Chapter

# Mathematical Model for $CO_2$ Emissions Reduction to Slow and Reverse Global Warming

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## Abstract

This chapter aims to provide climate policy makers with smooth patterns of global carbon dioxide ($CO_2$) emissions consistent with the UN climate targets. An accessible mathematical approach is used to design such models. First, the global warming is quantified with time to determine when the climate targets will be hit in case of no climate mitigation. Then, the remaining budget for $CO_2$ emissions is derived based on recent data. Considering this for future emissions, first proposed is an exponential model for their rapid reduction and long-term stabilization slightly above zero. Then, suitable interpolations are performed to ensure a smooth and flexible transition to the exponential decline. Compared to UN climate simulation models, the designed smooth pathways would, in the short term, overcome a global lack of no-carbon energy and, in the long term, tolerate low emissions that will almost disappear as soon as desired from the 2040s with no need for direct removal of $CO_2$.


**Keywords:** atmospheric carbon dioxide ($CO_2$), global $CO_2$ emissions, global warming, remaining $CO_2$ budget, time model, UN climate target

## 1. Introduction

The climate change has been declared as an urgent global threat [1] since the spread of devastating floods, severe droughts, and ravaging wildfires, due to rising temperatures especially in the past three decades [2–4]. In response to this threat, the UN parties adopted the *2015 Paris Agreement on Climate Change* along with its implementation by 2020. Was included 'holding the increase in global annual average temperature above the pre-industrial level well below 2°C and pursuing efforts to limit it to 1.5°C' ([1], Art.2). Was also comprised 'projecting global peaking of greenhouse gases emissions as soon as possible along with their rapid reduction' ([1], Art.4).

Since the last century, the atmospheric carbon dioxide ($CO_2$) has been largely dominating the other greenhouse gases [5, 6] due to increasing anthropogenic $CO_2$ emissions as a consequence of a growing global demand for fossil-fuel-based products. Subsequently, climate policies would include a massive reduction of these emissions by shifting to no-carbon energy and introducing gas capture/removal technologies.

Climate mathematical modelling has so far focused on the physics behind the global warming, and has therefore described the rise in global average temperature using a mathematical approach based on the law of conservation of energy (e.g., [7–10]). When it comes to the climate mitigation, the existing models were mostly produced by computer simulation, which involved rather climatologists. Among these are the representative concentration pathways (RCPs; [5, 11–13]), which were adopted by the Intergovernmental Panel on Climate Change (IPCC) to predict future annual $CO_2$ emissions by simulating representative mitigation scenarios of radiative forcing; 2.6, 4.5, 6, and 8.5 W $m^{-2}$, going from the highest to the lowest mitigation. In the same setting, the C4MIP as part of the CMIP (Coupled Model Inter-comparison Project) provided a set of earth system models, involving the carbon cycle [14], also adopted by the IPCC (AR5, WG I). Among these were included models that infer $CO_2$ emissions based on atmospheric $CO_2$ concentrations targets. More recently, mixed models were developed using a combination of simulation climate and socio-economic models [15] to limit the radiative forcing to 1.9 W $m^{-2}$, and hence to meet the 1.5°C target.

The purpose of this chapter is to provide climate policy makers with smooth patterns of global $CO_2$ emissions consistent with a prescribed UN climate target, i.e., a limit $\mathcal{L}$ (°C) to the rise in global average temperature above the pre-industrial level. Unlike in literature where modelling is often based on computer simulation, an accessible mathematical analysis is used to design such models. Basically, two parameters are required; an estimation of the emissions level in the beginning of their mitigation (fixed parameter) and the remaining $CO_2$ budget (dependent parameter) which, by definition, consists of the cumulative $CO_2$ emissions (from the starting time) that will raise the global average temperature up to the given climate target. These parameters will be determined using a very strong and highly significant linear regression involving recent data on the gas emissions [16], which also provides a time model for these emissions in case of no climate policy. Based on this model, the second parameter will be explicitly determined in terms of the climate target. Modelling future emissions to make them fit the given UN target would be nothing else but connecting their initial state (predicted level in the beginning of the mitigation) to their desired final state (zero or almost-zero emission). Naturally, an exponential interpolation would provide such a connecting way with a rapid reduction of the emissions over the first 50–60 years, their stabilization slightly above zero in the long term, along with their extinction in far future due to the asymptotic behavior of the exponential model. Another source of mathematical modelling with regards to climate mitigation is the transition to this exponential trend, which can provide more feasible patterns for $CO_2$ emissions. Indeed, an independent parameter is introduced as an arbitrary fraction of the remaining $CO_2$ budget expected to be used exponentially, which also gives an indication for the transition length. Then suitable quadratic interpolations are performed to smoothly connect the current linear trend to the exponential decline. As a result, an uncountable range of exponential pathways is designed with smooth and flexible transition, which will not only overcome a global shortage of no-carbon energy but also lead to the nearly-zero emission as soon as desired depending on the climate target. The graphical representation of the designed models will help to explore their similarities to the (IPCC) RCPs and no- and low-overshoot 1.5°C pathways [17].

The rest of the chapter is organized as follows. The required materials are presented in Section 2, including recent annual data on $CO_2$ (concentration in the air as well as emissions level) and the correlation between the global warming and the atmospheric $CO_2$. In Section 3, a time model for global warming is presented along with a formulation of the hitting time for a given UN climate target. Section 4 is devoted to the elaboration and discussion of smooth mathematical models for global $CO_2$ emissions consistent with the UN climate targets. The results are summarized in Section 5.

