] = \sum_{k=1}^n [\bar{Q}_{ij}]_k t_k \bar{z}_k \quad (14)$$

$$[D_{ij}] = \sum_{k=1}^n [\bar{Q}_{ij}]_k \left( t_k \bar{z}_k^2 + \frac{t_k^3}{12} \right) \quad (15)$$

$$\begin{Bmatrix} \hat{N} \\ \dots \\ \hat{M} \end{Bmatrix} = \begin{pmatrix} A & \vdots & B \\ \dots & \dots & \dots \\ B & \vdots & D \end{pmatrix} \begin{Bmatrix} \varepsilon^o \\ \dots \\ \kappa \end{Bmatrix} \quad (16)$$

## 2.2 Failure criteria

There are three major failure modes in the microscopic level of the FRP, i.e. the constituent materials and their interface:

- Failure of the matrix phase through crack initiation and propagation.
- Failure at the reinforcing phase, which is the fracture of one or more fibers.
- Failure at the interface of the two constituents, referred to as debonding, where the fibers detach from the matrix phase.

<sup>1</sup> All loading conditions, including thermal and hygral effects, are accounted for in  $\hat{N}$  and  $\hat{M}$ .

In each of the above degrade, the mechanical properties of the composite and affect the strength and the performance of the material in a different way. The fibers in FRPs, holding the load carrying capacity of the composite, constitute the phase that determines to a large degree the strength and stiffness of the material. The orientation of the fibers is crucial in stress and strain calculations, as has been previously shown through the discussion on CLT. As a result, the second failure mode, which concerns failure at the reinforcing phase, becomes of special interest, as is the one that interrupts the load carrying capacity of the fibers. It is also among the main concepts of this chapter, and will be given further attention in Section 3. Fractured fibers cannot be replaced or repaired and therefore, this failure mode permanently degrades the strength of the material.

The fracture of fibers becomes especially important when the load carrying capacity of the FRP structure is expected to be along one of the axis of the structure. Typically, fibers are chosen along or off this axis. Take for example a plate under bending. Such a plate may represent a flat beam spring (e.g. leaf springs in suspension systems) which is loaded and deflected under a bending moment. As this bending moment creates a stress distribution along the longitudinal axis of the beam, if the choice of material is FRP, the fibers are chosen parallel or at an angle to this longitudinal direction. This way the fibers hold the load carrying capacity of the beam, and the stacking sequence choice regulates the stress distribution and desired stiffness of the structure, as followed by the CLT equations. If due to failure, one or more of these fibers fracture, a discontinuity along the load carrying capacity in this longitudinal direction is generated. The specific lamina(e) with the fractured fibers become(s) responsible for the degradation of the mechanical properties of the composites, as it can no longer participate in the aforementioned CLT equations, which are exclusive to longitudinal continuous fibers.

There also exist other failure modes, such as delamination (i.e. the deboning at the lamina interface), or failure due to environmental factors (such as high moisture absorption [11, 12] or UV degradation of the matrix). In such cases, examining the failure mechanism to determine the extent to which the strength of the composite has been affected, requires investigation at the material level and its chemical composition.

Failure criteria, on the other hand, allow us to determine the effect of loading to the strength of the material. Such criteria may be used in conjunction with CLT to determine optimum stacking sequences that can guarantee a long life performance of the FRP structure at specific loading conditions before the occurrence of first ply failure.

The anisotropic nature of FRPs requires failure criteria that account for the interaction of stresses, and consequently material properties, in different directions. Such criteria are referred to as interactive failure criteria, as opposed to non-interactive ones, which focus on parameters in each direction separately (e.g. Tresca and von Mises) [6, 10, 13]. The interactive failure criteria may give a prediction of the onset of failure irrespective of the failure mode or any other conditions responsible for it (environment, thermal, etc.).

