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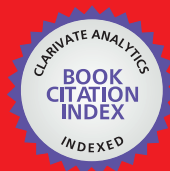
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Discipline, Task and Reader Characteristics of Introductory Physics Students' Graph Comprehension in Mathematics and Kinematics

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Abstract

Students' comprehension of graphs may be affected by the characteristics of the discipline in which the graph is used, the type of the task, as well as the background of the students who are the readers or interpreters of the graph. This research study investigated these aspects of the graph comprehension from 152 first year undergraduate physics students by comparing their responses to the corresponding tasks in the mathematics and physics disciplines. The discipline characteristics were analysed for four task-related constructs, namely coordinates, representations, area and slope. Students' responses to corresponding visual decoding and judgement tasks set in mathematics and kinematics contexts were statistically compared. The effects of the participants' gender, year of school completion and study course were determined as reader characteristics. The results of the empirical study indicated that participants generally transferred their mathematics knowledge on coordinates and representation of straight-line graphs to the physics contexts, but not in the cases of parabolic and hyperbolic functions or area under graphs. Insufficient understanding of the slope concept contributed to weak performances on this construct in both mathematics and physics contexts. Discipline characteristics seem to play a vital role in students' understanding, whilst reader characteristics had insignificant to medium effects on their responses.

Keywords: kinematics, algebra, graphs, interpret, coordinates, slope, straight line, parabolic and hyperbolic functions

1. Introduction

Graphic representation, a method used to show and represent values, increases, decreases, comparisons to either make predictions or show a report of how a certain situation was yesterday and how it is today, is an integral part of all scientific subjects. Scientific graphs visually communicate data and information about variables and their relationships and are often used in the analysis of data to determine patterns and relationships [11, 21]. Be that as it may, the specific purpose and usage

of graphs may differ tremendously, even in subjects as closely related as mathematics and physics [19]. Graph comprehension is thus subject specific, that is, it depends on the discipline characteristics of different subjects [6].

According to Redish and Gupta [19], it is important that physical meaning of mathematical symbols is attached when applying mathematical knowledge in physics. Meredith and Marrongelle [14] further explain this by stating that we interpret mathematical concepts in the context of physics; hence according to Redish [18], the blending of the mathematics symbols, structures and rules with physics concepts, principles and laws is significant to students. This is because the blending will help students to solve kinematics/physics equations and interpret graphs. Woolnough [25] discovered that students tend to interpret slope as a mathematical quantity and that it cannot be associated with units as in physics graphs.

The researchers are therefore investigating in the empirical study why participants' performances on similar tasks in mathematics and physics graphs yielded different responses. There are few investigations on students' application of mathematics knowledge in physics [12, 25], whilst most studies focussed on problem-solving (e.g., [5, 18]) and specific aspect interpretation like slope of graphs [17, 25]. The researchers also found out that less study has been conducted in the four qualitative and quantitative constructs' tasks on the effect of discipline, task and reader characteristics in the mathematics and physics contexts and hence this study.

2. Theoretical background

2.1 Graph comprehension

According to Okan et al. [15], graph literacy is a necessary skill for decision-making, and it has often been neglected. Szyjka [24] citing Fry [7] defined "graphs as two-dimensional representations of points, lines and spaces, where data are displayed through represented words and numbers." A student can show comprehension of graph by being able to read and interpret it, that is, derive its meaning [6, 8]. According to Dori and Sasson [3] and Friel et al. [6], by working with graphs, students acquire graph sense and graphical thinking skills, and they are also able to comprehend the nature of graphs presented to them and are able to give variables and their relationships meaning [11]. Students acquire graph sense by working with graphs, and they gain graphical thinking skills and are also able to comprehend the nature of graphs as well as give variables and their relationships a meaning [11].

Scott [20] reported variation in students' performance in questions set on different levels in a questionnaire with corresponding mathematics and chemistry questions. He conducted a study on the participants' use of mathematics on the mole concept. No significant difference occurred in the participants' responses to the easier questions; however, the more difficult questions yielded a significant difference with better performances in the mathematics questions than the chemistry ones. He argued that algorithmic approaches in mathematics contribute to students' difficulties with calculations in chemistry.

Stahley [22] reported that even though students may have a correct idea or procedure to comprehend and illustrate discipline, task and reader characteristics of a graph [6], their confidence in taking such a decision is lacking. Some of them may understand the concept but lack the principles, and they seem unable to demonstrate the procedure. In physics graphs, physical contexts embed both algebraic and graphical representations [9].

2.1.1 Students' difficulties with kinematics graphs

McDermott et al. [13] investigated difficulties students experience with graphs as used in kinematics, and in their findings, state that the students seem to lack the ability to abstract information from the graphs. This cannot just be due to inadequate mathematics preparation, because often, students that are able to construct and interpret graphs in mathematics cannot do the same for graphs in physics. The difficulties they experience are rather because of an inability to make connections between graphical representations and physical ideas. The difficulties found by McDermott et al. [13] were divided into two groups: connecting graphs to physics concepts and connecting graphs to the real world.

Concerning the difficulties students experience in connecting graphs to physical concepts, McDermott et al. [13] found that students often do not know whether to use the value of the graph or the gradient of the graph to subtract the information from. This is referred to as the (function) value/gradient confusion. Students are also confused between changes in the value of the graph and changes in the gradient. Changes in value are easier to see than changes in gradient. As mentioned earlier, students see a constant graph as a graph with a constant gradient (linear graph). When constructing one graph from another, students find it difficult to ignore the form of the original graph. Many students do not have the ability to differentiate between displacement-time, velocity-time, and acceleration-time graphs. This can be due to the confusion between graph value and gradient and/or the inability to connect the physical concepts to the different features of the graph.

