Chapter

Folding on the Chaotic Graph Operations and Their Fundamental Group

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Abstract

Our aim in the present chapter is to introduce a new type of operations on the chaotic graph, namely, chaotic connected edge graphs under the identification topology. The concept of chaotic foldings on the chaotic edge graph will be discussed from the viewpoint of algebra and geometry. The relation between the chaotic homeomorphisms and chaotic foldings on the chaotic connected edge graphs and their fundamental group is deduced. The fundamental group of the limit chaotic chain of foldings on chaotic. Many types of chaotic foldings are achieved. Theorems governing these relations are achieved. We also discuss some applications in chemistry and biology.

Keywords: chaotic graph, edge graph, chaotic folding, limit folding fundamental group

2010 Mathematics Subject Classification: 51H20, 57N10, 57M05, 14F35, 20F34

1. Introduction and definitions

During the past few decades, examinations of social, biological, and communication networks have taken on enhanced attention throughout these examinations; graphical representations of those networks and systems have been evident to be terribly helpful. Such representations are accustomed to confirm or demonstrate the interconnections or relationships between parts of those networks [1, 2].

A graph is an ordered G = (V(G), E(G)) where V(G) $\neq \varphi$, E(G) is a set disjoint from V(G), elements of V(G) are called the vertices of G, and elements of E(G) are called the edges. The foundation stone of graph theory was laid by Euler in 1736 by solving a puzzle called Königsberg seven-bridge problem as in **Figure 1** [1, 3].

There are many graphs with which one can construct a new graph from a given graph or set of graphs, such as the Cartesian product and the line graph. A graph G is a finite non-empty set V of objects called vertices (the singular is vertex) together with a set E of two-element subsets of V called edges. The number of vertices in a graph G is the order of G, and the number of edges is the size of G. To indicate that a graph G has vertex set V and edge set E, we sometimes write G = (V, E). To emphasize that V is the vertex set of a graph G, we often write V as V(G). For the same reason, we also write E as E(G). A graph H is said to be a subgraph of a graph G if V(H) $\subseteq$ V(G) and E(H) $\subseteq$ E(G). The complete graph with n-vertices will be denoted by K_n. A null graph is a graph containing no edges; the null graph with

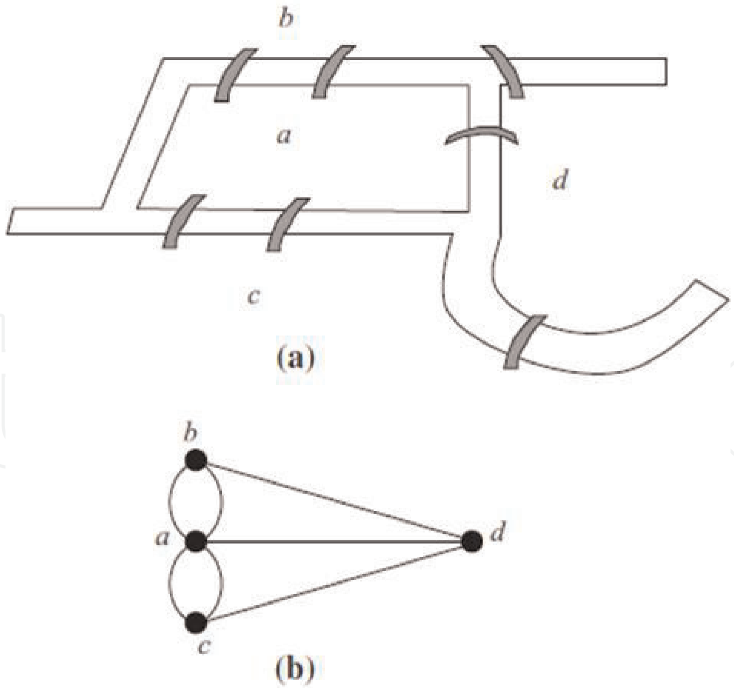

Figure 1.
Königsberg seven-bridge problem.

n-vertices is denoted by N_n. A cycle graph is a graph consisting of a single cycle, the cycle graph with n-vertices is denoted by C_n. The path graph is a graph consisting of a single path; the path graph with n-vertices is denoted by P_n [1–11]. Let G and H be two graphs. A function $\varphi : V(G) \to V(H)$ is a homomorphism from G to H if it preserves edges, that is, if for every edge $e \in E(G), f(e) \in E(H)$ [12, 13]. A core is a graph which does not retract to a proper subgraph. Any graph is homomorphically equivalent to a unique core [7].