The two criteria discussed in this chapter are the Tsai-Hill and Tsai Wu. They both operate on a comparison of the stress state in each lamina to the failure stress under stress plane conditions in order to determine the failure or not of a lamina. They concern therefore, similar to CLT, the lamina level. As the majority of failure criteria, they are polynomial expansions treating the stress tensor ( $\sigma_{ij}$ ) as the sole parameter to characterize the onset of failure. As polynomial expansions, they may be tailored to the case of transversely isotropic materials, thus reducing significantly the number of required material parameters [6]. However, because they are mere criteria, they should always be verified by experimental data, as they can only give a prediction for the onset of failure.

The stress tensor ( $\sigma_{ij}$ ) in these criteria is calculated using CLT. As a result, it refers to the stress distribution of the FRP structure along one of the directions of the lamina. Such a lamina is considered to contain as its reinforcing phase continuous longitudinal fibers. If this is not the case, and the fibers are either discontinued or fractured, the lamina is degraded and not included in the CLT calculations, which results, in its exclusion from the following criteria (Eqs. (17) and (18)). As a result, similar to CLT the criteria presented below may provide a prediction for the onset of failure in a lamina, provided that the lamina maintains its continuous unidirectional fibers. Therefore, they would not be appropriate for failure predictions in laminae with discontinuities due to which fibers are interrupted.

*Tsai-Hill Failure Criterion:*

$$\frac{\sigma_{11}^2}{X^2} - \frac{\sigma_{11}\sigma_{22}}{X^2} + \frac{\sigma_{22}^2}{Y^2} + \frac{\sigma_{12}^2}{S^2} < 1 \quad (17)$$

In the Tsai-Hill criterion (Eq. (17)) the longitudinal ( $\sigma_{11}$ ), transverse ( $\sigma_{22}$ ), and shear stresses ( $\sigma_{12}$ ) in each lamina are compared to the longitudinal tensile and compressive ( $X$  and  $X'$ ), transverse tensile and compressive ( $Y$  and  $Y'$ ), and shear ( $S$ ) ultimate strengths. These latter strength parameters are all material parameters that may be obtained from experimental results or material databases. From the total of 5 parameters required, only 3 are involved in the equation, i.e. the above criterion becomes specific to the type of loading. If the loading results in compressive stresses, Eq. (17) will be rewritten to include the longitudinal and transverse compressive ultimate strength ( $X'$  and  $Y'$ ), as well as the shear ultimate strength. In the format presented above, it addresses failure due to tensile stresses. In either case failure has occurred when the equation on the left hand side of the criterion equals to or is greater than 1.

*Tsai-Wu Failure Criterion:*

$$\begin{aligned} \left(\frac{1}{X} - \frac{1}{X'}\right)\sigma_{11} + \left(\frac{1}{Y} - \frac{1}{Y'}\right)(\sigma_{22} + \sigma_{33}) + \frac{\sigma_{11}^2}{XX'} + \frac{1}{YY'}(\sigma_{22} + \sigma_{33})^2 + 2F_{12}\sigma_{11}(\sigma_{22} + \sigma_{33}) \\ + \frac{1}{S'^2}(\sigma_{23}^2 - \sigma_{22}\sigma_{33}) + \frac{1}{S^2}(\sigma_{12}^2 + \sigma_{31}^2) < 1 \end{aligned} \quad (18)$$

The Tsai-Wu criterion also investigates failure at the lamina level and states that failure occurs when Eq. (18) is equal to 1. The equation contains 6 constants involving the material parameters of tensile and compressive ultimate strengths in the longitudinal and transverse directions, as well as shear ultimate strengths. The Tsai-Wu criterion does not address failure separately due to either tensile or compressive stresses, as it includes all ultimate strengths of the material irrespective of their directionality. Additionally, it addresses stresses in direction 3, as well as shear stresses in planes including direction 3. As a result, the Tsai-Wu criterion is not exclusive to the transversely isotropic materials examined using CLT. Therefore, this criterion requires a total of 7 material parameters. The Tsai-Wu criterion terms can be evaluated by the assumption of uniaxial tension and compression results, which is based on experimental data [6, 10]. The interaction parameter ( $F_{12}$ ) due to its interactive nature is an approximation that depends on the product of the products of tensile and compressive longitudinal ultimate strengths and tensile and compressive transverse ultimate strengths (Eq. (19)). It is often estimated from multiaxial stress data [6, 10].

$$F_{12} \leq \sqrt{\frac{1}{XX'} \times \frac{1}{YY'}} \quad (19)$$

The above strength data is obtained from experimental results on unidirectional FRPs with continuous fibers. This is one more reason, why the above criteria would fall short in accurately predicting failure in laminae with discontinuous fibers.