It was also found that students are not able to match the narrative information of the problem with the relevant features of the graph (McDermott, et-al. [13]). In the example used by McDermott et al. [13], the students had to determine the acceleration from a velocity-time graph over certain intervals. Many of them only used the coordinates of one of the endpoints of the line sector (y/x) instead of the change over the interval ($\Delta y/\Delta x$), despite the fact that they referred to the acceleration as change in velocity divided by the change in time. They were also asked to determine the acceleration of a part of the movement that was not included on the graph. Most of those who determined the acceleration on the given interval wrongly calculated acceleration for the part not given on the graph. This shows that they did not match the narrative description of the problem with the graph correctly.

In physics, students have to determine the area under graphs before they have done integration in mathematics. Although they have calculated areas of many two-dimensional figures, the idea that the area under a graph can be used to determine a physical quantity is very new and strange to them (McDermott et al. [13]). The fact that, for example, the area under a velocity-time is displacement is memorised and used. They do not realise that the area under the graph represents the functional relation $f(x)\Delta x$ and that, for example, the area under a velocity-time graph is $\Delta s = v\Delta t$. They further do not associate a positive area with displacement in the positive direction and a negative area with displacement in the negative direction. When asked to determine the position at a certain instant from a velocity-time graph, students found it hard to understand that they have to determine the displacement over an interval.

Problems which can be solved by simple recall can be done with ease by most students (McDermott et al. [13]). Students find it hard to solve problems where the detailed interpretation of a graph is needed. To be able to use graphical interpretation to solve problems requires more than just memorization, for example, the gradient of the velocity-time graph is the acceleration and that a constant gradient on a velocity-time graph means constant acceleration.

To determine to what extent students connect kinematics graphs with the real world in the study of McDermott et al. [13], balls were released to roll down different inclines, and the students had to register the instant a ball passes a certain point. From that information, displacement-time, velocity-time and acceleration-time graphs had to be drawn. When constructing the displacement-time graph, many students indicated the displacement per time interval instead of the displacement at a certain instant, drawing discontinuous graphs. Others indicated the displacement at certain instances correctly but did not connect the dots to indicate continuous motion. The students also struggled to separate the actual path of the ball from the form of the graph. In one of the movements, the ball rolls up an incline and down again. Many students did not represent the velocity as negative, indicating the ball was rolling in the opposite direction. When drawing the acceleration-time graphs, most students did not realise that a positive (negative) acceleration does not necessarily mean speeding up (down) the ball. They did not realise that when the ball was rolling up and down the incline, the acceleration was in the same direction. Many students also drew the displacement-time, the velocity-time and the acceleration-time graphs with similar shapes. They found it hard to accept that the same motion can be represented by graphs with different shapes.

According to a study done by Beichner [1] in which he used the Test of Understanding Graphs in Kinematics, similar difficulties and misconceptions were found. It was found that students struggled to determine gradient in the correct way especially if the graph did not run through the origin. Students considered the graph as a picture of the path followed by the object and not as an abstract mathematical representation of the movement. When answering the questions, the students did not distinguish between the variables' displacement, velocity and acceleration. As indicated above, they believe that the displacement-time, the velocity-time and the acceleration-time graphs have to look similar. Beichner [1] also found that the students did not recognise the meaning of the area under the different graphs. In the answering of many of the questions, the confusion between the graph value, the gradient and the area under the graph was clear.

Some of these misconceptions are caused by the fact that students do not connect what they learn in physics with their everyday experiences Brungardt and Zollman [2]. The difficulty students have with negative velocity can, in part, be because a speedometer only indicates positive speed. Students may associate the word "negative" with decreasing or lesser quantity. This then means that vocabulary also causes problems for the students. They use the word "constant" to refer to a linear graph with a constant gradient, whilst the words "up" and "down" are sometimes used to indicate an increase or decrease of magnitude or to indicate direction.

3. Aim and research questions

3.1 Research aim

The aim of the research is to investigate how discipline, task and reader characteristics influence physics students' graph comprehension in the corresponding mathematics and kinematics questions. The participants were 152 willing first year physics students enrolled at the Central University of Technology, Free State (CUT) in South Africa.

3.2 Research questions

The following research questions were addressed in the empirical study:

- What characteristics of graphic tasks (reading coordinates, connecting representations and interpreting the area under and the slope of a graph) hamper the participants' performances in mathematics and kinematics?
- What is the role of discipline and reader characteristics on the participants' comprehension of kinematics graphs?

4. Research design and methodology

In order to address the research questions, a questionnaire was designed, consisting of two sections, one section focusing on kinematics graphs and the other one focusing on corresponding graphs in mathematics. The kinematics questions were designed using Beichner's Test of Understanding Graphs in Kinematics (TUG-K) model (1994). The questions were based on the reading of coordinates, connection of representations, understanding and calculating the area under a graph and the gradient of a graph. Mathematics section was comprised of linear functions and graphs, the required skills and knowledge to solve kinematics graphs and equations. Validation of the content of questionnaires was done by two academics in the same research field. The questionnaires were further piloted using 30 first year physics students enrolled at Central University of Technology, Free State (CUT). Thereafter, changes necessary in the questionnaires were then effected. The final questionnaire showed a reliability with the Cronbach's alpha coefficients of 0.69 in the kinematics section and 0.75 in the section on mathematics.

The pairs of kinematics and mathematics questions are attached as Appendices. The corresponding mathematics and kinematics questions were not identical in order to prevent similarities in students' answers based on recognition of graphs in questions in the two sections. Discipline characteristics further necessitated differences. For example, in kinematics graphs, the independent variable, time, can only have positive values, whilst positive and negative x-values can be used in mathematics graphs. Still, care was taken that the corresponding mathematics and physics tasks in the questionnaire require the same judgement and similar visual decoding (as shown in **Table 1**).

