Chapter

Students with Mathematics Learning Disabilities and Their Ways of Thinking in Fraction Learning

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Abstract

This chapter presents the result of research on ways of thinking of students with mathematics learning disabilities in fraction learning. We conducted a class of fraction learning with Lesh translation model. From the class discussion, interview, and students' work, we then explore the students' ways of thinking when they learn fraction. In the class, students with mathematics learning disabilities perform two mental acts with corresponding ways of thinking and ways of understanding; those are interpreting and problem-solving. We find some interesting findings and they are: (1) students know the common denominator method in the addition of fractions; however, they incorrectly apply the method; (2) students use the common denominator approach (for fraction addition) in the multiplication of fraction; and (3) in the division of fraction, students mistakenly apply the invert multiply algorithm.

Keywords: students with mathematics learning disabilities, fraction learning, ways of thinking

1. Introduction

We may have heard the case of a student having difficulty in mathematics, but the student does not experience obstacles in other subjects in school. After further observation, it turns out that the IQ of the student was at an average level even above average. For cases like this, the student can be suspected of having dyscalculic symptoms or mathematics learning disabilities (MLD). Based on the results of the study, the number of people with MLD according to Strauss is 5–8% of school-age children [1], while according to Adler, the number of people with dyscalculia is 5–6% of all children [2].

Research on dyscalculia is still ongoing. Researchers, especially in the United Kingdom and the United States continue to conduct studies to study dyscalculia in greater depth. Therefore, the understanding and understanding of dyscalculia will continue to develop. The following are some of the dyscalculia definitions issued by both formal institutions and individual researchers. Definition of dyscalculia issued by the National Center for Learning Disabilities is as follows: dyscalculia is a term related to learning difficulties in mathematics. Although learning barriers differ from person to person, the general characteristics are as follows: difficulty in numerating, learning numbers, and doing mathematical calculations; difficulty in measurement, showing time, counting money, and estimating the number; problematic in mathematical intelligence and problem-solving strategies [3].

In general, dyscalculia is an umbrella term used for various difficulties in learning mathematics, such as developmental dyscalculia, mathematical difficulties, difficulty learning numerical concepts, and difficulties about learning number concepts.

There are many studies that discuss MLD students, with a different research focus: first, the research that focuses on the identification or criteria of MLD students; second, the research that focuses on how MLD students think in learning mathematics; and third, the research that focuses on finding solutions to learning mathematics in MLD students. The detailed of the research focus is as follows:

1.1 Research that focuses on the identification or criteria of MLD students

The study of the identification and criteria of MLD students has been carried out by several researchers, including the following: Geary described dyscalculia as a numerical and arithmetic difficulty caused by brain injury; he uses this term to describe a population of 5–8% of school-age children who have a cognitive disorder that affects their ability to learn concepts or procedures in one or more areas of mathematics [4].

Next the opinions of several experts about the criteria of MLD students will be described:

- students with an average IQ whose standardized test scores are below the 20th or 25th percentile [4];
- slower and often make mistakes in processing the representation of numbers, for example, the symbol number “3” and the equivalent of the non-symbol “◆◆◆” [5];
- make mistakes in comparing and estimating numbers [6];
- wrong in doing arithmetic calculations [7]; and
- wrong in solving numbers problems that are very easy, for example, 4 × 5 = 20 [8].

The researchers identified students with MLD using standardized test results, for example, the Woodcock-Johnson Test of Achievement and the Wide Range Achievement Test, by looking at students who were below the 20th or 25th percentile [9]. Lewis further tightens the criteria for identifying MLD students, which combines the following three criteria:

- students score below 25th percentile on standardized mathematics tests;
- the results of observations and interviews revealed that there was no influence of environmental or social factors on students’ inability in mathematics; and
- after being given treatment, the effect of the treatment on increasing mathematical ability is very less. To find this out, Lewis made a comparison with a control class whose members were not MLD students [10].