As mentioned above, failure criteria should be used in conjunction with experimental data for better understanding the onset of failure. Research has shown that the Tsai-Hill criterion tends to overestimate failure, while Tsai-Wu tends to underestimate failure [3–5, 14].

### **3. Accounting for geometric discontinuities in FRP laminates**

The above discussion shows the importance of laminae fiber orientation and therefore, the stacking sequence of laminates. However, the tools discussed in Section 2, CLT and the interactive failure theories, take into account unidirectional uninterrupted fibers in the laminae. Fiber fracture is considered as one of the failure modes in FRP composites, and is one of the most detrimental ones to the material. To approach therefore, a similar analysis on structures with geometric discontinuities the above methods should be combined with further analysis tools to address the high stresses in the area of the discontinuity and avoid working with interrupted fibers.

#### **3.1 Orienting fibers around a circular hole**

To predict failure and evaluate critical stresses around geometric discontinuities in FRP laminates, different approaches and models have been developed and presented in literature. Some of these models and theories focus on fiber failure, as is for example Hashin's theory [15], while other newer approaches look into the prediction of fiber and interfiber failure [16]. In the case of geometric discontinuities, such as notches, there exist the Waddoups-Eisenmann-Kamiski (WEK) model [17, 18] that evaluates the strength of notched composite specimens using the stress intensity factor. However, the above models only evaluate failure and do not address any predictions of its onset, which is important when designing against failure.

As previously mentioned an optimum stacking sequence can improve the onset of first-ply failure in FRP laminates. As a result, the importance of an appropriate stacking sequence around a geometric discontinuity becomes even more significant. CLT has been used by Goteti and Reddy in conjunction with the stress intensity around a circular hole to examine the effect of fiber orientation, hole size, and fiber volume fraction on the stress concentration around the hole [19]. A different approach using Muskhelishvili's complex variable method and fiber orientation as input was attempted by Sharma in determining the stress concentration around circular/elliptical/triangular cutouts [20]. On the other hand, other researchers, such as Huang and Haftka, instead of focusing on the stress intensity and concentration in the discontinuity region, attempted to determine the fiber orientation around it, while keeping the fiber orientation in the remaining lamina unidirectional [21].

The aforementioned work agrees that fiber orientation around a discontinuity is affected by the following parameters:

- Size of the discontinuity (eg. diameter of a hole).
- Load type and direction

- Volume fraction of the fibers, which has already been shown to affect the mechanical properties of the FRP material as determined by Rules of Mixtures.

This chapter discusses a case study of a slightly different approach to determine fiber orientation around circular discontinuities [22]. The approach focuses on the immediate vicinity of the discontinuity, where it attempts to determine an optimum fiber orientation. It will be shown that the approach is concerned only with the plastic region around the discontinuity, where the fibers will not be interrupted, and as a result will maintain their load carrying capacity from one end of the lamina to the other. Additionally, the fiber orientation outside the plastic region of the discontinuity will remain unidirectional, based on the orientation of the lamina in the stacking sequence.

The majority of the works in literature discussing approaches to optimum fiber orientation around discontinuities or the evaluation of stress intensity in such regions use axial loading conditions. The case study presented below will assume a three point bending loading condition on the FRP laminate. In such loading cases, the majority of the aforementioned work becomes non-applicable, as the fibers in order to maintain their longitudinal load carrying capacity in the structure should remain continuous and uninterrupted. The meaning of continuous fibers, disregards the concept of fibers starting at the rim of a central to the structure discontinuity, as this considers a lamina of two sets of continuous fibers, one on each side of the discontinuity but interrupted by it.

