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# Digital Algebraic Method for Processing Complex Signals for Radio Monitoring Systems

*Sergey G. Rassomakhin*

## Abstract

The methods of processing digital samples of complex structure signals with unknown parameters are considered. With the use of algebraic methods, the following tasks are sequentially solved: clock synchronization, determining the range of carrier frequencies, the multiplicity of phase modulation, and obtaining a stream of information bits. The methods for improving the quality of processing digital samples of signals based on solving special overdetermined systems of linear equations are proposed. The estimation of efficiency of the offered method is carried out by an imitation statistical modeling. The advantages of the proposed methods of signal processing for the telecommunications and radio monitoring systems are shown.

**Keywords:** orthogonal frequency-division multiplexing, digital sampling, linear algebraic equations, correlation convolutions

## 1. Introduction

The construction of effective information transmission systems is inextricably linked with the problem of intensifying the usage of the time and frequency-energy resource of communication channels. One of the ways to solve this problem is using the complex signals with combined types of modulation in combination with the methods of spectrum narrowing and noise-resistant coding [1–5]. In this connection, the structure of the signals used to transmit information is becoming complicated, and, consequently, the algorithms of their processing are becoming complicated as well.

The most promising type of signal code constructions in wireless networks is orthogonal frequency-division multiplexing (OFDM) [2–5]. The basic idea of building such signals is arranging a set of mutually orthogonal frequency subchannels so that, on the one hand, one subchannel does not interfere with the other and, on the other hand, the spectra of the subchannels overlap. Due to the orthogonality of the linear subchannels, each of them can be considered independently of the others. Errors caused by the interference in one of the subchannels do not lead to errors in the other. As a result, only a small part of the transmitted information is distorted. Error-correcting coding being used, the errors can be corrected. The structure of signals with multiple simultaneously operating subcarrier frequencies has well established itself in conditions of heterogeneity of the propagation medium. In recent years, the capabilities of systems with OFDM

signals have evolved significantly. Such signals began to be used in a wide variety of telecommunication systems operating in different radio frequency bands.

The complex structure of such signals and the a priori uncertainty of the channel properties cause significant difficulties in solving the problems of radio control and radio monitoring [6–11]. Similar tasks also arise when processing images and signals in biotechnical systems [12–16]. A distinctive feature of such tasks is the absence of data on the structure and informative parameters of the measured signals. This information should be obtained from the results of the study, with high accuracy and as soon as possible. Therefore, the tasks of developing mathematical methods for analyzing complex signals based on digital measurement sequences are highly relevant. This is evidenced by the intensive development of special software systems [17].

## 2. Mathematical model of OFDM signals

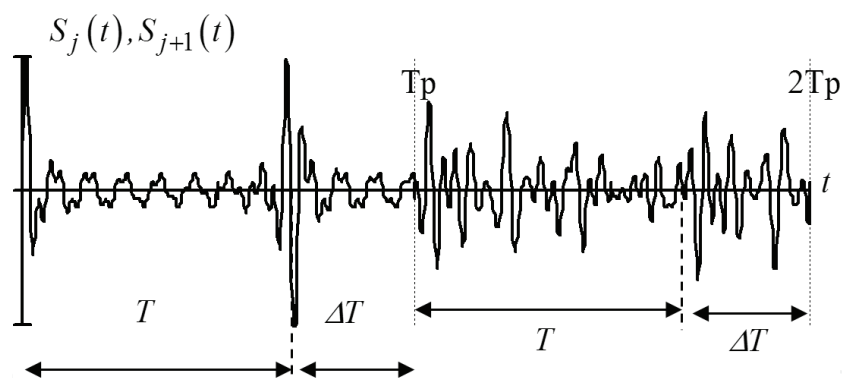
For correct choice of the methods for digital analysis of the primary parameters of OFDM signals, a brief description of their basic properties is necessary. Arbitrary OFDM signal  $S_j(t)$  on  $j$ -th modulation interval  $T_p$  is formed by algebraic summation of the several harmonic oscillations of the same amplitude. Each of the oscillations has  $m$  options of modulation phase shift. The value  $m$  determines the multiplicity of the used phase (PM) modulation and corresponds to the base of the numerical source code. Commonly,  $m = 2^k$  where  $k$  is the number of binary symbols (bits) represented by the elementary signal on one modulation interval. When using relative phase coding and a unit value of the amplitude of the oscillation subcarriers, the mathematical model of the signal can be represented as the following sequence:

$$S_j(t) = \sum_{i=0}^{n_f-1} \sin \left\{ 2\pi \left( f_0 + \frac{i}{T} \right) \cdot \left( t - T_p \left\lfloor \frac{t}{T_p} \right\rfloor \right) + \varphi_{j,i} \right\}, \quad (1)$$

where  $t$  is the current time,  $f_0$  is the lowest subcarrier frequency in the signal spectrum,  $T = 1/\Delta f$  is inverse of the minimum subcarrier spacing  $\Delta f$ ,  $n_f$  is the number of frequencies used, and  $\varphi_{j,i}$  is the value of the manipulation angle of  $i$ -th fluctuation on  $j$ -th modulation interval. This angle can take one of the  $m$  values depending on the manipulation code used. The informative features in the signal described by model (1) are relative phase jumps in carrier frequencies. These jumps are measured for each of the frequency subcarriers separately:  $\varphi_{j,i} - \varphi_{j-1,i}$ ,  $i = 0, \dots, n_f - 1$ . The time parameters of the modulation interval used in model (1) are tied by the relation

$$T_p = T + \Delta T = \frac{1}{\Delta f} + \Delta T, \quad (2)$$

where  $\Delta T$  is the duration of the prefix part of the signal. The prefix part (hereinafter the prefix) is a repeating (to the exact sign) initial part of the signal added at the end of the modulation interval  $T_p$ . Prefix structure is used to facilitate synchronization in the presence of channel irregularities. For OFDM synchronization violations, the signal can be correctly received and processed for any segment with the duration of  $T$  within the full modulation interval  $T_p$ . Commonly, the choice of the prefix duration on the modulation interval corresponds to the ratio  $\Delta T = (0, 1 \div 0, 5)T$ . The sign of the prefix depends on the value of the following parameter:



**Figure 1.**  
Example of the OFDM signal.

$$P = f_0 \bmod(\Delta f). \quad (3)$$

For existing OFDM standards, parameter (3) can take two values that determine the sign of the cyclic prefix: when  $P = 0$  the prefix is positive and when  $P = \Delta f/2$  inverse. **Figure 1** gives a qualitative idea of the form of the signal envelope constructed in accordance with model (1) on two adjacent modulation intervals with  $n_f = 16$  and  $T_p = 1, 47 \cdot T$ . At the end of each of the intervals  $T_p$ , the inverse cyclic continuation of the signal with duration  $\Delta T$  is located, which repeats, up to a sign, the shape of the initial segment of the signal on the modulation interval. For the example in question,  $P = \Delta f/2$ ; therefore the prefix part is the inverse of the initial part of the signal.

### 3. The main stages of the structural analysis of OFDM signals

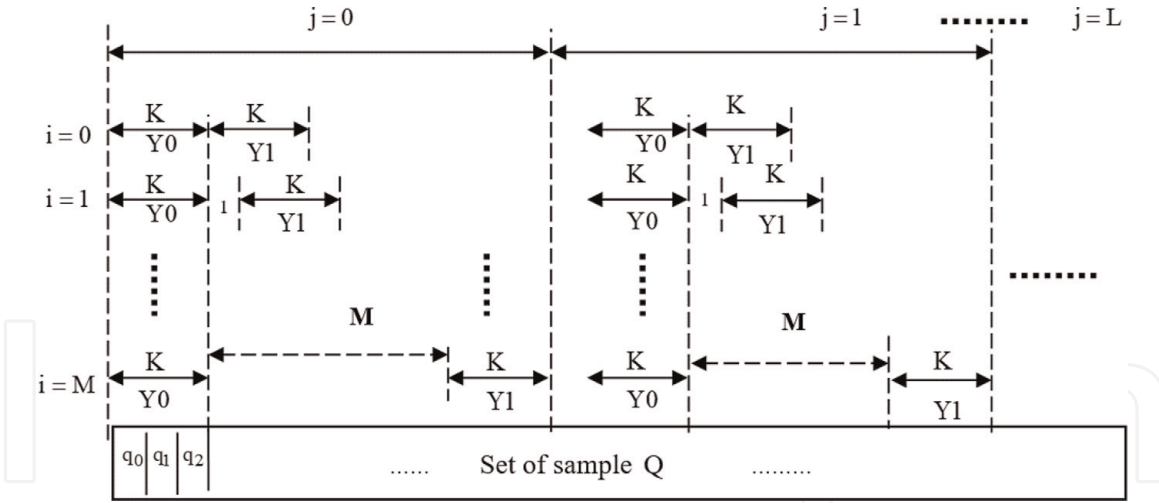
A comprehensive analysis of the properties of complex signals is advisable to implement on the basis of phased processing. At each stage, only a part of the signal parameters is determined. Given the fact that OFDM signals contain a prefix, it is advisable to use the correlation method for determining structural time parameters  $T_p$  and  $T$  at the first stage. This technique is based on the principle of “sliding” time window. This makes it possible to determine the following parameters of an OFDM signal: the value of the orthogonality interval, the duration of the modulation interval, and the value of the frequency spacing between the channels.

At the second stage of the analysis, the tasks of determining the number and the values of service and information channel frequencies, as well as, the signal phase demodulation.

The two-stage processing results in the possibility to extract an information flow from signals of an a priori unknown structure without using traditional fast Fourier transform (FFT) algorithms.

#### 3.1 Correlation method for determining the time parameters of OFDM signals

We propose a correlation method for determining structural time parameters  $T_p$  and  $T$ . The basis is the “sliding time window” principle. The most probable value of the time interval between the most correlated segments (with the “+” or “−” sign) of the segments from the digital sample of signal measurements is determined. The assessment of  $T_p$ —the most likely period of the emergence of “bursts” of



**Figure 2.**  
Calculation scheme.

correlation in the process of moving the viewing window on the samples of the array of measurements  $Q = \{q_0, q_1, \dots\}$ —is determined as well. The scheme of the calculation procedure is presented in **Figure 2**. For the correlation analysis, the two vectors, each containing  $K$  elements of array  $Q$  in two non-overlapping time observation windows of the signal, are formed:

$$\begin{aligned} Y0 &= \{q_j, q_{j+1}, \dots, q_{j+K-1}\}, \\ Y1 &= \{q_{j+i+K}, q_{j+i+K+1}, \dots, q_{j+i+2K-1}\}, \end{aligned} \quad (4)$$

removed from each other by  $i$ ,  $i = 0, \dots, M$ . The position of the second time window corresponding to the vector  $Y1$  is determined by the successive change in the offset index  $i = 0, \dots, M$  that ensures its “slip” along the signal sample  $Q$  at each of the values  $j = 0 \dots L$ .

For a wide range of analyzed signals, for example, for the 0.3–3.4 KHz frequency band with minimum quality ADC, the most universal limits of the values of these parameters, resulting in a quick and accurate assessment, are  $K = 10 \div 30$ ,  $M = 200 \div 300$ , and  $L = 1000$ .