The results of the questionnaire were statistically analysed using effect sizes, because no random sampling (only available sampling) was done. Effect sizes yield important results in any empirical study and can be used to give the practical significance of such results [10]. In this study, comparison between differences in proportions for mathematics and physics successes were interpreted according to Cohen's effect sizes

$$w = \sqrt{\frac{\chi^2}{n}} \quad (1)$$

where n is the total number of participants and the χ^2 -value with one degree of freedom is retained from the McNemar test [23]. This effect size determines whether there is a practically significant difference between the proportion of students who succeeded in answering the mathematics correctly and the proportion of students who succeeded in answering the physics correctly. The w -values are interpreted as follows:

- $w < 0.3$ is a small effect.
- $0.3 \leq w \leq 0.5$ is a medium effect.
- $w > 0.5$ is a large effect.

A w -value of > 0.5 indicated a practically significant difference between the two aspects considered. For this study, a small effect size indicates that the mathematics and physics questions were answered similarly, either both correct or both incorrect. A large effect size means that the mathematics and physics questions were answered differently, either the mathematics correctly and the physics incorrectly or vice versa.

| Tasks | Task characteristic | Mathematics knowledge | Physics knowledge |
|-------------------------------|---------------------|--|--|
| 1. Reading coordinates | Context | Coordinates in a Cartesian plane | Symbolic meaning of coordinates |
| | Visual decoding | Connecting a function value to the specified x (or t) value | |
| 2. Connecting representations | Context | Algebraic and graphical representations of functions | Algebraic and graphical representations of motion. Differentiate between constants and variables |
| | Visual decoding | Recognise the form and expression of graphs | |
| | Judgement | Differentiate between features such as positive/negative gradient and symmetry around x/y axis | |
| 3a. Area qualitative | Context | Comparison of areas of different geometric forms with the same base and height. | $s = \int v dt$ =area under v - t graph for interval dt , $v = \int a dt$ =area under a - t graph for interval dt . |
| | Visual decoding | Identify the geometric forms of the areas under the graphs | |
| | Judgement | Compare the geometric areas with the same basis and height. | |
| 3b. Area quantitative | Context | Formulas for calculation of areas of different geometric forms. | $s = \int v dt$ = area under v - t graph for interval dt $v = \int a dt$ =area under a - t graph for interval dt |
| | Visual decoding | Recognize the forms of the graphs and read from the graph features of the forms (e.g. length, width, height) | |
| | Judgement | Compare the areas of the figures | |
| 4a.Slope qualitative | Context | Meaning of the gradient of a graph | $v = ds/dt$ = gradient of s - t graph at time t |
| | Visual decoding | The steepness of the given line segments differ | |
| | Judgement | Compare the steepness of the line segments and connect it with the slopes. | |
| 4b. Slope quantitative | Context | Formulas for calculation of gradient of straight-lines. | $v = ds/dt$ =gradient of s - t graph at time t |
| | Visual decoding | Reading appropriate coordinates from the graph. | |
| | Judgement | Is the value of the gradient reasonable? | |

Table 1.
Task characteristics of questions.

Effect sizes of the reader characteristics on students' performances in the mathematics and physics sections of the questionnaire were statistically determined using Cohen's effect sizes [4]. The characteristics evaluated were the participants' gender, their study courses and whether they completed school the previous year or two or more years prior to the study. A gap between school and university physics may prevent knowledge retention and consequently lower performances. The statistical results are interpreted as follows for differences in average percentages in the mathematics and physics sections:

Effect size of 0.2 shows a small effect.

Effect size of 0.5 is medium but observable effect.

Effect size of 0.8 is large, that is, the difference is of practical significance.

5. Analysis of task characteristics of questions in the two disciplines

Before the empirical results are discussed, the characteristics of the tasks set in the mathematics and kinematics contexts were analysed on the level of the participants. This implies that this analysis may differ for more or less advanced participants. For example, more experienced participants may distinguish characteristic features of graphs by visual decoding only and consequently may not need to explicitly perform judgement.

As indicated in **Table 1**, each task (e.g., reading coordinates, etc.) requires different mathematical and kinematics contextual knowledge, although similar visual decoding and judgement are to be performed in both the contexts. The first task, reading coordinates, is the simplest and requires only contextual knowledge and visual decoding. The other graph tasks require contextual knowledge, visual decoding and judgement.

It is important to note that the kinematics tasks can only be done if the mathematics contextual knowledge is transferred and integrated with kinematics knowledge. In the first task (reading coordinates), participants should have contextual mathematics knowledge of Cartesian coordinates and integrate it with kinematics knowledge about the variables of position (s), velocity (v), acceleration (a) and time (t). Conventionally, the independent variable t is placed on the x-axis and the dependent (s , v or a) on the y-axis. In the questionnaire items, participants needed to connect the proper dependent variable (function value) to a given independent variable, using visual decoding.

The second task (called connecting representations) requires mathematical knowledge of the graphical representation and formula of straight-line, parabolic and hyperbolic functions. In the kinematics questions, participants needed to recognise the mathematical formats and graph forms of the given expressions containing kinematics variables, instead of mathematical symbols. Proper understanding further requires insight that the given kinematics equations and graphs represent functions of time. Without having and integrating this contextual mathematics and kinematics knowledge, the participants will not know which visual decoding and judgement tasks to perform.

In order to accomplish "area quantitative" and "area qualitative" tasks (tasks 3a and 3b in **Table 1**) on kinematics, participants must recall the kinematics relation $s = \int v dt$. Then they should know from mathematics that the integral is determined from the area under a line graph. Blending these kinematics and mathematics knowledge elements should result in understanding that displacement in interval dt is $s = \int v dt = \text{area under } v\text{-}t \text{ graph}$. Only then can the participants perform the expected visual decoding and judgement tasks.

Requirements for successful execution of the qualitative and quantitative tasks on slopes (tasks 4a and 4b) are similar to those for area. From mathematics, participants should know the meaning and formula for calculating the slope of a graph and be able to attach the kinematics meaning to it, that is, $v = ds/dt =$ gradient of s-t graph at time t. Thereafter, the visual decoding and judgement required by the different questions can be performed.

