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Saving Time in Portfolio Optimization on Financial Markets

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Abstract

The time management is important part for tasks in real-time operation of systems, automation systems, optimization in complex system, taking explicit consideration in time constraints, scheduling of tasks and operations, making with incomplete data, and time management in different practical cases. The limit in time for taking appropriate decisions for management and control is a strong constraint for the implementation of autonomic functionalities as self-configuration, self-optimization, self-healing, self-protection in computer systems, transportation systems, and distributed systems. Time is an important and expensive resource. The time management in financial domain is a prerequisite for high competitiveness and an increase in the quality of the investment activities. It is the popular phrase that time is money, and particularly, the portfolio optimization targets its implementation in real cases. This research targets the identification of portfolio parameters, which are strongly influenced by time. We restrict our considerations only on portfolio optimization task, and we identify cases, which are strongly influenced by time constraints. Thus, the portfolio optimization problem is discussed on position how the time can influence the portfolio characteristics and solutions. This chapter starts with the description of the object portfolio management, which provides the cases where time in explicit way influences the portfolio problem.

Keywords: data driven analysis, real-time portfolio optimization, decision making, automation in information systems

1. Introduction

The time management is important part for tasks in real time operation of systems, automation systems, optimization in complex system, taking explicit consideration in time constraints, scheduling of tasks and operations, making with incomplete data, time management in different practical cases. The limits in time for taking appropriate decisions for management and control is a strong constraints for the implementation of autonomic functionalities as self-configuration, self-optimization, self-healing, self-protection in computer systems, transportation systems, distributed systems. Time is an important and expensive resource.

The time management in financial domain is a prerequisite for high competitiveness and increase of the quality of the investment activities. It is the popular phrase that “time is money” and particularly the portfolio optimization targets its

implementation in real cases. This research targets the identification of portfolio parameters, which are strongly influenced by time. We restrict our considerations only on portfolio optimization task and we identify cases, which are strongly influenced by time constraints. Thus, the portfolio optimization problem is discussed on position how the time can influence the portfolio characteristics and solutions. This chapter starts with description of the object “portfolio management” which provides the cases where time in explicit way influences the portfolio problem.

2. Portfolio optimization problem

The task, which is resolved by the portfolio optimization of financial resources, is related with maximization of the return and simultaneously minimization of the investment risk. The portfolio optimization can be applied also to assets, which belong to the stock markets, because the same valued characteristics are used for portfolio optimization. The goal of the portfolio problem is to share the amount of investments among a set of securities, which are chosen to enter into the portfolio. The portfolio goal is to allocate in optimal manner the parts of the investment for buying securities. The time management problem initially arises with its complexity on the stage of the portfolio definition. The investment procedure has to be implemented at time t_0 (now). The assets’ characteristics can be evaluated for this time moment t_0 , **Figure 1**.

The portfolio management insists to make decision for buying (or selling) assets at the current time t_0 . Then after a period of time $\Delta t > 0$ at time moment $T = t_0 + \Delta t$ the investor has to sell (or buy) the assets from the portfolio and must receive positive return

$$Return(T) = \frac{Receipt(T) - Expenditure(t_0)}{Expenditure(t_0)} \quad (1)$$

The value of the *Receipt* is defined in the future time T and the *Expenditure*—on the current time t_0 . The portfolio problem arises according to the difference of the time moment $t_0 < T$. The investment decisions are based on the assets’ characteristics for the moment t_0 , $\mathbf{A}(t_0)$. But in time T these characteristics will be $\mathbf{A}(T)$ and in common case they will differ in values $\mathbf{A}(t_0) \neq \mathbf{A}(T)$. These differences strongly influence the portfolio return at time T . In general, the assets’ characteristics are the return and risk, $A_i(t_0) = A_i(Return_i(t_0), Risk_i(t_0))$, $i = 1, \dots, N$, N is the types of assets in the portfolio which are evaluated for the current time t_0 . But the portfolio return is evaluated at the end of the investment period T . Respectively, the assets’ characteristics at time T are different $A_i(T) = A_i(Return_i(T), Risk_i(T))$, $i = 1, N$. Hence, the final portfolio returns from (1) becomes

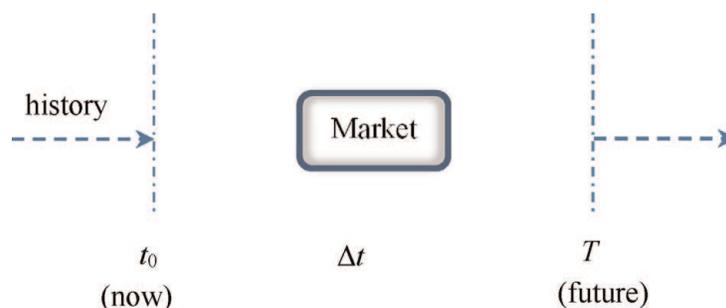


Figure 1.
Time schedule of the portfolio investment.

$$\text{Portfolio_Return} = \frac{\text{Return}[\mathbf{A}(T)] - \text{Expenditures}[\mathbf{A}(t_0)]}{\text{Expenditures}[\mathbf{A}(t_0)]} \quad (2)$$

Following (2) for the implementation of the portfolio investment, the investor has:

- to choose the types and number of assets N , which will participate in the portfolio;
- to assess the assets' characteristics $Risk_i(t_0)$ and $Return_i(t_0)$, $i = 1, \dots, N$ at the current time t_0 ;
- to choose the duration Δt of the investment, which defines the final investment time T ;
- to forecast the assets' characteristics $Risk_i(T)$ and $Return_i(T)$, $i = 1, \dots, N$ for the end of the investment period T ;
- to define and solve the portfolio optimization problem which will give the relative weights w_i , $i = 1, \dots, N$ of the investment, allocated for buying (selling) asset i . The relative values of weights introduce the analytical constraint

$$\sum_{i=1}^N w_i = 1 \quad (3)$$

and w_i are the solutions of the portfolio optimization problem. To move ahead about the time management problem and to recommend relations between t_0 , Δt and T there is a need to analyze the character of the portfolio optimization problem.

3. Modern Portfolio Theory

The Modern Portfolio Theory (MPT) was quantitatively introduced from Markowitz, with his seminal work [1]. The problems, introduced for the portfolio optimization are defined with two formal descriptions:

- maximization of portfolio *Return* by finding optimal values of the assets' weights w_i , $i = 1, \dots, N$, satisfying constraints about portfolio *Risk* to stay below a predefined value

$$\begin{array}{l} \max \\ \mathbf{w} \end{array} \left[\begin{array}{l} \mathbf{E}^T \mathbf{w} \\ \mathbf{w}^T \mathbf{\Sigma} \mathbf{w} \leq \sigma_{\max}^2 \end{array} \right] \quad (4)$$

- and/or minimization of portfolio *Risk* by finding optimal assets' weights w_i , $i = 1, \dots, N$, satisfying constraints about the portfolio *Return* to stay over a predefined value

$$\begin{array}{l} \min \\ \mathbf{w} \end{array} \left[\begin{array}{l} \mathbf{w}^T \mathbf{\Sigma} \mathbf{w} \\ \mathbf{E}^T \mathbf{w} \geq E_{\min} \end{array} \right]. \quad (5)$$

The notations used concern

E_i —the mean return of asset $i = 1, \dots, N$, $\mathbf{E}^T = (E_1, \dots, E_N)$,
 Σ — is the covariance matrix of the assets' returns, square symmetrical
 $N \times N$ matrix,
 σ_{max}^2 —the maximal portfolio risk, which the investor can afford for problem (4),
 E_{min} —the minimal portfolio return which the investor expects from the
investment,
 $\mathbf{w}^T = (w_1, \dots, w_N)$ —a vector of relative weights of the investment, which will be
allocated to asset $i = 1, \dots, N$, for buying or selling.

Particularly, additional nonnegative constraints are aided, $w_i \geq 0, i = 1, \dots, N$,
which means that asset i will be bought for the portfolio. The case with negative
weights, $w_i < 0$ means that the investor will sell asset i at time t_0 and at the end of
the investment period T the will buy these assets to recover his wealth. During these
operations the investor has to achieve positive portfolio return. The case of portfolio
optimization with negative weights is named "short sells" but it is allowed only for
special types of investors [2]. That's the reason that MPT mainly applies an addi-
tional constraint for nonnegative weights $\mathbf{w} \geq \mathbf{0}$ to problems (4) and (5).