At each value of index  $j$  (moving the window slip area), the  $M + 1$  dimensional vector is being formed:

$$V_j = \{v_0^j, v_1^j, \dots, v_M^j\}, \quad (5)$$

the elements of which are the coefficients of mutual correlation of vectors  $Y0$  and  $Y1$ . The calculations (according to **Figure 2**) are performed after centering and normalizing the vectors by the formulas

$$\begin{aligned} Y0_N &= \left[ Y0_j - \frac{1}{K} \sum_{i=0}^{K-1} Y0_i \right] \cdot \left[ \frac{1}{K} \sum_{i=0}^{K-1} Y0_i \right]^{-1}; \\ Y1_N &= \left[ Y1_j - \frac{1}{K} \sum_{i=0}^{K-1} Y1_i \right] \cdot \left[ \frac{1}{K} \sum_{i=0}^{K-1} Y1_i \right]^{-1}. \end{aligned} \quad (6)$$

The resulting vectors  $Y0_N$  and  $Y1_N$  in normalized space have the same length, equal to  $\sqrt{K}$ , and the cosine of the angle between these vectors is equal to the cross-correlation coefficient:



$$\cos(Y0_n, Y1_n) = \sum_{i=0}^{K-1} Y0_{ni} \cdot Y1_{ni} = r(Y0_n, Y1_n). \quad (7)$$

Since the prefix is a repetitive (up to sign) part of the OFDM signal, ideally the correlation coefficient between these parts is  $\pm 1$ .

**Figure 3** shows the distribution of the values of the elements of the vector  $V_j$  calculated according to a specific implementation OFDM signal (16 carrier frequencies, a modulation rate—75 bauds) at  $M = 200$ . The presence of pronounced extreme values which are close in magnitude to unity is obvious. According to the results of calculations when  $j = 0 \dots L$ , the two new vectors  $V1 = \{v1_0, v1_1, \dots, v1_L\}$  and  $V2 = \{v2_0, v2_1, \dots, v2_L\}$  are formed. Their elements are calculated according to the rules

$$\begin{aligned} v1_j &= \text{match}[\min(V_j), V_j]_0 + K; \\ v2_j &= \text{match}[\max(V_j), V_j]_0 + K; \end{aligned} \quad j = 0 \dots L. \quad (8)$$

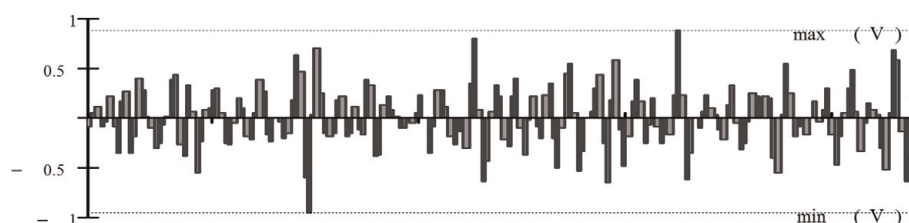
Here the function  $\text{match}[x, X]$  calculates the indices of the elements of the vector  $X$  equal to  $x$ , where the index 0 in the function (8) indicates a selection of the element with a minimum sequence number, if there are several such elements in the vector. The elements of the vector  $V1$  represent the number of sampling intervals that fit between the initial elements  $Y0$  and  $Y1$  with minimal (negative) correlation on  $j$ -th step of moving the observation window. Accordingly, the elements of the vector  $V2$  are calculated for the maximum (positive) correlation of the vectors  $Y0$  and  $Y1$ . Simultaneous determination of the maximum and minimum is necessary to reveal the value of function (3). It is obvious that the elements of the vectors  $V1$  and  $V2$  defined by expression (8) can take values only in the range  $K \dots K + M$ . To study the statistical distribution of the values of the elements, the histograms for the elements of the vectors  $V1$  and  $V2$  are formed:

$$\begin{aligned} H1 &= \{h1_0, h1_1, \dots, h1_{K+M}\}, \\ H2 &= \{h2_0, h2_1, \dots, h2_{K+M}\}. \end{aligned} \quad (9)$$

Obtaining distributions (9) gives an opportunity to estimate the most likely value  $Num$ —the number of sampling intervals between the initial measurements of the segments of the digital sample  $Q$  with maximal (positive or negative) correlation:

$$Num = \begin{cases} \text{match}[\max(H1), H1]_0; & \langle \max(H1) \geq \max(H2) \rangle \\ \text{match}[\max(H2), H2]_0. & \langle \max(H1) < \max(H2) \rangle \end{cases} \quad (10)$$

The value  $Num$  determines the number of sampling intervals that fit on the orthogonality interval of the signal  $T$ . It gives an opportunity to find two



**Figure 3.**  
 Cross-correlation coefficients.

interrelated OFDM signal parameters: the orthogonality interval and the minimum carrier frequency spacing:

$$T = \frac{Num}{f_\delta} \text{ and } \Delta f = T^{-1}. \quad (11)$$

Besides, choosing the corresponding method of detecting the  $Num$  value according to the condition specified in function (10) automatically determines the value of function (3) and, consequently, the ratio between the frequency parameters  $f_0$  and  $\Delta f$ . If the value  $Num$  is determined by the first line of expression (10), then  $f_0 = (k + \frac{1}{2}) \cdot \Delta f$ . Otherwise the minimal carrier frequency is multiplied to the spacing of the carrier frequencies:  $f_0 = k \cdot \Delta f$  where  $k$  is any positive integer.