The folding is a continuous function $f : G \to H$ such that for each $v \in V(G), f(v) \in V(H),$ and for each $e \in E(G),\ f(e) \in E(H)$ [14]. Let X be a space, and let I be the unit interval [0,1] in R, a homotopy of paths in X is a family $g_t : I \to X,\ 0 \le t \le 1$such that (i) the endpoints $g_t(0) = x_0$ and $g_t(1) = x_1$ are independent of t and (ii) the associated map $G : I \times I \to X$ defined by $G(s,t) = g_t(s)$ is continuous [15]. Given spaces X and Y with chosen points $x_0 \in X,$ and $y_0 \in Y,$ the wedge sum $X \vee Y$ is the quotient of the disjoint union $X \cup Y$ obtained identifying x_0 and y_0 to a single point [15]. Two spaces X and Y are of the "same homotopy type" if there exist continuous maps $f : X \to Y$ and $g : Y \to X$ such that $g \circ f \cong I_X : X \to X$ and $f \circ g \cong I_Y : Y \to Y$ [16]. The fundamental group briefly consists of equivalence classes of homotopic closed paths with the law of composition following one path to another. However, the set of homotopy classes of loops based at the point x_0 with the product operation $[f][g] = [f \cdot g]$ is called the fundamental group and denoted by $\pi_1(X, x_0)$ [4, 17–24]. Over many years, chaos has been shown to be an interesting and even common phenomenon in nature. Chaos has been shown to exist in a wide variety of settings: in fluid dynamics such as Raleigh-Bernard convection, in chemistry such as the Belousov-Zhabotinsky reaction, in nonlinear optics in certain lasers, in celestial mechanics, in electronics in the flutter of an overdriven airplane wing, some models of population dynamics, and likely in meteorology, physiological oscillations such as certain heart rhythms, as well as brain patterns [17, 24–30]. AI algorithms related to adjacency matrices on the operations of the graph are discussed in [31, 32].

2. The main results

First, we will introduce the following:

Definition 1. *The chaotic edge* $\bar{e}$ *is a geometric edge* e_1 *that carries many other edges* $(e_2, e_3, \dots)$*, each one of them homotopic to the original one as in* **Figure 2**. *Also the chaotic vertices of* $\bar{e}$ *are* $\bar{v} = (v_1, v_2, \dots)$ *and* $\bar{u} = (u_1, u_2, \dots)$. *For chaotic edge* $\bar{e}$*, we have two cases*:

Case 1 (1) $e_1, e_2, e_3, \dots$ *are of the same physical properties*.

Case 2 (2) $e_1, e_2, e_3, \dots$ *represent different physical properties; for example,* e_1 *represents density,* e_2 *represents hardness,* e_3 *represents magnetic fields, and so on*.

Definition 2. *A chaotic graph* $\overline{G}$ *is a collection of finite non-empty set* $\overline{V}$ *of objects called chaotic vertices together with a set* $\overline{E}$ *of two-element subsets of* $\overline{V}$ *called chaotic edges. The number of chaotic edges is the size of* $\overline{G}$.

Definition 3. *Given chaotic connected graphs* $\overline{G}_1$ *and* $\overline{G}_2$ *with given edges* $\bar{e}_1 \in \overline{G}_1$ *and* $\bar{e}_2 \in \overline{G}_2$*, then the chaotic connected edge graph* $\overline{G}_1 \ \ \overline{G}_2$ *is the quotient of disjoint union* $\overline{G}_1 \cup \overline{G}_2$ *acquired by identifying two chaotic edges* $\bar{e}_1$ *and* $\bar{e}_2$ *to a single chaotic edge (up to chaotic isomorphism) as in* **Figure 3**.

Definition 4. *A chaotic graph* $\overline{H}$ *is called a chaotic subgraph of a chaotic graph* $\overline{G}$ *if* $\overline{V}(\overline{H}) \subseteq \overline{V}(\overline{G})$ *and* $\overline{E}(\overline{H}) \subseteq \overline{E}(\overline{G})$.

Definition 5. *Let* $\overline{G}$ *and* $\overline{H}$ *be two chaotic graphs. A function* $\overline{\varphi} : \overline{V}(\overline{G}) \to \overline{V}(\overline{H})$ *is chaotic homomorphism from* $\overline{G}$ *to* $\overline{H}$ *if it preserves chaotic edges, that is, if for any chaotic edge* $[\bar{u}, \bar{v}]$ *of* $\overline{G}$, $[\overline{\varphi}(\bar{u}), \overline{\varphi}(\bar{v})]$ *is a chaotic edge of* $\overline{H}$.