To be able to solve problems (4) and (5) the parameters \mathbf{E} and Σ have to be
numerically evaluated. These parameters are strongly influenced by time. The esti-
mation of the mean assets' returns $E_i, i = 1, \dots, N$, has to be made for historical period.
The portfolio manager must use a time series of assets' returns

$$\begin{aligned} R_1 &= [R_1^{(1)}, R_1^{(2)}, \dots, R_1^{(n)}] \\ &\quad \dots \\ R_N &= [R_N^{(1)}, R_N^{(2)}, \dots, R_N^{(n)}] \end{aligned} \quad (6)$$

where $R_i^{(m)}$ is the return of asset i at time $m, i = 1, \dots, N, m = 1, \dots, n; n$ -discrete
points from the return history. These return values could be on daily, monthly,
weekly basis for a past period of time. Because for that case the time is defined as
integer number of days/months/weeks, the number n describes the length of the
historical period, taken by the portfolio manager to estimate the mean assets'
returns $E_i, 1, \dots, N$. The value of n is a discrete time and it influences the values of the
assets' characteristics. For a discrete time diapason $1 \div n$ the mean assets' returns are

$$E_i = \frac{1}{n-1} \sum_{m=1}^n R_i^{(m)}, \forall i = 1, \dots, N. \quad (7)$$

Having the values $E_i, i = 1, \dots, N$ from (7) the covariance matrix Σ is calculated as

$$\mathbf{COV}(\cdot) = \Sigma = \begin{vmatrix} c_{11} & \dots & c_{1N} \\ \dots & \dots & \dots \\ c_{N1} & \dots & c_{NN} \end{vmatrix}, c_{ij} = \frac{1}{n-1} \sum_{m=1}^n (R_i^{(m)} - E_i)(R_j^{(m)} - E_j), \forall i, j = 1, \dots, N. \quad (8)$$

The covariance coefficient c_{ij} has meaning, which defines how the time series of
the assets' returns i and j behave. The case of positive correlation $c_{ij} > 0$ means that
if the time series of returns R_i of asset increase (or decrease) the same simultaneous
change of increase (or decrease) takes place for the time series of returns R_j . For the
case of negative correlation $c_{ij} < 0$, the time series R_i and R_j move in opposite
directions. If the time series R_i increase (or decrease) the time series R_j decrease (or
increase). The negative correlation has advantage in usage by the portfolio

managers to decrease the total risk of the portfolio. Because $c_{ij} = c_{ji}$ from (8), the covariance matrix \sum is symmetrical. For the case $i = j$ the value c_{ii} is the variation of the row R_i , $c_{ii} = \sigma_i^2$, σ_i —standard deviation of row R_i . Thus, the covariance matrix on its diagonal gives the variation of the assets' returns. The components c_{ij} define the behavior of the time series of returns R_i and R_m . The portfolio theory applies the variation σ_i^2 as quantitative values of the risk of asset i . The graphical interpretation of mean return and risk of asset i is given in **Figure 2**, where.

R_i is the dynamically changed return of asset i ,

E_i —the mean value of return for the time period $[t_1, t_2]$,

σ_i —standard deviation of R_i towards E_i and give value of the risk of asset i .

The risk of the asset graphically represents the diapason $[\sigma_i, -\sigma_i]$ between which the real asset returns R_i generally stay around the mean value E_i . After definition of the vector of mean assets' returns $\mathbf{E}^T = (E_1, \dots, E_N)$ and the covariance matrix $\text{COV}(\cdot) = \sum$, the portfolio return E_p analytically is evaluated as

$$E_p = \sum_{i=1}^N w_i E_i = \mathbf{E}^T \mathbf{w} \text{ or } R_p = \sum_{i=1}^N w_i R_i = \mathbf{R}^T \mathbf{w}. \quad (9)$$

The value of the portfolio risk is calculated by the quadratic term

$$\sigma_i^2 = \sum_{i=1}^N \sum_{j=1}^n c_{ij} w_i w_j \text{ or } \sigma_p^2 = \mathbf{w}^T \sum \mathbf{w}. \quad (10)$$

The MPT uses and integration of the portfolio problems (4) and (5) by definition of a common optimization problem

$$\min_{\mathbf{w}} \left\{ \frac{1}{2} (1 - \Psi) \mathbf{w}^T \sum \mathbf{w} - \Psi \mathbf{E}^T \mathbf{w} \right\} \quad (11)$$

$$\sum_{i=1}^N w_i = 1, w_i \geq 0, i = 1, N.$$

The value of Ψ is the “risk aversion” coefficient, which is normalized for the numerical diapason $[0, 1]$.

- For the case $\Psi = 0$ the investor doesn't care about the portfolio return and his goal is to achieve minimal portfolio risk.

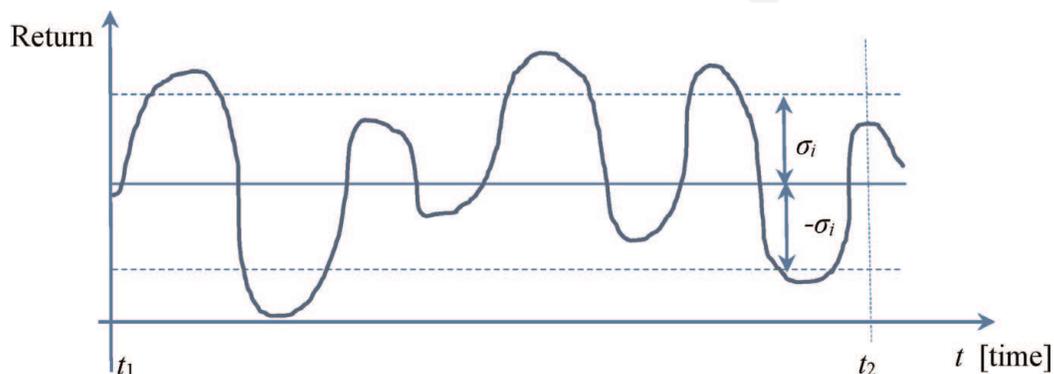


Figure 2.
 Graphical interpretation of the risk and mean return of asset.

- For the case $\Psi = 1$ the investor targets maximization of the portfolio return without considering the portfolio risk, because $\min (-\Psi \mathbf{E}^T \mathbf{w}) \equiv \max (+\Psi \mathbf{E}^T \mathbf{w})$.

By changing $\Psi \in [0, 1]$ different solutions $\mathbf{w}^{opt}(\Psi)$ are found from problem (11) which gives corresponding returns $E_p = \mathbf{E}^T \mathbf{w}^{opt}(\Psi)$ and risk $\sigma_p^2 = \mathbf{w}^{opt T}(\Psi) \Sigma \mathbf{w}^{opt}(\Psi)$ for the portfolios. These set of solutions can be presented as a set of points $[\sigma_p^2, E_p]$ in this space which in continuous case origins the “efficient frontier” curve, **Figure 3**.

The efficient frontier has quadratic character but it is not a smooth line [3]. This non-smooth character origins from the existence of non-negative constraints $w_i \geq 0$, $i = 1, \dots, N$ in problem (11). Hence, the MPT recommends to be defined and solved portfolio problem (11). Because the investors have different ability to undertake risk, the portfolio manager has to estimate the correct value of the “risk aversion” parameter. Because such identification is strongly subjective influenced, the MPT recommends to be evaluated the “efficient frontier” of portfolios. The investor can choose appropriate point from the frontier, which corresponds to the relation *Risk/Return*, which the investor is willing to accept. The portfolio, applied in problem (11) is named also “mean-variance” (MV) portfolio model. From the time management considerations, the cases which are influenced by the time, for the portfolio problems are summarized as:

- the portfolio manager has to choose the time for the portfolio implementation;
- he has to decide the duration of the investment $\Delta t = T - t_0$; T —final investment time;
- he has to choose the duration n of the historical period, which will be used for the evaluation of the mean assets’ returns E_i , $i = 1, N$ and the covariance matrix $\text{COV}(\cdot) = \Sigma$ of the assets’ returns. The diagonal values of matrix Σ gives assets’ risks σ_i^2 , $i = 1, \dots, N$.

Thus, the time is very important parameter, which influences the definition and implementation of the portfolio investment and optimization.

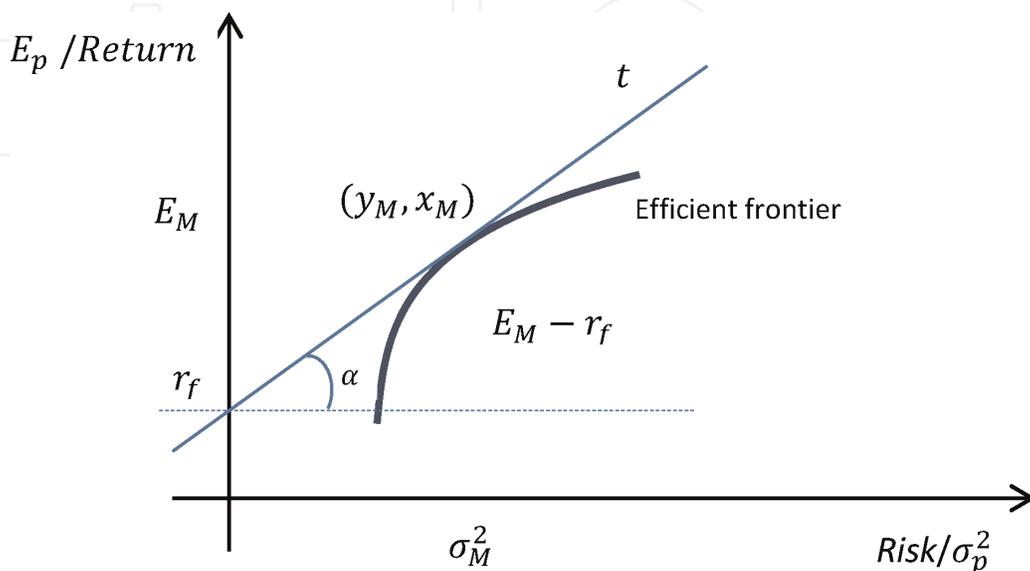


Figure 3.
The curve of “efficient frontier” and the market point.