As a result of processing histograms (9) according to function (10) based on the values of function (3), only one of the vectors  $V1$  or  $V2$  is left for the further analysis, hereinafter denoted  $V^*$ . This is possible because the prefix repetition sign is defined. Based on the elements of the vector  $V^*$ , another vector  $V3$  is formed for the analysis:

$$V3 = match[Num, V^*]. \quad (12)$$

The elements of this vector are equal to the numbers of the elements of the vector  $V^*$  in which the numbers  $Num$  are located. The feature of the vector  $V^*$ , provided that the analyzed signal belongs to the OFDM class, is that it contains a sequence of periodic series of numbers which are close or equal to  $Num$ . Therefore, the values of the elements of  $V3$  in order of increasing their indices will be the segments (series) of an ordinary positive integer sequence with some gaps in the sequence. Small gaps can be observed inside the series too. A possible approximation of the sequence of the elements of  $V3$  is illustrated by the following expression:

$$V3 = \left\{ \underbrace{11, 12, 14, 15, 16}_{\text{series.N}\hat{1}}, \underbrace{105, 107, 108, 110}_{\text{series.N}\hat{2}}, \dots, \underbrace{620, 621, 622, 623, 624}_{\text{series.N}\hat{n}_c} \right\}$$

The length of series of consecutive numbers (position numbers) may differ due to measurement errors, features of the signal envelope, and rounding during calculations. However, when the vector length is sufficient, averaging results converges to the true estimation in accordance with the law of large numbers. To exclude “fragmentation” of the series, small gaps between adjacent numbers of the series must be ignored. It has been empirically found that in most cases, the number  $N_i$  should be considered to belong to the current series of numbers if  $N_i - N_{i-1} \leq \varepsilon$  where  $\varepsilon = 3 \div 5$ . In general, the structure of the vector  $V3$  can be depicted as shown in **Figure 4**.

In **Figure 4**,  $v3_{n_j}$ ,  $v3_{k_j}$  are the initial and final elements of the  $j$ -th series of the consecutive numbers in the vector  $V3$ , and  $n_c$  is the total number of the identified

$$\left\{ \underbrace{v3_0, \dots, v3_{k_1}}_{\text{series.N}\hat{1}}, \underbrace{v3_{n_2}, \dots, v3_{k_2}}_{\text{series.N}\hat{2}}, \dots, \underbrace{v3_{n_{(n_c-1)}}, \dots, v3_{k_{(n_c-1)}}}_{\text{series.N}\hat{(n_c-1)}}, \underbrace{v3_{n_{(n_c)}}, \dots, v3_{k_{(n_c)}}}_{\text{series.N}\hat{n}_c} \right\}$$

**Figure 4.**  
Example of the  $V3$  series structure.

series. Using the presented structure and values of the elements of the vector  $V3$ , it is possible to determine the number of sampling intervals that fit between adjacent pairs of mutually correlated segments of signal measurements, i.e., a period of “bursts” of correlation:

$$Num1 = \frac{(v3_{k(n_c-1)} - v3_{k_2}) - (v3_{n(n_c-1)} - v3_{n_2})}{2(n_c - 2)} \quad (13)$$

Using the obtained value  $Num1$ , we can determine the average value of the modulation interval  $T_p$  and, therefore, the average modulation rate  $W$ :

$$T_p = \frac{Num1}{f_\partial}, W = T_p^{-1} \quad (14)$$

For the final determination of the time-frequency structure of the signal, we must find the number of carrier frequencies  $n_f$  and the vector of their nominal values  $F = \{f_0, \dots, f_{n_f-1}\}$ . This can be done on the basis of previously obtained values  $T_p$ ,  $T$ ,  $\Delta f$  when the position of the element of the array  $Q$  corresponding to the beginning of the first full modulation interval is determined correctly. The beginning of a reliably identified clock interval could be most correctly associated with the beginning of the second series of maximal responses of correlators in the vector  $V3$ , since, due to the randomness of the beginning of the observation, the first series may be incomplete. It should be taken into account that the beginning of a series of maximal responses of correlation of the segments from  $K$  samples must appear before the next modulation interval actually begins. Therefore, to fall within the interval with the duration  $T_p$  (taking into account that demodulation can be performed on any segment  $T$  within  $T_p$ ), it is necessary to add the number  $K/2$  to the starting sample, at least. Then the beginning of the modulation interval can be assumed to coincide with the next element number in the sequence

$$n_T^0 = round\left\{\left[v3_{n_2} mod\left(\frac{f_\partial}{W}\right)\right] + \frac{K}{2}\right\} \quad (15)$$

Here  $round(x)$  is the rounding function to the nearest integer. The lowest possible frequency  $f_0$  in the group of carrier frequencies is determined by the value of function (3) and the fulfillment of the corresponding condition in function (10):

$$f_0 = \begin{cases} \frac{1}{2}\Delta f, & \text{if } \max(H1) \geq \max(H2); \\ \Delta f, & \text{if } \max(H1) < \max(H2). \end{cases} \quad (16)$$

The maximal number of subcarrier frequencies (or half the number of quadrature components) that can fit in the channel band  $F_{ef}$  is

$$n_{fmax} = round\left(\frac{F_{ef} - f_0}{\Delta f} + 1\right) \quad (17)$$



### 3.2 Determining the amount and nominal values of subcarrier frequencies of OFDM signals

The correlation method, considered above, allows making a reliable assessment of the main structural parameters of OFDM— $T$  and  $T_p$ . The value of  $\Delta f = T^{-1}$  uniquely defines the spacing of adjacent subcarrier frequencies. The minimal value of the subcarrier frequency and the maximal possible number of subcarriers placed within the signal bandwidth  $n_{fmax}$  are determined from Eqs. (16) and (17).