## 2. Background materials

It is well-known that the global warming is due to the growing concentration of the greenhouse gases in the atmosphere, particularly the anthropogenic $CO_2$. Its quantification with time would therefore require the consideration of both; its correlation with and annual data of the atmospheric $CO_2$. On the other hand, recent data on global $CO_2$ emissions will be necessary to design appropriate pathways for these emissions in order to limit their warming effect to a prescribed UN target. Additionally, non-linear interpolations are inevitable to ensure a smooth transition from the current trend to the rapid decline of the emissions as urged by the UN.

### 2.1 Global warming vs. atmospheric $CO_2$

One of the key results in [10], reminded below, will be of great use in modelling with time the global warming. Based on the physics law of conservation of energy, this result states that the rise in global average temperature above the pre-industrial record is growing with the ratio $r$ of $CO_2$ concentration to the pre-industrial level. It can be seen as a generalization of the well-investigated climate response to doubling $CO_2$ concentration [5, 18, 19].

$$\Delta T(r) \approx \beta \frac{r-1}{r-k} \qquad (\beta \approx 5.84, k \approx -0.85) \tag{1}$$

### 2.2 Atmospheric $CO_2$ data (2000–2017)

The warming effect of the atmospheric $CO_2$ as quantified in (1), along with the trend of its concentration over the past two decades, will allow to describe the global warming through time. This trend will be estimated by linear regression of the annual average concentration of the gas based on the NASA monthly measurements from 2000 to 2017 [20].

### 2.3 Global $CO_2$ emissions data (2000–2013)

It is necessary to determine the trend of the global $CO_2$ emissions over the past two decades prior to modelling with time the desired effect of any projected mitigation in line with the UN climate goal. This trend will be estimated by linear regression of the annual gas emissions recorded by Carbon Dioxide Information Analysis Center (CDIAC) up to 2013 [16].

### 2.4 Quadratic interpolation

Classically, a quadratic interpolation consists of determining a quadratic function using the values that it takes on at exactly three particular values of its variable. The following result provides an original quadratic interpolation using also three given data on the parabola representing the function: its symmetry axis, one of its points (other than the vertex), and the slope of the tangent line at that point. This technique will be used to add a smooth transition to an exponential model for $CO_2$ emissions.

*If a parabola is symmetric about the line: $x = u$, passes through a point $(x_0, y_0)$, with $x_0 \neq u$, and is tangent at this point to the line of slope $m$, then an equation of this parabola is*:

$$
\begin{aligned}
y &= A(x-u)^2 + B \\
A &= m/2(x_0 - u) \\
B &= y_0 - m(x_0 - u)/2
\end{aligned}
\tag{2}
$$

Indeed, the form of the equation is due to the symmetry about the line $x = u$. The coefficient $A$ is determined by equating the slope $m$ with $\left.\frac{dy}{dx}\right|_{x=x_0} = 2A(x_0 - u)$. Then $B$ is deduced by plugging in $x_0$, $y_0$ and $A$ in Eq. (2).

## 3. Time model for global warming

As it can be seen from (1), an estimation of the atmospheric $CO_2$ concentration ratio through time will give a time model for the global warming. This estimation can be done by linear regression of the annual average ratio for $CO_2$, using the NASA dataset [20]. This leads to the following no-climate-mitigation model $r_0$, applicable from the year 2000 (i.e., $t = 0$)

$$
\begin{aligned}
&r_0(t) \approx a_0 t + b_0 \\
&a_0 \approx 0.0076, \quad b_0 \approx 1.3176
\end{aligned}
\tag{3}
$$

Such a linear regression was found to be statistically highly significant ($p < 10^{-21}$) and extremely strong ($r^2 \approx 0.99$). By composing the energy-balance-based model $\Delta T$ (given in (1)) with $r_0$, one gets the following climate-policy-free model $w_0$ for global warming, applicable from the year 2000 (i.e., $t = 0$)

$$
\begin{aligned}
\boldsymbol{w_0(t)} = \boldsymbol{\Delta T(r_0(t))} \approx \boldsymbol{\beta} \frac{\boldsymbol{t} + \boldsymbol{\lambda_0}}{\boldsymbol{t} + \boldsymbol{\mu_0}} \\
\boldsymbol{\lambda_0} = (\boldsymbol{b_0} - 1)/\boldsymbol{a_0}, \boldsymbol{\mu_0} = (\boldsymbol{b_0} - \boldsymbol{k})/\boldsymbol{a_0} \\
\boldsymbol{a_0}, \ \boldsymbol{b_0} \ \text{as in (3)}, \ \boldsymbol{\beta} \ \text{as in (1)}
\end{aligned}
\tag{4}
$$

The estimations based on this model, for the period 2005–2015, appear to be very close to the annual averages calculated using the NASA data [3] for the same period. According to (4), the rise in global average temperature will be estimated at

