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Quantum Walks in Quasi-Periodic Photonics Lattices

Dan Trung Nguyen, Daniel A. Nolan and Nicholas F. Borrelli

Abstract

We present construction rules for a new class of quasiperiodic photonics lattices (QPL) to realize localized quantum walks (LQWs) deterministically. The new quasiperiodic structures are constructed symmetrically with Fibonacci, Thue-Morse, and other quasiperiodic sequences. Our construction rules allow us to build the symmetrical quasiperiodic photonics lattices. As a result, LQWs with symmetrical probability distributions can be realized in these QPLs. Furthermore, the proposed QPLs are composed with different waveguides providing both on- and off-diagonal deterministic disorders. We show LQWs in the proposed QPLs are highly programmable and controllable.

Keywords: quantum walks, localized quantum walks, photonics lattices, quasiperiodic, Fibonacci sequence, Thue-Morse sequence

1. Introduction

Quantum walks (QWs) have been used to construct exponential speedup quantum algorithms [1–3] and quantum simulations [4, 5], to implement universal gates for quantum computers [6, 7], etc. For the last few decades, scientists have made tremendous progress on research and development of those areas which are the most promising for solving problems that are intractable by classical computers. Among different schemes (or approaches) quantum photonics has the advantage of highly advanced technology that can easily generate and manipulate almost any desired photons—the walkers, even in room-temperature conditions. Discrete-time QWs (DTQW) have been demonstrated using beam splitter arrays [8, 9], and continuous-time quantum walks (CTQWs) have been investigated both theoretically and experimentally in evanescently coupled parallel waveguide arrays [10–17]. Integrated photonics lattices consisting of evanescently coupled waveguides are perfectly suited for investigation of CTQWs, and in fact, laser-written waveguides were the first systems used to demonstrate quantum walks on a line with coherent light [10–13]. In those photonics lattices, the walking process occurs in the region of evanescent-coupled waveguides. As a result, spacing between waveguides in those lattices is close enough, typically on the order of several micrometers to ensure evanescent coupling to occur. It is well established both theoretically and experimentally that in a uniform and/or periodic array of coupled waveguides (photonics lattices), the probability distribution of single-photon QWs spreads across the waveguide lattice by coupling from one waveguide to its neighbors in a pattern characterized by two strong "ballistic" lobes [11, 12].

In contrast to the normal QWs in periodic photonics lattices (PLs), quantum walks can be localized in disordered ones. This phenomenon belongs to a more general concept commonly known as Anderson localization (AL). Moreover, AL is usually discussed in terms of coherent evolution in the presence of a randomly disordered medium. By breaking the order of structures and/or the periodicity of the QWs' evolution through spatially and temporally randomizing operations, localized QWs (LQWs) have been realized theoretically and experimentally [15–17]. For the last decade, there have been increased interests and efforts on exploration of LQWs for applications. The research works have been inspired by proposals of employing LQWs for quantum communications, such as secure transmission of quantum information [18], and quantum memory [19]. Besides, LQWs in photonics lattices are also a very effective approach to simulate the AL using quantum optics.

Anderson predicted that the wave function of a quantum particle can be localized in the presence of a static disordered potential in his famous paper [20]. As a result, evolution of a quantum particle with its dual wave-particle nature through a disordered medium can be strongly suppressed, depending on the degree of the disorder. More generally, in a static disordered medium, destructive interferences among different propagating paths of a quantum particle could generate AL. Although AL has been widely studied in quantum solid-state physics, localization of light has recently been explored with potentially important applications [21]. One unique phenomenon is that deviations from periodicity may result in higher complexity and many surprising effects would arise. For decades, such deviations have been investigated in optics in the realization of photonic quasicrystals: a class of structures made from optical elements that are arranged in different patterns but lack translational symmetry. A quasiperiodic structure is neither a periodic nor a random system, so it could be considered as an intermediate between the order and disorder systems. Furthermore, it has been demonstrated that quasiperiodic crystals can also lead to the localization of light [22–24]. Examples of such quasiperiodic structures constructed with the Fibonacci sequence include one-dimensional (1D) Fibonacci quasiperiodic dielectric multilayers [25], semiconductor quantum wells [26], 2D quasicrystals [27], and 3D quasicrystals [28].

It is important to stress here that AL has been conventionally realized in *randomly disordered* systems, and the effect of disorder-induced localization is quantified by averaging over a large number of realizations on many systems having the same degree of disorder. Similarly, LQWs in integrated photonics lattices have been realized on many randomly disordered waveguide lattices. The results of LQWs are averaged over all realizations in those waveguide lattices that are well controlled in a defined range of randomness of the disorder. Experimentally, it is not simple as is shown in [15]. Note that an experimental method has been proposed to determine localized modes systematically in 1D-disordered waveguide lattices [16], and the method is useful for investigating LQWs in randomly disordered lattices. As mentioned earlier, optically quasiperiodic structures or quasicrystals can provide deterministic disorders deviated from periodicity. Localization of light has been realized deterministically in those quasicrystals [23–25], and there is no need to average over a large number of structures. In that spirit, we propose a new class of quasiperiodic photonics lattices (QPL) to realize LQWs deterministically. As presented in this chapter, the new class of QPLs could be useful for quantum communication applications as proposed recently in [18, 19]. The new quasiperiodic structures are constructed *symmetrically* with Fibonacci, Thue-Morse, and other quasiperiodic sequences. Benefits of using the new class of QPL for realizing LQWs are twofold: (i) LQWs can be realized deterministically and therefore are highly programmable and optimizable, and (ii) it is much simpler for realization of LQWs in comparison with random disordered systems as there is no need to do averaging over many

realizations. Furthermore, our simple construction rules allow us to create symmetric QPLs. Consequently, LQWs with symmetric probability distributions can be realized deterministically in these structures. The proposed QPLs are simple and straightforward to make but would be potentially useful for many research areas in quantum communication as will be discussed in this chapter.

The chapter is organized as follows. Section 1 is the introduction. The new concept of QPLs based on Fibonacci and Thue-Morse sequences is described in Section 2. In Section 3, simulation results of QWs in periodic photonics lattices (PPLs) and LQWs in the new class of photonics lattices—the quasiperiodic photonics lattices (QPLs)—are presented. Section 4 is the discussion and conclusion.