4. Capital Market Theory

The MPT originated by the works of Markowitz has its further developments. The next important stage of MPT is the definition of the Capital Market Theory, [2]. The Capital Market Theory introduces a new point on the “efficient frontier,” named “market portfolio.” It has analogical portfolio characteristics as market return E_M and market risk σ_M^2 . This theory derives new analytical relations with the market characteristics, which are formal part of the Capital Asset Price Model (CAPM). This model added three additional linear relations named Capital Market Line (CML), Security Market Line (SML) and cHaracteristic Line (HL).

The graphical representation of the CML is given in **Figure 3**. It starts from the point $(0, r_f)$ which is a riskless asset with mean return r_f . The market point (E_M, σ_M^2) is a tangent one over the “efficient frontier.” The CML gives relations between the portfolios returns and risks for a particular market, assessed by the characteristics r_f, E_M, σ_M^2 . Analytically, the CML is a linear relation between E_p and σ_p ,

$$E_p = r_f + \frac{E_M - r_f}{\sigma_M} \sigma_p. \quad (12)$$

By estimating the market parameters r_f, E_M, σ_M^2 the investor has information about the level of risk σ_p^2 , which has to be undertaken by means to obtain portfolio return E_p . This prevents the investor to have unrealistic expectation about the potential mean return, which has to be achieved by a portfolio. The values of the market parameters, E_M, σ_M^2 are defined mainly according to the behavior of market index (S&P500, Dow Jones Industrial Average, NASDAQ Composite, NYSE Composite, FTSE100, Nikkei225, IPC Mexico, EURONEXT 100 and others). On each market the risk-free assets (deposits, long time bonds) has its own value r_f .

The SML introduces linear relations between the mean return of a particular asset E_i and the market return E_M

$$E_i = r_f + (E_M - r_f)\beta_i. \quad (13)$$

The coefficient “beta” (β_i) is a value of the relation

$$\beta_i = \frac{cov(i, M)}{\sigma_M^2}, \beta_i \in [-1, 1] \quad (14)$$

The “beta” coefficient takes normalized values from the diapason $[-1, 1]$. Numerically, it defines how strong the mean return value E_i is related with the market return E_M . If the return E_i is strongly related to the market behavior, the coefficient β_i has high value, close to 1, if the covariation coefficient $cov(i, M)$ is positive. The case of $\beta_i < 0$ means that the covariance between the series of returns R_i and R_M are in opposite directions.

The HL line makes additional clarification between the current value of the asset return R_i and market one R_M

$$R_i = r_f + \beta_i(R_M - r_f). \quad (15)$$

Relation (15) is timely influenced. If the market value R_M is changed/predicted, the corresponding asset return R_i of asset i can be estimated and/or predicted.

The CAPM does not apply explicit inclusion of time in its characteristics. Time explicitly influences only the values of the market return E_M and market risk σ_M . Applying the same considerations which take place for the evaluation of the assets' characteristics $E_i, \sigma_i, i = 1, \dots, N$ the historical period for the evaluation of the market characteristics is recommended to be the same, with n discrete historical values of the market return $R_M = [R_M^{(1)}, R_M^{(2)}, \dots, R_M^{(n)}]$.

5. Black-Litterman model for estimation of portfolio characteristics

The Black-Litterman (BL) model is based on both achievements of the MV portfolio model and CAPM. The idea behind the BL model is the ability to use additional information by means to estimate and to predict the assets' characteristics $E_i(T)$ and $\sigma_i(T), i = 1, \dots, N$ [4–6]. The difference and the added value N for the future time moment T when the portfolio investment will be capitalized e of the BL model is graphically interpreted in **Figure 4**.

The MV model estimates the assets characteristics $E_i, \sigma_i, i = 1, \dots, N$ using historical data from n discrete points of the assets' returns $R_i^{(m)}, i = 1, \dots, N, m = 1, \dots, n$. The BL model allows additional information to be used by means to modify the mean values of return $\mathbf{E}^T = (E_1, \dots, E_N)$ as the assets' risk characteristics, given by the covariation matrix Σ . The modification of \mathbf{E}^T to a new vector $\mathbf{E}_{BL}^T = (E_{BL1}, \dots, E_{BLN})$ is made by two additional numerical matrices \mathbf{P} and \mathbf{Q} . These matrices are evaluated from expert views, who make a subjective assessment about the future levels of assets' returns at time T , when the portfolio investment should be capitalized.

\mathbf{P} is a $k \times N$ matrix, which contains k expert views. The vector \mathbf{Q} defines quantitative information about the k -th expert view for increase or decrease the mean return E_i of i -th asset. The elements p_{ki} of \mathbf{P} defines the view of k -th expert about the change of the E_i return of asset i . The component p_{ki} takes value $+1$ for the case of increase, and respectively -1 for decrease.

The BL model added a new contribution to the MPT by introducing new characteristic of the portfolio asset: "implied return," $\Pi_i, i = 1, \dots, N$ ("implied excess return," when the return is evaluated according to the level of risk-free asset r_f). These returns differ from the historically evaluated mean returns $E_i, i=1, \dots, N$. The assumption behind these new "implied returns" is related with the existence of market point (E_M, σ_M) . For the case of market equilibrium, the CAPM asserts that all assets, which participate on this market should have appropriate mean returns $\mathbf{\Pi}^T = (\Pi_1, \dots, \Pi_N)$ and market weights $\mathbf{w}_M^T = (w_{M1}, \dots, w_{MN})$. Hence from the market values (E_M, σ_M) it follows exact values of $\mathbf{\Pi}$ and \mathbf{w} . But the market is a stochastic system and it endues a lot of noises, which change the values of the "implied

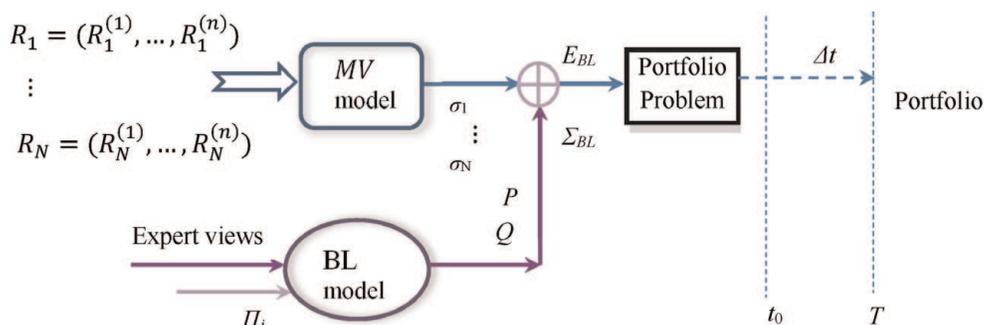


Figure 4. Additional modification of portfolio parameters by BL model.

returns.” These returns $\Pi_i, i = 1, \dots, N$ are values, which “should be.” But the noises make changes to Π_i and the BL model evaluates the unknown mean values \mathbf{E}_{BL} which are the “best approximation to Π_i .” These considerations origin the matrix linear relation in BL model

$$\mathbf{\Pi} = \mathbf{E}_{BL} + \boldsymbol{\varepsilon}, \mathbf{E}_{BL} = \begin{pmatrix} E_{BL1} \\ \vdots \\ E_{BLN} \end{pmatrix}, \quad (16)$$

where the noise $\boldsymbol{\varepsilon}$ is assumed to be with normal distribution, zero mean and volatility proportionally decreased from the historical covariance matrix, $\boldsymbol{\varepsilon} \sim \mathbf{N}(0, \tau\boldsymbol{\Sigma})$, $0 < \tau < 1$. The subjective views formally are introduced by the linear stochastic relation

$$\mathbf{Q} = \mathbf{P} \mathbf{E}_{BL} + \boldsymbol{\eta}, \quad (17)$$

where \mathbf{Q} is the quantitative assessment of the experts’ views about the value with which the historical returns will change; \mathbf{P} identifies which assets’ returns will be changed. The expert views contain also noise $\boldsymbol{\eta}$. Due to the independence of the expert views the noise $\boldsymbol{\eta}$ is assumed with zero mean and volatility $\boldsymbol{\Omega}$, $\boldsymbol{\eta} \sim \mathbf{N}(0, \boldsymbol{\Omega})$. The matrix $\boldsymbol{\Omega}$ is $k \times k$ square one with only diagonal elements because of the independence of the expert’s views. The matrix $\boldsymbol{\Omega}$ is presented mainly in the form [7].

$$\boldsymbol{\Omega} = \tau \text{diag} \left(\mathbf{P} \sum \mathbf{P}^T \right) \quad (18)$$

The goal of the BL model is the evaluation of the returns \mathbf{E}_{BL} which have to approximate in maximal level the stochastic relations (16) and (17). The values of the vectors and matrices $\mathbf{\Pi}$, \mathbf{Q} , \mathbf{P} , $\boldsymbol{\varepsilon}$, $\boldsymbol{\eta}$ are assumed to be known and/or estimated. The definitions of these parameters are given in the next paragraph.

