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Modeling of Turbulent Flows and Boundary Layer

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1. Introduction

Most flows occurring in nature and in engineering applications are turbulent. The boundary layer in the earth's atmosphere is turbulent; jet streams in the upper troposphere are turbulent; cumulus clouds are in turbulent motion. The water currents below the surface of the oceans are turbulent. The Gulf Stream is a turbulent wall-jet kind of flow. The photosphere of the sun and the photospheres of similar stars are in turbulent motion; interstellar gas clouds are turbulent; the wake of the earth in the solar wind is presumably a turbulent wake. Boundary layers growing on aircraft wings are turbulent. The study of turbulence clearly is an interdisciplinary activity, which has a very wide range of applications. In fluid dynamics laminar flow is the exception, not the rule: one must have small dimensions and high viscosities to encounter laminar flow.

Turbulence is the feature of fluid flow but not of fluids. Most of the dynamics of turbulence is the same in all fluids, whether they are liquids or gases, if the Reynolds number of the turbulence is large enough; the major characteristics of turbulent flows are not controlled by the molecular properties of the fluid in which the turbulence occurs. Since the equations of motion are nonlinear, each individual flow pattern has certain unique characteristics that are associated with its initial and boundary conditions. No general solution to the Navier-Stokes equations is known; consequently, no general solutions to problems in turbulent flow are available. Since every flow is different, it follows that every turbulent flow is different, even though all turbulent flows have many characteristics in common. Students of turbulence, of course, disregard the uniqueness of any particular turbulent flow and concentrate on the discovery and formulation of laws that describe entire classes or families of turbulent flows.

2. Turbulence

To begin with a question what is turbulence? The Reynolds number of a flow gives a measure of the relative importance of inertia forces and viscous forces. In experiments on fluid systems it is observed that at values below the so-called critical Reynolds number Re_{cri}, the flow is smooth and adjacent layers of fluid slide past each other in an orderly fashion. If the applied boundary conditions do not change with time the flow is steady. This regime is called laminar regime. At values of the Reynolds number above Re_{cri}, a complicated series of events takes place which eventually leads to a radical change of the flow character, in the final state the flow behavior is random and chaotic. The motion becomes intrinsically

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unsteady even with constant imposed boundary conditions. The velocity and all other flow properties vary in a random and chaotic way. This regime is called turbulent flow.
Turbulence is the state of fluid processing a non regular or irregular motion such that velocity at any point may vary both in magnitude and direction with time. Turbulent motion is also called as serious motion and is accompanied by the formulation of eddies and the rapid interchange of momentum in the fluid. The physical phenomenon of the irregularity or disorderliness is not simply the Turbulence. We do not have a clear-cut or final definition of Turbulence. It can also be stated as the irregular flow of fluid in which various quantities show a random variation with time and space. It is insufficient to define turbulent flow as irregular or chaotic only in time alone or in space alone.
According to von Karman, Turbulence can be generated by fluid flow past solid surfaces or by the flow of layers of fluids at different velocities past or over one another. Turbulence can be distinguished as wall Turbulence and free Turbulence. Wall Turbulence is the Turbulence generated by the viscous effects due to the presence of a solid wall. Free Turbulence is generated by the flow of layers of fluids at different velocities. Why we should study the theory of Turbulence is the another question we must answer. There are many problems of engineering importance such as boundary layer, heat transfer, friction and diffusion of fluids which cannot be estimated correctly without the consideration of Turbulence.
The Figure 1, which depicts a cross-sectional view of a turbulent boundary layer on a flat plate, shows eddies whose length scale is comparable to that of the flow boundaries as well as eddies of intermediate and small size. Particles of fluid which are initially separated by a long distance can be brought close together by the eddying motions in turbulent flows. As a consequence, heat, mass and momentum are very effectively exchanged.

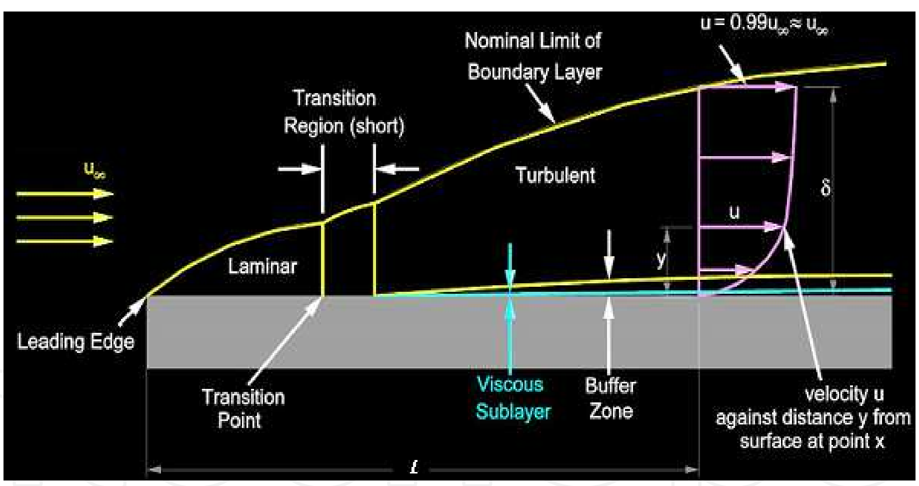

Fig. 1. Turbulent Boundary layer and Viscous Sublayer

For example, a streak of dye which is introduced at a point in a turbulent flow will rapidly break up and be dispersed right across the flow. Such effective mixing gives rise to high values of diffusion coefficients for mass, momentum and heat. The largest turbulent eddies interact with and extract energy from the mean flow by a process called vortex stretching. The presence of mean velocity gradients in sheared flows distorts the rotational turbulent eddies. Suitably aligned eddies are stretched because one end is forced to move faster than the other.
In the fluid flow, if the stress and velocity at a point fluctuate in a random fashion with time. Turbulence sets up greater shear stresses throughout the fluid and causes more

irreversibility or losses. Also the losses vary about 1.7 to 2 power of the velocity in laminar flow, they vary as the first power of the velocity. Turbulent flow occurs when the boundary and initial conditions that are characteristic of the flow led to the spontaneous growth of hydrodynamic instabilities which eventually decay to yield a random statistically fluctuating fluid motion. Now we shall also see that why there is turbulence. The first reason may be the occurrence of strong shear regions in the flow, the presence of wakes or boundary layers; it may be because of separated flow regions or because of Buoyancy flow. Turbulence is characterized by high levels of momentum, heat and mass transport due to turbulent diffusivity. Due to the energy is not supplied continuously in the high shear regions. At slightly higher Reynolds numbers the turbulence primarily gets initiated by the formation of two dimensional vertices and eventually breaks down to a fully three dimensional turbulent flow. If observed figuratively the laminar flow at very low Reynolds number say Re<5, the stream lines are shown in the figure 2; the seams separated and small vortex are shown in the figure 3, though the flow is separated the normal flow parameters would be the same and the stream lines are seen to be more organized and ordered.

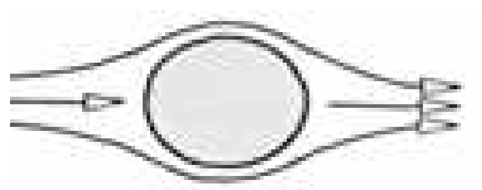

Fig. 2. Re < 5 Laminar attached Steady

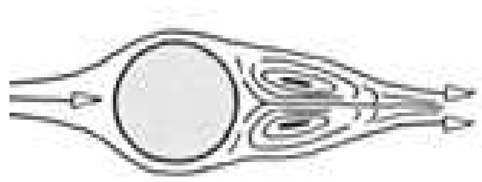

Fig. 3. 5 < Re < 40 Laminar Separated Steady

In Figure 4 it is clearly visible that the periodic and laminar separated flow stream lines are getting spread over the spatial scale and over the time period. In the similar way if the flow conditions are further changed to even higher Reynolds number a turbulent wake gets created and the same is shown in the figure 5. The fluid motion becomes turbulent and more chaotic at slightly higher Reynolds number; it is illustrated in the figure 6.