2. Periodic and quasiperiodic photonics lattices

In this section, we will present the general construction of photonics lattices (PLs) that will be used for our numerical investigation of single-photon quantum walks (QWs). **Figure 1** shows diagrams of several PLs that are the subject of our investigation. **Figure 1(a)** and **(b)** shows diagrams of two different types of PPLs: type-I consists of identical waveguide PPLs (IPPL), and type-II are lattices of two (or more) different waveguide PPLs (DPPL). **Figure 1(c)** and **(d)** shows two different QPLs, one with the Fibonacci sequence or Fibonacci QPLs (FQPL) and the other with the Thue-Morse sequence or Thue-Morse QPLs (TMQPL).

In **Figure 1**, the structures are arrays of single-mode (SM) waveguides having core diameter a, center-to-center distance between cores d, and index difference between core and clad Δn. In the structures composed of two different waveguides as in the cases of DPPL, FQPL, and TMQPL, each waveguide is characterized by V-number. For example, waveguides A and B are characterized by $V_A = \pi a_A NA_A / \lambda$ and $V_B = \pi a_B NA_B / \lambda$, respectively, where $a_{A(B)}$ stands for core diameter and $NA_{A(B)}$ is the numerical aperture of waveguide $A(B)$. Note that the numerical aperture NA can be determined by the index difference between core and clad Δn, and we will use Δn to characterize waveguides in our calculations. For example, the IPPL shown
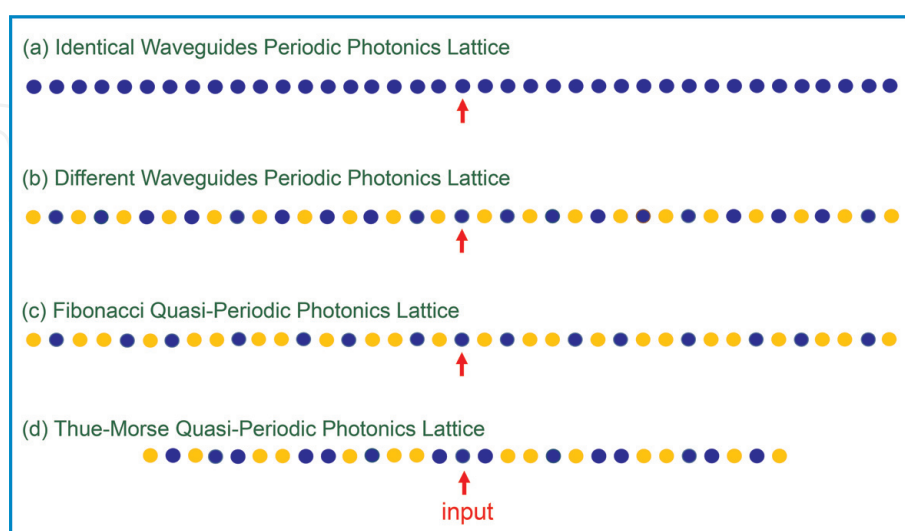

Figure 1.
Diagrams of different photonics lattices of single-mode waveguides having diameter a *and center-to-center distance* d. *(a) IPPL with all cores have the same index difference Δn. (b) DPPL is composed of two different cores A (blue) and B (yellow) having Δn_A and Δn_B, respectively. (c) Sixth-order FQPL of 39 cores composed of A and B waveguides. (d) Fourth-order TMQPL of 29 waveguides composed of A and B waveguides. The red arrows indicate the position of the input signal. The construction rules for FQPL and TMQPL are explained in the text.*

in **Figure 1(a)** is a regular array of 39 identical SM waveguides with a = 4 μm, d = 8 μm, and Δn = 0.0035. The DPPL of 39 cores in **Figure 1(b)** is a periodic array of two different waveguides A (blue) and B (yellow) with Δn_A = 0.0045 and Δn_B = 0.0035, respectively. Both A and B have the same a = 4 μm and d = 8 μm. Those parameters are used in all numerical simulations in this work. **Figure 1** also shows a Fibonacci QPL of 39 waveguides of sixth order and a Thue-Morse QPL of 29 waveguides of fourth order. Both FQPL and TMQPL are constructed with two different waveguides A and B whose properties are defined above.

Below we will describe the construction rules for our quasiperiodic photonics lattices based on Fibonacci and Thue-Morse sequences. It is worth mentioning that these rules have been first proposed to construct one-dimensional Fibonacci quasi-periodic multiple dielectric layers [22–25] and Fibonacci quasiperiodic arrays of waveguides [29, 30]. In general Fibonacci QPLs and Thue-Morse QPLs (TMQPL) are constructed recursively with Fibonacci and Thue-Morse sequences, respectively, starting with two different single-mode waveguides A and B. We can easily write down the formulae of the elements and their corresponding structures as in **Figure 2** for the first-order elements of FQPLs and TQPLs in the upper and lower panels, respectively.

We now define a new j^{th}-order quasiperiodic photonics lattice as [29, 30]

$$F_j = S_j S_{j-1} \cdots S_2 S_1 S_2 \cdots S_{j-1} S_j, \tag{1}$$

where S_1, S_2 ... S_j are Fibonacci and Thue-Morse elements defined in **Figure 2**. As an example, **Figure 1(c)** is a diagram of the sixth-order Fibonacci QPL with 39 cores. It is clear from **Figures 1** and **2** that the photonics lattices constructed with Eq. (1), with Sj being the elements of Fibonacci and Thue-Morse sequences, are quasiperiodic and symmetrical.