Definition 6. *A chaotic folding of a graph* $\overline{G}$ *is a chaotic subgraph* $\overline{H}$ *of* $\overline{G}$ *such that there exists a chaotic homomorphism* $\bar{f} : \overline{G} \to \overline{H}$, *called chaotic folding with* $\bar{f}(\bar{x}) = \bar{x}$ *for every chaotic vertex* $\bar{x}$ *of* $\overline{H}$.

Definition 7. *A chaotic core is a chaotic graph which does not chaotic retract to chaotic proper subgraph*.

Theorem 1. *Let* $\overline{G}_1$ *and* $\overline{G}_2$ *be two chaotic connected graphs. Then* $\bar{\pi}_1(\overline{G}_1 \veebar \overline{G}_2) = \bar{\pi}_1(\overline{G}_1) * \bar{\pi}_1(\overline{G}_2)$.

Proof. Let $\overline{G}_1$ and $\overline{G}_2$ be two chaotic connected graphs. Since $\overline{G}_1 \veebar \overline{G}_2$ and $\overline{G}_1 \vee \overline{G}_2$ are of same chaotic homotopy type, it follows that $\bar{\pi}_1(\overline{G}_1 \veebar \overline{G}_2) \approx \bar{\pi}_1(\overline{G}_1) * \bar{\pi}_1(\overline{G}_2)$. Hence, $\bar{\pi}_1(\overline{G}_1 \veebar \overline{G}_2) = \bar{\pi}_1(\overline{G}_1) * \bar{\pi}_1(\overline{G}_2)$.

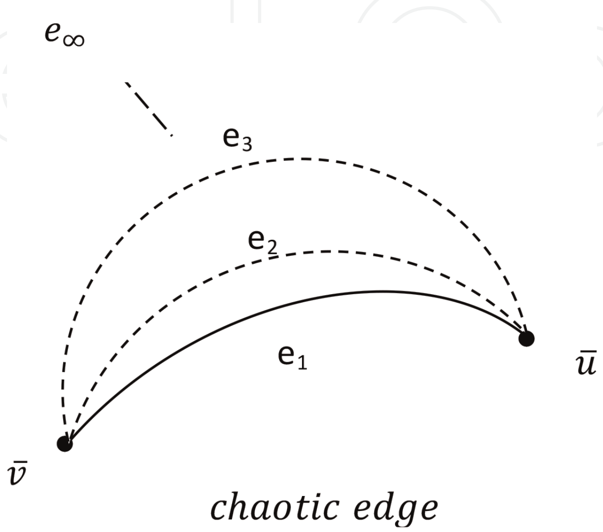

Figure 2.
Chaotic edge.