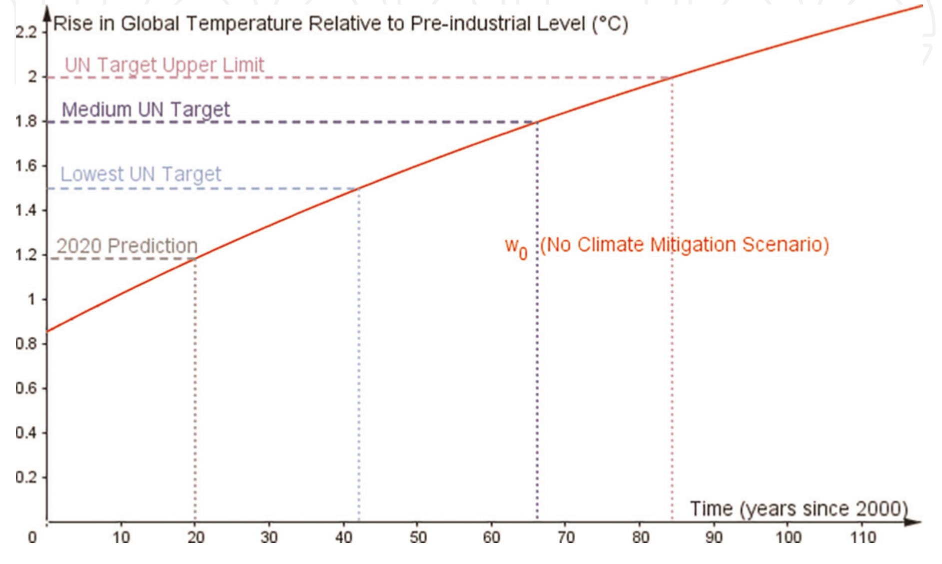


**Figure 1.**
*Estimated global warming since 2000 and expected trend in case of no global climate mitigation.*

1.19°C by 2020. Then, under the assumption of no climate mitigation, it will reach 1.5°C by 2041 and 2°C by 2084. More generally, any climate target $\mathcal{L}$ $(\mathcal{L} < \beta)$ will be hit by the year $2000 + [h]$ where $h$ is the hitting time:

$$h \approx \frac{\mu_0 \mathcal{L} - \lambda_0 \beta}{\beta - \mathcal{L}} \qquad (\lambda_0, \mu_0 \text{ as in } (4)) \tag{5}$$

which makes the year $d = 1999 + [h]$ the deadline for the implementation of the *2015 Paris Agreement*. See **Figure 1** for graphical estimation of $h$.

For the rest of the paper, $\mathcal{L}$ denotes any UN climate target $(1.5 \le \mathcal{L} < 2)$, and $h$ is its hitting time as formulated in (5).

## 4. Smooth pathways for $CO_2$ emissions to achieve the UN goal on climate change

The consistency of future $CO_2$ emissions with a prescribed UN target and their rapid reduction, as urged by the UN, are crucial in the elaboration of suitable pathways for the emissions. Prior to the modelling, however, two parameters need to be determined; an estimation of their level in the beginning of the mitigation and their expected cumulative amount during the mitigation (remaining $CO_2$ budget).

### 4.1 Remaining $CO_2$ budget

By definition, the $CO_2$ budget is the total amount of cumulative anthropogenic $CO_2$ emitted in the atmosphere since the industrial revolution up to the time $h$ when the UN climate target will be hit. To estimate the remaining budget at any time, future emissions need to be modelled explicitly with time in the scenario of no climate policy, which can be done by linear regression of the annual gas emissions since 2000 using CDIAC database [16]. This leads to the following no-climate-policy model $E_0$ (in $GtCO_2$), applicable from the year 2000 (i.e., $t = 0$):

$$E_0(t) \approx \alpha_0 t + \beta_0, \quad \alpha_0 \approx 0.91, \beta_0 \approx 24.47 \tag{6}$$

Such a linear regression was found to be statistically highly significant $(p < 10^{-11})$ and extremely strong $(r^2 \approx 0.98)$. As a consequence of (6), the remaining $CO_2$ budget $R(t)$, from time $t$ $(0 \le t < h)$, consistent with the $\mathcal{L}$-target, is estimated as follows:

$$\boldsymbol{R(t) \approx (h - t)(\alpha_0(h + t) + 2\beta_0)/2}, \quad \boldsymbol{h} \text{ as in } (5), \boldsymbol{\alpha_0, \beta_0} \text{ as in } (6) \tag{7}$$

Indeed, with no climate mitigation, the $CO_2$ emissions between times $t$ and $h$ would reach a total amount of $R(t) = \int_t^h E_0(x)dx$, which is nothing else but the area of a trapeze with bases $E_0(t)$ and $E_0(h)$ and height $h - t$, and this gives (7).

In particular, the remaining $CO_2$ budgets from 2020, to meet the targets 1.5 and 1.8°C, will be estimated at 1155 and 2929 ($GtCO_2$) respectively, and these represent about 63 and 81% of the corresponding remaining budgets from 2000.

### 4.2 $CO_2$ emissions pathways consistent with the UN climate targets

One obvious way to regularly reduce the $CO_2$ emissions would suggest a constant rate of reduction, which will definitely put an end to them at time $t = z$

(limiting therefore the rise in global temperature to the UN target $\mathcal{L}$), as described by the following piece-wise linear model, applicable from time $t = t_0 < h$:

$$\boldsymbol{L(t) \approx \begin{cases} \dfrac{E_0}{t_0 - z}(t - z), & t_0 \le t < z \\ 0, & t \ge z \end{cases}} \tag{8}$$

where $z = t_0 + 2R/E_0$ (with $E_0 = E_0(t_0)$, $R = R(t_0)$) is determined by solving the remaining $CO_2$ budget equation:

$$\int_{t_0}^{z} L(t)dt = (z - t_0)E_0/2 = R. \tag{9}$$

Nevertheless, the zero-emission ensured by the linear pattern will probably cause an environmental issue, as it could not be hit before the 2070s, for UN targets as low as 1.5°C, or late 2150s for even medium targets such as 1.8°C. In addition, a constant rate of reduction (annually 31% for 1.8°C and 79% for 1.5°C) will presumably not be compatible with a struggling switch to no-/low-carbon energy.