In general, we can apply the same rules, e.g., Eq. (1), to construct symmetrical quasiperiodic photonics lattices based on other quasiperiodic sequences, such as the
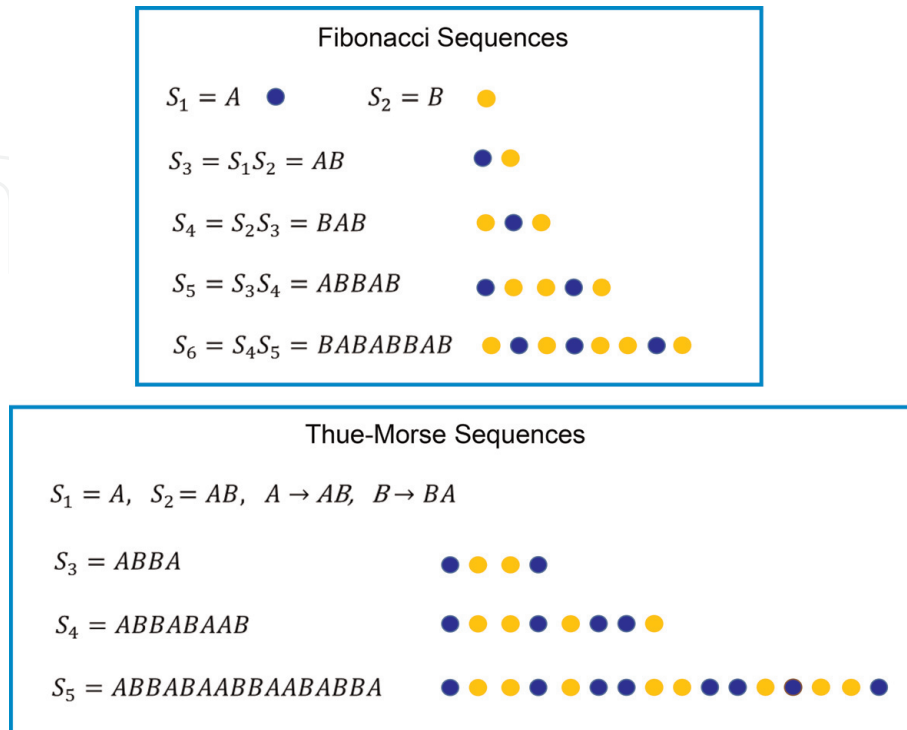

Figure 2.
First-order elements of FQPLs (upper) and TMQPLs (lower) composed by two waveguides A and B.

Rudin-Shapiro sequence, etc. Details of the new construction rules and the meaning of the deterministic disordered nature of such QPL can be seen in [29, 30]. Next, in Section 3, we will first present the method to simulate single-photon QWs in those PLs using the beam propagation method (BPM). We will then show in detail our simulations of QWs in IPPLs and DPPLs in comparison with FQPLs and TMQPLs. Especially, we will show unique localized quantum walks in FQPLs and TMQPLs with symmetrical probability distribution.

3. Localized quantum walks in quasiperiodic photonics lattices

The most well-known example of classical random walks on a line consist of a walker (e.g., a particle) walking to either the left or right depending on the outcomes of an unbiased toasting coin (probability system) with two mutually exclusive results, i.e., the walker moves according to a probability distribution. At each step of the walking process, an unbiased coin is tossed, and the walker makes consecutive left-or-right decisions depending on the result of the coin (up or down), respectively. For classical random walks, it is well known that both 1D and 2D distributions are Gaussian distributions [31]. For the quantum version of the random walk—the quantum walks (QWs)—the main components of discrete-time quantum walks (DTQWs) are a quantum particle, "the walker," a quantum coin, evolution operators for both walker and quantum coin, and a set of observables. QWs can also be in another form that has no classical counterpart, such as the CTQWs which have been extensively investigated in photonics lattices [10–17]. In contrast to DTQWs, CTQWs (QWs for short in the following) have no coin operations, and the walking evolutions are defined entirely in a position space where continuous coupling between vertices or lattice sites is required. Integrated photonics lattices consisting of evanescently coupled 1D and 2D arrays of waveguides are perfectly suited for the investigation of QWs. In such structures, spacing between waveguides typically is on the order of several micrometers or is in strong coupling regimes for evanescent couplings to occur.

In this chapter, we will restrict ourselves to the single-photon QWs in both periodic and quasiperiodic PLs. First, we would like to emphasize that single-photon QWs do not behave any differently from classically coherent wave propagation and the distribution of light intensity corresponds to the probability distribution of photons that can be detected [32]. It is important to stress, however, that in multiple-photon QWs (indistinguishable or entangled photons), truly nonclassical features will appear. Although single-photon QWs have limited features, the effects are still very important not only for understanding but also for quantum applications.

Let us briefly describe how single-photon QWs can be modeled mathematically and realized experimentally in PLs. Below, we follow the description from Refs. [11, 12, 32] which are excellent references on QWs in PLs. In general, the Hamiltonian description of the problem of quantum walks in a PL can be written as

$$H = \hbar \sum_n \left\{ \beta_n a_n^\dagger a_n + \sum_m \kappa_{nm} a_n^\dagger a_m \right\} \qquad (2)$$

where $a_n^\dagger (a_n)$ is the creation (annihilation) operator of a photon in the nth-waveguide, β_n is the propagation constant of the nth-waveguide, and κ_{nm} is the coupling coefficient between nearest neighbor sites $n = m \pm 1$. The propagation of a single photon can be described by the Heisenberg equation as below:

$$\frac{da_k^\dagger}{dz} = -\frac{i}{\hbar}\left[a_k^\dagger, H\right] = i\left(\beta_k a_k^\dagger + \kappa_{k,\,k+1} a_{k+1}^\dagger + \kappa_{k,\,k-1} a_{k-1}^\dagger\right) \qquad (3)$$

As described in detail in Refs. [11, 12, 32], since the system is conservative, and the Hamiltonian is explicitly time independent, we may formally integrate Eq. (3) to obtain the input-output relation for the mode operators as

$$a_k^\dagger(z) = \sum_{j=1}^{N} U_{j,k} a_j^\dagger(0) \qquad (4)$$

For the special case of a single photon coupled into site k of a uniform waveguide array ($\kappa_{k,k+1} = \kappa_{k,k-1} = \kappa$, *and* $\beta = \beta_k$), one can show that the probability amplitude at site j is analytically described by

$$U_{k,j} = i^{(k-j)} \exp\left(i\beta z\right) J_{k-j}(2\kappa z), \qquad (5)$$

where $J_{k-j}(z)$ is a Bessel function of the first kind and order (k-j). When a single photon is coupled to waveguide k, it will evolve to waveguide j with a probability $\eta_j = \left|U_{k,j}(z)\right|^2 = J_{k-j}^2(2\kappa z)$. From the solution of the single-photon QWs, e.g., Eq. (5), it is clear that the photon distribution spreads across the lattice by coupling from one waveguide to its neighbors in a unique pattern that is typically characterized by two strong "ballistic" lobes [12, 32].