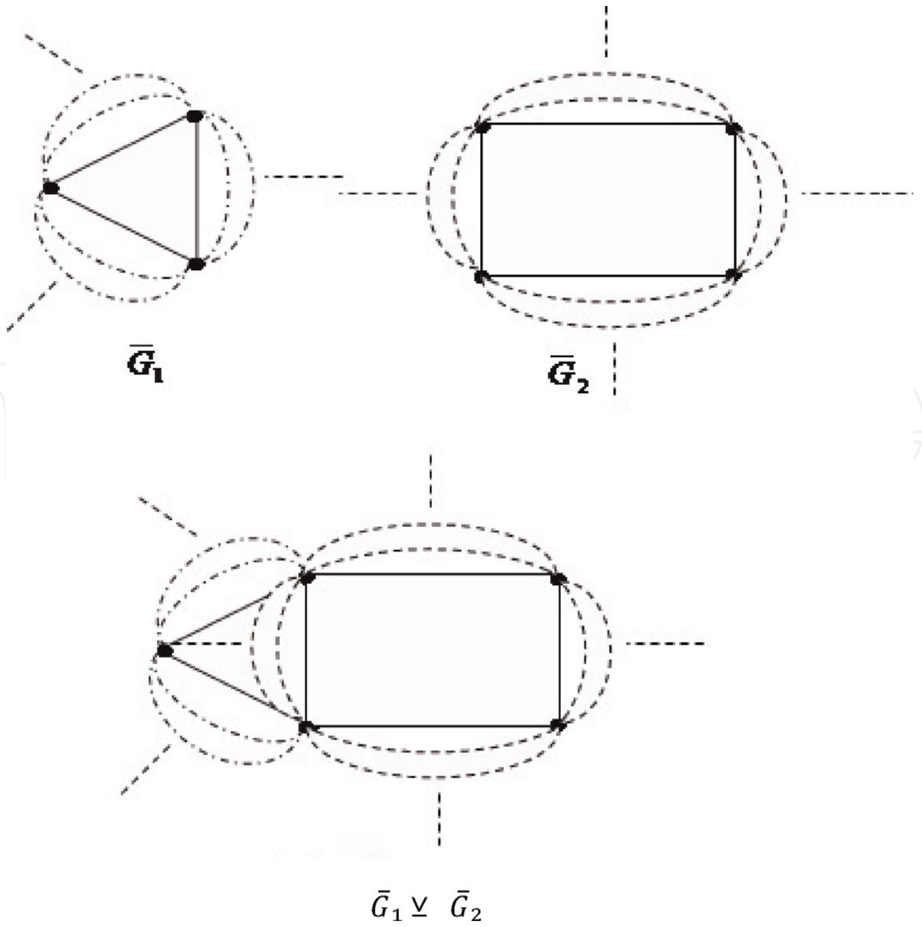

Figure 3.
Chaotic connected edge.

Theorem 2. *The chaotic graphs $\overline{G}_1$ and $\overline{G}_2$are chaotic subgraphs of $\overline{G}_1 \veebar \overline{G}_2$. Also, for any chaotic tree $\overline{G}_1$ and $\overline{G}_2$, $\overline{G}_1 \veebar \overline{G}_2$ is also chaotic tree and $\overline{\pi}_1\left(\overline{G}_1 \veebar \overline{G}_2\right) = \overline{0}$.*

Proof. The proof of this theorem is clear.

Theorem 3. *If $\overline{G}_1, \overline{G}_2, \dots, \overline{G}_n$ are connected graphs, and $\left\langle \overline{f}_1, \overline{f}_2, \dots, \overline{f}_n \right\rangle$ is a sequence of chaotic topological foldings of $\veebar_{i=1}^{n} \overline{G}_i$ into itself, then there is an induced sequence $\left\langle \overline{\overline{f}}_1, \overline{\overline{f}}_2, \dots, \overline{\overline{f}}_n \right\rangle$ of non-trivial chaotic topological folding $\overline{\overline{f}}_j : *_{i=1}^{n} \overline{\pi}_1\left(\overline{G}_{ii}\right) \to *_{i=1}^{n} \overline{\pi}_1\left(\overline{G}_{ii}\right)$, $j = 1, 2, \dots, n$ such that $\overline{\overline{f}}_j\left(*_{i=1}^{n} \overline{\pi}_1\left(\overline{G}_{ii}\right)\right)$ reduces the rank of $*_{i=1}^{n} \overline{\pi}_1\left(\overline{G}_{ii}\right)$.*

Proof. Consider the following sequence of topological foldings $\left\langle \overline{f}_1, \overline{f}_2, \dots, \overline{f}_n \right\rangle$, where $\overline{f}_1 : \veebar_{i=1}^{n} \overline{G}_i \to \veebar_{i=1}^{n} \overline{G}_i$, is a topological folding from $\vee_{i=1}^{n} \overline{G}_i$ into itself such that $\overline{f}_1\left(\veebar_{i=1}^{n} \overline{G}_i\right) = \overline{G}_1 \veebar \overline{G}_2 \veebar \dots \veebar \overline{f}_1\left(\overline{G}_s\right) \veebar \dots \veebar \overline{G}_n$ *for* $s = 1, 2, \dots n$.