To avoid the issue regarding the zero-emission, one could simply bring these emissions as close as possible to zero by considering a smooth non-linear pathway with an asymptotic behavior, such as the following power model:

$$\boldsymbol{P(t) \approx E_0.(t_0/t)^p, \quad t \ge t_0, \quad p = 1 + t_0E_0/R, \quad E_0 = E_0(t_0), \quad R = R(t_0)} \tag{10}$$

where the suitable power $p$, with $p > 1$ for integrability over $[t_0, \infty]$, that makes the model fit the given UN target, is determined by solving (for $p$) the associated remaining $CO_2$ budget equation:

$$\int_{t_0}^{\infty} P(t)dt = E_0 \ t_0^p \int_{t_0}^{\infty} t^{-p}dt = t_0E_0/(p-1) = R. \tag{11}$$

Unfortunately, the emissions could not be made as low as 0.1 ($GtCO_{2)}$ even for the 1.5°C target and after six centuries of reduction.

However, an exponential decrease of $CO_2$ emissions would not only ensure their rapid reduction (as recommended in the 2015 Paris Agreement, Art. 4), but will also tolerate very low emissions, relatively earlier (compared with the power model $P$), that will disappear in far future, which could maintain food production, especially in the regions where transition to no-carbon energy might be extremely challenging. This leads to the following 1-phase model for $CO_2$ emissions consistent with a prescribed UN climate target, applicable from time $t = t_0 < h$:

$$\boldsymbol{E_1(t) \approx E_0 e^{-\alpha_1(t - t_0)}, \quad t \ge t_0, \quad \alpha_1 = E_0/R, \quad E_0 = E_0(t_0), R = R(t_0)} \tag{12}$$

Indeed, to ensure an exponential decrease of the annual amount of $CO_2$ emissions from the initial level $E_0$, the model $E_1$ must satisfy the initial-value problem:

$$dE_1/dt = -\alpha_1 E_1 (\alpha_1 > 0), \quad E_1(t_0) = E_0 \tag{13}$$

which unique solution is in the form given in (10). Now, from the remaining $CO_2$ budget equation:

$$\int_{t_0}^{\infty} E_1(t)dt = R, \tag{14}$$

it follows that $E_0/\alpha_1 = R$, which gives $\alpha_1$ as in (12).

Although the model $E_1$ appears to better fit the UN climate goal, in comparison with the linear and power pathways, the abrupt reduction of $CO_2$ emissions may threaten fundamental industries such as food production. To overcome this risk, an alternate parametrized model $E_\gamma(0 < \gamma < 1)$ would start with a succession of two smooth parabolic junctions (ascending then descending) between the linear growth and the exponential decline as follows:

$$\boldsymbol{E_\gamma(t) \approx \begin{cases} A_1(t-u)^2 + B, & t_0 \le t < u \\ A_2(t-u)^2 + B, & u \le t < v \\ E_0\ e^{-\alpha(t-v)}, & t \ge v \end{cases}} \tag{15}$$

$$\begin{aligned} &\alpha = E_0/\gamma R, \quad R = R(t_0) \\ &A_1 = -\alpha_0/2\epsilon, \quad A_2 = -\alpha E_0/2\delta, \quad B = E_0 + \alpha_0\epsilon/2, \quad \alpha_0 \ \text{ as in } \ (6) \\ &u = t_0 + \epsilon, \quad v = u + \delta, \quad \epsilon = \alpha\delta E_0/\alpha_0 \\ &\delta = \left(-b + \sqrt{\Delta}\right)/2a, \quad \Delta = b^2 + 12a\alpha_0(1-\gamma)R \\ &a = \alpha E_0(\alpha E_0 + \alpha_0), \quad b = 3\alpha E_0(E_0 + 1) \end{aligned} \tag{16}$$

The free parameter $\gamma\,(0 < \gamma < 1)$ is introduced to split the remaining budget into two parts; one $(\gamma R)$ will go for the exponential reduction and the other $((1-\gamma)R)$ for the quadratic transition requiring two consecutive time periods; $\varepsilon$ to slow the emissions, then $\delta$ for their initial reduction. Let $u = t_0 + \epsilon$, $v = u + \delta$ be the ending times of these periods. The coefficient $\alpha$ can be determined the same way as $\alpha_1$ in (12) using $\int_v^\infty E_\gamma(t)dt = \gamma R$ (instead of $R$). On the other hand, the coefficients $A_1$, $A_2$, and $B$ follow immediately from (2) applied with $(x_0, y_0) = (t_0, E_0)$, $m = \frac{dE_\gamma}{dt}\big|_{t=t_0} = \frac{dE_0}{dt}\big|_{t=t_0} = \alpha_0$ (for a smooth slowdown) to get $A_1$ and $B$, as given in (16), then with $(x_0, y_0) = (v, E_0)$, $m = \frac{dE_\gamma}{dt}\big|_{t=v} = -\alpha E_0$ (for a smooth transition to the exponential decline) to get $A_2$ as in (16) and another formulation of $B\ (B = E_0 + \alpha\delta E_0/2)$. Then, by equating the two expressions of $B$, one gets the announced formula for $\epsilon$. As for $\delta$, it is found to be the unique positive solution of the following quadratic equation:

$$ax^2 + bx - 3\alpha_0(1-\gamma)R = 0 \tag{17}$$

which discriminant is given by: $\Delta = b^2 + 12a\alpha_0(1-\gamma)R$, where $a$ and $b$ are as announced with (16). Indeed, by evaluating the integrals in the remaining $CO_2$ budget equation:

$$\int_{t_0}^\infty E_\gamma(t)dt = \int_{t_0}^u E_\gamma(t)dt + \int_u^v E_\gamma(t)dt + \int_v^\infty E_\gamma(t)dt = R \tag{18}$$

one gets:

$$\left((A_1/3)\epsilon^3 + B\epsilon\right) + \left((A_2/3)\delta^3 + B\delta\right) + (\gamma R) = R \tag{19}$$

Then by plugging the expressions of $A_1$, $A_2$, $B$, and $\epsilon$ into (19) then simplifying, one gets the following quadratic equation in $\delta$:

$$\left(\alpha^2 E_0^2/(3\alpha_0) + \alpha E_0/3\right)\delta^2 + \left(E_0 + \alpha E_0^2/\alpha_0\right)\delta - (1-\gamma)R = 0 \tag{20}$$

or equivalently,

$$\alpha E_0(\alpha E_0 + \alpha_0)\delta^2 + 3\alpha E_0(E_0 + 1)\delta - 3\alpha_0(1-\gamma)R = 0 \tag{21}$$

which gives (17) with $x = \delta$.

On the other hand, $\Delta > 0$, and more precisely $\Delta > b^2$, and hence $\delta$, as given in (16), is the unique positive solution of Eq. (17).

Based on the model formulated in (15), **Table 1** provides an estimation of the expected reduction (in % below the 2000 level) of $CO_2$ emissions, due to a smoothly-implemented exponential mitigation starting by 2020, considering two different years (2050 and 2100), two climate targets (1.5 and 1.8°C), and two $\gamma$ values (0.4 and 0.6). For example, whereas the emissions consistent with the 1.8°C target (for both $\gamma$s) will be still above the 2000 record in 2050, the 1.5°C-pathway projects, for the same year, their reduction by 39% for $\gamma = 0.6$ and by 46% for $\gamma = 0.4$. However, the latter predicts for 2050 a similar reduction as half of the IPCC-1.5°C scenarios (70–90% below 2010 record, [5]) for $0.11 \le \gamma < 0.59$ (long to medium transition), and as the other half (95% or more below 2010 record, [17, 21]), for $\gamma < 0.11$ (long transition). More generally, as it can be seen from the rate of decline $(-\alpha E_0)$, with $\alpha$ as in (16), a lower target or a longer transition will require a faster reduction. See also **Figure 2** for the effect of transition length.

### 4.2.1 Notes

i. The parameter $\gamma$ represents the fraction of the remaining budget to be used during the exponential reduction of $CO_2$ emissions. Consequently, the remaining fraction $(\gamma - 1)$ will go for the transition period. Therefore, the closer to 1$\gamma$ is, the less $CO_2$ will be emitted during the transition, and the shorter the transition will be; about 14 years (resp. 4 years) with $\gamma = 0.9$, compared to about 25 years (resp. 7 years) with $\gamma = 0.8$, for the climate target 1.8°C (resp. 1.5°C). In the limit case where $\gamma = 0$ (no reduction because of no climate mitigation), the model $E_\gamma$ degenerates into the linear pathway $E_0$ (given in (6)). However, in the other limit case where $\gamma = 1$ (no transition), the model is simply reduced to the exponential pathway $E_1$ (given in (12)).

ii. For all pathways $E_\gamma$, the two transition phases cannot have the same duration, i.e., there is no parameter $\gamma$ for which $\epsilon = \delta$. Indeed, if this were the case, one would necessary have $\gamma = E_0^2/(\alpha_0 R)$, and this would imply that, for any climate target $\mathcal{L}$, the corresponding remaining budget $R$ (which is an increasing function of $\mathcal{L}$) would be bounded below (by the constant $E_0^2/\alpha_0$).

| UNCT (°C) | $R$ ($GtCO_2$) | Reduction by 2050 (%)[a] | | Reduction by 2100 (%)[a] | |
|---|---|---|---|---|---|
| 1.8 | 2929 | None[b] | (None)[c] | 35.2[b] | (46.2)[c] |
| 1.5 | 1155 | 39.0[b] | (45.8)[c] | 97.2[b] | (99.5)[c] |

[a]*Reduction (in %) of $CO_2$ emissions below 2000 level.*
[b]*40% of remaining budget to be used for transition to exponential decline* ($\gamma = 0.6$).
[c]*60% of remaining budget to be used for transition to exponential decline* ($\gamma = 0.4$).