6. Definition of the “implied excess returns”

Using [8, 9] the assumption is made that the “implied excess return” $\mathbf{\Pi}$ must satisfy the market portfolio. The goal function of the portfolio problem for that case is

$$\min_{\mathbf{w}} \left\{ \frac{1}{2} \lambda \mathbf{w}_M^T \sum \mathbf{w}_M - \mathbf{w}_M^T \mathbf{\Pi} \right\}, \quad (19)$$

where $\lambda = \frac{1-\psi}{\psi}$ is not normalized value of the risk aversion coefficient, $\lambda \in (0, \infty)$. Because the market point is used in (18) according to the CAPM the relation $\mathbf{w}_M^T \cdot \mathbf{1} = 1$ is satisfied, $\mathbf{1}^T = (1, \dots, 1)$ is a unity $N \times 1$ vector. The unconstrained minimization of (19) gives solution

$$\lambda \sum \mathbf{w}_M - \mathbf{\Pi} = 0. \quad (20)$$

By multiplication from left of the both sides of (20) with market capitalization weights \mathbf{w}_M^T it follows

$$\lambda \mathbf{w}_M^T \sum \mathbf{w}_M = \mathbf{w}_M^T \mathbf{\Pi}. \quad (21)$$

The right component of (21) contains the market “excess return” $E_M - r_f$, according to (9). The left side gives the market volatility (risk) σ_M^2 , (10) or

$$\lambda = \frac{E_M - r_f}{\sigma_M^2}. \quad (22)$$

Substituting (22) in (21) the “implied excess return” results as

$$\mathbf{\Pi} = \frac{E_M - r_f}{\sigma_M^2} \mathbf{\Sigma} \mathbf{w}_M. \quad (23)$$

The “implied return” $\mathbf{\Pi}^*$ is the value of $\mathbf{\Pi}$ to which the riskless return is added

$$\mathbf{\Pi}^* = \mathbf{\Pi} + r_f. \quad (24)$$

This manner of definition of $\mathbf{\Pi}$ is known as “inverse optimization” because the market risk and return are known, but we need to calculate the asset returns.

7. Definition of P and Q from scientific views

Following [10] absolute and relative manner for the formalization of the expert views are applied. The explanation of these forms of formalization is given with a simple example. Let's the portfolio contains $N = 4$ assets. Assuming that an expert expects that the first asset will increase its return with 2%; a second expert makes conclusion that the fourth asset will decrease its return with 3% the formalization of \mathbf{P} , \mathbf{Q} are

$$\mathbf{P} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}, \quad \mathbf{Q} = \begin{vmatrix} 2 \\ -3 \end{vmatrix}. \quad (25)$$

The relative form of views applies comparisons between the assets' returns. Let's the first expert expects that the first asset will outperform the third one with 2.5%; the second expert makes view that the second asset will outperform the fourth one with 3.5%. The formalization of matrices \mathbf{P} and \mathbf{Q} are

$$\mathbf{P} = \begin{vmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{vmatrix}, \quad \mathbf{Q} = \begin{vmatrix} 2.5 \\ 3.5 \end{vmatrix}. \quad (26)$$

The two types of formalization of expert views is widely mention in references dealing with the BL model [7, 10]. A new form of expert views has been developed in [11]. It has been applied a weighted form for the definition of matrix \mathbf{P} , where its components can take values different from ± 1 . To provide this new formalization of the expert views the matrix $\mathbf{\Omega}$ is analyzed. This matrix formalizes the variation of the expert views. Using relation (18) let's assume that the portfolio contains three assets, $N = 3$ and two experts $k = 2$ make views in relative form formalized in the matrices \mathbf{P} and \mathbf{Q}

$$\mathbf{P} = \begin{vmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{vmatrix}, \quad \mathbf{Q} = \begin{vmatrix} 2 \\ 4 \end{vmatrix}. \quad (27)$$

Hence it follows

$$\mathbf{\Omega} = \tau \text{diag} \left(\mathbf{P} \sum \mathbf{P}^T \right) = \tau \text{diag} \left(\left| \begin{array}{cc|cc} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{array} \right| \sum \left| \begin{array}{c} 2 \\ 4 \end{array} \right| \right), \quad (28)$$

where \sum is a symmetrical 4×4 matrix $\left| \begin{array}{cccc} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_4^2 \end{array} \right|$. The matrix multiplications results in 2×2 matrix $\mathbf{\Omega} = \left| \begin{array}{cc} \omega_1 & \mathbf{0} \\ \mathbf{0} & \omega_2 \end{array} \right|$ where

$$\omega_1 = \tau(\sigma_1^2 + \sigma_3^2 - 2\sigma_{13}), \omega_2 = \tau(\sigma_2^2 + \sigma_4^2 - 2\sigma_{24}). \quad (29)$$

Relations (29) have analytical structure with the risk relation of portfolio with two assets, $N = 2$, and negative correlation, [2] $\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 - 2w_1 w_2 \sigma_{12}$ where σ_1^2 and σ_2^2 are the volatilities of the two assets, σ_p^2 is the volatility of the portfolio, σ_{12} is the covariation between the two returns. Assuming equal weights in the portfolio, $w_1 = w_2$, the portfolio risk is evaluated as

$$\sigma_p^2 = 0.25(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}). \quad (30)$$

The comparisons between relations (29) and (30) can be interpreted that in (29) ω_1 and ω_2 are the values of risks of two virtual portfolios. The first one contains assets one and three. The second portfolio has the second and fourth assets. Thus, the values ω_i $i = 1, 2$, which formalize the risk of expert views are proportional to virtual portfolios with corresponding two assets, which have negative correlations and equal weights.

Now let's assume that the matrix \mathbf{P} contains weighted components α_i , which differ from the values ± 1 . To simplify the formal notations we assume that the matrix \mathbf{P} is on the form.

$$\mathbf{P} = \left| \begin{array}{cccc} \alpha_1 & 0 & -\alpha_3 & 0 \\ 0 & -\alpha_2 & 0 & \alpha_4 \end{array} \right|. \text{ The weighted coefficients satisfy the equalities}$$

$$|\alpha_1| + |\alpha_3| = 1 \text{ and } |\alpha_2| + |\alpha_4| = 1. \quad (31)$$

For that case the corresponding values of the components ω_i $i = 1, 2$ are

$$\omega_1 = \tau(\alpha_1^2 \sigma_1^2 + \alpha_3^2 \sigma_3^2 - 2\alpha_1 \alpha_3 \sigma_{13}), \omega_2 = \tau(\alpha_2^2 \sigma_2^2 + \alpha_4^2 \sigma_4^2 - 2\alpha_2 \alpha_4 \sigma_{24}). \quad (32)$$

Relations (32) interpret that for the weighted form $\mathbf{P}(\alpha)$ of the expert views the corresponding components ω_i $i = 1, 2$ of the variation of the expert views are proportional to the risk of a portfolio with two assets and negative correlation, and the assets weights α are normalized because equalities (31) hold. The ability to define matrix \mathbf{P} with components different to ± 1 allows the expert views to be generated not only by subjective assessments, but also with additional considerations, which are based on objective criteria, estimations and assessments.

This research makes several additions to the numerical definition of \mathbf{P} and \mathbf{Q} matrices.

1. Formalization $\mathbf{P}(\alpha)$ based on the difference $\Pi_i - E_i$, $i = 1, \dots, N$, normalized by the i -th volatility.

Following [11] a row of matrix \mathbf{P} concerning the view of an expert is defined in the form $p_s = |0 \dots \alpha_i \ 0 \dots 0 - \alpha_j \dots 0|$, $1 \times N$ vector. The values α_i and α_j must satisfy the normalization equation $|\alpha_i| + |\alpha_j| = 1$. The value α_i is chosen from the maximal difference

$$\alpha_i > 0 \equiv \max_i \left(\frac{\Pi_i - E_i}{\sigma_i^2} \right), i = 1, \dots, N. \quad (33)$$

Relation (33) presents that the mean history' return of asset i , E_i , is lower from its "implied excess return" and the investor has to expect that the return of asset i has to increase. The same considerations, but for decrease of the mean return E_j is made from the difference

$$\alpha_j < 0 \equiv \min_j \left(\frac{\Pi_j - E_j}{\sigma_j^2} \right), j = 1, \dots, N. \quad (34)$$

Asset j is over performed and the investor has to expect decrease of the historical mean return E_j towards the level of the "implied excess return" Π_j .

The value of the component from matrix \mathbf{Q} is

$$q = \min_{i,j} (|\Pi_i - E_i|, |\Pi_j - E_j|). \quad (35)$$

2. Formalization $\mathbf{P}(\Pi - E)$ based on the difference $\Pi_i - E_i$, $i = 1, \dots, N$ without normalization with volatilities.

For that case relations (33) and (34) are slightly modified with lack of volatility normalization

$$\alpha_i > 0 \equiv \max_i \left(\frac{\Pi_i - E_i}{|\Pi_i - E_i| + |\Pi_j - E_j|} \right), i, j = 1, \dots, N \quad (36)$$

$$\alpha_j < 0 \equiv \min_j \left(\frac{\Pi_j - E_j}{|\Pi_i - E_i| + |\Pi_j - E_j|} \right), i, j = 1, \dots, N$$

3. Formalization of $\mathbf{P}(\Pi)$ based only on the value of Π_i , $i = 1, \dots, N$.

$$\alpha_i > 0 \equiv \max_i \left(\frac{\Pi_i}{|\Pi_i| + |\Pi_j|} \right), i, j = 1, \dots, N \quad (37)$$

$$\alpha_j > 0 \equiv \max_j \left(\frac{\Pi_j}{|\Pi_i| + |\Pi_j|} \right), i, j = 1, \dots, N$$

4. A particular case can arise when all differences $\alpha_i = \Pi_i - E_i$, $i = 1, N$ have equal sign (+) or (-). Hence all assets' returns have to be increased, when $\alpha_i > 0$ or decreased if $\alpha_i < 0$.