Since *size*$\left(\overline{f}_1\left(\overline{G}_s\right)\right) \le$ *size*$\left(\overline{G}_s\right)$ and $\overline{\overline{f}}_1\left(\overline{\pi}_1\left(\overline{G}_i\right)\right) = \overline{\pi}_1\left(\overline{f}_1\left(\overline{G}_i\right)\right)$, it follows that $\text{rank}\left(\overline{\overline{f}}_1\left(\overline{\pi}_1\left(\overline{G}_s\right)\right)\right) = \text{rank}\left(\overline{\pi}_1\left(\overline{f}_1\left(\overline{G}_s\right)\right)\right) \le \text{rank}\left(\overline{\pi}_1\left(\overline{G}_s\right)\right)$, and so $\overline{\overline{f}}_1$ reduces the rank of $*_{i=1}^{n} \overline{\pi}_1\left(\overline{G}_{ii}\right)$. Also, if $\overline{f}_2\left(\veebar_{i=1}^{n} \overline{G}_i\right) = \overline{G}_1 \veebar \overline{G}_2 \veebar \dots \veebar \overline{f}_2\left(\overline{G}_s\right) \veebar \dots \veebar \overline{f}_2\left(\overline{G}_k\right) \veebar \dots \veebar \overline{G}_n$ *for* $k = 1, 2, \dots n$ and $s < k$ and *size*$\left(\overline{f}_2\left(\overline{G}_s\right)\right) \le$ *size*$\left(\overline{G}_s\right)$ *and size*$\left(\overline{f}_2\left(\overline{G}_k\right)\right) \le$ *size*$\left(\overline{G}_k\right)$, we haverank$\left(\overline{\overline{f}}_2\left(\overline{\pi}_1\left(\overline{G}_s\right)\right)\right) = \text{rank}\left(\overline{\pi}_1\left(\overline{f}_2\left(\overline{G}_s\right)\right)\right) \le \text{rank}\left(\overline{\pi}_1\left(\overline{G}_s\right)\right)$, $\text{rank}\left(\overline{\overline{f}}_2\left(\overline{\pi}_1\left(\overline{G}_k\right)\right)\right) = \text{rank}\left(\overline{\pi}_1\left(\overline{f}_2\left(\overline{G}_k\right)\right)\right) \le \text{rank}\left(\overline{\pi}_1\left(\overline{G}_k\right)\right)$; thus $\overline{\overline{f}}_2$ reduces the rank of $*_{i=1}^{n} \overline{\pi}_1\left(\overline{G}_{ii}\right)$. Moreover, by continuing with this procedure if $\overline{f}_n\left(\veebar_{i=1}^{n} \overline{G}_i\right) = \veebar_{i=1}^{n}\left(\overline{f}_n\left(\overline{G}_i\right)\right)$, then $\overline{\overline{f}}_n\left(*_{i=1}^{n} \overline{\pi}_1\left(\overline{G}_{ii}\right)\right) = \overline{\pi}_1\left(\overline{f}_n\left(\veebar_{i=1}^{n} \overline{G}_i\right)\right) = \overline{\pi}_1\left(\veebar_{i=1}^{n} \overline{f}_n\left(\overline{G}_i\right)\right) \approx *_{i=1}^{n} \overline{\pi}_1\left(\overline{f}_n\left(\overline{G}_{ii}\right)\right)$. Hence, $\overline{\overline{f}}_n$ reduces the rank of $*_{i=1}^{n} \overline{\pi}_1\left(\overline{G}_{ii}\right)$.

Theorem 4. *Let $\overline{G}_1$ and $\overline{G}_2$ be two chaotic graphs; then there is a chaotic homomorphism $\overline{f}:\overline{G}_1\to\overline{G}_2$ which induces $\overline{\overline{f}}:\pi_1(\overline{G}_1)\to\pi_1(\overline{G}_2)$ if $\pi_1(\overline{G}_2)$ is a chaotic folding of $\pi_1(\overline{G}_1\veebar\overline{G}_2)$.*

Proof. Let $\overline{f}:\overline{G}_1\to\overline{G}_2$ be a chaotic homomorphism. Since $\overline{G}_2$ is chaotic subgraph of $\overline{G}_1\veebar\overline{G}_2$, then there exists a chaotic homomorphism $\overline{f}:\overline{G}_1\veebar\overline{G}_2\to\overline{G}_2$ with $\overline{f}(\overline{v})=\overline{v}$ for any chaotic vertex $\overline{v}$ of $\overline{G}_2$ which induces $\overline{\overline{f}}:\pi_1(\overline{G}_1)\to\pi_1(\overline{G}_2)$. What follows from $\overline{G}_2$ is a chaotic folding of $\overline{G}_1\veebar\overline{G}_2$ in that $\pi_1(\overline{G}_2)$ is a chaotic folding of $\pi_1(\overline{G}_1\veebar\overline{G}_2)$. Conversely, assume that $\overline{G}_2$ is a chaotic folding of $\overline{G}_1\veebar\overline{G}_2$; thus $\overline{f}:\overline{G}_1\veebar\overline{G}_2\to\overline{G}_2$ is a chaotic homomorphism with $\overline{f}(\overline{v})=\overline{v}$ for any chaotic vertex $\overline{v}$ of $\overline{G}_2$, and so there is a chaotic homomorphism $\overline{f}:\overline{G}_1\to\overline{G}_2$ which induce $\overline{\overline{f}}:\pi_1(\overline{G}_1)\to\pi_1(\overline{G}_2)$.