This case study examines first CLT and the Tsai-Hill failure criterion to determine a minimum moment required to cause first ply failure in a given FRP laminate in the absence of a discontinuity, and second, the geometric stress concentration factor under bending, to determine the moment to cause failure in the presence of a circular hole. The optimum fiber orientation in the area of the hole will be determined when this minimum moment is applied. The above approach therefore, uses the aforementioned theories and criteria solely for a unidirectional lamina, and introduces linear fracture mechanics to account for the discontinuity effect.

### 3.1.1 Laminate beam model and discontinuity region

A six layer GFRP (Glass Fiber Reinforce Plastic) laminate with no discontinuities is being considered, at first. The laminate has a symmetric, general stacking sequence  $([0/45/0]_s)$ , where laminae 1 and 6 both have fibers parallel to the global horizontal dimension of the laminate (i.e. at  $0^\circ$ ). For simplicity each layer is designed to have a thickness of 1 mm.<sup>2</sup>

A medium to high stiffness GFRP laminate material, S2 glass/fiber epoxy (**Table 1**), is selected with the fibers occupying 55% of the composite material volume. Each lamina is transversely isotropic, with direction 1 along the fibers and plane 23 as the isotropy plane. Therefore, information only in the 1 and 2 directions is required for CLT and Tsai-Hill calculations.

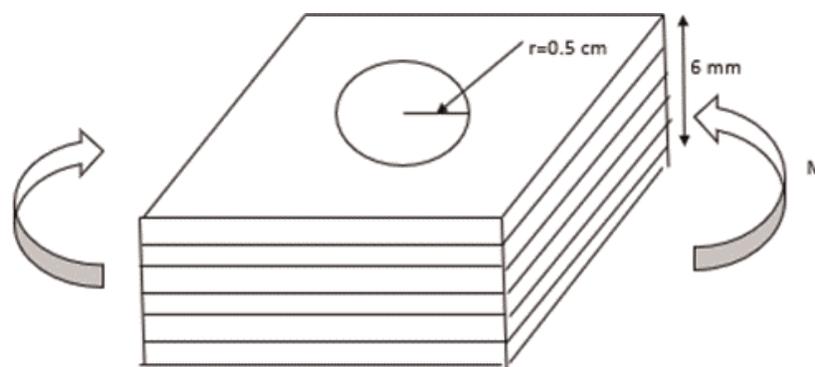
The above laminate will be compared to an identical laminate of the exact same dimensions (6 layers each at 1 mm thickness), same stacking sequence, and a central circular hole of 1 cm diameter. This second laminate will constitute the structure with the discontinuity (**Figure 2**). The loading moment  $M$ , will be determined using the Tsai-Hill failure criterion.

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<sup>2</sup> GFRP laminae tend to be thinner than 1 mm. However, to simplify calculations this exaggerated thickness is chosen here.

$E_1$	34 GPa
$E_2$	8.9 GPa
$G_{12}$	4.5 GPa
$\nu_{12}$	0.27
X (Longitudinal tensile strength)	2000 MPa
X' (Longitudinal compressive strength)	1240 MPa
Y (Transverse tensile strength)	49 MPa
Y' (Transverse compressive strength)	158 MPa
S (Shear strength)	63 MPa

**Table 1.**  
 Mechanical properties of S2 glass fiber/epoxy.



**Figure 2.**  
 Laminate with circular discontinuity of 1 cm diameter loaded by at the three point bending [22].

A plastic zone approach is used to approximate the region surrounding the geometric discontinuity that is affected by a maximum stress concentration. To determine the radius of the plastic zone ( $r_y$ ) a very small crack is assumed to exist on the verge of the hole. This crack could potentially propagate in the matrix of the GFRP and lead to catastrophic failure of the matrix (one of the FRP failure modes mentioned in Section 2.2). The assumption of a very small crack allows the crack length ( $\alpha$ ) to the hole radius ( $r$ ) ratio approach zero. As a result, Eq. (20) is used to estimate the Mode I (opening mode) stress intensity factor ( $K_I$ ). If the stress intensity factor becomes equal to the critical stress intensity factor ( $K_C$ ), the crack will begin to propagate with a radius of the plastic zone given by Eq. (21).