The number of samples  $N$ , taken into account when analyzing a signal on one modulation interval, as well as the harmonic quadrature ( $2 \cdot n_{fmax}$ ), define the dimensions of the matrix of the linear algebraic equations system (SLAE) which can be compiled and solved to estimate the frequency range. Depending on the ratio of the vertical and horizontal dimensions of the matrix of coefficients, the system of equations can be overdetermined ( $N > 2 \cdot n_{fmax}$ ), determined ( $N = 2 \cdot n_{fmax}$ ), or underdetermined ( $N < 2 \cdot n_{fmax}$ ). The simplest one is the ( $N = 2 \cdot n_{fmax}$ ) case because then the SLAE is a joint one almost every time. The number of equations that matches the number of used elements of the digital sample  $Q$  equals to the number of unknowns ( $2 \cdot n_{fmax}$ ) determining the amplitudes of quadrature components in the spectrum of carrier frequencies OFDM. For the correct solution of SLAU ( $2 \cdot n_{fmax}$ ), uniformly spaced sample counts  $Q$  starting from the point of beginning of the observation of the first complete clock interval of signal  $n_T^0$  should be selected on  $i$ -m modulation interval. For this the following rule is used:

$$n_T^i = n_T^0 + \text{round}(i \cdot \Delta) \text{ where } \Delta = T_p/t_\delta. \quad (18)$$

Square matrix of coefficients for unknown SLAE with size  $(2 \cdot n_{fmax}) \times (2 \cdot n_{fmax})$  composed for quadrature components of subcarrier frequencies is formed according to the rule

$$\begin{aligned} A_1 &= \|a_{i,j}\|, \quad i, j = 0, \dots, (2 \cdot n_{fmax} - 1); \\ a_{i,j} &= \text{Sin}[\omega_j \cdot t_i], \quad 0 \leq j \leq n_{fmax} - 1; \\ a_{i,j} &= \text{Cos}[\omega_j \cdot t_i], \quad n_{fmax} \leq j \leq 2 \cdot n_{fmax} - 1; \end{aligned} \quad (19)$$

where  $\omega_j = \omega_{(j+n_{fmax})} = 2\pi(f_0 + j \cdot \Delta f)$ ,  $j = 0, \dots, n_{fmax}$ ,  $t_i = n_T^i + i \cdot t_\delta$ .

The column matrix of free members is formed as a vector of signal measurements on the duration of one orthogonality interval:

$$B_1 = \{b_0, \dots, b_{(2 \cdot n_{fmax} - 1)}\}; b_i = q_i, \quad i \cdot t_\delta \in T_p^i \quad (20)$$

Normal solution of normally defined SLAE

$$A_1 \cdot X_1 = B_1 \Rightarrow X_1 = A_1^{-1} \cdot B_1 \quad (21)$$

gives an estimation of the amplitude vector of quadrature components  $X_1 = \{x_0^1, \dots, x_{(2 \cdot n_{fmax} - 1)}^1\}$  which corresponds to the permissible values of carrier frequencies.

On the basis of this solution, it is possible to determine the power distribution vector of the signal between the harmonic oscillations of the carrier frequencies:

$$Y = \{y_0, \dots, y_{(n_{fmax}-1)}\}, \quad y_i = (x_i^1)^2 + \left(x_{(i+n_{fmax})}^1\right)^2, \quad i = 0, \dots, (n_{fmax} - 1) \quad (22)$$

The case of insufficiently defined SLAE ( $N < 2 \cdot n_{fmax}$ ) is interesting for analyzing small samples of the signal. To solve such a SLAE, the pseudoinverse matrix method Moore-Penrose can be used. It is known that there is a normal solution of an underdetermined SLAE, and it is the only one. It is found by  $X_1 = A_1^+ \cdot B_1$  where  $A_1^+$  is the Moore-Penrose pseudoinverse matrix of size  $2 \cdot n_{fmax} \times 2 \cdot n_{fmax}$  which is determined by the ratio  $A_1 \cdot A_1^+ \cdot A_1 = A_1$ . In practice  $A_1^+$  can be found by the formula

$$A_1^+ = C^+ \cdot D^+ = C^* \cdot (C \cdot C^*)^{-1} \cdot (D \cdot D^*)^{-1} \cdot D^*.$$

The representation of the matrix  $A_1^+$  in the form of a product of two matrices with the size of  $N \times r$  and  $r \times N$  is used:

$$A_1 = D \cdot C = \begin{pmatrix} d_{1,1} & \dots & d_{1,r} \\ \vdots & \ddots & \vdots \\ d_{N,1} & \dots & d_{N,r} \end{pmatrix} \cdot \begin{pmatrix} c_{1,1} & \dots & c_{1,N} \\ \vdots & \ddots & \vdots \\ c_{r,1} & \dots & c_{r,N} \end{pmatrix}.$$

With various skeletal decompositions of the matrix  $A$ , the same solution for  $A^+$  which can be written in the form  $X_1 = A_1^+ \cdot B_1$  is derived. It is a pseudosolution giving a zero residual:  $\|X_1 - A_1^+ \cdot B_1\| = 0$ .

The case of ( $N > 2 \cdot n_{fmax}$ ) is the most advantageous for the maximal recording of signal information on the modulation interval. Due to using additional signal measurements from the sample  $Q$ , the system which contains more equations with the same number of unknowns is formed. To form the matrix  $A_2$  and the vector  $B_2$ , the maximal number of signal measurements determined by  $Num \approx T_P/t_\delta$  on the duration  $T_P$  is used:

$$A_2 = \|a_{i,j}\|, \quad i = 0, \dots, (Num - 1), \quad j = 0 \dots (2 \cdot n_{fmax} - 1);$$

$$a_{i,j} = \sin[\omega_j \cdot t_i], \quad 0 \leq j \leq n_{fmax} - 1; \quad (23)$$

$$a_{i,j} = \cos[\omega_j \cdot t_i], \quad n_{fmax} \leq j \leq 2 \cdot n_{fmax} - 1;$$

$$B_2 = \{b_0, \dots, b_{(Num-1)}\}, \quad b_k = q_k, \quad k = 0, \dots, (Num - 1). \quad (24)$$

SLAE has the form

$$A_2 \cdot X_2 = B_2 \quad (25)$$

and, as a rule, has many solutions. To select the only one, we need to use some criteria. In practice, the maximum likelihood criterion is used more often. In the case of a normal distribution of the vector  $B_2$ , it is equivalent of the least square's criterion:

$$X_2^* = (A_2^T \cdot A_2)^{-1} A_2^T \cdot B_2 \quad (26)$$

An approximate solution of system (26) gives a more accurate result than a strict solution of system (25). The noise immunity of the solution is achieved by averaging the disturbing effect of interference when the number of signal measurements exceeds the required minimum. The obtained vector of amplitudes of the quadrature components  $X_2^*$  as well as  $X_1^*$  gives a possibility to calculate the power

distribution signal in carrier frequencies using expression (22), wherein  $x_i^2$  is used instead of  $x_i^1$ .