In identifying students with MLD, Lewis [9] suggests that if researchers use self-developed identification instruments, it is also necessary to include the results of standardized measuring instruments as a comparison. The next suggestion is to apply a cutoff under the 10th percentile; observing longitudinal data showing that learning difficulties in mathematics are long-standing, and researchers must distinguish the difficulty of learning mathematics is the result of cognitive or non-cognitive factors. To do this it is recommended to conduct a demographic analysis of the respondents, for example, socioeconomic status, ethnicity, and mother tongue. This can also be done with qualitative methods, such as interviews, questionnaires, observation of students, parents, and teachers, to find out the factors that lead to the low mathematical achievement of students.

1.2 Research that focuses on MLD students' way of thinking in learning mathematics

The study of how MLD students think in learning mathematics has been carried out by several researchers, including the following:

Lewis states that students with MLD have a different mindset in understanding fractions, she looked at students with MLD does not mean they have deficiencies in understanding the concept of fractions, but there are differences in the way of thinking in understanding fractions [11]. Then Lewis states that students with MLD experience obstacles in learning fractions, especially on the topic of fraction comparison, both fraction comparisons with the same denominator, as well as in fractions comparisons involving fractions of half; in this study Lewis suggested examining students' understanding of the quantity of fractions [12].

Hunt et al. [13] state that MLD students have obstacles in mastering the concept of fractions by learning part-whole models. Newton et al. [14] state that the main error pattern in understanding fractions in MLD students is the use of traditional algorithms that are wrong.

1.3 Research that focuses on finding solutions for MLD students in learning mathematics

The study of alternative mathematical learning solutions for MLD students has been carried out by several researchers, including the following:

Shin and Bryant state that good fraction teaching by MLD students must involve the following 5 aspects: real objects and visual representations such as pictures and number lines, explicit and systematic learning, various time frames and sets of examples, heuristic strategies, and use real problem [15].

Mazzocco et al. state that visual models can be used as alternatives when helping MLD students understand fractions [16]. Gersten et al. [17] state that in assisting MLD students, practitioners are expected to take the following steps: (a) teach students with diverse teaching examples; (b) directing students to say the thoughts and solutions of a problem; (c) teach students to visualize math problems that they face; (d) teach students with diverse/heuristic strategies; (e) the teacher prepares a partner/discussion partner for MLD students; (f) teach MLD students with explicit instructions; (g) the teacher prepares the correct variety and sequence of examples;

Shin and Bryant [15] state that the use of a computer program, Fun Fraction, can help MLD students solve problem-solving in the form of stories. Virtual manipulation in Fun Fraction helps problem-solving skills because students are assisted by this program in representing the problem stories they are dealing with.

Finally, Tian, Jing, and Siegler, state that the use of an optimal number line model can help MLD students understand fraction size and calculation [18].

In this chapter, we focus on students' ways of thinking in fractions learning. It is needed as an essential first step toward effective instructional methods. We use the theory of mental act, ways of thinking, and ways of understanding from

Harel. Furthermore, we also analyze the error pattern of MLD students when they learn fractions. The results of this study are expected to add to the discourse of educational scholarship, especially on the teaching and learning mathematics in an inclusive setting for students with MLD.

2. Fraction learning

Fractional topics include material in mathematics that is difficult to explain. This is because fraction is one of the topics in mathematics that requires high-level and complex thinking. Definition of fractions according to Clarke et al. [19]:

"Fractions are symbolic-shaped expressions that represent the quotient of two numbers $\frac{a}{b}$(where b is not equal to zero). So all rational numbers expressed in terms $\frac{a}{b}$ are fractions, but rational numbers 1.45 are not fractions. Rasonals 1.45 can be called a fraction if written $\frac{145}{100}$. So that all rational numbers can be written as fractions, but there are some important fractions that are not rational numbers, for example: $\frac{a}{b}$ or $\frac{a}{b}$" (p. 15).