It is worth noting that such analytical solutions, e.g., Eq. (5) above, can be obtained only in uniformly regular PLs consisting of identical SM waveguides. In general, simulations of QWs in different PLs, even in the periodic arrays that are composed of different waveguides, are not simple and become extremely difficult in *irregular* PLs. Simulation of LQWs in randomly disordered structures becomes more challenging even just only for the single-photon QW problems. Here we present an effective approach to simulate single-photon QWs. As stated earlier, single-photon QWs do not behave any differently from classically coherent wave propagation, and the distribution of light intensity corresponds to the probability distribution of photons. Therefore, we can use the beam propagation method (BPM), one of the most effective methods of light propagation simulation in complicated structures for simulating single-photon QWs in different and irregular PLs. The method of BPM has been well developed for decades [33, 34], and commercial software is available. Below, we describe briefly the BPM, and we show the results of BPM simulation of QWs in periodic PLs, in quasiperiodic FQPLs, and also in randomly disordered PLs.

The wave equation in paraxial approximation for the slowly varying electric field that propagates along the z-axis in a general structure can be written as [33–35]:

$$\frac{d}{dz} E(x, y, z) == \left(\hat{D} + \hat{V}\right) E(x, y, z) \qquad (6)$$

The diffraction $\hat{D}$ and inhomogeneous operators $\hat{V}$ are given by:

$$\hat{D} = \frac{i}{2k}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right), \quad \hat{V} = \left(ik\Delta n(x, y, z) - \alpha(x, y, z)\right) \qquad (7)$$

In Eq. (7) $k = n_0 k_0 = \frac{n_0 \omega}{c} = 2\pi n_0/\lambda$ where n_0 is the background or reference refractive index and λ is the free-space wavelength, $\Delta n = n(x, y, z) - n_0$ is the

refractive index profile relative to the reference refractive index, and λ is the power absorption/loss of the waveguide. A small propagation step is implemented using the following approximation:

$$E(x,y,z+\Delta z) = e^{(\hat{D}+\hat{V})\Delta z}E(x,y,z) \approx e^{\hat{D}\Delta z/2}e^{\hat{V}\Delta z}e^{\hat{D}\Delta z/2}E(x,\mathrm{y},\mathrm{z}) \qquad (8)$$

where $\exp\left(\hat{D}\Delta z/2\right)$ means taking a half step of diffraction alone and $\exp\left(\hat{V}\Delta z\right)$ means taking the whole step of linear propagation alone. This approximation operation is third-order accurate in the propagation step size, and the change by each step is required to be small when compared to unity. The BPM solution, e.g., Eq. (8), can be solved very effectively by fast Fourier transformation (FFT) algorithm [33, 34]. The method has been successfully applied to simulate Er/Yb co-doped multicore fiber amplifiers [35] and Yb-doped multicore fiber lasers [36]. We have developed our own MATLAB programs, and the simulation results of single-photon QWs in different PLs are presented in the following paragraphs.

Figure 3 shows simulation results of single-photon QWs in the photonics lattices that are described in **Figure 1**. **Figure 3(a)** and **(b)** shows QWs in periodic lattices: an IPPL with 39 identical SM waveguide B ($\Delta n = 0.0035$) and a DPPL with 39 SM waveguides A ($\Delta n = 0.0045$) and B ($\Delta n = 0.0035$). **Figure 3(c)** and **(d)** shows QWs in quasiperiodic lattices, sixth-order FQPL with 39 waveguides A and B, and fourth-order TMQPL with 29 waveguides A and B as described in **Figure 2** and Eq. (1). All waveguides have the same core size of 4 μm, center-to-center separation d = 8 μm, and λ = 1.55 μm. The simulation results of QWs for each waveguide structure in **Figure 3** show in the order from bottom to top: top-view, front-view, and probability distribution of photon at the end of the walks.

The results in **Figure 3** show probability distributions of photons of QWs in IPPL structures spread across the lattice by coupling from one waveguide to its neighbors in a pattern characterized by two strong "ballistic" lobes [11, 12, 32]. Interestingly, QWs in DPPLs also have those two lobes at the edges of the waveguide lattice but have no localization. Note that DPPLs are composed of two different waveguides A and B that are used to construct the FQPLs and TMQPLs. In contrast with periodic lattices IPPL and DPPL, the simulation results show clearly LQWs in quasiperiodic lattices FQPL and TMQPL. Furthermore, LQWs with the
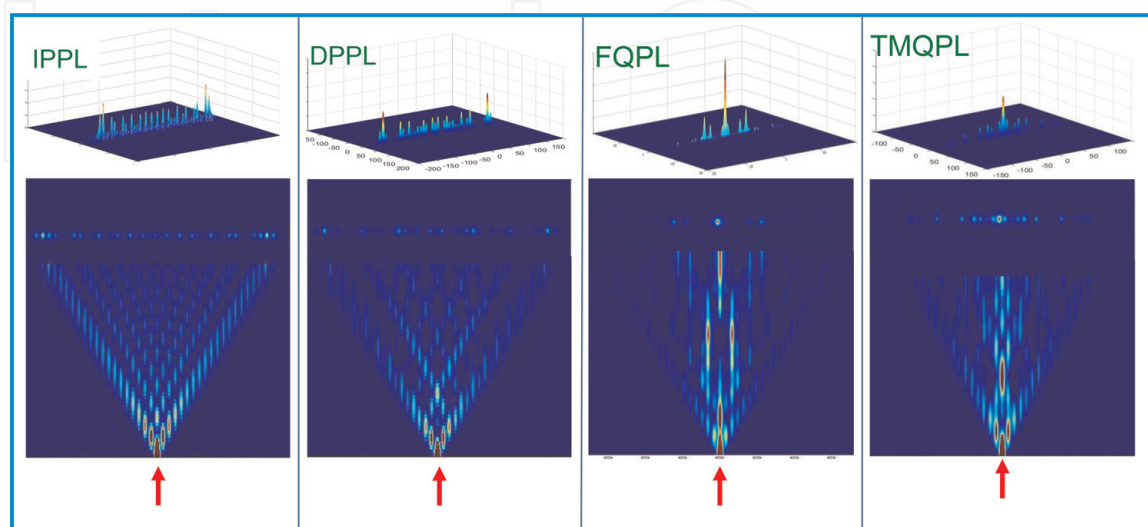

Figure 3.
QWs in periodic photonics lattices IPPL (a) and DPPL (b) and in quasiperiodic lattices FQPL (c) and TMQPL (d). From bottom to top: top-view, front-view, and probability distribution of photon (in the same scale). Lattices (a), (b), and (c) have 39 cores, and (d) has 29 cores. Red arrows indicate the position of input signal.

symmetrical probability distribution can be realized in quasiperiodic structures constructed symmetrically, e.g., Eq. (1). It is worth noting that conventional LQWs have been realized in randomly disordered structures in which symmetrical LQWs are impossible to achieve. On the other hand, LQWs have recently been proposed for secure quantum memory applications [19]. To achieve symmetrically distributed LQWs, the authors of [19, 37] have proposed to use temporally disordered operations in spatially ordered systems. However, their approach requires multiple quantum coins for temporally disordered operation which could be extremely difficult in practice.