Fig. 4. 40 < Re < 200 Laminar Separated Periodic

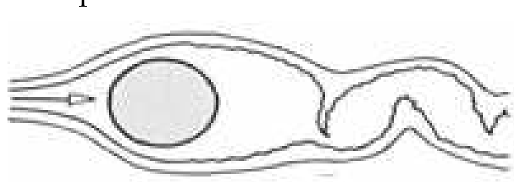

Fig. 5. 200 < Re < 350K Turbulent Wake Periodic

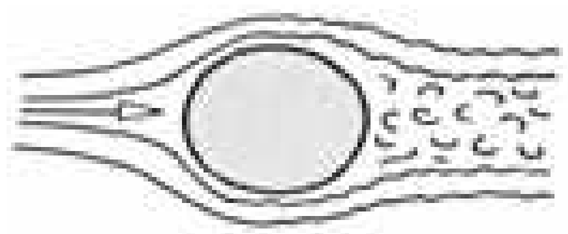

Fig. 6. 350K < Re Turbulent Separation Chaotic

Reynolds was the first person to study the Turbulence experimentally. He used dye experiment to investigate the transition from laminar to turbulent flow. He observed and concluded with his experimental results that transition from laminar to turbulent flow in pipes is occurring at nearly same Reynolds number. He established critical Reynolds number at which laminar regime breaks down to Turbulence for a particular flow conditions. The transition occurs as a result of external disturbances. It is proposed experimentally that there is a definite limit below which all the initial disturbances in the flow will be damped out, and laminar flow becomes stable. The time and space dependent disturbance are analyzed by scientists after Reynolds and in the two dimensional flow plate problems, it is observed that the flow is considered to be stable if the disturbance decay with time. It is very essential to study and understand the nature of Turbulence.
A typical point velocity measurement might exhibit the form shown in Figure below in Fig 7

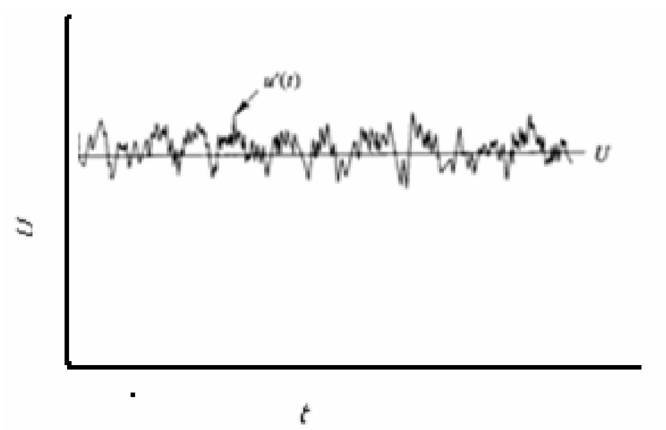

Fig. 7. Point velocities in turbulent flow

The random nature of a turbulent flow precludes computations based on a complete description of the motion of all the fluid particles. Instead the velocity in Figure 1.7 can be decomposed into a steady mean value U with a fluctuating component u'(t) superimposed on it: $U(t) = U + U'(t)$. In general, it is most attractive to characterize a turbulent flow by the mean values of flow properties (U, V, W, P etc.) and the statistical properties of their fluctuations. Even in flows where the mean velocities and pressures vary in only one or two space dimensions, turbulent fluctuations always have a three-dimensional spatial character. Furthermore, visualizations of turbulent flows reveal rotational flow structures, so-called turbulent eddies, with a wide range of length scales.

2.1 Characteristics of the turbulent flow

***Highly unsteady*:** A plot of velocity as a function of time would appear random to an observer unfamiliar with these flows.

Irregularity: It is another characteristic of Turbulence which makes the deterministic approach to Turbulence problems impossible. One should relay on statistical approach.
Diffusivity: If the flow pattern is random but does not exhibit spreading of velocity fluctuations through the surrounding fluid then it is not turbulent. The mixing is accomplished by diffusion. This process is also named as turbulent diffusion. The diffusivity of turbulence the single most important feature as far as applications are concerned.
Three dimensional: Turbulence is three dimensional and rotational.
Dissipative: The turbulent lows are always dissipative. The viscous shear stresses perform deformation work which increases the internal energy of the fluid at the expense of kinetic energy of turbulence.
Higher Reynolds number: Turbulence in the fluid flow always occurs at high Reynolds numbers. The instabilities are related to the interaction of viscous terms and non linear inertia terms in the equations of fluid motion.

2.2 Transition from laminar to turbulent flow

The initial cause of the transition to turbulence can be explained by considering the stability of laminar flows to small disturbances. A sizeable body of theoretical work is devoted to the analysis of the inception of transition: hydrodynamic instability. In many relevant instances the transition to turbulence is associated with sheared flows. Linear hydrodynamic stability theory seeks to identify conditions which give rise to the amplification of disturbances. Of particular interest in an engineering context is the prediction of the values of the Reynolds numbers at which disturbances are amplified and at which transition to fully turbulent flow takes place. Fig 8 is shown to illustrate the phenomenon. The subject matter is fairly complex but its confirmation has led to a series of experiments which reveal an insight into the physical processes causing the transition from laminar to turbulent flow. Most of our knowledge stems from work on two-dimensional incompressible flows. All such flows are sensitive to two- dimensional disturbances with a relatively long wavelength, several times the transverse distance over which velocity changes take place.

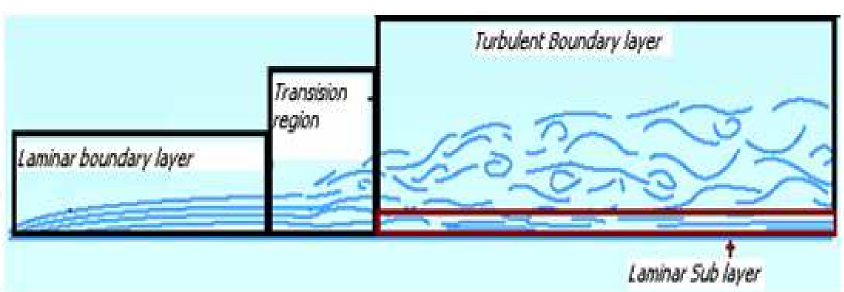

Fig. 8. Transition from Laminar to Turbulent flow

The point where instability first occurs is always upstream of the point of transition to fully turbulent flow. The distance between the points of instability where the Reynolds number equals critical Reynolds and the point of transition depends on the degree of amplification of the unstable disturbances. The point of instability and the onset of the transition process can be predicted with the linear theory of hydrodynamic instability. There is, however, no comprehensive theory regarding the path leading from initial instability to fully turbulent flows. Below we describe the main, experimentally observed, characteristics of three simple flows: jets, flat plate boundary layers and pipe flows.

3. General governing equations of fluid flow

The unsteady Navier stokes equations are considered as the governing equations of turbulent flows. These unsteady Navier stokes equations are not easy to solve sometimes next to impossible. The basic Navier-Stokes equations are presented here, as they are the basic equations of fluid flow it becomes essential to the reader to know the equations so that the modifications can easily be understood.