In many classes, fractions are taught only in a procedural way. The teacher usually teaches fractions by applying the method of equalizing the denominator, by calculating the Least Common Multiples (LCM). On the other hand, according to Hiebert and Wearne [20], with this procedural method, students will only gain procedural understanding or syntax thinking. Students will not understand the relationship between fractions, in other words, students' conceptual understanding (semantic thinking) will be weak.

How can students gain a conceptual understanding of fractional material? Riccomini suggests two teaching strategies for better fraction learning; the two strategies are learning fractions by using number lines and the use of diverse representations [21]. The use of number lines and paper folding as representations is also suggested by Wyberg et al. [22].

Several other research results also support the use of diverse representations. Dey and Dey suggest the use of geometry representations; addition, subtraction, multiplication, and division operations can be represented geometrically [23]. Furthermore, Clark and Roche suggest the use of games in fraction learning; the game is done like a monopoly game using a kind of broken board, dice, and involves all students in the class [24].

The use of image representation is suggested by de Castro [25]. The same representation, using colored art drawings was suggested by Scaptura et al. [26]. Fractional learning using technology was suggested by Mendiburo and Hasselbring; they also prove that teaching fractions with technology are as effective as teaching fractions that use physical manipulation [27].

Other researchers, Lesh, Posh, and Behr stated that students gain a better understanding when they can identify and model mathematical concepts through various representations [28]. Furthermore, the Principle and Standards for School Mathematics suggest that students represent their mathematical ideas so that mathematical ideas make sense according to students [29]. One learning model that offers the use of diverse representations is the Lesh Translation Model.

3. Lesh translation model

Lesh Translational Model states that basic mathematical ideas can be represented in 5 ways: real (manipulative) objects, images, real-world contexts, verbal symbols, and written symbols. This model is illustrated by the following **Figure 1**:

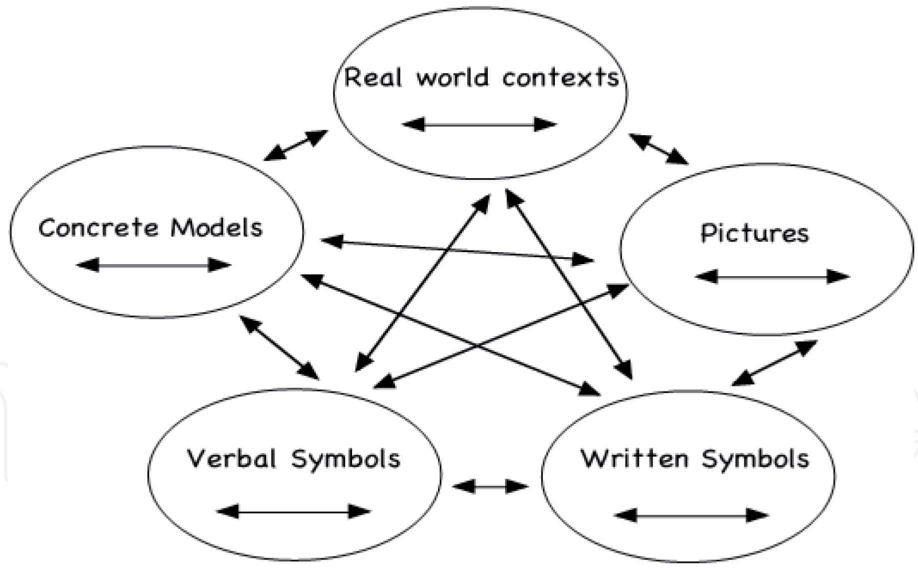

Figure 1.
Lesh translation model [30].

Lesh Translational Model emphasizes interactions within and between representations. The arrows between one representation and another represent the intermodal translation, while the arrows in one mode represent the translation in the mode itself. This model suggests that a good understanding of mathematical ideas requires experience from various modes (ways) and the experience of making connections between and within these modes of representation. A translation requires interpretation of ideas that differ from one mode to another. This activity with its intellectual relations activity reflects dynamic learning.