Theorem 5. *For any chaotic path graphs $\overline{P}_n, \overline{P}_m, n, m\geq 2,$ there is a sequence of topological foldings with variation curvature $\left\{\overline{f}_i : i=1,2,...k\right\}$ on $\overline{P}_n\veebar\overline{P}_m$ which induce a sequence of topological foldings $\left\{\overline{\overline{f}}_i : i=1,2,...k\right\}$ such that $\overline{\overline{f}}_k\left(\overline{\overline{f}}_{k-1}\left(...\left(\overline{\overline{f}}_1\right.\right.\right.$ $(\pi_1(\overline{P}_n\veebar\overline{P}_m)...\}=\overline{Z}$ and $\lim_{k\to\infty}\left(\overline{\overline{f}}_k\left(\overline{\overline{f}}_{k-1}\left(...\left(\overline{\overline{f}}_1\left(\pi_1(\overline{P}_n\veebar\overline{P}_m)...\right)\right)\right)\right)\right)=\overline{0}$.*

Proof. Consider the following sequence of chaotic topological foldings with variation curvature, $\overline{f}_1:\overline{P}_n\veebar\overline{P}_m\to(\overline{P}_n\veebar\overline{P}_m)_1$, where $(\overline{P}_n\veebar\overline{P}_m)_1$ is a chaotic subgraph with decreasing inner curvature between every two adjacent chaotic edges in $\overline{P}_n\veebar\overline{P}_m$ and $\overline{f}_2:\overline{f}_1(\overline{P}_n\veebar\overline{P})_m\to\overline{f}_1((\overline{P}_n\veebar\overline{P}_m)_1)$ where $\overline{f}_2\left(\overline{f}_1((\overline{P}_n\veebar\overline{P}_m)_1)\right)$ is a chaotic subgraph with decreasing inner curvature between every two adjacent chaotic edges in $\overline{f}_1((\overline{P}_n\veebar\overline{P}_m)_1)$, and so on, such that $\overline{f}_k\left(\overline{f}_{k-1}\left(\overline{f}_{k-2}\left(...\left(\overline{f}_1(\overline{P}_n\veebar\overline{P}_m)\cdots\right)\right.\right.\right.=$ $\overline{C}_{n+m-2}$ and $\lim_{k\to\infty}\left(\overline{f}_k\left(\overline{f}_{k-1}\left(\overline{f}_{k-2}\left(...\left(\overline{f}_1(\overline{P}_n\veebar\overline{P}_m)...\right)\right.\right.\right.\right.=\overline{N}_1,$ thus $\overline{\overline{f}}_k\left(\overline{\overline{f}}_{k-1}\left(\overline{\overline{f}}_{k-2}(...\right.\right.$ $\left(\overline{\overline{f}}_1(\pi_1(\overline{P}_n\veebar\overline{P}_m)...\right)=\pi_{11}(\overline{C}_{n+m-2})=\overline{Z}$. Also, $\lim_{k\to\infty}\left(\overline{\overline{f}}_k\left(\overline{\overline{f}}_{k-1}\left(\overline{\overline{f}}_{k-2}\left(...\left(\overline{\overline{f}}_1(\pi_1\right.\right.\right.\right.\right.$ $(\overline{P}_n\veebar\overline{P}_m)...)=\pi_1(\overline{N}_1)=\overline{0}$.

Theorem 6. *For every two chaotic connected graphs $\overline{G}_1$ and $\overline{G}_2$, the fundamental group of the limit of chaotic topological folding of $\overline{G}_1\veebar\overline{G}_2=\overline{0}$.*

Proof. Let $\overline{G}_1$ and $\overline{G}_2$ be two chaotic connected graphs; then we have two cases:

Case (1): If $\overline{f}_1:\overline{G}_1\veebar\overline{G}_2\to\overline{G}_1\veebar\overline{G}_2$ is a chaotic topological folding such that $\overline{f}_1(\overline{G}_1\veebar\overline{G}_2)$ consists of chaotic cycles, so we can define a sequence of chaotic topological folding $\overline{f}_2:\overline{f}_1(\overline{G}_1\veebar\overline{G}_2)\to\overline{f}_1(\overline{G}_1\veebar\overline{G}_2)$ where $\overline{f}_2\left(\overline{f}_1(\overline{G}_1\veebar\overline{G}_2)\right)$ is a chaotic tree with $\overline{n}$chaotic edges, $\overline{f}_3:\overline{f}_2\left(\overline{f}_1(\overline{G}_1\veebar\overline{G}_2)\right)\to\overline{f}_2\left(\overline{f}_1(\overline{G}_1\veebar\overline{G}_2)\right)$, such that $\overline{f}_3\left(\overline{f}_2\left(\overline{f}_1(\overline{G}_1\veebar\overline{G}_2)...\right)\right.$ is a chaotic tree with $\overline{k}<\overline{n}$ chaotic edges, chaotic edges by continuing this process we get $\overline{f}_k:\overline{f}_{k-1}\left(\overline{f}_{k-2}\left(...\left(\overline{f}_1(\overline{G}_1\veebar\overline{G}_2)\cdots\right)\to\right.\right.$ $\overline{f}_{k-1}\left(\overline{f}_{k-2}\left(...\left(\overline{f}_1(\overline{G}_1\veebar\overline{G}_2)\cdots\right)\right.\right.$ such that $\lim_{k\to\infty}\left(\overline{f}_k\left(\overline{f}_{k-1}\left(\overline{f}_{k-2}\left(...\left(\overline{f}_1(\overline{G}_1\veebar\overline{G}_2)...\right)\right.\right.\right.\right.$ is a chaotic edge, and so $\pi_1\left(\lim_{k\to\infty}\left(\overline{f}_k\left(\overline{f}_{k-1}\left(\overline{f}_{k-2}\left(...\left(\overline{f}_1(\overline{G}_1\veebar\overline{G}_2)...\right)\right)\right)\right)\right)\right)=\overline{0}$.