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}= \tag{1}$$

$$\begin{aligned}
\rho\left[\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right] &= -\frac{\partial P}{\partial x}+\mu\left[\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right]\\
\rho\left[\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right] &= -\frac{\partial P}{\partial y}+\mu\left[\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right]\\
\rho\left[\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right] &= -\rho g\vec{k}-\frac{\partial P}{\partial z}+\mu\left[\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right]
\end{aligned} \tag{2}$$

The N-S equations are presented as under in the most common form of their practice

$$\begin{aligned}
\rho\frac{Du}{Dt} &= X-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left[\mu\left(2\frac{\partial u}{\partial x}-\frac{2}{3}\,div\,\omega\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\left[\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial \omega}{\partial x}+\frac{\partial u}{\partial z}\right)\right]\right]\\
\rho\frac{Dv}{Dt} &= Y-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left[\mu\left(2\frac{\partial v}{\partial y}-\frac{2}{3}\,div\,\omega\right)\right]+\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v}{\partial z}+\frac{\partial \omega}{\partial y}\right)\right]+\left[\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]\right]\\
\rho\frac{D\omega}{Dt} &= Z-\frac{\partial p}{\partial x}+\frac{\partial}{\partial z}\left[\mu\left(2\frac{\partial \omega}{\partial z}-\frac{2}{3}\,div\,\omega\right)\right]+\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial \omega}{\partial x}+\frac{\partial u}{\partial z}\right)\right]+\left[\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial v}{\partial z}+\frac{\partial \omega}{\partial y}\right)\right]\right]
\end{aligned} \tag{3}$$

In the form of tensor notation the Navier-Stokes equations is given in the equation [4]

$$\frac{\partial \rho u_i}{\partial t}+\frac{\partial(\rho u_i u_j)}{\partial x_j}=-\frac{\partial(p)}{\partial x_i}+\frac{\partial}{\partial x_j}\left(\mu\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right]\right)-\frac{\partial}{\partial x_i}\left(\tfrac{2}{3}\mu\frac{\partial u_k}{\partial x_k}\right)+S_i \tag{4}$$

<u>Equation of Energy</u>

$$\begin{aligned}
&\frac{\partial(\rho\varepsilon)}{\partial t}+\frac{\partial(\rho u_j e)}{\partial x_j}=-p\frac{\partial u_j}{\partial x_j}+\frac{\partial}{\partial x_j}\left(k\frac{\partial T}{\partial x_j}\right)+\Phi+S_e\\
&where,\\
&\Phi=\mu\frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\tfrac{2}{3}\mu\left(\frac{\partial u_k}{\partial x_k}\right)^2
\end{aligned} \tag{5}$$

If turbulence is entirely irregular and chaotic, it would be inaccessible to any kind of mathematical treatment; instead the irregularity of the turbulence can be described by the laws of probability to a great extent. Through the physics of the turbulence is problem specific, almost all of the situations of the turbulent fluid motion can be mathematically modeled.

Generally the irregular flow would decay if there are no external sources of energy for the continuous generating of turbulence.

The smallest length scale in the study of turbulence is still several orders of magnitude larger than the molecular mean free path. Therefore the continuum approximation is still valid in the study of turbulence. A point here to mention is Navier stokes equations do represent the turbulence without any approximation.

To solve numerically or analytically the full Navier stokes equations we face little difficulty. One such difficulty is computing. For example N-S equation is fineness of spatial and temporal resolution required to represent the smallest length and scales of velocity and pressure fluctuations. Therefore obviously the modern powerful supercomputers also may sometimes fail.

Also when we generally attach a physical problem to solve numerically, there will be eventually round off errors in the methods. Now when we go for finer and finer meshes or grid points the round off errors will accumulate and thereby destroy the accuracy of the solution. However this drawback can be overcome by highly accurate numerical schemes. Numerical methods for the turbulent flows may be classified as empirical correlations, integral equation, averaging equations. This approach is also known as one point closure which leads to a set of partial differential equations called as Reynolds averaged Navier-stokes (or RANS) equations two-point closure, the Fourier transform equations will be evolved. LES and DNS are the other very important methods. As mentioned above the computation of turbulence is with the determination of time averaged velocity, pressure and temperature profiles and effect of time dependent fluctuations on them. The wall shear stress, heat transfer rates and points of separation could be determined from the time averaged flow properties.

Nevertheless it is quite clear that turbulence is characterized by random fluctuations, the statistical methods are studied extensively rather than deterministic methods. In this approach the time averaging of variables is carried out in order to separate the mean quantities from the fluctuations. As a result of this new unknown variable appears in the governing equations.

Therefore additional equations are needed to close the system, this process is known as turbulence modeling. The turbulence modeling is also known as Reynolds averaged Navier-stokes (or RANS) methods. This approach will help in modeling the large and small scales of turbulence so the requirements of DNS such as refined mesh can be ignored. Large eddy simulation which has become more popular in recent years is actually a compromise between DNS and RANS.

Turbulent flows contain great deal of velocity. It is rotational and moreover three dimensional. Velocity dynamics plays an essential role in the description of turbulent flows. In fact the intensity of turbulence is increased by the mechanism known as vertex stretching, as such the vertex stretching is absent in two dimensional flows.

The instantaneous field fluctuations rapidly in all three spatial dimensions because Turbulence is composed of high level fluctuating vertex. The velocity dynamics also plays an important role in the description of turbulent flows. Turbulent flows always exhibit high

levels of fluctuating vorticity. Turbulent motion is random and irregular, if has a broad range of length scales. It is next to impossible to obtain theoretical solutions by solving three dimensional, time dependent problems. Therefore we are forced to restrict ourselves to go on some averaged quantities; there are two types of averaging procedures.
Conventional time averaging (Reynolds averaging) and Mass – weighed time averaging (Favre averaging). In most of the flow situations the instantaneous values do not satisfy Navier poisson law. There will be additional shear stress due to the turbulence in the fluid flow also to be considered
There are many ways of averaging flow variables such as time averages, spatial averages and mass averages etc.

Time averages

Any variable f assumed to be the sum of its mean quantity $\bar{f}$ and its fluctuating part f'

$$f(x,t) = \bar{f}(x,t) + f'(x,t)$$

$\bar{f}$ is the time average of f'

$$\bar{f}(x,t) = \frac{1}{\Delta t}\int_{t}^{t+\Delta t} f(x,t)\,dt$$

$$\bar{f}(x,t) = \frac{1}{\Delta t}\int_{t}^{t+\Delta t} f'\,dt = 0$$

Spatial Averages

When the flow variable is uniform on the average such as in homogeneous turbulence,

$$\lim_{\varphi\to\infty}\frac{1}{\varphi}\int_{\varphi} f(x,t)\,d\varphi$$

Mass Averages:

The mass average instead of time averages is preferred for compressible flows.

$$F = \bar{f} + f''$$

Where the mean quantity $\bar{f}$ is defined

$$\bar{f} = \frac{\overline{\rho f}}{\rho} = \bar{f} + \frac{\overline{\rho f'}}{\rho}$$

$$\overline{\rho f''} = 0$$

Where as $$\overline{f''} = -\frac{\overline{\rho\, f'}}{\bar{\rho}} = 0$$

$\bar{f}$ is known as force fluctuation field.

Ensemble Averages:

If N identical experiments are carried out f(x, t) = f_n (x, t) we may determine the average

$$\bar{f}(x,t) = \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} f_n(x,t)$$

4. Reynolds Averaged Navier-Stokes (RANS) equations

As described above, turbulence models seek to solve a modified set of transport equations by introducing averaged and fluctuating components. RANS are time-averaged equations of motion for fluid flow. These equations can be used with approximations based on knowledge of the properties of flow turbulence to give fairly accurate averaged solutions to the Navier–Stokes equations.

For example, Reynolds decomposition refers to separation of the flow variable (like velocity *u*) into the mean (time-averaged) component ($\bar{u}$) and the fluctuating component (u').