For that case absolute views can be assign. The matrix \mathbf{P} will be square $N \times N$

identity matrix. $\begin{vmatrix} 1 & \dots & 0 \\ \dots & 1 & \dots \\ 0 & \dots & 1 \end{vmatrix} N \times N$. The \mathbf{Q} , $N \times 1$ vector will have components equal to $\alpha_i = \Pi_i - E_i$, $i = 1, \dots, N$.

Thus, for the formalization of p. 2 the matrices \mathbf{P} and \mathbf{Q} are

$$\mathbf{Q} = \begin{vmatrix} \Pi_1 - E_1 \\ \dots \\ \Pi_N - E_N \end{vmatrix} N \times 1, \mathbf{P} = \begin{vmatrix} 1 & \dots & 0 \\ \dots & 1 & \dots \\ 0 & \dots & 1 \end{vmatrix} N \times N. \quad (38)$$

$$\text{or } \mathbf{Q} = \mathbf{\Pi} = \begin{vmatrix} \Pi_1 \\ \dots \\ \Pi_N \end{vmatrix} \quad (39)$$

for the case of p. 3. These four forms of weighted formalization of matrix $\mathbf{P}(\alpha)$ allows to be overcome the need to have subjective expert views. With these formalizations the assets' characteristics are evaluated not only by historical returns and covariances but by adding data, which in this case concerns differences from the "implied returns." The BL model incorporates such additional source of information, **Figure 4**. The formalism $\mathbf{P}(\alpha)$ allows to be compared portfolio solutions, based on MV model and BL one because subjective influences in BL model now are missing. The BL model integrates different sources of information, concerning future assets' characteristics, but this information is not subjectively generated and it origins from real and actual behavior of the market.

8. BL modification of the assets' characteristics

Using relations (22) and (23) the BL returns \mathbf{E}_{BL} are found by means to approximate in best way these two linear stochastic relations. For simplicity additional notation are used in the next matrix relations

$$\mathbf{Y} = \mathbf{X}\mathbf{E}_{BL} + \boldsymbol{\psi}, \quad (40)$$

where

$$\mathbf{Y} = \begin{vmatrix} \mathbf{\Pi} \\ \mathbf{Q} \end{vmatrix}, \mathbf{X} = \begin{vmatrix} \mathbf{I} \\ \mathbf{P} \end{vmatrix}, \boldsymbol{\psi} = \begin{vmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\eta} \end{vmatrix}, \overline{\boldsymbol{\Psi}} = \begin{vmatrix} \tau\boldsymbol{\Sigma} & 0 \\ 0 & \boldsymbol{\Omega} \end{vmatrix}. \quad (41)$$

The general least square method with the minimization of the Mahalanobis distance

$$\mathbf{E}_{BL}^{\min} \equiv \arg \left\{ \min_{\mathbf{E}_{BL}} [(\mathbf{Y} - \mathbf{X}\mathbf{E}_{BL})^T \overline{\boldsymbol{\Psi}}^{-1} (\mathbf{Y} - \mathbf{X}\mathbf{E}_{BL})] \right\} \quad (42)$$

gives solution

$$\mathbf{E}_{BL} = [(\tau\boldsymbol{\Sigma})^{-1} + \mathbf{P}^T \boldsymbol{\Omega}^{-1} \mathbf{P}]^{-1} [(\tau\boldsymbol{\Sigma})^{-1} \mathbf{\Pi} + \mathbf{P}^T \boldsymbol{\Omega}^{-1} \mathbf{Q}] \quad (43)$$

and volatility $\text{Vol}(\mathbf{E}_{BL}) = \Delta_{BL} = [(\tau\boldsymbol{\Sigma})^{-1} + \mathbf{P}^T \boldsymbol{\Omega}^{-1} \mathbf{P}]^{-1}$.

Taking into account the riskless return, the final BL assets' returns and covariance matrix are

$$\mathbf{E}_{BL}^{\text{final}} = \mathbf{E}_{BL} + r_f \text{ and } \boldsymbol{\Sigma}_{BL}^{\text{final}} = \boldsymbol{\Sigma} + \Delta_{BL} \quad (44)$$

Using these modified assets' characteristics, the portfolio problem (11) is solved and appropriate point from the efficient frontier is chosen. It is recommended the best portfolio to be taken with weights w_i^{opt} , $i = 1, \dots, N$, which belongs to portfolios with characteristics

$$\text{Maximal Sharp excess ratio, } \max_w \left(\frac{E_p - r_f}{\sigma_p^2} \right) \quad (45)$$

$$\text{or maximal information ratio, } \max_w \left(\frac{E_p}{\sigma_p} \right) \quad (46)$$

9. Numerical simulations and comparisons between MV and BL portfolios solutions

The numerical simulations are performed with real data from the Bulgarian Stock Exchange [12]. The riskless investment for several years gives very low or even negative return. That is, the reason for the investors to start to apply portfolio optimization with risky assets. Currently, the risky investments are performed with a set of about 125 mutual funds in Bulgaria nowadays. The mutual funds are operated by different business and economics entities. The goal of all mutual funds is to manage their portfolios by means to achieve positive return or to decrease the losses in nonfriendly behavior of the financial market. The success or not successful management of the mutual funds can be seen by their historical data about achieved returns and risks in their investments. Thus, our portfolio simulations will start with historical return data of a set of chosen Bulgarian mutual funds. It has been chosen

month 2018	CONCORD	ELANA	PROFIT	TEXIM	LIDER	PATRIM	GROWTH	
January	0.7298	0.312	0.132	0.197	-0.054	-0.021	0.791	
February	-0.2581	-0.414	0.134	0.181	-0.244	-0.924	-0.917	
March	-0.411	-0.197	-0.206	-0.304	-0.494	-0.899	-0.448	
April	0.0193	0.159	0.147	0.256	0.107	0.174	-0.063	
May	-0.4491	-0.362	0.046	-0.136	-0.024	0.416	-0.299	
June	-0.062	0.173	-0.062	-0.134	-0.246	-0.041	-0.429	
July	-0.0869	0.045	0.033	0.108	0.086	0.205	0.473	
August	0.0446	-0.055	-0.03542	-0.251	-0.151	0.176	-0.079	
September	0.2544	0.174	0.105	0.264	-0.135	-0.339	-0.322	
October	-0.0728	0.148	-0.026	-0.003	-0.121	-0.553	-0.661	
November	-0.5829	-0.525	-0.378	-0.393	0.109	0.379	0.099	
December	0.7244	0.111	0.58	0.758	-0.308	-1.021	-0.038	
E	-0.012525	-0.03592	0.039132	0.04525	-0.12292	-0.204	-0.15775	
STD	0.417379	0.273612	0.229691	0.318346	0.18279	0.526381	0.469805	
COV	CONCORD	ELANA	PROFIT	TEXIM	LIDER	PATRIM	GROWTH	
	CONCORD	0.159688	0.08155	0.066914	0.094008	-0.00751	-0.05295	0.080864
	ELANA	0.08155	0.068625	0.026653	0.040608	-0.00075	-0.00987	0.040523
	PROFIT	0.066914	0.026653	0.048362	0.063314	-0.0061	-0.04621	0.007817
	TEXIM	0.094008	0.040608	0.063314	0.092899	-0.00322	-0.06888	0.017825
	LIDER	-0.00751	-0.00075	-0.0061	-0.00322	0.030628	0.070669	0.039516
	PATRIM	-0.05295	-0.00987	-0.04621	-0.06888	0.070669	0.253988	0.113633
	GROWTH	0.080864	0.040523	0.007817	0.017825	0.039516	0.113633	0.202324

Figure 5. Monthly and annual returns, and the covariance matrix of the mutual funds for 2018.

seven mutual funds to participate in the portfolio: Concord Asset Management (CONCORD), Elana Asset Management (ELANA), Profit Asset Management (PROFIT), Texim (TEXIM), Central Cooperative Bank Lider (LIDER), Asset Management UBB Patrimonium (PATRIM), Asset Management DSK Growth (GROWTH). They invest both in currencies and shares. The Bulgarian Association of Asset Management Companies [13] and the Government Financial Supervision Commission [14] regularly record and update the activities of the Bulgarian mutual funds. For the simulation experiments it has been taken the mean monthly return of these 7 mutual funds for 2018-year, **Figure 5**.

The calculations in this research have been performed in MATLAB environment. The mean years returns and the covariance matrix are given also in **Figure 5**. The simulations apply multiperiod investment policy, described in **Figure 6**.

9.1 Initial evaluation of historical data

The monthly mean returns of the mutual funds for the first 8 months of 2018 were taken as historical period. It has been calculated the average return for each fund for this historical period, $n = 8$. The average returns and the corresponding covariance matrix are given in **Figure 7**.

The portfolio manager has to pay attention for the different values of mean returns and covariance, given in **Figures 5** and **7**. The first case is evaluated for $n = 12$, 12 time period. While the second evaluations are made for a shorter period, $n = 8$. That is, a case where the time management is important for the estimation of the assets' characteristics.