**Table 1.**
*Remaining $CO_2$ budget (R) from 2020 and projected reduction of $CO_2$ emissions for 2050 and 2100 due to a global mitigation (starting by 2020) consistent with the UN climate target (UNCT).*

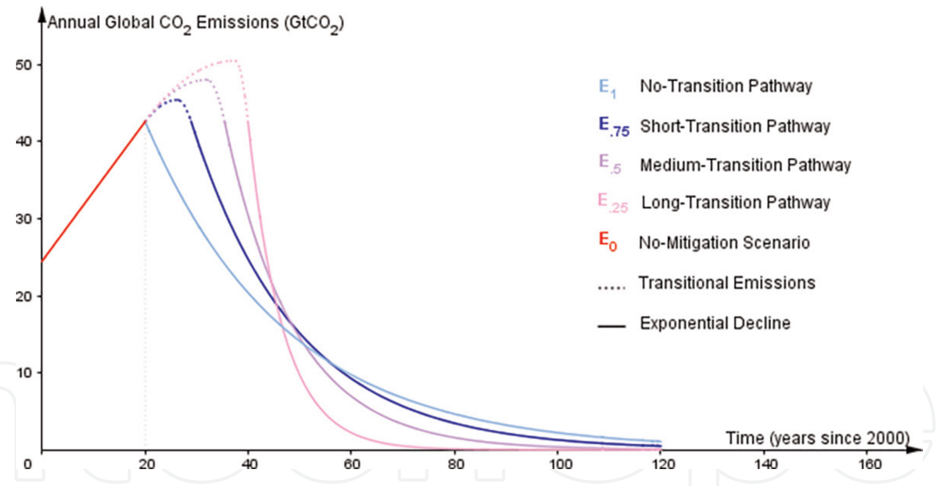


**Figure 2.**
*$CO_2$ emissions pathways consistent with the UN climate target 1.5°C (mitigation starting by 2020).*

| Pathway $E_\gamma$ | Slowdown (years) | 2080 | 2090 | 2100 |
|---|---|---|---|---|
| $\gamma = 1$ | 0 | 4.7 | 3.2 | 2.2 |
| $.7 \leq \gamma \leq .9$ | 3–7 | 3.1–4.3 | 1.8–2.8 | $1.1 - 1.9$ |
| $.4 \leq \gamma \leq .6$ | 10–14 | 0.8–2.4 | $.3 - 1.3$ | $.1 - .7$ |
| $.1 \leq \gamma \leq .3$ | 16–21 | $3(10^{-5}) - .9$ | $8(10^{-7}) - .08$ | $2(10^{-8}) - .04$ |

**Table 2.**
*Projected level of $CO_2$ emissions ($GtCO_2$) consistent with the 1.5°C target by the end of the current century (global mitigation starting by 2020).*

But this would be in contradiction with the case when $\mathcal{L} \leq 1.65$, for which $R < 2001 < E_0^2/\alpha_0$. More generally, for any pathway $E_\gamma$, the times it will take the two transition phases cannot be proportional. This is due to the fact that their ratio $\delta/\epsilon$ is a non-constant function of the climate target $\mathcal{L}$, as it can be seen from the formula: $\epsilon/\delta = E_0^2/(\alpha_0 \gamma R)$. This same formula shows that, for a long transition $(0 < \gamma < 0.3)$, the first phase (quadratic slowdown) will be much longer than the second (quadratic reduction). However, for a short transition $(0.7 < \gamma < 1)$, the first phase will be longer for low targets (e.g., 5 vs. 2.3 years, for 1.5°C and $\gamma = 0.8$) but shorter for higher targets (e.g., 11.5 vs. 13.4 years, for 1.8°C and $\gamma = 0.8$). The case of the 1.5°C target is illustrated in **Table 2** and **Figure 2**, where one can see that the longer the transition (decreasing $\gamma$) the longer the slowdown (increasing $\epsilon$) and the higher the peak of emissions (coefficient B).

iii. Whereas the model $E_\gamma$ for the 1.8°C target seems to be close to the (IPCC) RCP4.5, the 1.5°C version appears to be similar to the (IPCC) RCP2.6 and no- and low-overshoot over the first 30 years (see **Figure 2**), with low emissions (<1 $GtCO_2$), e.g., by 2080 for $\gamma < 0.43$ (see **Table 2**), the same way as the no-overshoot and half of the RCP2.6 models [17, 21], or earlier, e.g., by 2050 for $\gamma < 0.08$) with the advantage of predicting the nearly-zero emission (<0.01 $GtCO_2$), e.g., by 2090 for $\gamma < 0.22$ (see **Table 2**), or even as early as 2050 for $\gamma < 0.03$.

### 4.3 Ideal smooth pathways for $CO_2$ emissions consistent with a prescribed UN climate target

Whereas the model $E_\gamma$ $(0 < \gamma < 1)$ is designed to fit a prescribed UN climate target $\mathcal{L}$°C $(1.5 \leq \mathcal{L} < 2)$, in the sense that the cumulative $CO_2$ emissions will not raise the global average temperature by more than $\mathcal{L}^{\circ}$C above the pre-industrial level, its consistency turns out to be higher for more binding targets. Indeed, a numerical investigation (**Table 3**), based on specific criteria, shows that the lower the target the better the fit with the target. More precisely, there are (uncountable) many more $E_\gamma$s, compatible with lower targets, that peak below 2.05 times (if not twice) the 2000 record (Criterion $C_1$) project a reduction by at least 50% in 2050 (relative to 2000 level) (Criterion $C_2$) and predict nearly-zero emission ( $\leq 0.01$ $GtCO_2$) by 2100 (Criterion $C_3$). From an analytical point of view, what is said about criterion ($C_1$) is due to the decrease of the peak (coefficient $B$ in the model $E_\gamma$) with decreasing remaining $CO_2$ budget $R$, and therefore with decreasing target $\mathcal{L}$. As for the comparison based on ($C_2$) and ($C_3$), this can be explained by the fact that the exponential decline will start more rapidly for a lower remaining budget $R$ (due to a lower target), as it can be seen from the formula of the initial speed of the exponential reduction (at time $t = v$):

$$\left|\frac{dE_\gamma}{dt}\right| = \alpha E_0 = E_0^2/(\gamma R) \tag{22}$$

(using the formula of $\alpha$ given in (16)).