Thus the average component is given as

$$u(\mathbf{x},t) = \bar{u}(\mathbf{x}) + u'(\mathbf{x},t)$$

If *f* and *g* are two flow variables, ***viz*** pressure (*p*), velocity (*u*),density (ρ) and ***s*** is one of the independent variables, independent of space and time then,

$$\bar{\bar{f}} = \bar{f}\ ,\ \overline{f+g} = \bar{f} + \bar{g}$$

$$\overline{\bar{f}g} = \bar{f}\bar{g}\ ,\ \overline{fg} \neq \bar{f}\bar{g} \text{ and } \overline{\frac{\partial f}{\partial s}} = \frac{\partial \bar{f}}{\partial s}$$

If we consider the incompressible N-S equations now and substitute the average values in the equation then we will be obtaining the new form of equation which can conveniently capture the turbulence to certain flow characteristics' and boundary conditions.

$$\rho\frac{\partial \bar{u}_i}{\partial t} + \rho\frac{\partial \bar{u}_j\bar{u}_i}{\partial x_j} = \rho\bar{f}_i + \frac{\partial}{\partial x_j}\left[-\bar{p}\delta_{ij} + 2\mu\bar{S}_{ij} - \rho\overline{u_i'u_j'}\right] \tag{6}$$

where $\bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$ is the mean rate of strain tensor

4.1 Effect of turbulence on time-averaged Navier-Stokes equations

The critical difference amid visualizations of laminar and turbulent flows is the appearance of eddying motions of a wide range of length scales in turbulent flows. We would need computing meshes of 10^9 up to 10^{12} points to be able to describe processes at all length scales. For instance the direct simulation of a turbulent pipe flow at a Reynolds number of 500000 requires 10 million times faster super computer than CRAY, the fastest supercomputer now in the world.

As the computing power is increasing enormously it may now be possible to track the dynamics of eddies in very simple flows at transitional Reynolds number. The computing requirements for the direct solution of the time dependent Navier-Stokes equations of fully turbulent flows at high Reynolds numbers are truly phenomenal and must await major developments in computer hardware, in the meantime, engineers need to work on computational procedures which can supply adequate information about the turbulent flow processes, but which can avoid the need to predict the effects of each and every eddy in the flow. We examine the effects of the appearance of turbulent fluctuations on the mean flow properties.

4.2 Reynolds equations

First we define the mean Γ of a flow property φ as follows

$$\Gamma = \frac{1}{\Delta t}\int_0^{\Delta t} \varphi(t)\, dt$$

In theory we should take the limit of time interval Δt approaching infinity, but Δt is large enough to hold the largest eddies if it exceeds the time scales of the slowest variations of the property Γ.The general equations of the fluid flow with all kinds of considerations are represented by the Navier stokes equations along with the continuity equation.
The time average of the fluctuations $\overline{\Gamma}'$ is given as

$$\overline{\Gamma'} = \frac{1}{\Delta t}\int_0^{\Delta t} \varphi'(t)\, dt$$

The following rules govern the time averaging of the fluctuating properties used to derive the governing equations of the turbulent fluid flow.

$$\overline{\varphi'} = \overline{\psi'} = 0,\ \overline{\Gamma} = \Gamma_i,\ \frac{\partial\overline{\varphi}}{\partial s} = \frac{\partial\varphi}{\partial s},$$

$$\overline{\int \varphi\, ds} = \int \Gamma\, ds,\ \ \overline{\varphi'} = \overline{\psi'} = \Gamma, \overline{\varphi+\psi} = \Gamma + \Omega \text{ and } \overline{\varphi\psi} = \Gamma\,\Omega + \overline{\varphi'\psi'}$$

The root mean square of the fluctuations is given by the equation
The kinetic energy associated with the turbulence is

$$\varphi_{rms} = \sqrt{\overline{(\varphi')^2}} = \left[\frac{1}{\Delta t}\int_0^{\Delta t} (\varphi')^2\, dt\right]^{1/2} \qquad k = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$$

To demonstrate the influence of the turbulent fluctuations on the mean flow, we have to consider the instantaneous continuity and N-S equations.

$$\begin{aligned}
&div\ \mathbf{u} = 0\\
&\frac{\partial u}{\partial t} + div(u\mathbf{u}) = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\ div\ grad\ u\\
&\frac{\partial v}{\partial t} + div(v\mathbf{u}) = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\ div\ grad\ v\\
&\frac{\partial w}{\partial t} + div(w\mathbf{u}) = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\ div\ grad\ w
\end{aligned} \tag{7}$$

The flow variables u and p are to be replaced by their sum of the mean and fluctuating components.

$$\mathbf{u} = \mathbf{U} + \mathbf{u}'; u = U + u';\ v = V + v';\ w = W + w';\ p = P + p'$$

Continuity equation is $div\,\mathbf{U} = 0$

The time averages of the individual terms in the equation are as under

$$\overline{\frac{\partial u}{\partial t}} = \frac{\partial U}{\partial t}; \qquad \overline{div(u\mathbf{u})} = div(U\mathbf{U}) + div\left(\overline{u'\mathbf{u}'}\right)$$

$$\overline{-\frac{1}{\rho}\frac{\partial p}{\partial x}} = -\frac{1}{\rho}\frac{\partial P}{\partial x}; \qquad \overline{\nu\, div\, grad\, u} = \nu\, div\, grad\, U$$

Substitution of the average values in the basic derived equation would yield the following momentum conservation equations, the momentum in x- y- and z- directions.

$$\frac{\partial U}{\partial t} + div(U\mathbf{U}) + div\left(\overline{u'\mathbf{u}'}\right) = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\, div\, grad\, U \tag{8}$$

$$\frac{\partial V}{\partial t} + div(V\mathbf{U}) + div\left(\overline{v'\mathbf{u}'}\right) = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\, div\, grad\, V \tag{9}$$

$$\frac{\partial W}{\partial t} + div(W\mathbf{U}) + div\left(\overline{w'\mathbf{u}'}\right) = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \nu\, div\, grad\, W \tag{10}$$

In time-dependent flows the mean of a property at time t is taken to be the average of the instantaneous values of the property over a large number of repeated identical experiments. The flow property cp is time dependent and can be thought of as the sum of a steady mean components and a time-varying fluctuating components with zero mean value; hence p(t) = p + p'(t).

The non zero turbulent stresses usually large compared to the viscous stresses of turbulent flow are also need to be incorporated into the Navier Stokes equations, they are called as the Reynolds equations as shown below in the Euqations [11-13]

$$\frac{\partial U}{\partial t} + div(U\mathbf{U}) = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\, div\, grad\, U + \left[-\frac{\partial \overline{u'^2}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y} - \frac{\partial \overline{u'w'}}{\partial z}\right] \tag{11}$$

$$\frac{\partial V}{\partial t} + div(V\mathbf{U}) = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\, div\, grad\, V + \left[-\frac{\partial \overline{u'v'}}{\partial x} - \frac{\partial \overline{v'^2}}{\partial y} - \frac{\partial \overline{v'w'}}{\partial z}\right] \tag{12}$$

$$\frac{\partial W}{\partial t} + div(W\mathbf{U}) = -\frac{1}{\rho}\frac{\partial P}{\partial z} + v\ div\ grad\ W + \left[-\frac{\partial \overline{u'w'}}{\partial x} - \frac{\partial \overline{v'w'}}{\partial y} - \frac{\partial \overline{w'^2}}{\partial z}\right] \quad [13]$$

4.3 Modeling flow near the wall

Experiments and mathematical analysis have shown that the near-wall region can be subdivided into two layers. In the innermost layer, the so-called viscous sub layer, as shown in the figure 1 (indicated in blue)the flow is almost laminar-like, only the viscosity plays a dominant role in fluid flow. Further away from the wall, in the logarithmic layer, turbulence dominates the mixing process. Finally, there is a region between the viscous sublayer and the logarithmic layer called the buffer layer, where the effects of molecular viscosity and turbulence are of equal importance. Near a no-slip wall, there are strong gradients in the dependent variables. In addition, viscous effects on the transport processes are large. The representation of these processes within a numerical simulation raises the many problems. How to account for viscous effects at the wall and how to resolve the rapid variation of flow variables which occurs within the boundary layer region is the important question to be answered.