For any type of SLAE, determining the actual list of carrier frequencies in the OFDM spectrum is performed by comparing the elements of power distribution histograms with a threshold value. The obtained nominal values of frequencies determine the last structural time-frequency parameter of the analyzed signal—the vector of working subcarrier frequencies  $F$ .

Thus, the previously obtained signal parameters  $T_p$ ,  $T$ ,  $\Delta f$ ,  $W$ , and the obtained in this subsection vector  $F$  identify completely their structural properties and make signal demodulation possible.

## 4. Group algebraic demodulation

### 4.1 Determination of phase modulation multiplicity

OFDM standards as an informative parameter on subcarriers assume the use of relative phase shift keying. The number of variants of phase angles can vary from 2 to 16 or more [2–5]. As a rule, absolute phase modulation methods are not applied because they are critical to the correctness of the subcarrier phase restoration, which is highly dependent on the accuracy of determining the beginning of the modulation interval. In case of digital demodulation, when a sample of signal measurements is formed for time points asynchronously with respect to a periodic sequence of modulation interval boundaries, the usage of absolute phase modulation methods is practically impossible, since it is theoretically impossible to provide absolutely accurate clock synchronization of modems. Therefore, as an axiomatic assumption for developing the stages of secondary analysis, the hypothesis of using the relative phase coding methods is adopted. Depending on the multiplicity of the applied modulation method  $m = 2^\alpha$ ,  $\alpha = 1, 2, 3, \dots$  is an integer, during the transition from interval to interval; on each of the carriers, a phase jump occurs, the value of which determines the corresponding combination of information symbols. Jumps occur even in the case of complete coincidence of combinations of symbols on adjacent intervals.

The general idea of a preliminary analysis of the PM multiplicity is to statistically identify the number of observed fixed values of the phase of harmonic oscillations in carrier frequencies. To perform the analysis, let us use the algebraic method for the case of overdetermined SLAEs considered in the previous section. Let  $Num$  be the maximum number of measurements performed on the modulation interval and  $n_f$  the number of carriers detected during the primary analysis. Then the degree of overdetermination of SLAE is calculated from the expression

$$K_{\Pi} = \frac{Num}{2 \cdot n_f} > 1 \quad (27)$$

In accordance with the findings, the increase of  $K_{\Pi}$  leads to the increase in the noise immunity of the system solution. SLAE, compiled for the entire spectrum of subcarriers, is determined by a rectangular (size  $Num \times 2 \cdot n_f$ ) matrix of coefficients for the following unknowns:

$$A = \|a_{i,j}\|; \quad a_{i,j} = \begin{cases} \sin(2\pi f_j(i \cdot \Delta T)), & i = 0, \dots, Num - 1, \quad j = 0, \dots, n_f - 1; \\ \cos(2\pi f_j(i \cdot \Delta T)), & i = 0, \dots, Num - 1, \quad j = n_f, \dots, 2 \cdot n_f - 1 \end{cases}, \quad (28)$$

as well as a column matrix of  $Num$  signal measurements

$$B = \{b_0, \dots, b_{Num-1}\}, \quad b_i = y_{J+i}, \quad J = n_T^0 + \text{round}\left(N_u \frac{f_\delta}{W}\right) \quad (29)$$

Here  $f_\delta = (t_\delta)^{-1}$  is the sampling frequency of the digital signal sample ( $t_\delta$  is the sampling interval),  $Y = \{y_0, \dots, y_k\}$  is the digital sample of measurements of the OFDM signal,  $F = \{f_0, \dots, f_{n_f-1}\}$  is the vector of nominal values of the detected subcarrier frequencies,  $N_u$  is the number of the analyzed modulation interval,  $W$  is the modulation rate in bauds, and  $n_T^0$  is the initial measurement number (in the digital signal sampling), corresponding to the reliably detected beginning of the first complete modulation interval. The value  $n_T^0$  is one of the results of the stage of the primary correlation analysis using the “sliding time window” method discussed in the previous section.

To find the solution of the overdetermined SLAE (28), (29) the method of least squares is used to solve the following system:

$$A^T \cdot A \cdot Z = A^T \cdot B \quad (30)$$

where  $A^T$  is the transposed matrix (28) and  $Z = \{z_0, \dots, z_{n_f-1}, z_{n_f}, \dots, z_{2n_f-1}\}$  is the vector of the amplitudes of the harmonic quadrature components for all  $n_f$  subcarriers of the signal frequencies.