For all tasks, the discipline characteristics of the question thus determine what visual decoding and judgement tasks have to be done. Inability to perform the correct contextual tasks is expected to prohibit execution of correct visual decoding and judgement.

6. Results

6.1 Results: reader characteristics

The number of students and the average percentages obtained by each group are given in **Table 2** for gender, **Table 3** for the last school year and **Table 4** for the faculty in which they are enrolled.

The effect sizes for differences between groups are medium (≥ 0.5) for gender, small for last school year and insignificant for faculty. In all cases, the effect size values were larger for mathematics than physics.

6.2 Results: task characteristics

Table 5 summarises the average percentages correctly obtained by the participants as well as the results of the McNemar test for each question pair (refer to Appendix). The questions are categorised in constructs according to the tasks to be

| Gender | % of students | Mathematics Average % | Physics average percentage |
|--------------|---------------|--------------------------|-------------------------------|
| Male | 53.4 | 67.4% | 38.3 % |
| Female | 46.6 | 58.1 % | 31.7 % |
| Effect sizes | | 0.58 | 0.52 |

Table 2.
Gender performances.

| Last school year | % of students | Mathematics Average % | Physics average percentage |
|------------------|---------------|--------------------------|-------------------------------|
| Previous | 51.3 | 66.3 | 36.4 |
| Before | 48.7 | 59.3 | 33.4 |
| Effect sizes | | 0.43 | 0.23 |

Table 3.
Performance by last schooling attended.

| Faculty of study | % of students | Mathematics Average % | Physics average percentage |
|--|---------------|--------------------------|-------------------------------|
| Humanities | 23.1 | 60.7 | 35.2 |
| Engineering and Information Technology | 23.8 | 64.2 | 34.5 |
| Health and Environmental Sciences | 53.1 | 63.3 | 35.1 |
| Effect sizes between pairs of faculties | | ≤ 0.24 | ≤ 0.07 |

Table 4.
Performances by faculties.

| Task | Paired questions | Mathematics correct (%) | Physics correct (%) | w-value |
|------------------------------|------------------|----------------------------|------------------------|---------|
| 1. Coordinates | M_C1 x P_C1 | 93.4 | 92.1 | 0.04 |
| | M_C1 x P_C2 | 93.4 | 84.2 | 0.22 |
| 2. Representations | M_R1 x P_R1 | 67.1 | 65.1 | 0.03 |
| | M_R2 x P_R2 | 72.4 | 29.0 | 0.58** |
| | M_R3 x P_R3 | 69.8 | 38.2 | 0.44* |
| 3a. Area qualitative | M_A1 x P_A1 | 72.4 | 41.4 | 0.42* |
| | M_A2 x P_A2 | 69.1 | 27.0 | 0.63** |
| 3b. Area quantitative | M_A3 x P_A3 | 56.9 | 29.6 | 0.43* |
| | M_A4 x P_A4 | 63.8 | 12.5 | 0.67** |
| 4a. Gradient qualitative | M_S1 x P_S1 | 46.7 | 34.2 | 0.18 |
| 4b. Gradient quantitative | M_S2 x P_S2 | 82.9 | 64.48 | 0.30* |
| | M_S3 x P_S3 | 16.5 | 7.90 | 0.22 |

Table 5.
Results of the effect sizes for paired mathematics and physics questions.

performed, that is, reading coordinates, connecting representations, area (qualitative and quantitative) and slope (qualitative and quantitative). In **Table 5**, the label “M” is used for the mathematics questions, whilst “P” indicates physics (kinematics) questions. The percentages of participants who had the specific question correct are given in **Table 5** as well as the *w*-values calculated from the McNemar test, indicating the effect size of differences in responses. Medium effect sizes ($0.3 \leq w \leq 0.5$) are marked with a single star (*) and large effect sizes ($w > 0.5$) with a double star (**). Large effect sizes imply that the pair of questions were answered significantly different, that is, either the mathematics question correct and the physics incorrect or vice versa. The *w*-values that are not marked show a small effect size ($w < 0.3$), that is, the pair of questions were answered similarly, that is, either both correct or both incorrect.

Four additional physics questions aided in the interpretation of the results of **Table 2**. These questions are incorporated in the Appendix, and participants’ performances are given in **Table 6**.

| Task | Question | Percentage correct |
|-------------------|----------|--------------------|
| Area qualitative | P_A5 | 53.5 |
| Area quantitative | P_A6 | 51.7 |
| Gradient | P_S4 | 48.7 |
| | P_S5 | 59.2 |

Table 6.
Additional physics questions.

For all four tasks, the results (**Table 5**) show that participants performed better in the mathematics questions than the corresponding physics questions. Comparison of the average percentages and w-values between the tasks shows differences in how participants performed. Their responses thus seem to depend on the characteristics of the tasks, as discussed below:

6.2.1 Coordinates (task 1)

In task 1, the reading of coordinate values from the given graphs in the mathematics and physics contexts was assessed (questions M_C1, P_C1 and P_C2). The participants performed well in this task (>80% correct), and the low w-values (0.04 and 0.22) indicate consistency in responses, that is, the majority of participants answered correctly in both pairs of questions. It therefore, seems that the participants effectively transferred their mathematics knowledge about coordinates in a Cartesian plane to the kinematics domain. The lowest average performance (84.2%) obtained in the second kinematics question (P-C2) is probably due to the need to estimate the position (y) value by using the scale, which seems to be more difficult than reading values from intersections of grid lines as is the case in the other questions.

6.2.2 Representations (task 2)

In both sets of mathematics and physics questions on the representation task, five graphs of different forms were given (see Appendix). In the three pairs of questions, the participants had to match a straight-line, hyperbolic and quadratic function to one of the given mathematics graphs and linear motion equations to kinematic graphs.