Consequently, the $E_\gamma$s ($.24 < \gamma < .27$) appear to be the most consistent smooth pathways with the 1.5°C target, and among these, the $E_{.26}$ would be the ideal one as it satisfies the three criteria and predicts the lowest peak of emissions (by 2037), with a (constant) relative rate of exponential decline estimated at $\alpha \approx 14.2\%$. However, for more binding climate targets such as 1.4°C, which corresponding global mitigation needs to be implemented before 2034 (according to (5)), it turns out that half of the models $E_\gamma$, namely those with $0.01 < \gamma < 0.51$, meet all criteria to ideally fit this target, and the ideal one of them would be the $E_{.5}$, for the same reason as the $E_{.26}$ for the 1.5°C target, with a peak of emissions by 2028 and a (constant) relative rate of exponential decline estimated at $\alpha \approx 11.8\%$. See **Figure 3** for graphical illustration.

Nevertheless, if the 'zero' emission timing is prioritized over the peaking threshold, it is found that, among the $E_\gamma$s, those with $\gamma < 0.00067$ (resp. 0.00019) project the earliest 'zero' emission; by 2043 (resp. 2035) for the 1.5°C (resp. 1.4°C) target, with a peak estimated at 2.14 times (resp. twice) the 2000 level. But the interval between the peaking and the almost-zero moments seems to be extremely short; 2 months and 10 days respectively, making the curve look like a vertical line over this interval, which sounds rather unrealistic. However, feasible ideal $E_\gamma$s for the earliest 'zero' emission could be found by considering higher values of parameter $\gamma$ and time $z$ (as close as possible to the unrealistic 'zero' emission moment, as found previously) that satisfy the following conditions:

$$z - u > \max(\delta, \epsilon/3), \quad E_\gamma(z) < 0.01 \tag{23}$$

| UNCT (°C) | ($C_1$)-Pathways[a] | ($C_2$)-Pathways[b] | ($C_3$)-Pathways[c] |
|---|---|---|---|
| 1.6 | $\gamma > 0.55$ (0.46) | None | None |
| 1.5 | $\gamma > 0.37$ (0.24) | $\gamma < 0.34$ | $\gamma < 0.27$ |
| 1.4 | $\gamma > 0.01$ (All) | All | $\gamma < 0.51$ |

[a] *Pathways $E_\gamma$ for $CO_2$ emissions peaking below twice (2.05 times) 2000 level.*
[b] *Pathways $E_\gamma$ for $CO_2$ emissions reduced by at least 50% in 2050 (rel. to 2000 level).*
[c] *Pathways $E_\gamma$ for $CO_2$ emissions reduced to almost zero (below 0.01 $GtCO_2$) by 2100.*

**Table 3.**
*Ideal smooth pathways for $CO_2$ emissions by UN climate target (UNCT) (global mitigation starting by 2020).*

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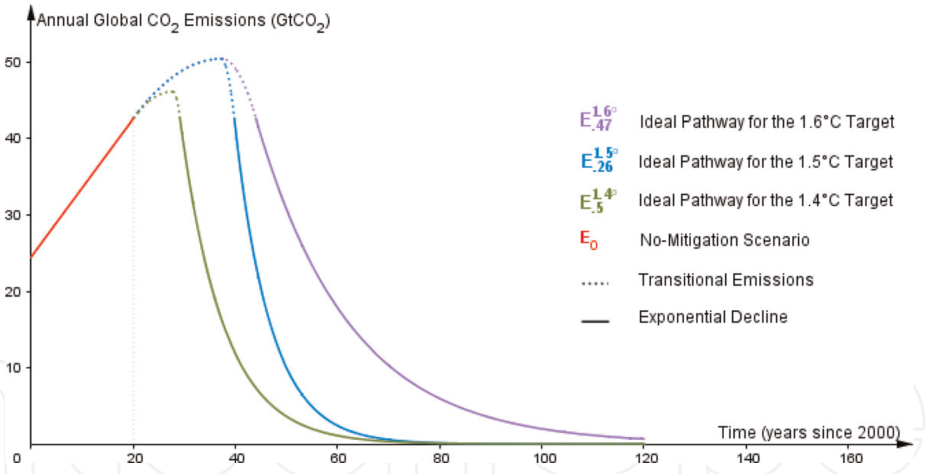


**Figure 3.**
*Ideal smooth pathways for $CO_2$ emissions peaking below 2.05 times (if not twice) 2000 level and shrinking below 0.01 $GtCO_2$ by 2100 (preferably); climate targets 1.6, 1.5 and 1.4°C (mitigation starting by 2020).*