As mentioned earlier in the introduction, AL has been conventionally realized in *randomly disordered* systems, and the effect of disorder-induced localization is quantified by averaging over a large number of realizations on many systems having the same degree of disorder. The results of LQWs are averaged over all realizations in those waveguide lattices that are well controlled in a defined range of randomness of the disorder, and experimental realization is not simple as is shown in [15]. Meanwhile, quasiperiodic systems or quasicrystals provide deterministic disorder deviated from periodicity resulting in localization of light deterministically in those systems [22–25], meaning there is no need to do averaging over many samples or systems. Benefits of using the new class of QPL for realizing LQWs are twofold: (i) LQWs can be realized deterministically and therefore are highly programmable and optimizable, and (ii) it is much simpler for the realization of LQWs than random disordered systems as there is no need to do averaging over many realizations. Furthermore, our simple rules allow us to construct quasiperiodic FQPLs and TMQPLs symmetrically. As a result, symmetrical LQWs can be realized deterministically in these QPLs. The proposed QPLs in general, and FQPLs and TMQPLs in particular, are simple and straightforward to make but would be potentially useful for different applications in quantum communication.

Figure 4 below illustrates realizations of LQWs in randomly disordered photonics lattices. **Figure 4** shows simulation results of single-photon QWs in IPPLs of 49 identical SM waveguides: IPPL without random disorder (a) and examples of QWs in IPPLs with randomly disordered waveguide positions $\Delta_i = 0.1d$ (b–d). All waveguides have the same core size of 4 μm, index difference between core and cladding $\Delta n = 0.0035$, center-to-center separation $d = 8$ μm, and $\Delta = 1.55$ μm. Note that in **Figure 4**, the probability distributions of all cases are plotted in the same scale so that the differences between periodic and quasiperiodic lattices and also between the two QPLs can be clearly seen. It is worth noting that the results from BPM simulation of QWs in regular lattices of identical waveguides (IPPL) are the same as the well-known analytical solutions of Eq. (5). The BPM simulations show that photon distribution spread across the lattice by evanescent coupling from one waveguide to its nearest neighbors are characterized by two strong "ballistic" lobes [12, 32]. Meanwhile, the results in **Figure 4(b)–(d)** show localizations of QWs in randomly disordered IPPLs are completely different even with the same degree of random disorder in those structures. The results of LQWs in randomly disordered IPPLs are examples of the need to do averaging over a large number of realizations for quantifying LQWs in randomly disordered systems.

As can be seen from **Figures 3** and **4**, LQWs in quasiperiodic structures of FQPLs and TMQPLs are controllable in contrast with the ones in spatially random disordered structures. Furthermore, LQWs with the *symmetrical probability distribution* can be conveniently realized in our new class of quasiperiodic photonics lattices. It is important to note that the authors in Ref. [19] have recently proposed a new scheme that employs symmetrical LQWs for storage and retrieving quantum information for quantum memory applications. The authors proposed using temporally disordered operations in spatially ordered systems to achieve symmetrically
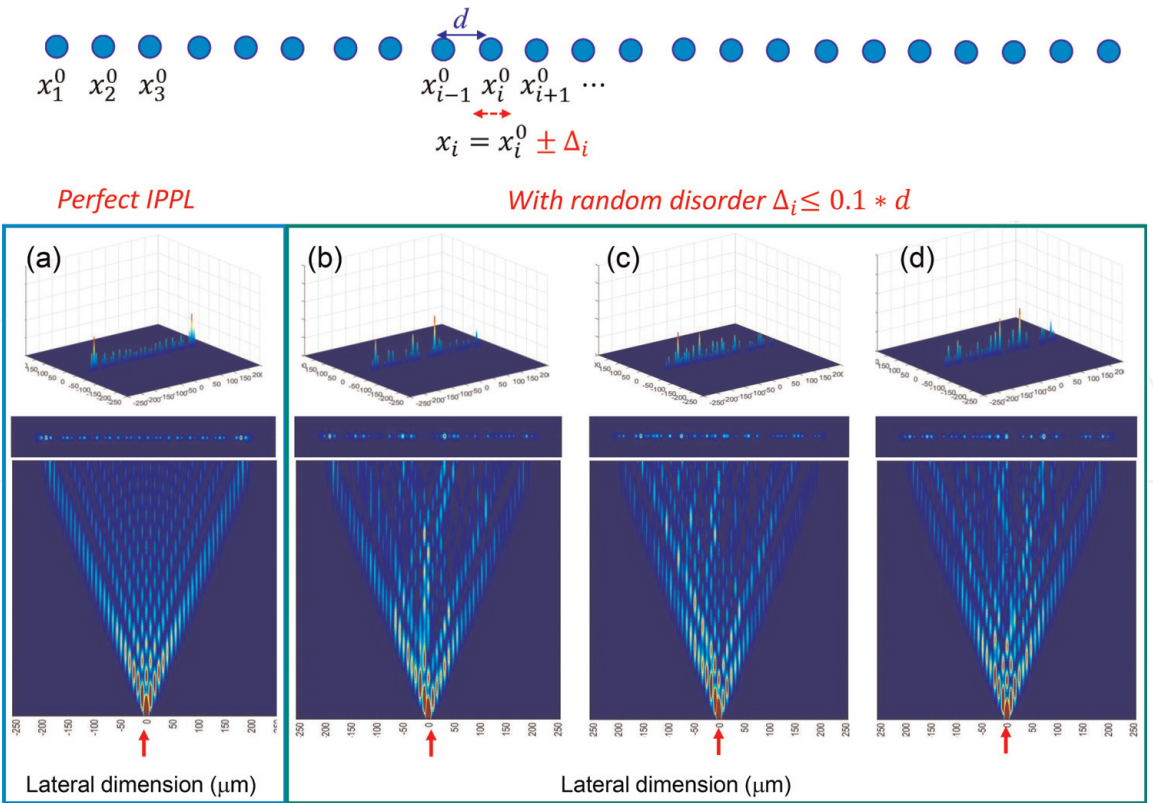
Figure 4.
QWs in IPPLs of 49 identical SM waveguides with and without spatially random disorder. (a) IPPL without disorder of waveguide positions; (b, c, d) IPPLs with random disorder of positions of 10%. From bottom to top: top-view, front-view, and 3D probability distribution of photon (in the same scale). Upper: diagram of array of waveguides.
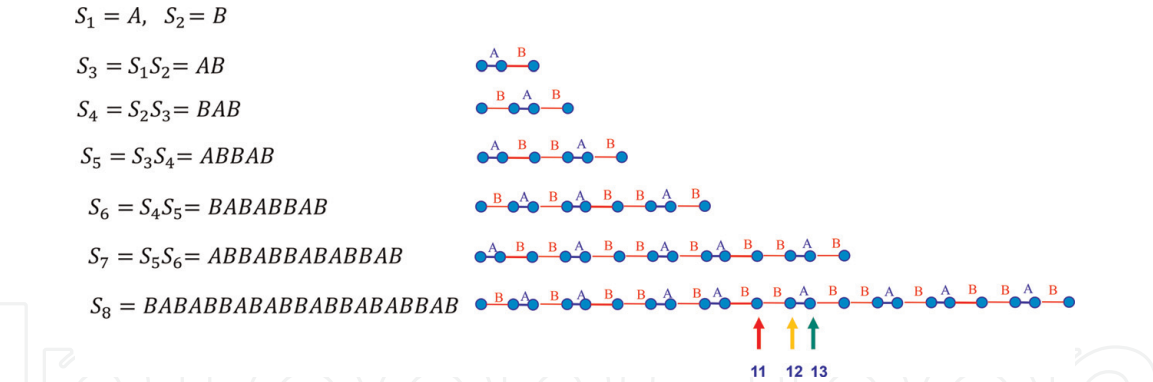