4. Mental act, ways of thinking, and ways of understanding

According to Harel [31], human reasoning involves many mental actions such as interpreting, guessing, concluding, proving, explaining, compiling, generalizing, applying, predicting, classifying, searching and solving problems. He states that way of understanding is a certain cognitive product of mental actions carried out by an individual. For example, after seeing the symbol $\frac{3}{4}$, one can interpret (one mental action) to produce meaning for the symbol $\frac{3}{4}$. The resulting interpretation is one's Ways of Understanding of the symbol $\frac{3}{4}$. This way of understanding can be different depending on the context, and if judged by an observer, can be considered right or wrong. For example, in a context one can interpret the symbol $\frac{3}{4}$ as "3 objects out of 4 objects," and another person can interpret as "repeated sums: $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$." Others might be able to produce sophisticated Ways of Understanding such as equivalent classes ($\frac{3n}{4n}$ where n is a non-zero integer) and naive Ways of Understanding, such as "two numbers with a bar between them."

Ways of Thinking is a cognitive characteristic of the Mental Act. The cognitive characteristics of the Mental Act are inferred from observations of Ways of Understanding (cognitive products of mental actions). For example, a teacher who follows students' mathematical behavior might conclude that students' interpretations of mathematical symbols are inflexible, there are absolutely no quantitative views, or for example, students' interpretations of symbols are flexible and connected with other concepts. Another example, the teacher can conclude that students' proof of mathematical statements is based on empirical evidence, or based on deductive reasoning [31].

5. Mental act, ways of thinking, and ways of understanding of MLD student

Here are the results of the data analysis from three students with MLD; we found mental acts, ways of understanding, and ways of thinking as follows:

5.1 Problem solving

Here is one example of student work that used mental act problem-solving (**Figure 2**).

In **Figure 2**, the student solves a problem: a tailor receives $\frac{2}{3}$ m of white cloth with floral motifs to make a handkerchief. Each handkerchief requires $\frac{1}{6}$ m of fabric. How many handkerchiefs can be made?. To solve this problem, the student wrote: $\frac{2}{3}:\frac{1}{6} = \frac{2}{3} \times \frac{6}{1} = 4$ handkerchiefs. To answer this word problem, the student performs mental act problem-solving by modeling mathematical word problem into fraction division operation. Then he solves the problem of dividing the fraction using the invert multiple algorithm method [32].

A problem-solving approach is a cognitive characteristic of mental act problem-solving. From the results of the analysis of the answers, it was found that 8 students did the problem-solving approach. In the answers above, it appears that students understand the questions and answer them using a problem-solving approach, in the form of an invert multiple algorithm (IMA) strategy in fraction division operations.

The solution is a cognitive product of mental act problem-solving. From the results of the analysis of answers, obtained student answers are examples of the way of understanding solution.

5.2 Interpreting

The second identifiable mental act of MLD students is interpreting. The example of student work is as follows (**Figure 3**).

In **Figure 3**, the student is asked to describe fractions $\frac{1}{2}$ and $\frac{2}{5}$ in two different ways. Students have been able to interpret $\frac{1}{2}$ with two different interpretation, which is the rectangle and triangle picture. In the rectangle picture which is divided into two parts; one part is shaded and the other part is not shaded. In the triangle picture which is divided into two parts; one part is shaded and the other part is not shaded.

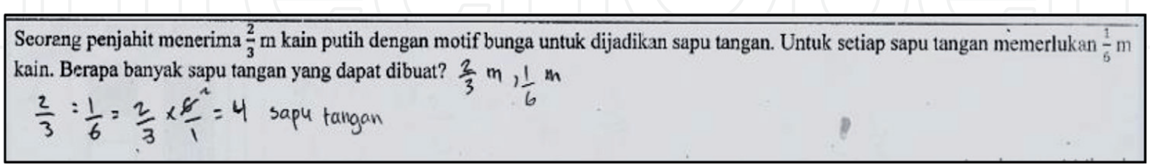

Figure 2.
Mental act problem-solving of MLD student.