Case (2): If $\overline{g}_1:\overline{G}_1\veebar\overline{G}_2\to\overline{G}_1\veebar\overline{G}_2$ is a chaotic topological folding such that. $\overline{g}_1(\overline{G}_1\veebar\overline{G}_2)$ has no chaotic cycles, then clearly $\lim_{k\to\infty}(\overline{g}_k(\overline{g}_{k-1}(\overline{g}_{k-2}(...(\overline{g}_1$ $(\overline{G}_1\veebar\overline{G}_2)...)$ is a chaotic edge and $\pi_1(\lim_{k\to\infty}(\overline{g}_k(\overline{g}_{k-1}(\overline{g}_{k-2}(...(\overline{g}_1(\overline{G}_1\veebar\overline{G}_2)...))=\overline{0}$.

Theorem 7. *If $\overline{G}_1$ and $\overline{G}_2$ are chaotic connected and not chaotic cores graphs, then $\pi_1\left(\lim_{n\to\infty}\overline{f}_n(\overline{G}_1\veebar\overline{G}_2)\right)=\pi_1\left(\lim_{n\to\infty}\overline{f}_n(\overline{G}_1)\right)*\pi_1\left(\lim_{n\to\infty}\overline{f}_n(\overline{G}_2)\right)$.*

Proof. If $\overline{G}_1$ and $\overline{G}_2$ are chaotic connected and not chaotic cores graphs, then we get the following chaotic induced graphs $\lim_{n\to\infty}\overline{f}_n\left(\overline{G}_1 \underline{\vee}\, \overline{G}_2\right)$, $\lim_{n\to\infty}\overline{f}_n\left(\overline{G}_1\right)$, $\lim_{n\to\infty}\overline{f}_n\left(\overline{G}_2\right)$, and each of them are isomorphic to $\overline{k}_2$. Since $\overline{k}_2 \approx \overline{k}_2 \,\underline{\vee}\, \overline{k}_2$ it follows that $\lim_{n\to\infty}\overline{f}_n\left(\overline{G}_1 \underline{\vee}\, \overline{G}_2\right) = \lim_{n\to\infty}\overline{f}_n\left(\overline{G}_1\right) \underline{\vee}\ \lim_{n\to\infty}\overline{f}_n\left(\overline{G}_2\right)$ and $\overline{\pi}_1\left(\lim_{n\to\infty}\overline{f}_n\left(\overline{G}_1 \underline{\vee}\, \overline{G}_2\right)\right) = \overline{\pi}_1\left(\lim_{n\to\infty}\overline{f}_n\left(\overline{G}_1\right)\right) * \overline{\pi}_1\left(\lim_{n\to\infty}\overline{f}_n\left(\overline{G}_2\right)\right)$.

3. Some applications

i. A polymer is composed of many repeating units called monomers. Starch, cellulose, and proteins are natural polymers. Nylon and polyethylene are synthetic polymers. Polymerization is the process of joining monomers. Polymers may be formed by addition polymerization; furthermore, one essential advance likewise polymerization is mix as in **Figure 4**, which happens when the polymer's development is halted by free electrons from two developing chains that join and frame a solitary chain. The accompanying chart portrays mix, with the image (R) speaking to whatever remains of the chain.

ii. Chemical nature of enzymes, all known catalysts are proteins. They are high atomic weight mixes made up primarily of chains of amino acids connected together by peptide bonds as in **Figure 5**.

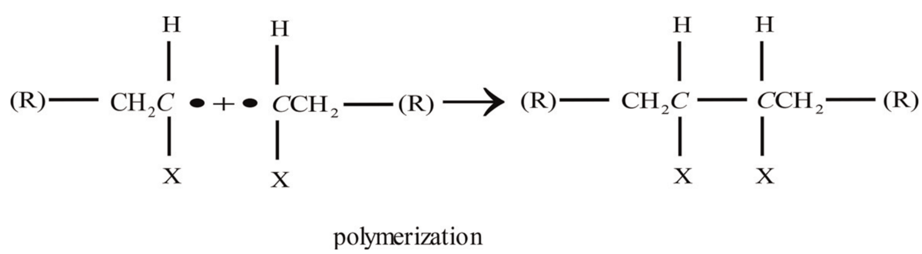

Figure 4.
Polymerization.