The vast majority of participants knew that the mathematics function in item M_R1 is a straight-line graph and chose either the correct one, option 1 (67.6%), or the additional straight-line, option 2 (21.2%). With regard to the hyperbolic and parabolic functions $g(x)$ in item M_R2 and $h(x)$ in item M_R3, respectively, more than 70% of participants related each to the correct graphs. In both latter cases, the second largest contingent of participants (about 20%) connected the hyperbolic function to the parabolic graph or vice versa. These participants seem to confuse the representations of hyperbola and parabola in the mathematics contexts.

With regard to the physics items on this task, the largest correct percentage (65.1%) was also obtained for the straight-line representation (P_R1). The small w-value of 0.03 indicates transfer of these participants' mathematics knowledge to kinematics. For the hyperbolic and parabolic equations only, small percentages of participants succeeded (about 38 and 29%, respectively). The large w-values (0.58 and 0.44) imply medium to practically significant differences in responses to the

mathematics and physics questions, indicating that participants who managed the mathematics tasks could not do the kinematics tasks.

6.2.3 Area qualitative and quantitative (tasks 3a and 3b)

The average percentages in the four mathematics questions on comparison or calculations of the area under graphs (M_A1, M_A2, M_A3 and M_A4) ranged from 56.9 to 72.4%. Higher percentages were obtained in the qualitative than the quantitative questions in the corresponding physics questions on area under kinematics graphs (P_A1, P_A2, P_A3 and P_A4), where the participants obtained low percentages ($\leq 40\%$), indicating that they did not apply their existing mathematics knowledge.

The w -values for the corresponding pairs of questions on area were all medium to large, confirming inconsistencies in the students' responses. Students who were successful in the mathematics contexts generally failed to transfer their mathematics knowledge to the kinematics context. Practically significant differences in answers were obtained when comparing qualitative questions M_A2 (largest area under graph) and P_A2 (largest displacement from v - t graph), as well as quantitative questions M_A4 (calculation of area under section of x - y graph) and P_A4 (calculation of change of velocity from an acceleration-time graph).

Possible reasons for the poor performances in the physics questions on the area were investigated by additional qualitative item P_A5 and quantitative item P_A6. In P_A5, the participants were asked whether displacement can be obtained from the area or slope of velocity-time or acceleration-time graphs. Only half of the students (53.5%) knew that the option "area under a velocity-time graph" is the way to determine displacement. Approximately a quarter of the participants chose the incorrect option "gradient of a velocity-time graph," showing area-slope confusion. The slope-area confusion was confirmed in the additional question P_A6 that assessed the participants' understanding of what task should be performed and how it should be performed to determine the displacement in a straight-line velocity-time graph over an interval starting at the origin. Only 51.7% had P_A6 correct, and a large number of students ($\sim 30\%$) indicated that they would calculate the slope making the same slope-area mistake as in P_A5. Both these additional questions indicate that a lack of physics conceptual knowledge contributed to participants' failure in the kinematics questions on area.

6.2.4 Slope qualitative and quantitative (tasks 4a and 4b)

Mathematics item M_S1 and physics item P_S1 required students' judgement of intervals where the slope and the instantaneous velocity (on a position-time graph), respectively, are the highest. In both questions, $< 50\%$ of the students chose the correct answer. According to the small w -value (0.18), the majority of students were unsure in both the mathematics and physics questions. It seems as if a lack of mathematics knowledge and understanding of the concept of slope is transferred from mathematics to physics. This deduction was confirmed in the additional physics questions P_S4 and P_S5, in which the participants had to identify the intervals on a velocity-time graph, where the gradient and acceleration, respectively, are negative. The w -value for these two questions is 0.17, indicating that the participants who did not know where the slope is negative, did not also know where the acceleration of the v - t graph is negative. In both questions, the option chosen by the second-most participants was DE, the interval with both negative function values and negative slope. This shows that students struggle to discriminate

between function values (velocity) and slope (acceleration), which corresponds to the height-slope confusion reported by McDermott et al. [13] and Beichner [1].

In both the mathematics and physics quantitative contexts, the students performed much better in calculating the positive slopes starting at the origin (M_S2 and P_S2) than the zero slopes in later intervals (M_S3 and P_S3). According to the w -values, these pairs of questions were answered differently with small to medium effect, that is, similar mistakes were made. A reason for the very weak performances (16.5 and 7.9% correct) in items with zero slopes may be that the students do not understand that slope is the ratio of the *change* in y -values to the *change* in x -values. This is evident from the result that the majority of students (66.4% in M_S3 and 57.7% in P_S3) chose option 3 in these items where y/x instead of $\Delta y/\Delta x$ is used for the slope. In the first pair of quantitative items (i.e., M_S2 and P_S2), $y/x = \Delta y/\Delta x$ is valid, and the majority of participants (82.9 and 64.5%) consequently chose the correct option, even though they might have made the same error. Furthermore, area-slope confusion and slope/height confusion occurred amongst some of the participants. It thus seems that deficiencies in understanding the concept of gradient in mathematics has been transferred to the physics graphs.

7. Discussion of results

7.1 Reader characteristics

Of the three reader characteristics evaluated (gender, last school year and faculty), none showed a practically significant difference in how the groups of students performed in the mathematics or the physics sections of the questionnaire. With regard to gender, male students outperformed female students in both the mathematics and physics sections with medium effect. Although the effects of the last school year were smaller, a larger effect was obtained for mathematics than physics. This result implies that students who had a gap of one or more year since their previous studies of mathematics performed observably weaker than those who did mathematics at school the previous year, although both groups performed badly in physics. An interesting result is the indifference of the faculty the students were enrolled in; engineering students performed similar to students from the humanities as well as from health and environmental sciences faculty.

7.2 Task characteristics

The characteristics, namely context, visual decoding and judgement, of the tasks in the questionnaire are analysed in **Table 1**, and the results of the empirical investigation thereof are given in **Tables 2** and **5**. The main trends that were revealed are now discussed.