Assuming that the logarithmic profile reasonably approximates the velocity distribution near the wall, it provides a means to numerically compute the fluid shear stress as a function of the velocity at a given distance from the wall. This is known as a 'wall function' and the logarithmic nature gives rise to the well known 'log law of the wall.' Two approaches are commonly used to model the flow in the near-wall region:

The wall function method uses empirical formulas that impose suitable conditions near to the wall without resolving the boundary layer, thus saving computational resources. The major advantages of the wall function approach is that the high gradient shear layers near walls can be modeled with relatively coarse meshes, yielding substantial savings in CPU time and storage. It also avoids the need to account for viscous effects in the turbulence model.

When looking at time scales much larger than the time scales of turbulent fluctuations, turbulent flow could be said to exhibit average characteristics, with an additional time-varying, fluctuating component. For example, a velocity component may be divided into an average component, and a time varying component.

In general, turbulence models seek to modify the original unsteady Navier-Stokes equations by the introduction of averaged and fluctuating quantities to produce the Reynolds Averaged Navier-Stokes (RANS) equations. These equations represent the mean flow quantities only, while modeling turbulence effects without a need for the resolution of the turbulent fluctuations. All scales of the turbulence field are being modeled. Turbulence models based on the RANS equations are known as Statistical Turbulence Models due to the statistical averaging procedure employed to obtain the equations.

Simulation of the RANS equations greatly reduces the computational effort compared to a Direct Numerical Simulation and is generally adopted for practical engineering calculations. However, the averaging procedure introduces additional unknown terms containing products of the fluctuating quantities, which act like additional stresses in the fluid. These terms, called 'turbulent' or 'Reynolds' stresses, are difficult to determine directly and so become further unknowns.

The Reynolds stresses need to be modeled by additional equations of known quantities in order to achieve "closure." Closure implies that there is a sufficient number of equations for all the unknowns, including the Reynolds-Stress tensor resulting from the averaging procedure. The equations used to close the system define the type of turbulence model.

5. Turbulance governing equations

As it has been mentioned earlier the nature of turbulence can well be analyzed comprehensively with Navier-stokes equations, averaged over space and time.

5.1 Closure problem

The need for turbulence modeling the instantaneous continuity and Navier-Stokes equations form a closed set of four equations with four unknowns' u, v, w and p. In the introduction to this section it was demonstrated that these equations could not be solved directly in the foreseeable future. Engineers are content to focus their attention on certain mean quantities. However, in performing the time-averaging operation on the momentum equations we throw away all details concerning the state of the flow contained in the instantaneous fluctuations. As a result we obtain six additional unknowns, the Reynolds stresses, in the time averaged momentum equations. Similarly, time average scalar transport equations show extra terms. The complexity of turbulence usually precludes simple formulae for the extra stresses and turbulent scalar transport terms. It is the main task of turbulence modeling to develop computational procedures of sufficient accuracy and generality for engineers to predict the Reynolds stresses and the scalar transport terms.

6. Turbulence models

A turbulence model is a computational procedure to close the system of flow equations derived above so that a more or less wide variety of flow problems can be calculated adopting the numerical methods. In the majority of engineering problems it is not necessary to resolve the details of the turbulent fluctuations but instead, only the effects of the turbulence on the mean flow are usually sought.
The following are one equation models generally implemented; out of the mentioned three spalart-Allmaras model is used in most of the cases.

- Prandtl's one-equation model
- Baldwin-Barth model
- Spalart-Allmaras model

The Spalart-Allmaras model was designed specifically for aerospace applications involving wall-bounded flows and has been shown to give good results for boundary layers subjected to adverse pressure gradients. It is also gaining popularity for turbo machinery and internal combustion engines also. Its suitability to all kinds of complex engineering flows is still uncertain; it is also true that Spalart-Allmaras model is effectively a low-Reynolds-number model.
In the two equations category there are two most important and predominant models known as k-epsilon, k-omega models. In the k-epsilon model again there are three kinds. However the basic equation is only the k-epsilon, the other two are the later corrections or improvements in the basic model.

6.1 K-epsilon models

- Standard k-epsilon model
- Realisable k-epsilon model
- RNG k-epsilon model

Launder and Spalding's the simplest and comprehensive of turbulence modeling are two-equation models in which the solution of two separate transport equations allows the turbulent velocity and length scales to be independently determined.

6.2 Standard k-ε model

The turbulence kinetic energy, ***k*** is obtained from the following equation where as

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_i}(\rho k u_i)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+P_k+P_b-\rho\epsilon-Y_M+S_k$$

rate of dissipation, ϵ can be obtained from the equation below.

$$\frac{\partial}{\partial t}(\rho\epsilon)+\frac{\partial}{\partial x_i}(\rho\epsilon u_i)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial x_j}\right]+C_{1\epsilon}\frac{\epsilon}{k}\left(P_k+C_{3\epsilon}P_b\right)-C_{2\epsilon}\rho\frac{\epsilon^2}{k}+S_\epsilon$$

The term in the above equation $P_k=-\rho\overline{u'_i u'_j}\frac{\partial u_j}{\partial x_i}$ represents the generation of turbulence kinetic energy due to the mean velocity gradients.

$P_b=\beta g_i\frac{\mu_t}{\mathrm{Pr}_t}\frac{\partial T}{\partial x_i}$ is the generation of turbulence kinetic energy due to buoyancy and

Y_M Represents the contribution of the fluctuating dilatation in compressible turbulence to the overall dissipation rate

6.3 Realisable k- ε model

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_j}(\rho k u_j)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+P_k+P_b-\rho\epsilon-Y_M+S_k$$

$$\frac{\partial}{\partial t}(\rho\epsilon)+\frac{\partial}{\partial x_j}(\rho\epsilon u_j)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial x_j}\right]+\rho C_1 S\epsilon-\rho C_2\frac{\epsilon^2}{k+\sqrt{\nu\epsilon}}+C_{1\epsilon}\frac{\epsilon}{k}C_{3\epsilon}P_b+S_\epsilon$$

Where $C_1=\max\left[0.43,\frac{\eta}{\eta+5}\right],\quad \eta=S\frac{k}{\epsilon},\quad S=\sqrt{2S_{ij}S_{ij}}$

The realizable ***k***- ϵ model contains a new formulation for the turbulent viscosity. A new transport equation for the dissipation rate, ϵ, has been derived from an exact equation for the transport of the mean-square vorticity fluctuation

In these equations, G_k represents the generation of turbulence kinetic energy due to the mean velocity gradients, and G_b is the generation of turbulence kinetic energy due to buoyancy.

Y_M represents the contribution of the fluctuating dilatation in compressible turbulence to the overall dissipation rate. And some constants *viz* C_2 $C_{1\varepsilon}$ and also the source terms S_k and S_ε

6.4 RNG k-ε model

The RNG k-ε model was derived using a statistical technique called renormalization group theory. It is similar in form to the standard k-ε model, however includes some refinements

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j}\left(\alpha_k \mu_{\text{eff}} \frac{\partial k}{\partial x_j}\right) + G_k + G_b - \rho\epsilon - Y_M + S_k$$

$$\frac{\partial}{\partial t}(\rho\epsilon) + \frac{\partial}{\partial x_i}(\rho\epsilon u_i) = \frac{\partial}{\partial x_j}\left(\alpha_\epsilon \mu_{\text{eff}} \frac{\partial \epsilon}{\partial x_j}\right) + C_{1\epsilon}\frac{\epsilon}{k}(G_k + C_{3\epsilon}G_b) - C_{2\epsilon}\rho\frac{\epsilon^2}{k} - R_\epsilon + S_\epsilon$$

The RNG model has an additional term in its ε equation that significantly improves the accuracy for rapidly edgy flows. The effect of spin on turbulence is included in the RNG model, enhancing accuracy for swirling flows. The RNG theory provides an analytical formula for turbulent Prandtl numbers, while the standard k- ε model uses user-specified, constant values. while the standard k- ε model is a high-Reynolds-number model, the theory provides an analytically-derived differential method for effective viscosity that accounts for low-Reynolds-number effects.