Figure 5.
Quasiperiodic elements based on the Fibonacci spacing sequence (FSS) with two fundamental distances A and B between nearest identical waveguides. Three arrows indicate different input cores 11, 12, and 13 used in simulation (core 1 is the first from the left).

distributed LQWs for quantum memory. However, their scheme using multiple quantum coins for temporally disordered operation would be extremely difficult in practice. As can be seen in **Figures 4** and **5** in this section, symmetrical LQWS can be realized conveniently in our proposed FQPLs and TMQPLs and in other QPLs as well that can be used for similar applications.

At this point, we want to stress that localization of light has been investigated theoretically and experimentally in 1D and 2D arrays of identical waveguides that are constructed with the Fibonacci sequence *in distances* [38, 39]. The quasiperiodic arrays of waveguides considered in those works are defined as the Fibonacci elements in **Figure 2**; however, the waveguides *A* and *B* were defined as the two distances 1 (unit) and τ = 1.618 (golden ratio of the Fibonacci sequence) between identical SM waveguides [37, 38]. Since the quasiperiodic properties of these

elements are determined by the Fibonacci sequences in spacing of waveguides, they can be classified as PLs with the Fibonacci spacing sequence (FSS). It is important, however, to point out that there are important differences between our proposed QPLs and the lattices of Fibonacci spacing sequence FSS in [38, 39]. The FSS lattices are constructed with the Fibonacci spacing sequence of all identical waveguides. As a result, the FSS lattices have a quasiperiodic distribution of coupling coefficients or *off-diagonal* deterministic disorder since all waveguides are identical. Meanwhile, our proposed QPLs are constructed with quasiperiodic sequences in general and in particular with the Fibonacci and Thue-Morse sequences with *two different wave-guides*. Therefore, in general, our proposed QPLs, FQPLs, and TMQPLs have both propagation constants, and coupling coefficients are quasiperiodic distributions. In other words, the QPLs have both *on- and off-diagonal* deterministic disorders. Experimentally, localizations have been realized in on-diagonal disordered lattices [16] and in off-diagonal disordered arrays of waveguides [17]. Our proposed QPLs in general, and in particular the FQPLs and TMQPLs, would offer systems possessed both on- and off-diagonal deterministic disorders for the investigation of different properties. Furthermore, the new proposed j^{th}-order QPLs is constructed as an orderly sequence chain of all quasiperiodic elements up to j^{th} order, and the construction rules are convenient to make QPLs symmetric as presented earlier in this section.

As an example, we show in **Figure 5** the concept of quasiperiodic lattices based on FSSs with two fundamental distances $A = 1$ (unit) and $B = \tau$.

In **Figure 6**, we show simulation results of QWs in the eighth-order FSS element with 22 waveguides for three different inputs. It is clear from **Figure 6** that localizations of QWs can also be realized in this lattice. The results show the behavior of the QWs is very different depending on the input position. The results confirm
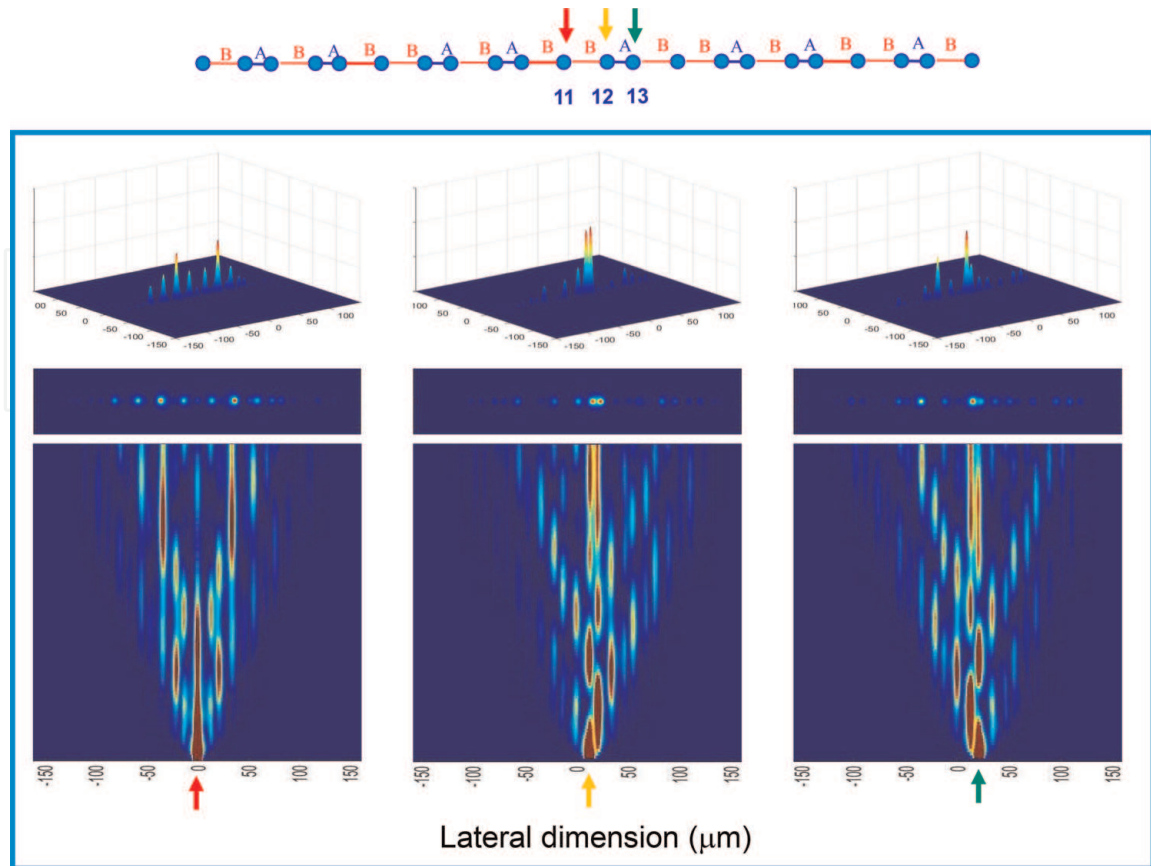