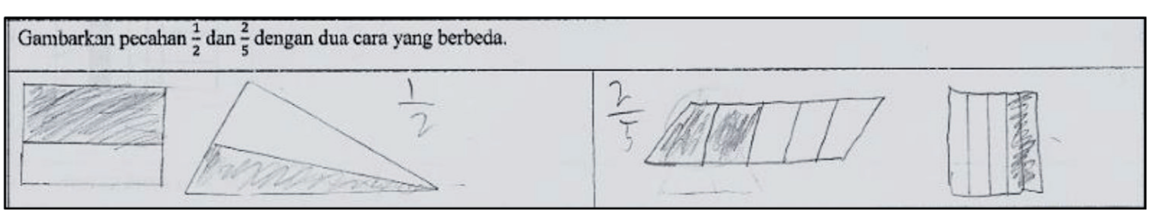

Figure 3.
Mental act interpreting of MLD student.

Mental act	Way of understanding	Way of thinking
Problem-solving	Solution	Problem-solving approach: invert multiply algorithm
Interpreting	Interpretation	Multiple interpretations (as pictures of the square, rectangle, etc.)

Table 1.
The mental act, way of understanding, and way of thinking.

There is something interesting in the triangle picture, students divide the triangle in the centerline, with a horizontal triangle position. Next, students interpret $\frac{2}{5}$ with pictures of parallelograms and squares, each of which is divided into five parts; two parts are shaded and the other is not shaded.

Diverse interpretation of mathematical symbols is a cognitive characteristic of mental act interpreting (way of thinking). From the analysis of MLD student test result data, it was found that he made a fractional interpretation in the form of images, namely rectangular and circular images, as shown above. Interpretation is a cognitive product of mental act interpreting. From the results of the analysis of MLD student answers, it is an embodiment of the way of understanding interpreting, namely interpretation. The students' interpretation of the fractions $\frac{1}{2}$ and $\frac{2}{5}$ is a picture of a rectangle, triangle, parallelogram, and square, as shown above.

We summarize these findings in **Table 1**.

6. The error pattern of MLD students in fractions learning

Some patterns of errors made by MLD students are as follows:

6.1 Students know the common denominator method in the addition of fractions; however, they incorrectly apply the method

The pattern of mistakes of the three students is wrong in applying the denominator equalization procedure. Here is a picture showing this (**Figure 4**).

In the questions, participants are asked to solve two fraction addition questions. In the first problem (part a), students are asked to solve questions $\frac{1}{3} + \frac{1}{3} = ...$ This question aims to reveal students' understanding of the fraction addition operation with the same denominator. For this problem, students give the correct answer: $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$.

In the second problem (part b), students are asked to solve questions $\frac{1}{3} + \frac{1}{2} = ...$ This problem aims to reveal students' understanding of the sum of fractions with different denominators. In this problem, students give answers: $\frac{1}{3} + \frac{1}{2} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$. Learners already know the procedure to do the denominator in the addition operation of

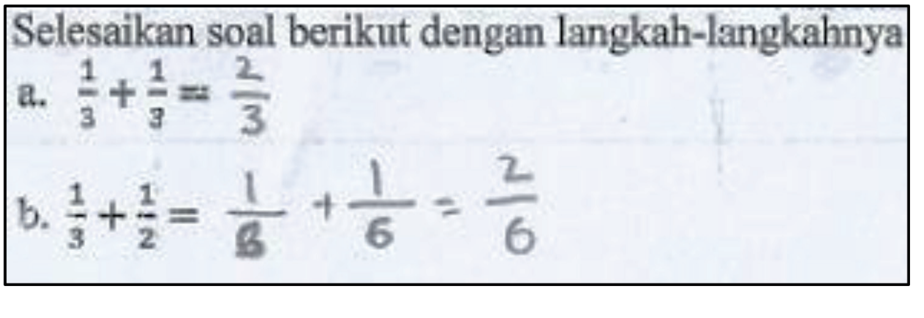

Figure 4.
Example of an error pattern in applying the denominator equalization procedure to the fraction addition operation.