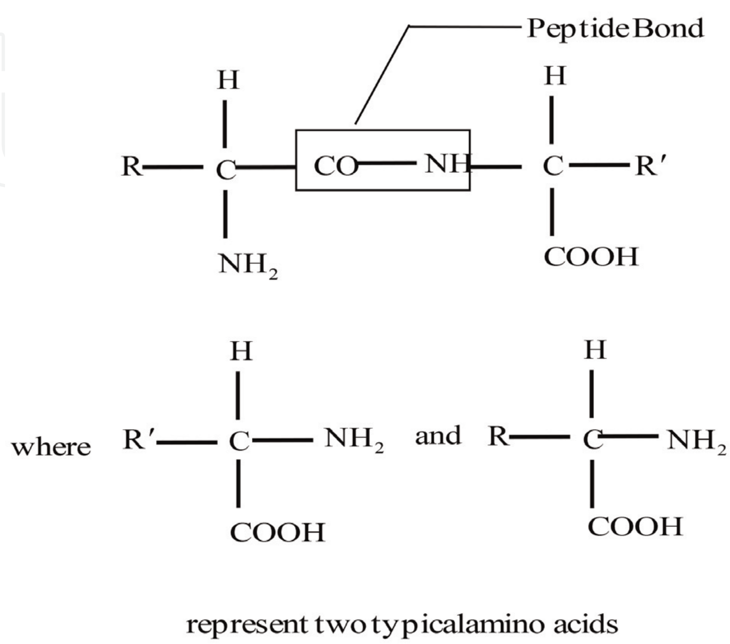

Figure 5.
Typical amino acids.

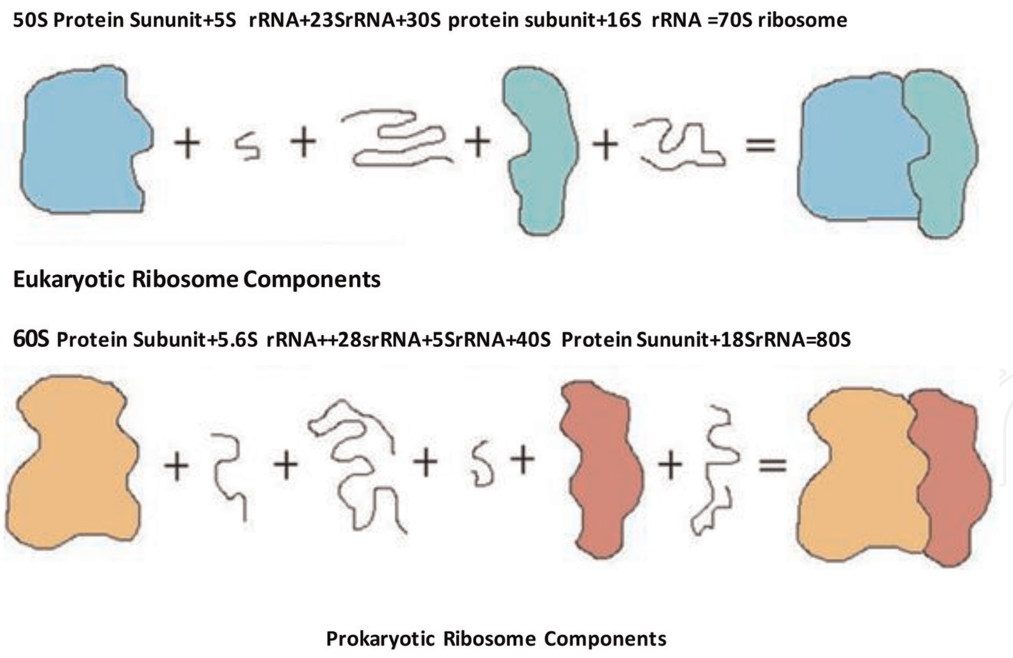

Figure 6.
Prokaryotic ribosome components.

iii. There are two types of the subunit structure of ribosomes as in **Figure 6** which is represented by the different connected types of protein subunit and rRNA to form a new type of ribosomes.

4. Conclusion

In this chapter, the fundamental group of the limit chaotic foldings on chaotic connected edge graphs is deduced. Also, we can deduce some algorithms from a new operation of a graph by using the adjacency matrices.

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