Figure 6.
LQWs in the eighth-order FSS element of 22 waveguides with different input cores as shown in diagram on top.

important findings of our work above that LQWs can be realized in deterministic disordered QPLs.

It is worth noting that the quasiperiodic properties of the FSS elements are originated from the quasiperiodic distribution in spacing of waveguides, not because of the golden ratio τ. Therefore, we do not need to restrict ourselves to the ratio $\tau = 1.618$ between distances A and B as in [38, 39] for realization of localizations in these structures. This is important since we now have more freedom in choosing these two fundamental distances. **Figure 7** shows results of LQWs in the eighth-order FSS element but with a different ratio $\tau = 1.5$, 1.618 (golden ratio), and 1.7.

The simulation results of the eighth-order FSS element with different ratios τ show an unsymmetrical probability distribution of photons. However, the photon distribution looks quite symmetrical near the input, core-11 (core order is from the left to the right). That is because the cores are symmetrical near the input but unsymmetrical for others that are father from the input. It is worth mentioning here that the distances for completion of QW process (QW distance or D_{QW} for short) are slightly different for different ratios of τ as shown in **Figure 7**. That is because couplings between nearest cores are dependent on core-to-core distances. The results in **Figure 7** are with $\tau = 1.5$ and $D_{QW} = 4.5$ mm, $\tau = 1.62$ and $D_{QW} = 4.56$ mm, and $\tau = 1.7$ and $D_{QW} = 4.62$ mm.

Finally, the wavelength dependence of QWs in photonics lattices is well-known [29, 30], and that is another interesting and controllable property in such systems. We show in **Figure 8** the simulation results of wavelength-dependent QWs in DPPL (left panel) and LQWs in FQPL (right panel). Note that both DPPL and FQPL in **Figure 8** are composed of the same 39 A and B waveguides, but one is periodic (DPPL), and the other is quasiperiodic lattice (FQPL).
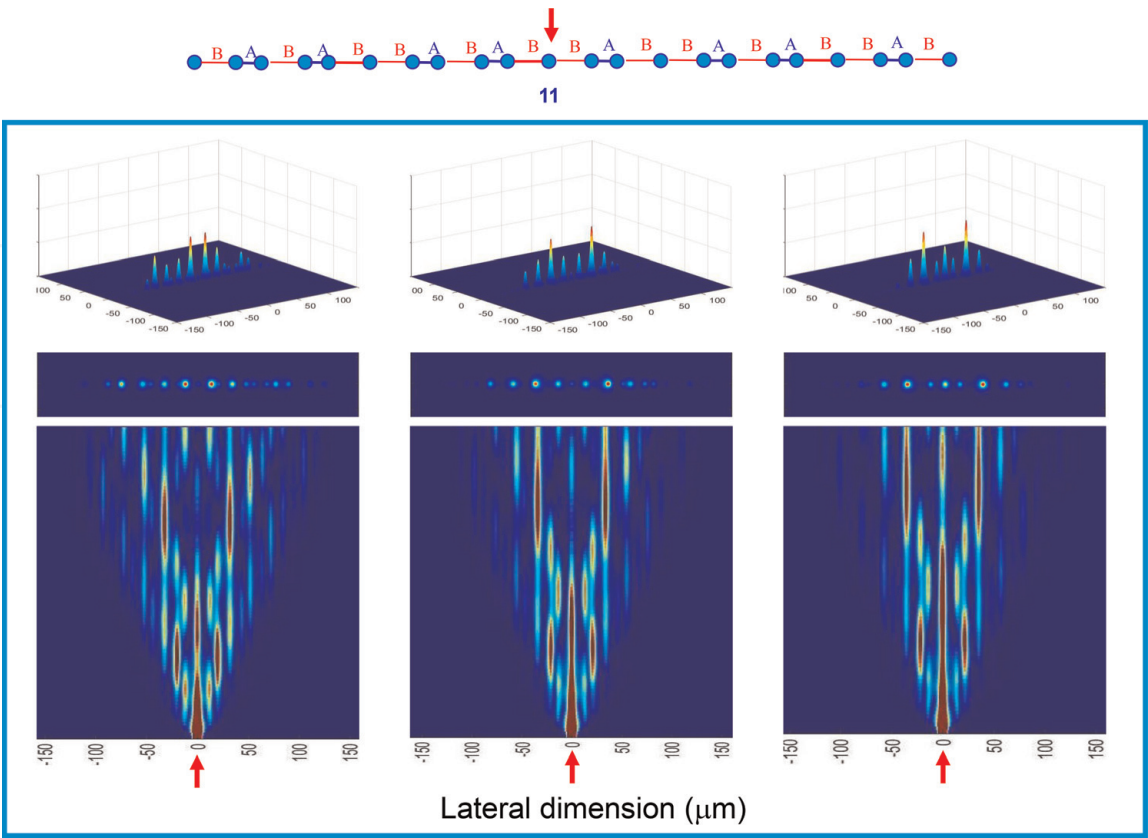
Figure 7.
LQWs in eighth-order FSS element of 22 waveguides with different ratio τ between A and B (core 11 is input). $\tau = 1.5$ (left), 1.618 (middle), and 1.7 (right).
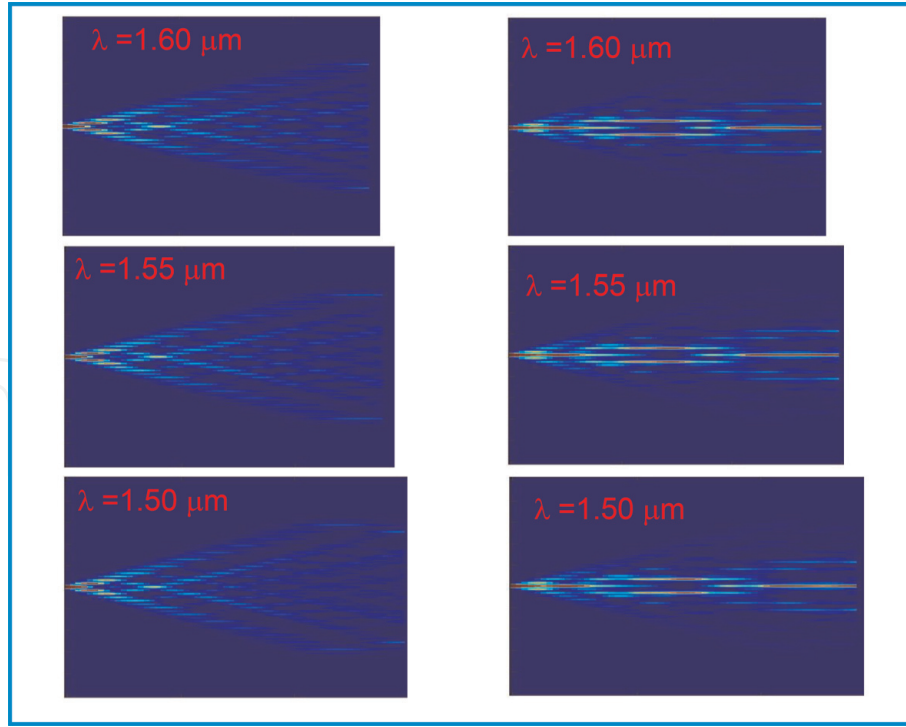
Figure 8.
Wavelength-dependent QWs in DPPL (left) and sixth-order FQPL (right). Both DPPL and FQPL are composed of the same A and B waveguides defined above, and both have 39 cores.