$$K_I = \sigma \sqrt{\pi \alpha} f(\alpha/r) \quad (20)$$

$$r_y = \frac{1}{2\pi} \left( \frac{K_c}{\sigma_y} \right)^2 \quad (21)$$

The radius of the plastic zone, as calculated in Eq. (21), begins at the crack tip. Based on the previous assumption of a very small crack length, this radius will begin on the rim of the hole, and therefore, the distance of the critical region around the discontinuity may be determined. This is the region of high stress concentration, where the reinforcement of the stacking sequence should be modified in order to strengthen the laminate. Equations (20) and (21) clearly show the effect that the size of the discontinuity has on the selection of this region.

Equation (12) in Section 2.1 calculates the stress distribution in the  $k^{\text{th}}$  lamina of a laminate. This stress refers to a lamina of unidirectional fibers and no geometric

discontinuity. Therefore, the stress concentration factor ( $K_t$ ) is used to multiply this stress in order to maximize it and account for the presence and effect of a discontinuity (Eq. (22)).

$$\sigma_{\max} = K_t \sigma_k \quad (22)$$

The magnitude of the stress concentration factor depends on the dimensions of the laminate, discontinuity, and loading condition. In this case study  $K_t = 2.7$ . It should be noted that the size of the hole and laminate dimensions directly affect this value.

### 3.1.2 Optimizing the stacking sequence around the discontinuity

Using CLT on a semi-infinite beam with no discontinuity, as the one described above, and following the analysis with the Tsai-Hill failure criterion, the minimum moment to cause first ply failure is determined. **Table 2** shows the minimum moment to cause failure in each lamina of the laminate. It can be observed from the values shown that the symmetry of the laminate and the three point bending loading conditions give a symmetry of the absolute minimum moments to cause failure in each lamina. The laminae at  $0^\circ$  fiber orientation are the strongest layers of the structure.

As shown in **Figure 2**, the discontinuity is at the center of the laminate and consequently each lamina, which for CLT purposes is modeled as a semi-infinite plate. Therefore, each lamina may be further modeled as symmetric in both the x and y directions of its plane, and a single quadrant of the hole may be considered for analysis. Around the quadrant of the hole a number of points are selected about which the optimum fiber orientations will be determined. The selection of these points is made based on the finite element concept of seeds around geometric discontinuities. A minimum of 16 seeds around a circular hole is recommended, and as a result, a total of 4 seeds is selected around the quadrant considered here. The analysis follows Huang and Haftka's [21] model of fiber orientation prediction, where the orientation of the fibers outside the plastic zone remains the same as the one originally prescribed to the lamina. In this study, the orientation of the fibers outside the plastic zone remains the same as that originally prescribed for the lamina in question (i.e.  $0$  or  $45^\circ$ ). Depending on the accuracy required, the number of seeds around the hole may be increased. Additionally, the optimization process followed should be repeated for all laminae in the laminate, in order to provide a stacking sequence around the hole. In this case study analysis is performed for Lamina 3 at  $0^\circ$  fiber orientation.