9.2 Evaluation of the efficient frontier with MV model for the first 8 months

By changing the values of $\Psi \in [0, 1]$ the portfolio problem (11) is repeatedly solved. The interim values of the portfolio return, risk and portfolio weights are stored in working arrays in MATLAB environment. The evaluation step of changing

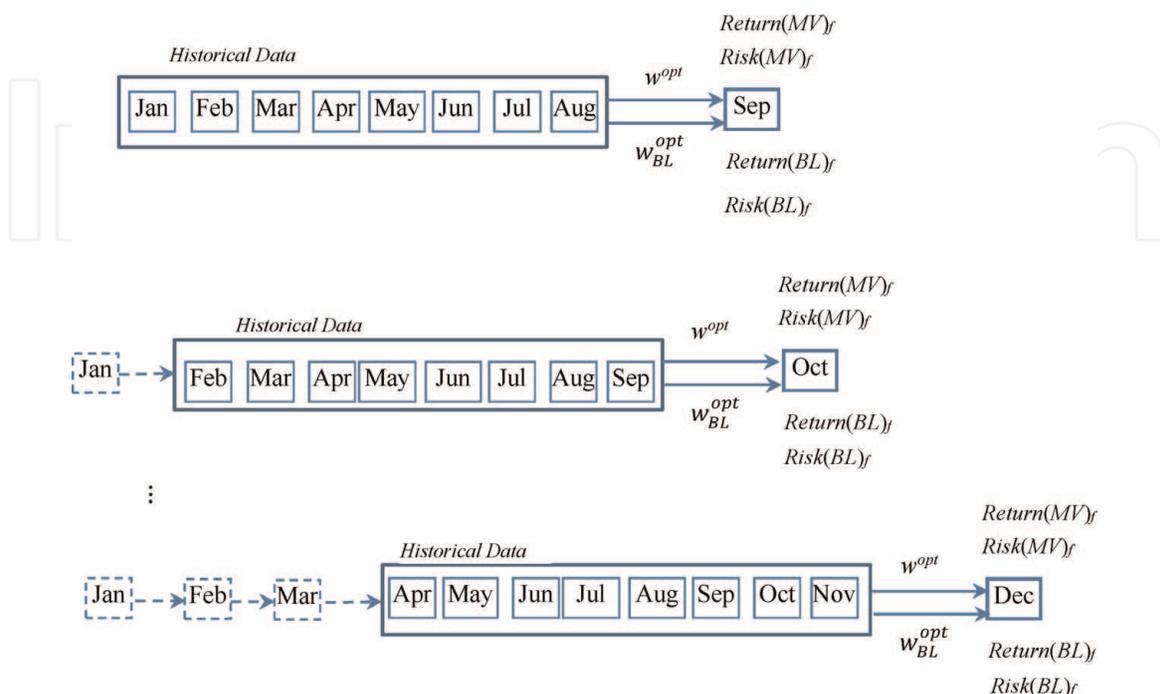


Figure 6.
 Multi period investment with flowing historical window.

$$E^T = [-0.0592; -0.0424; 0.0238; -0.0105; -0.1277; -0.1141; -0.1216];$$

$$\Sigma = [$$

0.1365	0.0778	0.0199	0.0388	0.0258	0.0497	0.1507
0.0778	0.0694	0.0048	0.0185	0.0192	0.0550	0.1034
0.0199	0.0048	0.0148	0.0239	0.0178	0.0183	0.0170
0.0388	0.0185	0.0239	0.0487	0.0281	0.0092	0.0386
0.0258	0.0192	0.0178	0.0281	0.0400	0.0807	0.0626
0.0497	0.0550	0.0183	0.0092	0.0807	0.2626	0.1556
0.1507	0.1034	0.0170	0.0386	0.0626	0.1556	0.2939];

Figure 7.
Mean returns and covariance matrix for the first 8 months of 2018.

Ψ was chosen $\Psi = 0.01$ resulting in 100 solutions of problem (11). The graphical presentation of the MV “efficient frontier” is given in **Figure 8**.

The Sharpe excess ratio (45) and the information ratio (46) are presented in **Figure 9**.

It is estimated the maximum Sharpe_excess_ratio = 4.321. This value corresponds to a portfolio with characteristics:

$$Return = 0.0218, Risk = 0.0143, w^{optT} = [0; 0.0304; 0.9696; 0; 0; 0; 0]. \quad (47)$$

These results recommend that the portfolio manager has to allocate his investment only in two mutual funds: the second in the portfolio (ELANA) and the third one (PROFIT). This recommendation is valid for the investment month of September 2018.

9.3 Evaluation of the assets’ characteristics for the BL model

9.3.1 Definition of the risk-free return r_f

In this research for the risk-free return r_f has been used an official index, evaluated and maintained by the National Bank of Bulgaria. The index is named

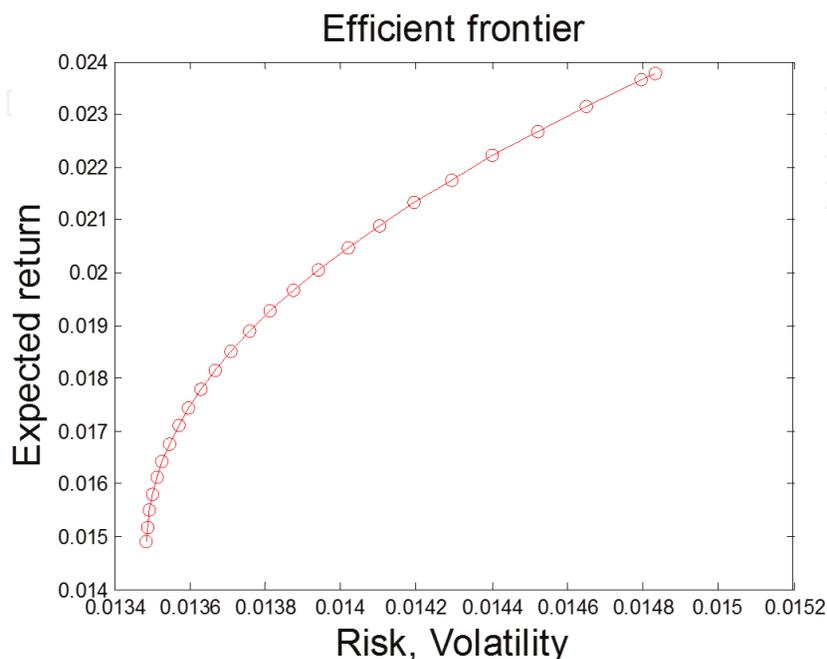


Figure 8.
Graphical presentation of the “efficient frontier” with historical data.

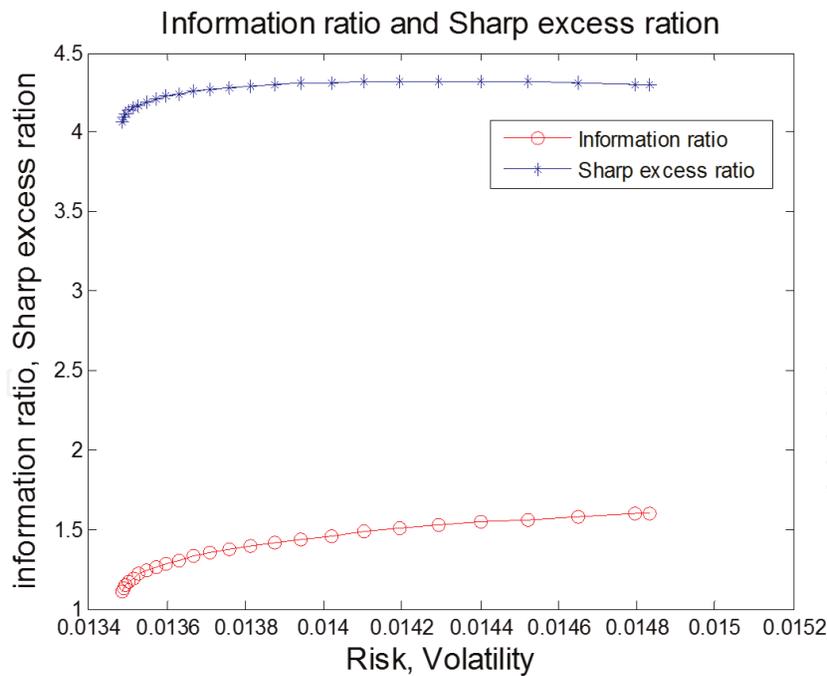


Figure 9.
 Graphical presentation of Sharpe excess ratio and information ratio.

LEONIA+ which is abbreviation of Lev (the name of the National currency) Over Night Index Average. This index is used by the mutual funds to take or giving loans for overnight activities on the financial market. This index is recommendation from the Bulgarian National Bank for all financial institution and authorities in Bulgaria dealing in overnight deposits with Bulgarian currency [15]. For this research the risk-free value is negative on monthly basis, $r_f = -0.4$.

9.3.2 Evaluation of the market point

The characteristics of the market point are the mean return E_M and the risk, numerically estimated by the standard deviation σ_M . The market point is found as a tangent one where the CML (Capital Market Line) makes over the “efficient frontier.” Additionally, the CML must pass through the riskless point $(0, r_f)$. The CML cannot be presented in analytical way because the “efficient frontier” is not analytically given. The last have been found numerically as a set of points in the plane (*Risk/Return*) from the multiple solutions of portfolio problem (11), given in p. 2. This research makes a quadratic approximation of the “efficient frontier” and finds analytical description of the “approximated efficient frontier.” Then with algebraic calculations using the linear equation of the CML and the approximated efficient frontier the tangent point is evaluated. The coordinates of the market point give the mean market return E_M and the market risk σ_M . For these market values the market capitalization weights \mathbf{w}_M are found from the working arrays when problem (11) has been solved in p. 2. The “approximated efficient frontier” is a quadratic curve of the form

$$y = a_2 x^2 + a_1 x + a_0 \quad (48)$$

where $a_2 = -3980.6$; $a_1 = 118.40$; $a_0 = -0.9$, $x = Risk$, $y = Return$.