6.5 K-ω models

- Wilcox's k-omega model
- Wilcox's modified k-omega model
- SST k-omega model

6.5.1 Wilcox's k-omega model

The K-omega model is one of the most common turbulence models. It is a two equation model that means, it includes two extra transport equations to represent the turbulent properties of the flow. This allows a two equation model to account for history effects like convection and diffusion of turbulent energy.

Kinematic eddy viscosity

$$\nu_T = \frac{k}{\omega}$$

Turbulence Kinetic Energy

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \tau_{ij}\frac{\partial U_i}{\partial x_j} - \beta^* k\omega + \frac{\partial}{\partial x_j}\left[(\nu + \sigma^*\nu_T)\frac{\partial k}{\partial x_j}\right]$$

Specific Dissipation Rate

$$\frac{\partial \omega}{\partial t} + U_j \frac{\partial \omega}{\partial x_j} = \alpha\frac{\omega}{k}\tau_{ij}\frac{\partial U_i}{\partial x_j} - \beta\omega^2 + \frac{\partial}{\partial x_j}\left[(\nu + \sigma\nu_T)\frac{\partial \omega}{\partial x_j}\right]$$

The constants are mentioned as under

$$\alpha = \frac{5}{9} \qquad \beta = \frac{3}{40} \qquad \beta^* = \frac{9}{100}$$
$$\sigma = \frac{1}{2} \qquad \sigma^* = \frac{1}{2} \qquad \varepsilon = \beta^*\omega k$$

6.5.2 Wilcox's modified k-omega model

Kinematic eddy viscosity

$$\nu_T = \frac{k}{\omega}$$

Turbulence Kinetic Energy

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \tau_{ij} \frac{\partial U_i}{\partial x_j} - \beta^* k\omega + \frac{\partial}{\partial x_j}\left[(\nu + \sigma^* \nu_T) \frac{\partial k}{\partial x_j}\right]$$

Specific Dissipation Rate

$$\frac{\partial \omega}{\partial t} + U_j \frac{\partial \omega}{\partial x_j} = \alpha \frac{\omega}{k} \tau_{ij} \frac{\partial U_i}{\partial x_j} - \beta \omega^2 + \frac{\partial}{\partial x_j}\left[(\nu + \sigma \nu_T) \frac{\partial \omega}{\partial x_j}\right]$$

The constants are mentioned as under

$$\alpha = \frac{13}{25} \qquad \beta = \beta_0 f_\beta \qquad \beta^* = \beta_0^* f_{\beta^*} \qquad \sigma = \frac{1}{2} \qquad \sigma^* = \frac{1}{2}$$

$$\beta_0 = \frac{9}{125} \qquad f_\beta = \frac{1 + 70\chi_\omega}{1 + 80\chi_\omega} \qquad \chi_\omega = \left|\frac{\Omega_{ij}\Omega_{jk}S_{ki}}{(\beta_0^*\omega)^3}\right| \qquad \beta_0^* = \frac{9}{100} \qquad f_{\beta^*} = \begin{cases} 1, & \chi_k \le 0 \\ \frac{1+680\chi_k^2}{1+80\chi_k^2}, & \chi_k > 0 \end{cases}$$

6.6 Standard and SST k- ω models theory

The standard and shear-stress transport k- ω is another important model developed in the recent times. The models have similar forms, with transport equations for k and ω. The major ways in which the SST model differs from the standard model are as follows:
Gradual change from the standard k- ω model in the inner region of the boundary layer to a high-Reynolds-number version of the k- ω model in the outer part of the boundary layer
Modified turbulent viscosity formulation to account for the transport effects of the principal turbulent shear stress. The transport equations, methods of calculating turbulent viscosity, and methods of calculating model constants and other terms are presented separately for each model.

6.7 v^2-f models

The v^2- f model is akin to the standard k-ε model; besides all other considerations it incorporates near-wall turbulence anisotropy and non-local pressure-strain effects. A limitation of the v^2- f model is that it fails to solve Eulerian multiphase problems. The v^2- f model is a general low-Reynolds-number turbulence model that is suitable to model turbulence near solid walls, and therefore does not need to make use of wall functions.

6.7.1 Reynolds stress model (RSM)

The Reynolds stress model is the most sophisticated turbulence model. Abandoning the isotropic eddy-viscosity hypothesis, the RSM closes the Reynolds-averaged Navier-Stokes equations by solving transport equations for the Reynolds stresses, together with an equation for the dissipation rate. This means that five additional transport equations are required in two dimensional flows and seven additional transport equations must be solved in three dimensional fluid flow equations. This is clearly discussed in the following pages. In view of the fact that the Reynolds stress model accounts for the effects of streamline swirl, curvature, rotation, and rapid changes in strain rate in a more exact manner than one-equation and two-equation models, one can say that it has greater potential to give accurate predictions for complex flows $\rho\overline{u_i'u_j'}$ is known as the transport of the Reynolds stresses

$$\frac{\partial}{\partial t}(\rho\,\overline{u_i'u_j'}) + \frac{\partial}{\partial x_k}(\rho u_k\overline{u_i'u_j'}) = -\frac{\partial}{\partial x_k}\left[\rho\,\overline{u_i'u_j'u_k'} + \overline{p\left(\delta_{kj}u_i' + \delta_{ik}u_j'\right)}\right]$$
$$+\frac{\partial}{\partial x_k}\left[\mu\frac{\partial}{\partial x_k}(\overline{u_i'u_j'})\right] - \rho\left(\overline{u_i'u_k'}\frac{\partial u_j}{\partial x_k} + \overline{u_j'u_k'}\frac{\partial u_i}{\partial x_k}\right) - \rho\beta(g_i\overline{u_j'\theta} + g_j\overline{u_i'\theta})$$
$$+\overline{p\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)} - 2\mu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}$$
$$-2\rho\Omega_k\left(\overline{u_j'u_m'}\epsilon_{ikm} + \overline{u_i'u_m'}\epsilon_{jkm}\right) + S_{\text{user}}$$

The first part of the above equation local time derivative and the second term is convection term; the right side of the equation is turbulent and molecular diffusion and buoyancy and stress terms.

6.7.2 Large eddy simulation

As it is noted above turbulent flows contain a wide range of length and time scales; the range of eddy sizes that might be found in flow is shown in the figures below. The large scale motions are generally much more energetic than the small ones. Their size strength makes them by far the most effective transporters of the conserved properties. The small scales are usually much weaker and provide little of these properties. A simulation which can treat the large eddies than the small one only makes the sense. Hence the name the large eddy simulation. Large eddy simulations are three dimensional, time dependent and expensive.

LES models are based on the numerical resolution of the large turbulence scales and the modeling of the small scales. LES is not yet a widely used industrial approach, due to the large cost of the required unsteady simulations. The most appropriate area will be free shear flows, where the large scales are of the order of the solution domain. For boundary layer flows, the resolution requirements are much higher, as the near-wall turbulent length scales become much smaller.LES simulations do not easily lend themselves to the application of grid refinement studies both in the time and the space domain. The main reason is that the turbulence model adjusts itself to the resolution of the grid. Two simulations on different grids are therefore not comparable by asymptotic expansion, as they are based on different levels of the eddy viscosity and therefore on a different resolution of the turbulent scales. However, LES is a very expensive method and systematic grid and time step studies are prohibitive even for a pre-specified filter. It is one of the disturbing facts that LES does not lend itself naturally to quality assurance using classical methods. This property of the LES also indicates that (non-linear) multigrid methods of convergence acceleration are not suitable in this application.