In the mathematics questions, the majority of participants were successful on reading coordinates (>90% correct), connecting representations (~70% correct) and on qualitative and quantitative area tasks (~65% correct). These participants showed conceptual understanding and effectively performed visual decoding and judgement tasks in the mathematics contexts. However, the majority of participants struggled with the tasks on slope, seemingly due to lack of conceptual understanding of the mathematical concept and calculation of slope.

In the physics domain, the majority of participants transferred and integrated their correct mathematics knowledge and skills on the reading coordinate task (>80% correct) as well as the representation of straight-line graphs (65% correct). In all other tasks, the average percentage was 50% or below, that is, the majority of

the participants could not perform the tasks successfully. It is therefore deduced that characteristics of tasks had an influence on the students' graph comprehension.

With regard to the task on reading coordinates, the participants successfully performed the required visual decoding skill in both contexts. In the physics context, they attached conceptual meanings (position and time) to the x and y coordinates on the Cartesian plane. This elementary task underlies all other kinematics graph tasks. A problem that a minority of participants experienced was to estimate a value using a scale.

In the mathematics questions on representation tasks, most students successfully performed the visual decoding task of identifying and connecting the form and the equation of the three types of graphs. However, some experienced problems to correctly judge which one of the two given straight-line graphs resembles the hyperbolic function $f(x)$ and which of the two parabolas are represented by the quadratic equation $h(x)$. Hyperbolic-parabolic confusion that occurred amongst a minority of students also reveals judgement errors.

The participants' mathematics knowledge and understanding of matching expressions to types of graphs were only transferred to the physics domain in the case of straight-line graphs. With regard to the physics questions on parabolic and hyperbolic graphs, the majority of participants probably did not recognise correspondences in the kinematics expressions or graphs with the standard mathematical formats. This visual decoding problem may be based on the contextual task error, namely lack of understanding that the given kinematic equations are indeed functions, that is, $s(t)$ and $v(t)$. Consequently, their responses in the physics questions differed with medium to practical significance from those in the mathematics domain.

The results on the area tasks indicate that the majority of participants have mathematical contextual knowledge related to areas of geometric forms and can execute the tasks of visual decoding (know what part on the graph is the area under the graph) and judgement (comparing the areas). In the corresponding physics questions, the participants firstly had to take the kinematics context of the questions into account before deciding what visual decoding and judgement tasks had to be done. The poor performance of the participants in the physics tasks indicated that they encountered problems in accomplishment of the contextual tasks. They seemed to lack knowledge and conceptual understanding of kinematics quantities and graphs, namely how to obtain the change in velocity from an acceleration-time and the change in position from a velocity-time graph. This knowledge deficiency was confirmed in the additional items on the area. Contextual difficulties in interpretation of the area under kinematics graphs were also found by Beichner [1], McDermott et al. [13] and Palmquist [16].

Although participants' responses to questions on calculations of the slope of a straight line starting at the origin were correct, the other questions revealed deficiencies in the basic conceptual understanding of slopes in mathematics, namely that slope is the ratio of the *change* in y-values to the *change* in x-values. This hindered success in both contexts (with practical significance) in the tasks on the qualitative comparison of magnitudes of gradients as well as the understanding and application of negative and zero gradients. In these tasks, function value/slope confusion occurred, which was also reported by Beichner [1] and McDermott et al. [13]. This can be a contextual task error, but since the same confusion was encountered in the corresponding mathematics and physics questions, it is here also considered as a judgement error.

Comparison of the performances in the corresponding mathematics and physics tasks shows the following main trends causing success or failure in the physics questions:

1. Participants have the correct mathematics knowledge and conceptual understanding and transfer it to the physics domain, for example, when reading coordinates.
2. Participants reveal the mathematics knowledge but lack the necessary physics knowledge and conceptual understanding, for example, in the kinematics tasks on area and slope, they seem not to know which kinematics relation to use and what to calculate.
3. Participants are unable to blend mathematics and physics knowledge, for example, they do not perceive the kinematic equations as quadratic and represented by a parabola or as a hyperbolic expression and graph.
4. Participants transferred their misconceptions or insufficient knowledge in mathematics to the physics domain. This is evident in the height-slope confusion, area-slope confusion and parabola-hyperbola confusions that occurred in both the mathematics and physics domains. Inaccurate knowledge of the slope as the ratio of change in variables was to a large extent transferred from mathematics to kinematics.

7.3 Discipline characteristics

The results indicated that the majority of participants have an understanding of the physics discipline characteristics with regard to the use of kinematics concepts as variables that can be presented as coordinates on Cartesian planes. In the physics tasks on reading coordinates, they attached symbolic meanings (position and time) to the x and y coordinates. They also recognised correspondences between a linear motion equation and the standard mathematical format for straight lines in a representation task. However, they seem not to have the insight that kinematics relationships can be represented as functions, especially with regard to quadratic (parabolic) and hyperbolic functions. In addition, students failed to attach physical meaning to the area under graphs and slopes of graphs in the kinematics contexts.

In order for the participants to solve the physics questions correctly, they did not only have to know the discipline characteristics concerning kinematics graphs but also the discipline characteristics of graphs in mathematics. There are practices that are similar for mathematics and physics, for example, using the Cartesian coordinate system and placing the dependent variable on the vertical axis. Also, concepts such as slope and area are calculated the same in both contexts. Discipline characteristics that differ are, for example, that in mathematics, variables are abstract and have no units, whilst in physics variables, area under graphs and gradients all have physical meanings and units. Another difference is that in mathematics, the horizontal axis has a positive and negative side, whereas in kinematics, the concept time as the independent variable is on the horizontal axis and starts from zero only. The latter difference probably contributed to the significant differences in students' responses on the hyperbolic and parabolic representations. The kinematics graphs only showed the parts of the hyperbola or parabola for which the x-coordinate (time) is positive, which might have prevented students from recognising the graph form.