The numerical values of the market point are:

$$E_M = 0.0222, \sigma_M^2 = 0.0143, \lambda = 4.3462 \text{ (according to (22)).}$$

The graphical presentation of the CML, the “efficient frontier” and its approximation and the market point are given in **Figure 10**.

9.3.3 Evaluation of the implied excess returns $\Pi_i, i = 1, \dots, N$.

Using relation (23) the “implied excess returns” $\Pi_i, i = 1, \dots, N$ are:
 $\Pi^T = [0.0523; -0.0126; 0.0235; 0.0635; 0.0375; 0.0433; 0.0427]$.
 Respectively, the “implied returns” is $\Pi^* = \Pi + r_f$ or.
 $\Pi^{*T} = [0.0923; 0.0274; 0.0635; 0.1035; 0.0775; 0.0833; 0.0827]$.

9.3.4 Definition of the characteristics of the expert views P and Q

The portfolio parameter, which is used for the estimation of matrices P and Q is the difference between the implied returns Π and the mean assets’ historical returns E , $(\Pi - E)$. These values are as follows:

$$\begin{aligned} \Pi^{*T} &= [0.0923; 0.0274; 0.0635; 0.1035; 0.0775; 0.0833; 0.0827]; \\ E^T &= [-0.0592; -0.0424; 0.0238; -0.0105; -0.1277; -0.1141; -0.1216]; \\ (\Pi^* - E)^T &= [0.1115; 0.0298; -0.0003; 0.0741; 0.1652; 0.1575; 0.1643]. \end{aligned}$$

Because the value of the third component of $(\Pi^* - E)^T$ is less than 0.1% it is assumed to be zero. All differences $(\Pi^* - E)$ have positive sign, which means that the assets are underestimated and their implied returns are higher. Hence, the portfolio manager has to expect an increase of the mean returns of the assets in the portfolio. This case of differences between implied and mean returns defines the usage of relation (39) for the definition of matrices P and Q . The option (39) is also applied in this simulation work. The calculations have been performed with 7×7 identity

matrix P , $\begin{vmatrix} 1 & \dots & 0 \\ \dots & 1 & \dots \\ 0 & \dots & 1 \end{vmatrix} 7 \times 7$ and two types of matrices Q :

$$Q = (\Pi^* - E) \text{ and } Q = \Pi^* . \tag{49}$$

9.3.5 Evaluation of the BL returns E_{BL} and the BL covariance matrix Σ_{BL}

The evaluations of the modified mean assets’ returns E_{BL} according to the BL model are done according to relations (43) and (44). The value of the covariance

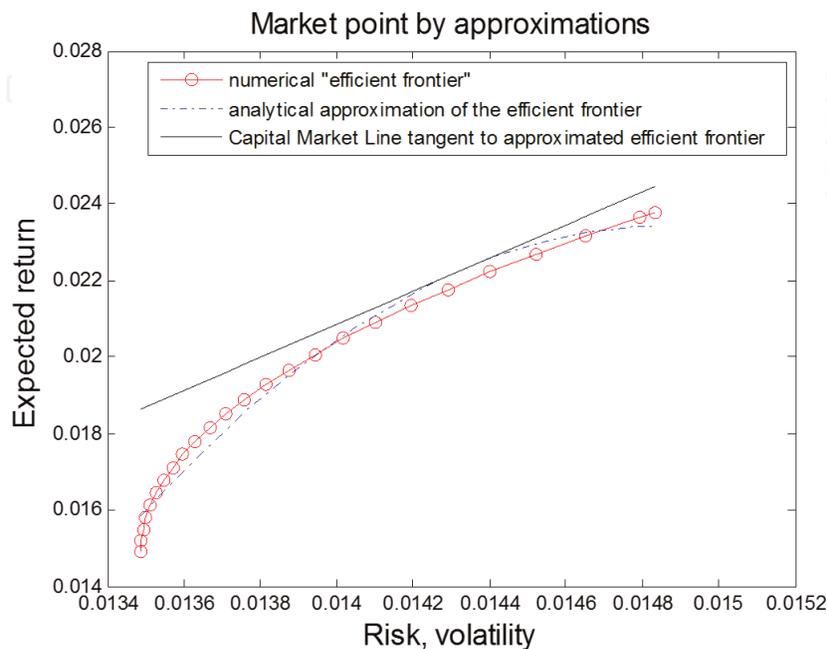


Figure 10. CML and approximated efficient frontier

matrix of the expert views is assumed to be as the historical covariance Σ but the values of its components are decreased with equal value τ . Thus the covariance matrix of the expert views is $\tau \Sigma$ where the value of τ must be between 0 and 1. From practical recommendations [7, 16, 17], this research uses $\tau = 0.5$. The BL model evaluations are.

$$E_{BL}^T = [0.0523, -0.0126; 0.0235; 0.0635; 0.0375; 0.0433; 0.0427]; \quad (50)$$

$$\Sigma_{BL} = \begin{bmatrix} 0.1588 & 0.0873 & 0.0219 & 0.0416 & 0.0252 & 0.0490 & 0.1675 \\ 0.0873 & 0.0816 & 0.0040 & 0.0199 & 0.0198 & 0.0593 & 0.1130 \\ 0.0219 & 0.0040 & 0.0172 & 0.0272 & 0.0198 & 0.0193 & 0.0163 \\ 0.0416 & 0.0199 & 0.0272 & 0.0566 & 0.0310 & 0.0044 & 0.0398 \\ 0.0252 & 0.0198 & 0.0198 & 0.0310 & 0.0456 & 0.0921 & 0.0681 \\ 0.0490 & 0.0593 & 0.0193 & 0.0044 & 0.0921 & 0.3113 & 0.1696 \\ 0.1675 & 0.1130 & 0.0163 & 0.0398 & 0.0681 & 0.1696 & 0.3424 \end{bmatrix} \quad (51)$$

9.3.6 Solution of portfolio problem with E_{BL}^T and Σ_{BL}

The portfolio problem (11) is repetitively solved by changing $\Psi \in [0, 1]$ with the BL evaluations of the assets' characteristics E_{BL}^T and Σ_{BL} . The new BL "efficient frontier" is found as a set of numerically evaluated points (100 points). For illustration purposes both "efficient frontiers" with historical data (MV model) and BL data (BL model) are given in **Figure 11**.

9.3.7 Evaluation of the BL weights w_{BL}^{opt}

The portfolio which has maximum Sharpe excess ratio is identified. This maximum is found from the numerically evaluated points of the BL "efficient frontier." The needed portfolio parameters are stored in the arrays in MATLAB, during the

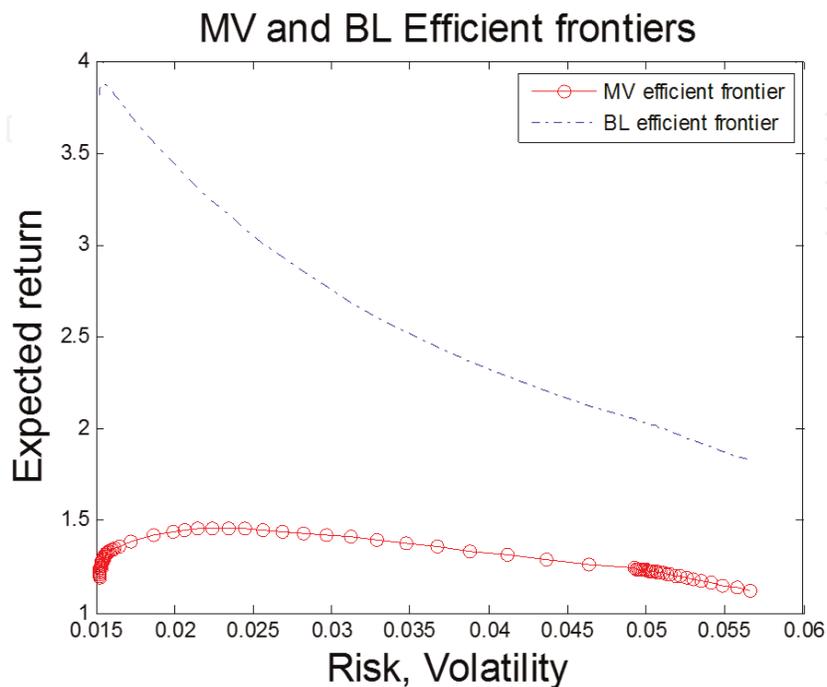


Figure 11.
 Efficient frontiers with MV and BL models

sequential solutions of problem (11). The Sharpe excess ratio evaluated from (45) gives:

$$Return(BL) = 0.0201, Risk(BL) = 0.0155, \mathbf{w}_{BL}^{optT} = [0;0.0924;0.9076;0;0;0;0]. \quad (52)$$

The difference between \mathbf{w}_{BL}^{opt} and \mathbf{w}^{opt} shows a bit increase of the weight for the second asset (PROFIT) for the BL portfolio.