The governing equations employed for LES are obtained by filtering the time-dependent Navier-Stokes equations in either Fourier (wave-number) space or configuration (physical) space. The filtering process effectively filters out the eddies whose scales are smaller than the filter width or grid spacing used in the computations. The resulting equations thus govern the dynamics of large eddies.

A filtered variable is defined by $\bar{\phi}(\mathbf{x}) = \int_D \phi(\mathbf{x}')G(\mathbf{x},\mathbf{x}')d\mathbf{x}'$

When the Navier stokes equations with constant density and incompressible flow are filtered, the following set of equations which are similar to the RANS equations.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho \bar{u}_i) = 0$$

$$\frac{\partial}{\partial t}(\rho \bar{u}_i) + \frac{\partial}{\partial x_j}(\rho \bar{u}_i \bar{u}_j) = \frac{\partial}{\partial x_j}\left(\mu \frac{\partial \sigma_{ij}}{\partial x_j}\right) - \frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j}$$

The continuity equation is linear and does not change due to filtering.

$$\frac{\partial}{\partial x_i}(\rho \bar{u}_i) = 0$$

6.8 Example

Wall mounted cube as an example of the LES; the flow over a cube mounted on one wall of a channel. The problem is solved using the mathematical modeling and the Reynolds number is based on the maximum velocity at the inflow. The inflow is fully developed channel flow and taken as a separate simulation, the outlet condition is the convective condition as given above. No-slip conditions all wall surfaces. The mesh is generated in the preprocessor and the same is exported to the solver. The time advancement method is of fractional step type. The convective terms are treated solved by Runge-Kutta second order method in time. The pressure is obtained by solving poisson equation.

The stream lines of the time averaged flow in the region close to the wall is observed. The simulation post processed results and plots are presented. The stream line of the time-averaged flow in the region is depicting the great deal of information about the flow. The

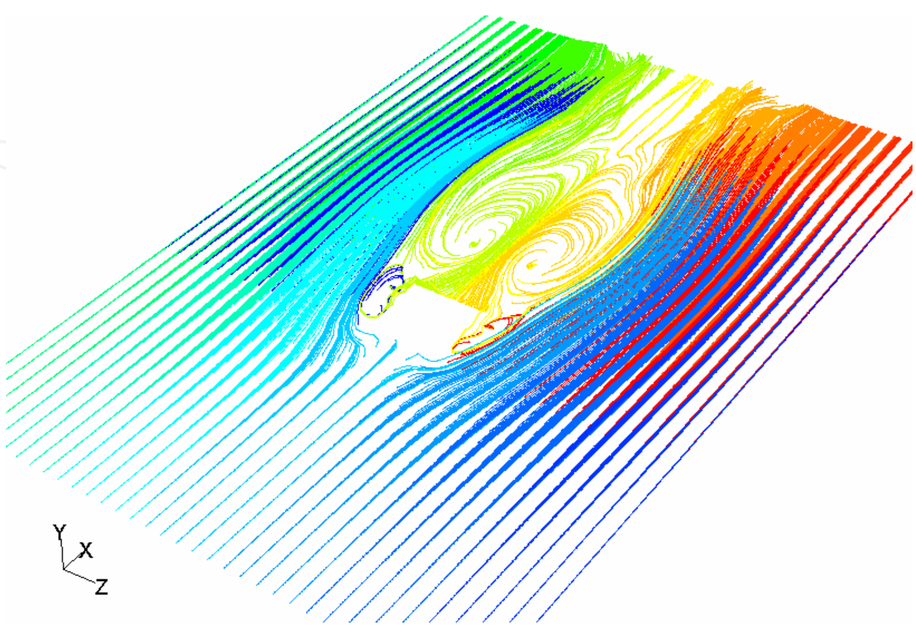

Fig. 9. Stream Lines from the top view

Figure 9 is showing the stream lines and it is clearly visible that the flow is not separated at the incoming and if it is closely observed that there is a secondary separation and reattachment in the flow just afterwards. There are two areas of swirling flow which are the foot prints of the vortex. Almost all the features including the separation zone and also horseshoe vortex.
It is significant to note down that the instantaneous flow looks very different than the time averaged flow.; the arch vortex does not exit in and instantaneous since; there are vortices in the flow but they are almost always asymmetric as shown in the figure figure 10. Indeed, the near-symmetry of figure 10 is an indication that the averaging time is long enough.
Performance of such a simulation has more practical importance and experimental support to such mathematical modeling would help to understand the real time problems.

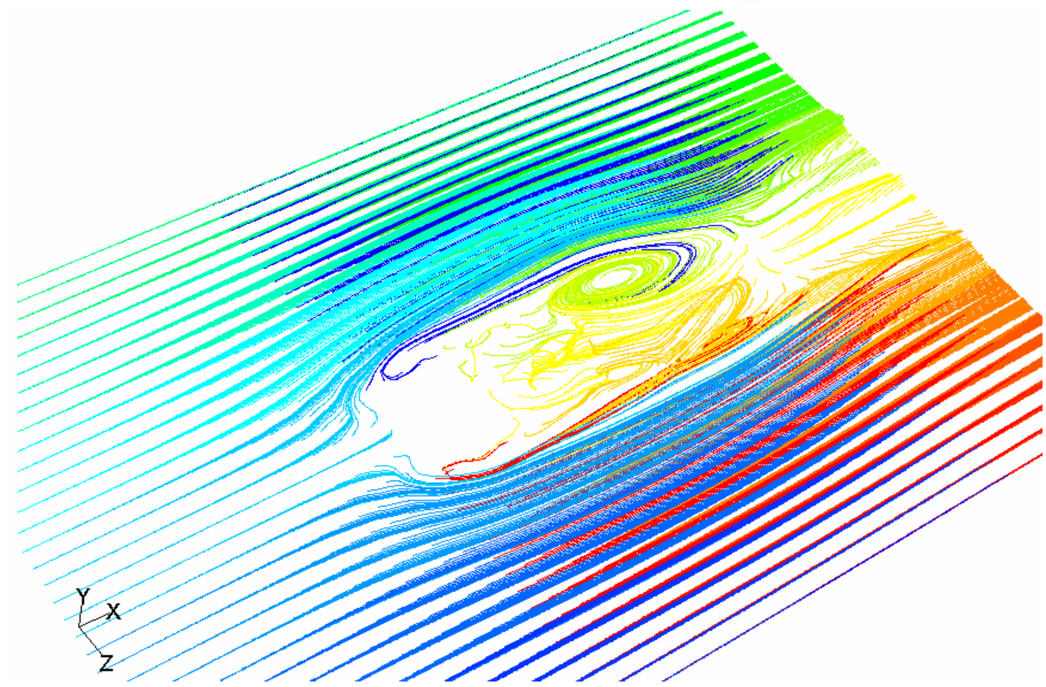

Fig. 10. Stream lines in the region close to the cube to trace the large eddies

7. Direct Numerical Simulation (DNS)

A direct numerical simulation (DNS) is a simulation of fluid flow in which the Navier-Stokes equations are numerically solved without any turbulence model. This means that the whole range of spatial and temporal scales of the turbulence must be resolved. Closure is not a problem with the so-called direct numerical simulation in which we numerically produce the instantaneous motions in a computer using the exact equations governing the fluid. Since even when we now perform a DNS simulation of a really simple flow, we are already overwhelmed by the amount of data and its apparently random behavior. This is because without some kind of theory, we have no criteria for selecting from it in a single lifetime what is important.
DNS using high-performance computers is an economical and mathematically appealing tool for study of fluid flows with simple boundaries which become turbulent. DNS is used to compute fully nonlinear solutions of the Navier-Stokes equations which capture important phenomena in the process of transition, as well as turbulence itself. DNS can be