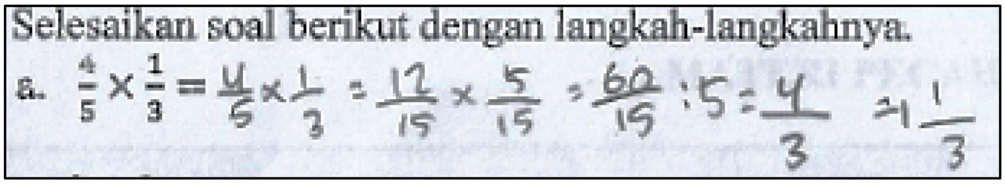

Figure 5.
Example of error pattern applying the denominator equalization procedure to multiplication operations.

fractions. So when he sees the question $\frac{1}{3}+\frac{1}{2}=\ldots$, he performs the denominator equalization procedure by changing 3 to 6 in the first term and changing 2 to 6 in the second term. However, students do not make numerator changes. So, participants already know the denominator equalization procedure, but do not make adjustments to the numerator. In other words, students mistakenly understand the denominator equalization procedure in fraction addition operations.

6.2 Students use the common denominator approach (for fraction addition) in the multiplication of fraction

The second error pattern is very interesting, namely, students apply the denominator equalization procedure in multiplication operations. Here is a picture showing this (**Figure 5**).

In the problem, students are asked to solve questions $\frac{4}{5}\times\frac{1}{3}=\ldots$.This problem aims to reveal students' understanding of fraction multiplication. In this problem, students give answers: $\frac{4}{5}\times\frac{1}{3}=\frac{12}{15}\times\frac{5}{15}=\frac{60}{15}\div 5=\frac{4}{3}=1\frac{1}{3}$. There is an interesting thing, students apply the denominator equalization procedure (supposed to be the sum operation) on the fraction multiplication operation. So when he saw the problem $\frac{4}{5}\times\frac{1}{3}=\ldots$, he did the procedure of equating the denominator in the first syllable by changing 5 to 15 and in the second syllable changing 3 to 15. There were other interesting things done by students. He only did the multiplication, namely: $\frac{12}{15}\times\frac{5}{15}=\frac{60}{15}$. He then divides $\frac{60}{15}$ by 5 to produce $\frac{4}{3}$ fractions. The interesting thing is that students apply the denominator equalization procedure in fraction multiplication operations.

6.3 In the division of fraction, students mistakenly apply the invert multiply algorithm

The third error pattern is very interesting, namely, students turn the first syllable in a fraction division operation. Here is a picture showing this (**Figure 6**):

In the second problem (part b), students are asked to solve questions $\frac{9}{4}\div\frac{3}{5}=\ldots$. This question aims to reveal students' understanding of fraction distribution operations. In this problem, students seem to already know the procedure of division

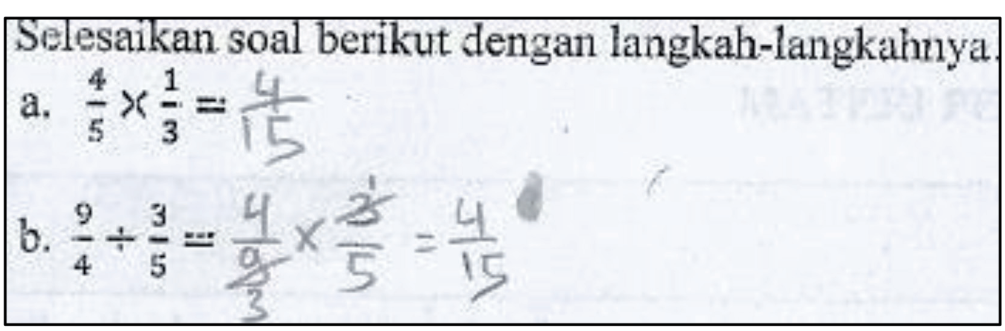

Figure 6.
Example of the first syllable error pattern in a fraction division operation.