9.4 Comparison of the MV solution \mathbf{w}^{opt} and the BL one \mathbf{w}_{BL}^{opt}

The optimal weights \mathbf{w}_{BL}^{opt} and \mathbf{w}^{opt} are assumed to be implemented as portfolio solutions in the beginning of month of September 2018. At the end of this month we can estimate the actual mean returns of the assets for month of September \mathbf{E}_f and the modified actual covariation matrix \sum_f which is calculated again for 8 months history but from February to September 2018.

- For the case when the MV weights \mathbf{w}^{opt} are invested the investor results will be

$$Return(MV)_f = \mathbf{E}_f^T \mathbf{w}^{opt}, Risk(MV)_f = \mathbf{w}^{optT} \sum_f \mathbf{w}^{opt}. \quad (53)$$

- For the case when \mathbf{w}_{BL}^{opt} weights are applied the investor results will be

$$Return(BL)_f = \mathbf{E}_f^T \mathbf{w}_{BL}^{opt}, Risk(BL)_f = \mathbf{w}_{BL}^{optT} \sum_f \mathbf{w}_{BL}^{opt}. \quad (54)$$

Then these portfolio results will be compared in the space $Risk(Return)$. The portfolio point which is situated far on the Nord-West direction of the $Risk(Return)$ space is the preferable portfolio. Such assessment will prove which portfolio model MV or BL gives more benefit and efficiency.

9.5 Multiperiod portfolio optimization

Following **Figure 6** a next portfolio investment with MV and BL models is done by moving the history period 1 month ahead. The portfolio evaluations are done for a history period from February till September 2018. The evaluated weights \mathbf{w}_{BL}^{opt} and \mathbf{w}^{opt} are applied for the month of October. For this case of 8 months historical period and available data for all 12 months of 2018 such multiperiod investment policy will evaluate 4 portfolios using the two models MV and BL. This research did three modifications of the BL model, concerning the evaluation of the matrices \mathbf{P} and \mathbf{Q} , related to the views for changing the assets characteristics:

- $\mathbf{P}(\boldsymbol{\alpha})$, weighted procedure, according to relations (33), (35);
- $\mathbf{P}(\boldsymbol{\Pi} - \mathbf{E})$, weighted procedure according to relations (35), (36);
- $\mathbf{P}(\boldsymbol{\Pi})$, weighted procedure according to relations (35), (37).

For the cases when all components $(\boldsymbol{\Pi} - \mathbf{E})$ or $\boldsymbol{\Pi}$ have same sign, the procedures (32) or (33) are applied. The obtained results are given in **Table 1**.

MV model		BL model					
		P(α)		P($\Pi - E$)		P(Π)	
Return (MV) _f	Risk (MV) _f	Return (BL) _f	Risk (BL) _f	Return (BL) _f	Risk (BL) _f	Return (BL) _f	Risk (BL) _f
0.1080	0.0133	0.1017	0.0132	0.1055	0.0132	0.1122	0.0129
-0.0187	0.0117	-0.0931	0.0120	-0.0632	0.0111	-0.0221	0.0106
-0.4011	0.0282	-0.3861	0.0263	-0.3793	0.0255	-0.4088	0.0292
-0.3525	0.0240	-0.2313	0.0114	-0.1523	0.0080	-0.2028	0.0106
Mean values							
-0.1661	0.0193	-0.1552	0.0157	-0.1223	0.0145	-0.1304	0.0158

Table 1.
 Results of multi-period portfolio management with MV and BL models.

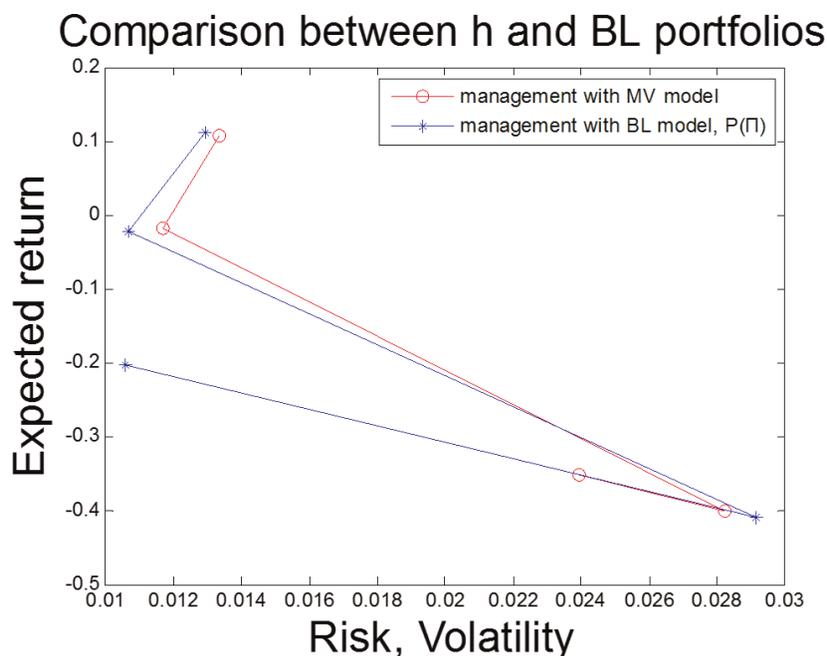


Figure 12.
 Comparison of multiperiod MV and BL(P(Π)) portfolio optimization.

The graphical presentation of the comparison of the multiperiod portfolio management between MV and BL with P(Π) modification is given in **Figure 12**.

The common results prove that the market situation in 2018 does not allow the mutual funds to achieve positive return. The results are negative but this negative value is less than the riskless return value $r_f = -0.4$. Hence, the portfolio management allows reduction of the losses. Particularly, all three modifications of the BL model give better results in comparison with the classical MV portfolio model. The mean values of the returns with BL model are very close to the returns of the MV model. But the risk values are considerably lower, which means that the probability to be closer to the mean values of BL returns is higher than the case of MV model.

10. Time management considerations for the portfolio investments

This research illustrates that the task of portfolio investment is quite complicated. The meaning of portfolio optimization concerns the definition and solution

of portfolio problem. In both these tasks the time is a prerequisite for successful portfolio investment.

10.1 Time requirements for the stage of definition of the portfolio problem

The content in the paragraph “Portfolio optimization problem” explicitly asserts that the investor has to choose the duration of the historical period. This duration, n is in discrete form. It has to be chosen in a way that can refer to the investment period $(T-t_0)$. Obviously, high number of n will give influence for the slow changes in the market behavior. Respectively, the active portfolio management will not benefit with long duration of the historical period n .

The active management needs to follow the current dynamics of the market. The relations between n and $(T-t_0)$ cannot be derived on theoretical basis. Only practical considerations could be useful. The authors’ experience recommends duration of the historical period to be considered between 6 and 8 months. Such history period can be used for multiperiod portfolio management from 1 to 3 months ahead in the future.

An unexpected problem has been met by the authors, concerning the relation between the historical discrete points n and the number N of the assets, included in the portfolio. The two parameters n and N participate both for the evaluation of the covariance matrix Σ . This matrix should be in full rank by means that the portfolio problem (11) must generate regular solutions. If the rank of Σ is less than N problem (11) gives unrealistic solutions. To keep Σ with rank N it is needed its components to be evaluated with historical data $n > N$. The practical minimal case is $n + 1 = N$ but before solving the portfolio problem the investor has to check the rank of Σ . As practical consideration, if the portfolio contains many assets and N is high, the data from the historical period n have to be also high. For that case one can use not only monthly returns but also weekly average data. Thus, the value of n can increase.

10.2 Time requirements for the solution of the portfolio problem

The solution of the portfolio problem (11) gives unique set of weights, which have to be implemented for the portfolio investment. Because the market behavior changes, reasonable policy is to perform repeatedly definition and solution of the portfolio problem. Potential beneficial strategy can be the multiperiod portfolio management, presented in **Figure 6**. It incorporates the multiperiod management and adopts the portfolio parameters with up to date market data. The relation between the duration of the historical period and the investment period is still an open question. But making additional simulations with 1, 2, 3 or more months (time) ahead the portfolio manager can change his decision on each investment step.

11. Conclusions

This research identifies in explicit way the influence of the time for the definition and solution of portfolio problems. These time requirements are considerably related with the estimation of the parameters of the portfolio problem. Respectively, the time requirements insist the portfolio management to be performed in multiperiod investment.

This research makes an analysis of the development of the portfolio theory. Starting with the Markowitz formalization, the MV portfolio problems are based

only on historical data about mean returns and covariances between the returns. The development of CAPM gives new relations, originated from a new “market” point. The last gives additional information about the values of the parameters of the portfolio problem. Finally, the BL model introduces a new set of points, “implied excess returns,” which originate from the market point. As a result, new values for the parameters of the portfolio problem are found. Respectively, the portfolio problem gives weights of the assets, which are not sharp cut, which decreases the risk of the investment.

This research introduces new modifications of the BL model for the part of definition of expert views. Particularly the experts are substituted by additional data, which origins from the dynamical behavior of the assets’ returns. Thus, not only mean returns and covariances are taken into consideration, but also the difference between objective parameters as implied and historical mean returns. These modifications allow the portfolio model MV and these based on BL one to be compared on a common basis and to assess their performances. Such comparison cannot be made if subjective experts are used, because their mutual views will be different for the same historical data and with changes the members of the experts.

This research gives also a practical added value with the analysis of the behavior of the market with mutual funds in Bulgaria. This gives additional experience and bases for future comparisons and assessments of the different portfolio models.

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