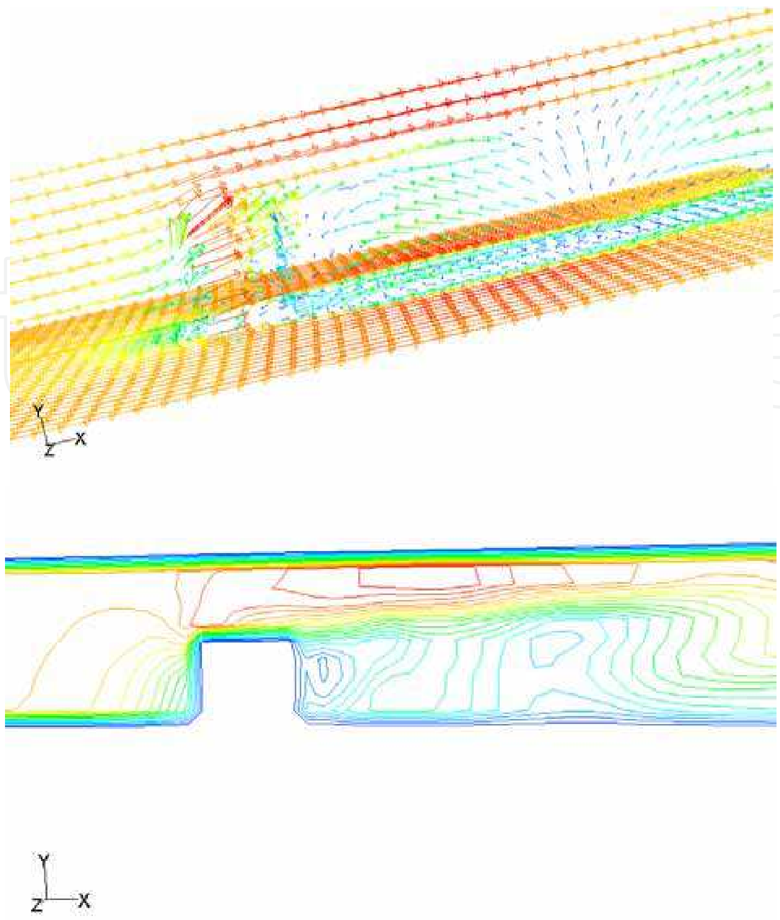

Fig. 11. Vector plots and the stream lines over the cube

used to compute a specific fluid flow state. It can also be used to compute the transient evolution that occurs between one state and another. DNS is mathematical, and therefore, can be used to create simplified situations that are not possible in an experimental facility, and can be used to isolate specific phenomena in the transition process.

All the spatial scales of the turbulence must be resolved in the computational mesh, from the smallest dissipative scales known as Kolmogorov scales, up to the integral scale L, and the kinetic energy.

Kolmogorov scale, η, is given by

$$\eta = (\nu^3/\varepsilon)^{1/4}$$

where v is the kinematic viscosity and ε is the rate of kinetic energy dissipation.

$Nh > L$, so that the integral scale is contained within the computational domain, and also

$h \leq \eta$, so that the Kolmogorov scale can be resolved

Since

$$\varepsilon \approx u'^3/L$$

where u' is the root mean square of the velocity, the previous relations imply that a three-dimensional DNS requires a number of mesh points N^3 satisfying

$$N^3 \geq \mathrm{Re}^{9/4} = \mathrm{Re}^{2.25}$$

where Re is the turbulent Reynolds number:

$$\mathrm{Re} = \frac{u'L}{\nu}$$

The memory storage requirement in a DNS grows very fast with the Reynolds number. In addition, given the very large memory necessary, the integration of the solution in time must be done by an explicit method. This means that in order to be accurate, the integration must be done with a time step, Δt, small enough such that a fluid particle moves only a fraction of the mesh spacing h in each step. That is,

$$C = \frac{u'\Delta t}{h} < 1$$

C is here the Courant number

The total time interval simulated is generally proportional to the turbulence time scale τ given by

$$\tau = \frac{L}{u'}.$$

Combining these relations, and the fact that h must be of the order of η, the number of time-integration steps must be proportional to L /η. By other hand, from the definitions for Re, η and L given above, it follows that

$$\frac{L}{\eta} \sim \mathrm{Re}^{3/4}$$

and consequently, the number of time steps grows also as a power law of the Reynolds number.

The contributions of DNS to turbulence research in the last decade have been impressive and the future seems bright. The greatest advantage of DNS is the stringent control it provides over the flow being studied. It is expected that as flow geometries become more complex, the numerical methods used in DNS will evolve. However, the significantly higher numerical fidelity required by DNS will have to be kept in mind. It is expected that use of non-conventional methodologies (e.g. multigrid) will lead to DNS solutions at an affordable cost, and that development of nonlinear methods of analysis are likely to prove very productive.

8. The Detached Eddy Simulation model (DES)

In an attempt to improve the predictive capabilities of turbulence models in highly separated regions, Spalart proposed a hybrid approach, which combines features of classical RANS formulations with elements of Large Eddy Simulations (LES) methods. The concept has been termed Detached Eddy Simulation (DES) and is based on the idea of covering the boundary layer by a RANS model and switching the model to a LES mode in detached regions. Ideally, DES would predict the separation line from the underlying RANS model, but capture the unsteady dynamics of the separated shear layer by resolution of the developing turbulent structures. Compared to classical LES methods, DES saves orders of magnitude of computing power for high Reynolds number flows. Though this is due to the moderate costs of the RANS model in the boundary layer region, DES still offers some of the advantages of an LES method in separated regions.

9. Final remarks

This chapter provides a first glimpse of the role of turbulence in defining the wide-ranging features of the flow and of the practice of turbulence modeling. Turbulence is a phenomenon of great complexity and has puzzled engineers for over a hundred years. Its appearance causes radical changes to the flow which can range from the favorable to the detrimental. The fluctuations associated with turbulence give rise to the extra Reynolds stresses on the mean flow. What makes turbulence so difficult to attempt mathematically is the wide range of length and time scales of motion even in flows with very simple boundary conditions. It should therefore be considered as truly significant that the two most widely applied models, the mixing length and k-ε models, succeed in expressing the main features of many turbulent flows by means of one length scale and one time scale defining variable. The standard k-ε model still comes highly recommended for general purpose CFD computations. Although many experts argue that the RSM is the only feasible way forward towards a truly general purpose standard turbulence model, the recent advances in the area of non-linear k-e ε models are very likely to re- revitalize research on two-equation models.
Large eddy simulation (LES) models require great computing resources and are used as general purpose tools. Nevertheless, in simple flows LES computations can give values of turbulence properties that cannot be measured in the laboratory owing to the absence of suitable experimental techniques. Therefore LES models will increasingly be used to guide the development of classical models through comparative studies. Although the resulting mathematical expressions of turbulence models may be quite complicated it should never be forgotten that they all contain adjustable.
DNS data is extensively used to evaluate LES results which are an order of magnitude faster to obtain. The availability of this detailed flow information has certainly improved our understanding of physical processes in turbulent flows which thus emphasizes the importance of DNS in present scientific research. Due to the very good correlation between the DNS results and the experimental data, DNS has become synonymous with the term “Numerical Experiment”. CFD calculations of the turbulence should never be accepted without the validation with the high quality experiments.

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This book is intended to serve as a reference text for advanced scientists and research engineers to solve a variety of fluid flow problems using computational fluid dynamics (CFD). Each chapter arises from a collection of research papers and discussions contributed by the practiced experts in the field of fluid mechanics. This material has encompassed a wide range of CFD applications concerning computational scheme, turbulence modeling and its simulation, multiphase flow modeling, unsteady-flow computation, and industrial applications of CFD.