From the results of this study, it is clear that if students know the underlying mathematics, it does not imply that they can use it in another context. There is no automatic transfer from the mathematics domain to the physics domain when using mathematics to solve a physics problem. For a student to be able to solve a certain physics problem, he/she has to know and understand the underlying mathematics

as well as the physics concepts and principles. Only then they may be able to blend the knowledge effectively.

9. Recommendation

In the physics classroom, the students have to be taught how to use their existing mathematics to solve the problems at hand. The instructor has to revise the relevant existing mathematics as well as physics knowledge and draw analogies between aspects such as geometric figures, expressions and graphic representations of functions, etc. Differences in discipline characteristics need to be discussed with the students so that they understand the purpose and applications of graphs in the two contexts.

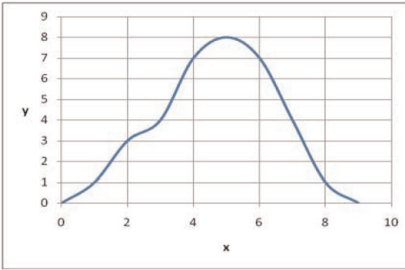
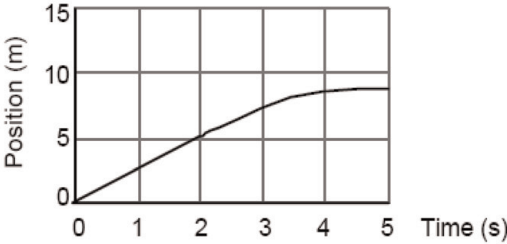
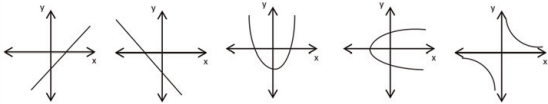
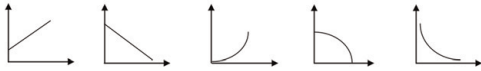
Further research can be conducted for follow-up years after specific interventions have been done to specifically address the problems identified. This questionnaire can also be used by other lecturers for research purposes or to test their students' abilities and identify areas of concern and come up with intervention strategies thereof.

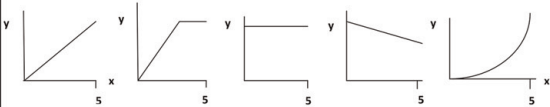
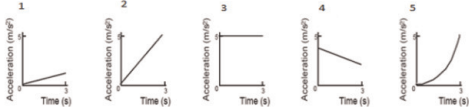
It is thus recommended that lecturers of undergraduate introductory physics should emphasise the knowledge and skills of algebraic graphs in teaching and learning of kinematics, especially kinematics graphs. This will enable these students to collect data, analyse it, plot graphs and interpret graphs based on this knowledge and relate it to and show physics understanding and knowledge.

Acknowledgements

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Appendix

| Mathematics | Physics |
|---|--|
| Reading coordinates | |
| <p>Use the graph below and answer the question that follows:</p>  <p>M_C1 What is the value of y if x = 4?</p> <div><div>1</div><div>2</div><div>3</div><div>4</div><div>5</div><div>4</div><div>0</div><div>7</div><div>3</div><div>8</div></div> | <p>The position-time graph below shows the straight-line motion of an object. Answer the following questions:</p>  <p>P_C1 The position at the 2 second point in the position-time graph is most nearly:</p> <div><div>1</div><div>2</div><div>3</div><div>4</div><div>5</div><div>0.4 m</div><div>2.0 m</div><div>2.5 m</div><div>5.0 m</div><div>9.0 m</div></div> <p>6.3 The position at the 5 second point in the position-time graph is most nearly:</p> <div><div>1</div><div>2</div><div>3</div><div>4</div><div>5</div><div>0.0 m</div><div>0.56 m</div><div>1.8 m</div><div>5.0 m</div><div>9.0 m</div></div> |
| Connecting representations | |
| <p>Choose the answers of the questions below from the following graph forms.</p>  | <p>Choose the answers of questions below from the following graph forms.</p>  |

| | |
|--|---|
| <p>M_R1 Which one of the graphs shows the function $f(x) = x - 1$?</p> <p>1 a 2 b 3 c 4 d 4 e</p> <p>M_R2 Which one of the graphs shows the function $g(x) = \frac{2}{x}$?</p> <p>1 a 2 b 3 c 4 d 5 e</p> <p>M_R3 Which one of the graphs shows the function $h(x) = x^2 - 1$?</p> <p>1 a 2 b 3 c 4 d 5 e</p> | <p>P_R1 What is the form of the v versus t graph if $v = u + at$ is plotted with u and a positive constants?</p> <p>1 a 2 b 3 c 4 d 5 e</p> <p>P_R2 What is the form of the v-t graph if $v = \frac{s}{t}$ is plotted with s a positive constant?</p> <p>1 a 2 b 3 c 4 d 5 e</p> <p>P_R3 What is the form of the s versus t graph if $s = ut + \frac{1}{2}at^2$ is plotted with u and a positive constants?</p> <p>1 a 2 b 3 c 4 d 5 e</p> |
| <p>Area qualitative</p> | |
| <p>Choose the answers of questions below from the following graphs. In all these graphs the maximum values for y are the same.</p> <p></p> <p>M_A1 Which one of the graphs has the <u>smallest</u> area under the graph from $x=0$ to $x=5$?</p> | <p>Five objects move according to the following acceleration versus time graphs.</p> <p></p> <p>P_A1 Which object had the smallest change in velocity during the three second interval?</p> <p>1 1. 2 2. 3 3. 4 4. 5 5.</p> |

1a

2b

3c

4d

5e

M_A2 Which one of the graphs has the largest area under the graph from $x = 0$ to $x = 5$?