Lamina/fiber orientation	Moment (Nm/m) Tsai-Hill = 1
1/ $0^\circ$	269
2/ $45^\circ$	72
3/ $0^\circ$	808
4/ $0^\circ$	808
5/ $45^\circ$	72
6/ $0^\circ$	269

**Table 2.**

*Absolute values of minimum moment to cause failure in individual lamina.*

Possible angle values in the area of the discontinuity	8°
	14°
	40°
	71°
	72°
	81°

**Table 3.**  
 Possible fiber orientations near the hole for lamina 3 at 0° fiber orientation.

Away from the plastic zone the load is carried along the unidirectional orientation of the fibers, which remains unchanged based on the given stacking sequence. However, within the plastic zone around the hole, the fiber orientation will be constantly changing and will be following the four new orientations prescribed by the number of seeds selected. It was previously mentioned that based on symmetry, one quadrant of the plate will be considered. Therefore, the possible orientations will vary between 0 and 90°. The results obtained from the analysis may then be mirrored to the remaining quadrants in order to obtain a complete image of the fiber orientations around the hole.

$$\left(\frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2}\right)\rho^2 - 1 = 0 \quad (23)$$

The Tsai-Hill failure criterion is used in its polynomial form with a positive load factor ( $\rho$ ) calculated as the root of the polynomial at the onset of failure, i.e. when Tsai-Hill is equal to 1 (Eq. (23)). The load factor accounts for the effect of the constantly changing fiber orientations.

A range of possible orientations near the hole for Lamina 3 are given in **Table 3**. All values are in the range from 0 to 90°. To narrow the selection of possible orientations a genetic algorithm may be applied to determine the appropriate orientations based on more specific information of the lamina and its loading. Repeating the analysis for the remaining laminae at 0° fiber orientation (laminae 1,4, and 6), is observed that that similar results are obtained, which are explained by the symmetric nature of the laminate.

#### 4. Conclusion

FRP laminates have entered the industry world as strong and lighter material alternatives to metals, while they offer the option of an excellent material solution to many emerging technologies.

FRP laminates fail due to degradation of their mechanical properties through a range of failure modes. When designing FRPs against failure care should be taken which of the many traditional and newer approaches in predicting first-ply failure is chosen.

The stacking sequence of an FRP laminate is of great significance in determining the stress distribution in the laminate as well as predicting first ply failure. Using CLT and interactive failure criteria an optimum stacking sequence may be determined for specified loading conditions, or the loads to cause first ply failure can be calculated when the stacking sequence of the laminate is known. However, the

above theories and criteria are limited to addressing unidirectional and continuous laminates with no geometric discontinuities.

To perform a similar analysis on FRP laminate structures with holes or other geometric discontinuities, the above methods should be combined with other techniques or models that account for the presence of a discontinuity. In this chapter a case study is used to show such an approach in an attempt to determine the stacking sequence around a circular hole. The limitations of CLT and the interactive failure criteria are overcome with the use of fracture mechanics and more specifically the concepts of stress intensity and stress concentration factors. The approach uses CLT and the Tsai-Hill criterion to predict the loads and lamina of first ply failure, and then fracture mechanics to determine a plastic zone around the discontinuity and maximize the stresses in this region. As a result, a multitude of new possible fiber orientations are calculated, which can be used as the extension of the lamina fiber orientation around the hole to strengthen the lamina in that region and prevent or delay failure, without interrupting the fibers and consequently the load carrying capacity of the FRP.

## Appendices and nomenclature

$E_{ijkl}$	Young's Modulus
$G_{ij}$	shear modulus
$K$	bulk modulus
$f$	volume fraction
$\nu_{ij}$	Poisson's ratio
$\sigma_{ij}$	stress tensor
$\epsilon_{kl}$	strain tensor
$\kappa_{ij}$	curvature
$\epsilon_{ij}^o$	mid-surface strains
$\alpha_{ij}$	coefficient of thermal expansion
$\beta_{ij}$	hygroscopic coefficient
$\overline{Q}$	stiffness matrix.
$A_{ij}$	extension stiffness matrix
$B_{ij}$	extension-bending coupling matrix
$D_{ij}$	bending stiffness matrix
$z$	position of layer in laminate
$X$ and $X'$	longitudinal tensile and compressive strength
$Y$ and $Y'$	transverse tensile and compressive strength
$S$	shear strength
$K_I$	mode I stress intensity factor
$K_C$	critical stress intensity factor
$K_t$	stress concentration factor
$\alpha$	crack length
$r$	hole radius
$r_y$	plastic zone radius
$\sigma_y$	applied yield stress
$\rho$	function of the orientations around the discontinuity