The difference between QW distances D_{QW} in the same structure but different wavelengths is originated from the different couplings. For the same lattice of SM waveguides, the shorter the wavelength, the weaker the evanescent couplings between the nearest cores. As a result, the light takes a longer distance to fully couple from one core to the other. This property would give us another parameter to control and optimize the QWs in both PLs and QPLs.

4. Conclusions

As stated earlier, QWs and LQWs are important for many applications in quantum computing and quantum communication. In particular, recent investigations show symmetrical LQWs can be potentially employed for storage and retrieve quantum information for a secure quantum memory. The authors in Ref. [19] have pointed out that symmetrical LQWs are required for quantum memory applications. Because it is impossible to achieve symmetrical LQWs in spatially randomly disordered systems, the authors proposed temporally disordered operations using multiple quantum coins. Furthermore, the concept of localization due to temporally disordered operations has been later expanded to consider multiple quantum coins with quasiperiodic sequences that include Fibonacci, Thue-Morse, and Rudin-Shapiro sequences [37]. Although the idea of employing symmetrical LQWs for quantum memory is very interesting and worthy of further exploration, we would like to stress that experimental implementation of QWs is not simple even with only one quantum coin; therefore the idea of using multiple quantum coin operations would be extremely difficult in practice.

As presented in this chapter, our results show that symmetrical QPLs can be conveniently constructed with quasiperiodic sequences. As a result, symmetrical LQWs can be achieved in the proposed QPLs in general and in particular in FQPLs and TMQPLs. The results are significant in two aspects:

i. Symmetrical LQWs can be achieved in QPLs. That way is much simpler than using temporal disorder due to multiple quantum coin operations.

ii. The proposed QPLs are *deterministic* disordered systems; therefore the systems are controllable and optimizable in contrast with randomly disordered systems.

We therefore believe that the quasiperiodic photonics lattices in general and in particular the proposed FQPLs and TMQPLs would have advantages for applications that require symmetrical LQWs, especially in comparison with the cases in which multiple quantum coins are used.

In conclusion, in this work we have presented construction rules of a new class of quasiperiodic photonics lattices constructed with Fibonacci and Thue-Morse sequences to realize deterministic LQWs. We would like to stress here that the construction rules can be applied to other quasiperiodic sequences such as Rudin-Shapiro and other sequences to realize deterministic LQWs. Although the structures of the proposed QPLs are straightforward to make, the outcome results of LQWs are programmable and predictable, in contrast with randomly disordered systems. We have also presented an effective method of simulation of single-photon QWs in complicated structures based on beam propagation method (BPM). Furthermore, results of LQWs in the proposed symmetrical QPLs and in the Fibonacci spacing sequence (FSS) lattices are presented in detail. The unique features of QPLs that are of potential use for applications requiring symmetrical LQWs have also been discussed.

Finally, as stated earlier single-photon QWs play an important role not only for understanding the QWs effects and quantum simulations but also in many quantum applications. However, the application of single-photon states in general is limited in comparison with multiphoton states. Although our simulation results in this work are about single-photon QWs, the underlying results of our work show that symmetrical LQWs in the proposed QPLs can still be potentially important in general for quantum applications. For example, the advantage of the QPL scheme relates to the fact that the output photons localized around a particular output waveguide can be predictable and controllable with an important wavelength dependence. Also, regarding quantum communications in general, we can design the output localized around a particular waveguide using the concept of QPLs of different waveguides that can also enable low loss unique components. For example, low loss wavelength filters that are not possible with periodic structures but only with quasiperiodic structures can be realized.

Conflict of interest

The authors declare no competing financial interests and any conflict of interest.

Thanks

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