1a

2b

3c

4d

5e

P_A2 Velocity versus time graphs for five objects are shown below. All axes have the same scale. Which object had the greatest change in position (displacement) during the interval?

(A)

(B)

(C)

(D)

(E)

1(A)

2(B)

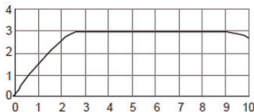
3(C)

4(D)

5(E)

Area quantitative

M_A3 Consider the following graph:



The area under the graph in the x -interval (4, 8) is:

10

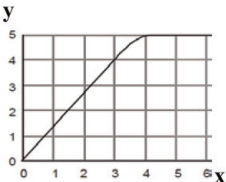
21.33

34.0

412.0

524.0

M_A4 For the graph below, answer the following question:



What is the area under the graph for $0 < x < 3$?

10.75

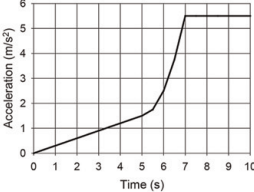
21.33

34.0

46.0

512.0

The acceleration-time graph below represents the motion of an object travelling in a straight-line. Answer the following questions.



P_A3 What is the change in velocity of the object from 7 seconds to 10 seconds?

15.5 m/s

216.5 m/s

31.833 m/s

40.545 m/s

555.0 m/s

P_A4 What is the change in velocity of the object from 0 seconds to 5 seconds?

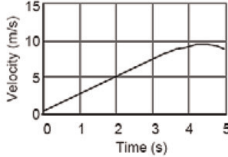
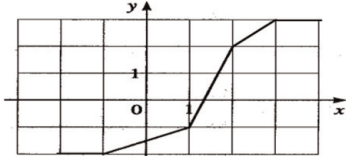
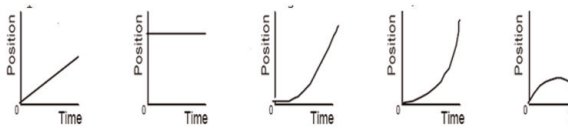
11.5 m/s

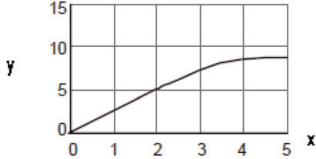
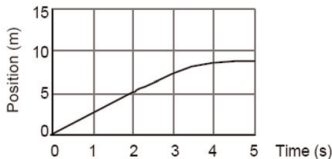
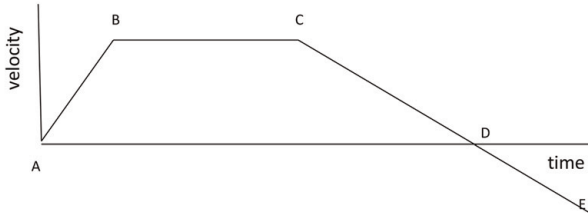
25 m/s

33.33 m/s

43.75 m/s

57.5 m/s

| | |
|---|--|
| Area additional items | |
| <p>P_A5 Displacement can be obtained from the:</p> <ol style="list-style-type: none"> 1 gradient of an acceleration-time graph 2 gradient of a velocity-time graph 3 area under an acceleration-time graph 4 area under a velocity-time graph <p>P_A6 If you wanted to know the distance covered during the interval from $t = 0$ s to $t = 2$ s, from the graph below you would:</p> |  <ol style="list-style-type: none"> 1 Read 5 directly off the vertical axis. 2 Find the area between that line segment and the time axis by calculating $(5 \times 2)/2$. 3 Find the slope of that line segment by dividing 5 by 2. 4 Find the slope of that line segment by dividing 15 by 5. 5 Not enough information to answer. |
| Gradient qualitative | |
| <p>M_S1. In which of the intervals is the gradient of the graph the largest?</p>  <ol style="list-style-type: none"> 1 $-2 < x < -1$ 2 $-1 < x < 1$ 3 $1 < x < 2$ 4 $2 < x < 3$ 5 $3 < x < 4$ | <p>P_S1 Position versus time graphs for five objects are shown below. All axes have the same scale. Which object has the highest instantaneous velocity in the interval shown?</p>  <ol style="list-style-type: none"> 1 a 2 b 3 c 4 d 5 e |
| Gradient quantitative | |
| <p>What is the gradient of the graph when</p> | <p>The position-time graph below shows the straight-line motion of an object.</p> |

| | |
|---|---|
| <div></div> <div><p>M_S2 $x = 2?$</p><div><div>1 0.4</div><div>2 2.0</div><div>3 2.5</div><div>4 5.0</div><div>5 10.0</div></div></div> <div><p>M_S3 $x = 4.5?$</p><div><div>1 5</div><div>2 0</div><div>3 $8 / 4.5$</div><div>4 $4.5 / 8$</div><div>5 8</div></div></div> | <div></div> <div><p>P_S2 The velocity at the 2 second point in the position-time graph is most nearly:</p><div><div>1 0.4 m/s</div><div>2 2.0 m/s</div><div>3 2.5 m/s</div><div>4 5.0 m/s</div><div>5 10.0 m/s</div></div></div> <div><p>P_S3 The velocity at the 5 second point in the position-time graph is most nearly:</p><div><div>1 0.0 m/s</div><div>2 0.56 m/s</div><div>3 1.8 m/s</div><div>4 5.0 m/s</div><div>5 9.0 m/s</div></div></div> |
| <p>Gradient additional items</p> | |
| <p>The figure below shows a velocity-time graph of an object's motion.</p> <div></div> | |
| <div><p>P_S4 Where is the gradient negative?</p><div><div>1 AB</div><div>2 BC</div><div>3 CD and DE</div><div>4 CD only</div><div>5 DE only</div></div></div> | <div><p>P_S5 Where is the acceleration negative?</p><div><div>1 AB</div><div>2 BC</div><div>3 CD and DE</div><div>4 CD only</div><div>5 DE only</div></div></div> |

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