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## References

- [1] Gürdal Z, Haftka RT, Hajela P. Design and Optimization of Laminated Composite Materials. 1st ed. New York, NY: Wiley-Interscience; 1999
- [2] Suresh S. Fatigue of Materials. Cambridge, UK: Cambridge University Press; 1998
- [3] Fragoudakis R, Saigal A. Predicting the fatigue life in steel and glass fibre reinforced plastics using damage models. *Journal of Materials Science and Applications*. 2011;2:596-604
- [4] Fragoudakis R, Saigal A. Using damage models to predict fatigue in steel and glass fibre reinforced plastics. *Journal of Materials Science and Engineering With Advanced Technologies*. 2011;3:53-65
- [5] Fragoudakis R. In: Aly A, editor. Failure Concepts in Fiber Reinforced Plastics, Failure Analysis and Prevention. Croatia: Intech; 2018. ISBN: 978-953-51-5230-9
- [6] Christensen RM. Mechanics of Composite Materials. Mineola, NY: Dover; 2005
- [7] Barbero EJ. Introduction to Composite Materials Design. 2nd ed. Boca Raton, FL: CRC Press; 2010
- [8] Dvorak G. Micromechanics of Composite Materials. New York: Springer; 2013
- [9] Vasiliev V, Morozov EV. Advanced Mechanics of Composite Materials and Structural Elements. 3rd ed. UK: Elsevier; 2013
- [10] Staab GH. Laminar Composites. Woburn, MA: Butterworth-Heinemann; 1999
- [11] Dhakal HN et al. Effect of water absorption on the mechanical properties of hemp fibre reinforced unsaturated polyester composites. *Composites Science and Technology*. 2007;67:1674-1683
- [12] Wang W. Study of moisture absorption in natural fiber plastic composites. *Composites Science and Technology*. 2006;66:379-386
- [13] Jones RM. Mechanics of Composite Materials. 2nd ed. New York, NY: Taylor & Francis, Inc.; 1999
- [14] Fragoudakis R. Predicting the optimum stacking sequence of fiber reinforced plastic laminated beams under bending. In: SAMPE Seattle 2017; 22–24 May 2017; Seattle, WA; 2017
- [15] Hashin Z. Failure for unidirectional fiber composites. *Journal of Applied Mechanics*. 1980;47:329-334
- [16] Ribeiro MC et al. Finite element analysis of low velocity impact on thin composite disks. *International Journal of Composite Materials*. 2013;3:59-70
- [17] Waddoups ME, Eisenmann JR, Kaminski BE. Macroscopic fracture mechanics of advanced composite materials. *Journal of Composite Materials*. 1971;5:446-451
- [18] Kannan VK, Rajadurai A, Nageswara Rao BN. Residual strength of laminated composite after impact. *Journal of Composite Materials*. 2010;45:1031-1043
- [19] Goteti C, Reddy S. Influence of fiber volume fraction, fiber angle and hole size in the stress concentration around the circular hole of an orthotropic lamina under unidirectional in plane loading. *International Journal of Applied Science & Engineering*. 2014;2:1-12
- [20] Sharma D. Stress concentration around circular/elliptical/triangular

cutouts in infinite composite plate. In:  
Proceedings of the World Congress on  
Engineering, III; 2011

[21] Huang J, Haftka RT. Optimization  
of fiber orientations near a hole for  
increased load- carrying capacity of  
composite laminates. *Structural and  
Multidisciplinary Optimization*. 2005;  
**30**:335-341

[22] Fragoudakis R. A numerical  
approach to determine fiber  
orientations around geometric  
discontinuities in designing against  
failure of GFRP laminates. *International  
Journal of Structural Integrity*. 2019;**10**:  
371-379. DOI: 10.1108/IJSI-10-2018-  
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