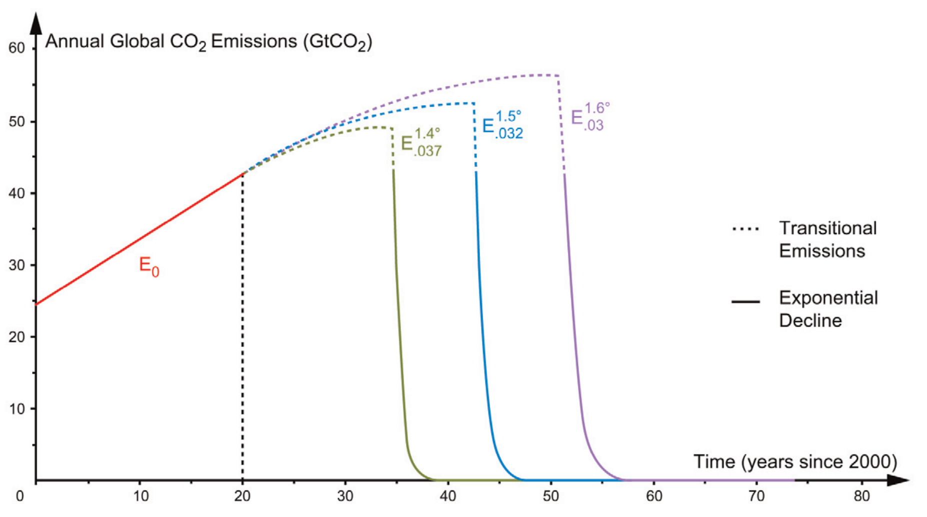


**Figure 4.**
*Ideal smooth pathways, for the earliest feasible 'zero'-$CO_2$-emission; climate targets 1.6, 1.5 and 1.4°C (mitigation starting by 2020).*

As a result, the earliest feasible almost-zero emission will occur by 2061, 2050, and 2040 for the respective climate targets 1.6, 1.5, and 1.4°C, with the $E_\gamma$s induced by $.0179 < \gamma < .0301$, $.0163 < \gamma < .033$, and $0 < \gamma < .038$ respectively. **Figure 4** shows the most feasible of these pathways, with time periods between the peaking and the almost-zero moments estimated at 11, 8, and 6 years respectively.

An alternative ideal pathway could be made by juxtaposing the lowest restrictions of $E_1$ and one of the ideal $E_\gamma$s consistent with the same climate target. But the resulting pattern would include two singularities (cusp shape); one in the beginning of the mitigation and another at the junction between $E_1$ and $E_\gamma$.

## 5. Conclusions

Smooth pathways for $CO_2$ emissions are designed taking into consideration not only their consistency with the UN climate targets but also the rapidity that has been urged by the UN for their reduction. Unlike the existing models, mostly produced by computer simulation such as the (IPCC) RCPs, a mathematical modelling, as an ideal host of interpolation and smoothing techniques, is presented throughout this chapter. First, the global warming is quantified with time to determine the moment when a prescribed UN climate target will be hit (in case of no climate mitigation), which is then used to explicitly determine the remaining $CO_2$ budget; crucial parameter in emissions modelling. Naturally, an exponential pattern is proposed at first for its rapid decline and long-term stabilization slightly above zero. Then, by means of quadratic interpolations, a parametrized collection of flexible pathways $E_\gamma$ $(0 < \gamma < 1)$ is derived to ensure more feasibility by including a smooth transition to the exponential trend, which will help compensate a certain lack of no-carbon energy. It turns out that the no-transition (exponential) and no-mitigation (linear) models correspond to the limit values of the involved parameter $\gamma$ introduced as an arbitrary fraction of the remaining $CO_2$ budget expected to be used during the exponential phase, which also gives an indication for the transition length.

Graphically, the $E_\gamma$s are comparable to the corresponding IPCC pathways; similar to the RCP4.5, for targets between 1.5 and 2°C, and to the RCP2.6 and no- and low-overshoot, for the 1.5°C target. However, they have the advantage of predicting the nearly-zero emission ($<0.01$ $GtCO_2$), e.g., by 2090 for $\gamma < 0.22$, or even as early as 2050 for $\gamma < 0.03$, with no need for $CO_2$ removal. Such similarities could be improved by using the IPCC estimation for the remaining $CO_2$ budget (though determined with high uncertainties), which may lead to more representative pathways by involving further greenhouse gases.

Another virtue of the designed $E_\gamma$s is their flexibility with regards to the constraints that would come with the climate target, which would provide climate policy makers with an uncountable set of ideal smooth pathways enlarging with decreasing target. For instance, whereas $E_\gamma$s with $0.24 < \gamma < 0.27$ are recommended for the 1.5°C target, based on specific criteria including the peaking threshold, those with $0.01 < \gamma < 0.51$ are recommended for a more binding target; the 1.4°C one. When it comes to the projection of the earliest feasible 'zero' emission, are recommended the $E_\gamma$s with $0.017 < \gamma < 0.033$ and $0 < \gamma < 0.038$, for the respective climate targets 1.5, and 1.4°C, which would result in the near extinction of $CO_2$ emissions by 2050 and 2040 respectively.

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## Conflict of interest

The author declares no conflict of interest.

## Abbreviation

| | |
|---|---|
| CDIAC | carbon dioxide information analysis center |
| $CO_2$ | carbon dioxide |

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| | |
|---|---|
| $GtCO_2$ | gigatons of $CO_2$ |
| IPCC | Intergovernmental Panel on Climate Change |
| NASA | National Aeronautics and Space Administration |
| RCP | representative concentration pathway |
| UN | United Nations |
| UNCT | UN climate target |

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