operations on fractions. But there is an interesting thing, students use the method of multiplying with the inverse (invert multiply algorithm), but what is reversed is not the second term, but the first term. Consider the following illustration of student answers: $\frac{9}{4} \div \frac{3}{5} = \frac{4}{9} \times \frac{3}{5} = \frac{4}{15}$. So that the answers obtained are reversed, the answer should be $\frac{15}{4}$, students get $\frac{4}{15}$.

7. Discussion

MLD students solve fractions problem procedurally, they apply common denominator approach, drawing a picture, direct multiplied strategy, and invert multiply algorithm in solving fractions problems. They cannot practice the other strategies like using a benchmark or residual which demands the ability to infer and explain. Therefore, we conclude MLD students only perform two mental acts, which are problem-solving and interpreting. They could not develop other mental acts like explaining or inferring.

Some interesting findings when MLD students solve fractions problem are: (1) they know the procedure of common denominator approach in fraction addition operation, however, they mistakenly apply the procedure; (2) in multiplication and divisions operation, they are familiar with the procedure, however, they mistakenly apply the procedure. The two finding is in line with Newton et al. research, they revealed that the main pattern of error in fraction understanding on MLD students is the use of traditional false algorithms [14]. These findings also in accordance with the research of Mazzocco et al., which show that the difficulties in fraction learning are still felt by MLD students until they are in grade 8 [16]. Other researchers also had the same research result, which stated that MLD students make a mistake in performing arithmetic calculations [7].

Another previous research explained that students with MLD have a different ways of thinking in understanding fractions. Lewis considered that the MLD students did not mean to have a lack of understanding of fractions; however, they had different ways of thinking in understanding fractions [11]. We find that MLD students have different ways of thinking in understanding fractions addition operation; they differently understand the common denominator approach, they do not multiply the numerator by the same number with the denominator.

The other research findings deduced that adolescent MLD students are experiencing difficulties in fraction comparison subjects, either fractions comparisons with the same denominator or in fractions comparisons involving a half fraction [12]. Lewis suggested to investigating younger MLD students as the subject. We involved younger students with MLD in our research, a similar result is found, that is MLD students have difficulties in solving fractions comparison problems [33].

In our finding, partitioning activities, which are beneficial for regular students, but not necessarily helpful to MLD students; this may happen because MLD students do not follow a developmental pattern like their regular peers. In accordance with our findings, Lewis explained that partitioning activity was probably the root of understanding the quantity of fractions in regular students; MLD students may not follow this pattern of development [10].

According to Brousseau, the appearance of learning obstacle in mathematics can be caused by three obstacles, namely ontogenic obstacle (mental learning readiness), didactical obstacle (obstacle from teacher instruction or teaching material), and epistemological obstacle (students' knowledge which has limited application context) [34]. In the context of Brousseau theory, the three error patterns of the MLD students in fractions learning is prone to the type of epistemological obstacle, that is MLD students already know fractions concept, however, they have limited application context to the other fractions problems [35].

8. Conclusion

We found only two mental acts with corresponding WoU and WoT, namely problem-solving and interpreting. On the analysis of MLD students, it was found an interesting thing in the mental act problem solving, i.e., the student knew the common denominator approach in the operation of fraction addition, but the practice is still wrong. The same thing is also found in multiplication and division operation. Surprisingly, students use the common denominator approach in the fraction multiplication. In the division of fraction, students mistakenly apply the invert multiply algorithm.

The results of this study can be used by the teachers as a guideline when teaching fractions to students. Future research is recommended to analyze the error patterns of MLD students with other topics in mathematics, such as geometry.

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