The decision vector  $Z$  contains the amplitudes of sines (first  $n_f$  elements) and cosines (last  $n_f$  elements) in frequencies determined by the vector  $F$ . It gives an opportunity to calculate the elements of the phase vector (expressed in degrees) of harmonic oscillations  $\Phi = \{\phi_0, \dots, \phi_{n_f-1}\}$  in the subcarrier frequencies:

$$\phi_i = \text{mod} \left[ \left( \frac{180}{\pi} \text{Atan}(z_i, z_{i+n_f}) + 360 \right), 360 \right], \quad i = 0, \dots, n_f - 1 \quad (31)$$

In this expression, the following functions are used:

- $\text{mod}(a, b)$ —to calculate the value of the number  $a$  by modulo  $b$
- $\text{Atan}(x, y)$ —to calculate the angle in radians from the interval  $[-\pi, \pi]$ , between the axis  $Ox$  and the vector drawn from point  $(0, 0)$  to point  $(x, y)$

The process of calculations by expressions (30) and (31) is the absolute phase demodulation of discrete measurements on an interval  $T$  located within the segment of modulation interval  $T_P$ . It should be noted that with relative phase shift keying, the calculated absolute values of the phases do not matter for correct demodulation of the bit stream. Therefore, in this case, the error in determining the beginning of the orthogonality interval  $T$  within the modulation interval  $T_P$  is not significant. The calculation of the sampling number of the next interval is based on the known modulation rate  $W$ , the starting number of the initial interval  $n_T^0$ , and the sampling frequency  $F_\delta$  and performed by using the rounding formula from expression (29).

The factor that reduces the effectiveness of group digital demodulation is the asynchrony of the sampling procedure when a sample of signal measurements is being obtained with the duration of repeated demodulation intervals. In general, the

value  $T/t_\delta = f_\delta \cdot T$  is not an integer. This leads to a variation of the value of “indentation”  $dT_i$  from the beginning of the  $i$ -th modulation interval. This phenomenon is illustrated in **Figure 5**. Differences in the values of  $dT_i$  lead to the fact that the phase error periodically repeated on a combination of several adjacent modulation intervals is added to the correct solution of the system. The significant “blurring” of phase increments in subcarrier frequencies, as well as the departure of their average values from standardized nominal values, is an external manifestation of this asynchronous effect. Therefore, the phase errors of asynchronous sampling are an additional source of disturbing noise, which must be taken into account and, if possible, compensated.

A known correlation between magnitudes  $W$  and  $t_\delta$  allows us to estimate the magnitude of these errors. On an arbitrary  $i$ -th interval, the error in determining the initial measurement is

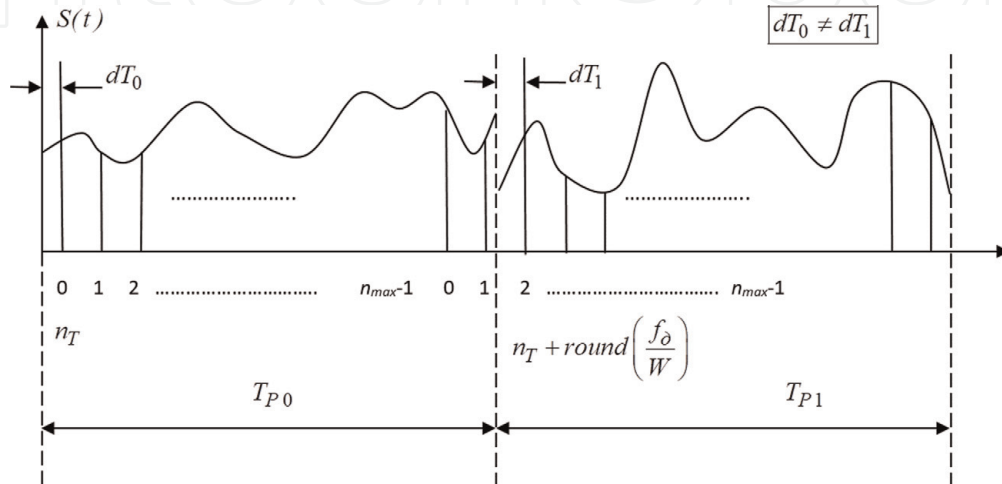
$$dT_i = t_\delta \left[ i \frac{f_\delta}{W} - \text{round} \left( i \frac{f_\delta}{W} \right) \right] \quad (32)$$

Obviously, for different subcarrier frequencies, the same time shift  $dT$  will cause phase determination errors which are different in magnitude. By knowing the range of carrier frequencies of the vector  $F$ , we can precisely calculate the corrections to the calculations for the current phase of the  $L$ -th carrier frequency on  $i$ -th modulation interval as follows:

$$d\phi_{L,i} = 2\pi \cdot f_L \cdot dT_i = 360 \cdot f_L \cdot dT_i \quad (33)$$

The multiplicity of the phase modulation used in the observed signal is uniquely determined by the minimal difference between the closest values of the vector  $\Phi$  elements calculated on the basis of expression (31), taking into account corrections (32) and (33). Under the conditions of a priori uncertainty, the multiplicity of relative PM (RPM) applied in the subcarrier frequencies can be detected only by a statistical method based on an analysis of a series of SLAE solutions from the sample of consecutive modulation intervals which has a sufficiently large size  $N$ . For this the vector is created:

$$\Psi = \left\{ \psi_0^0, \dots, \psi_{N-1}^0; \psi_0^1, \dots, \psi_{N-1}^1; \dots; \psi_0^{n_f-1}, \dots, \psi_{N-1}^{n_f-1} \right\} \quad (34)$$



**Figure 5.**  
Asynchronous sampling illustrations.



The vector contains  $N$  series of  $n_f$  elements. Each element in the  $\psi_i^j$  series is the phase increment measurement of the  $j$ -th subcarrier oscillation on the  $i$ -th modulation interval and is calculated (in degrees) as follows:

$$\psi_i^j = \text{mod} \left[ \left( \phi_{i+1}^j - \phi_i^j \right), 360 \right] \quad (35)$$

where  $\phi_i^j$  is the absolute phase measurement of the  $j$ -th subcarrier on the  $i$ -th modulation interval. Since the multiplicity of RPM in all frequencies must be the same, the vector  $\Psi$  can be considered as a single statistic characterizing the entire OFDM signal. Based on the vector of the statistics of phase increments  $\Psi$ , we form a vector histogram  $\bar{\Psi}$ , the elements of which are proportional to the frequency of occurrence of various increments. When rounding calculations up to  $1^\circ$ , the histogram  $\bar{\Psi}$  will contain 360 elements:

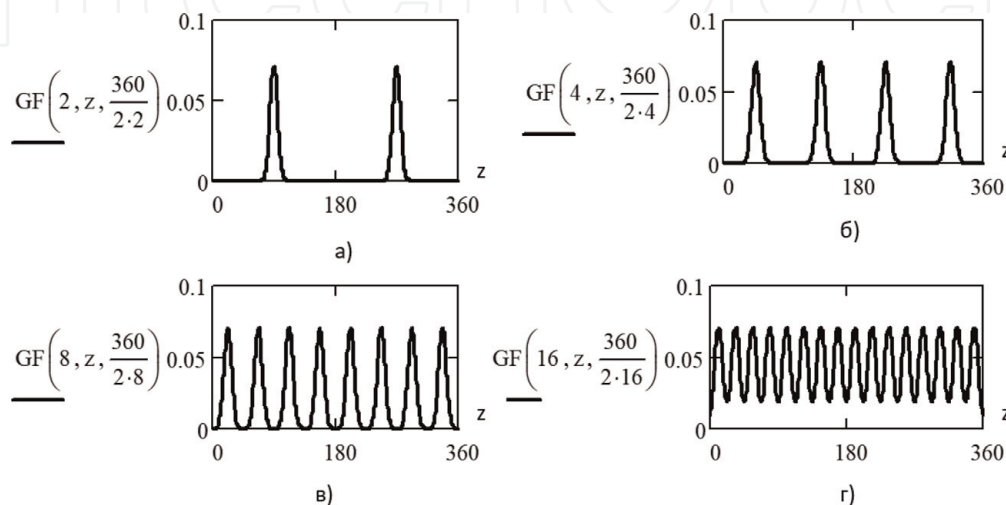
$$\bar{\Psi} = \{\bar{\psi}_0, \dots, \bar{\psi}_{359}\}, \quad \bar{\psi}_i = n\psi_i, \quad (36)$$

where  $n\psi_i$  is the number of elements of the vector  $\Phi$  that fall inside the interval  $[i - 0,5; i + 0,5]$ .

A visual analysis of the histogram (36) lets us reliably identify the type of modulation used. However, the automatic analysis requires the formal mathematical one. The cross-correlation which is the most reliable mathematical tool for calculating the degree of similarity of two functions should be the natural basis of such algorithm. A tuple of reference functions for calculating mutual correlation with a histogram vector  $\bar{\Psi}$  can be obtained using Gaussian probability density function based on the following expression:

$$GF(m, w, d) = \frac{1}{\sigma\sqrt{2\pi}} \sum_{i=0}^{m-1} \exp \left( -\frac{(w - d \frac{360}{m})^2}{2\sigma^2} \right), \quad (37)$$

where  $m$  is the multiplicity of RPM,  $w$  the phase value in degrees, and  $d$  the offset parameter of the “reference comb” of the histogram maxima on the interval  $[0 - 360^\circ]$ . This expression has no probabilistic sense, since the Gaussian distribution is used only as the convenient analytical definition of the multimodal structure of reference functions. An example of generation of references for the multiplicity of modulation  $m = 2, 4, 8$  u  $16$  is presented in **Figure 6**.



**Figure 6.**  
Tuple of reference functions.

The structure of the references specified by Eq. (37) ensures the symmetry of the structure of functions relative to the average value of the phase  $\pi$ . For the given multiplicity  $m$  based on Eq. (37), the vector  $x$  for different values of the shift of the reference comb is formed:

$$x = \{x_0, \dots, x_{359}\}, \quad x_i = GF(m, i, j), \quad i = 0, \dots, 359; \quad j = 0, \dots, 180 \quad (38)$$

For each shift value  $j$ , the cross-correlation coefficient is calculated:  $X_j = \frac{x \times \bar{\Psi}}{|x| \cdot |\bar{\Psi}|}$  where “ $\times$ ” denotes a scalar product.

The maximal obtained value  $Y_m = \max(X_j)$  is taken as an indicator of the similarity of the histogram  $\bar{\Psi}$  with the reference of the given multiplicity. Then the next value  $m$  is taken and calculations are repeated. After a complete exhaustion of all possible values of multiplicity, the largest calculated value  $Y_m$  is determined. The index of this element corresponds to the detected multiplicity of phase modulation  $m$ . The described computational process for calculating the correlation proximity is conveniently implemented using the mechanism of nested cycles. It is easy to program, and the numerous tests show that it guarantees the correct finding of the modulation multiplicity even under conditions of significant distortion of signal samples.

## 4.2 Algebraic demodulation

For the final demodulation, the SLAE of reduced dimensionality is compiled with a rectangular matrix of coefficients with unknowns:

$$\begin{aligned} A &= \|a_{i,j}\|, \quad i = 0, \dots, Num - 1, \quad j \in N; \\ B &= \{b_0, \dots, b_{Num-1}\}, \quad b_i = y_{J+i}, \quad J = n_T^0 + \text{round}((N \cdot f_\delta)/W). \end{aligned} \quad (39)$$

The solution of the system  $A^T \cdot A \cdot Z = A^T \cdot B$  for several modulation intervals gives a vector of absolute values of the phases which gives a sequence of corrected phase increments after transformations. To identify the combinations of binary symbols in accordance with the obtained modulation multiplicity and the corresponding keying code, it is necessary to calculate the decision boundaries. Since all combinations of binary symbols are assumed to be equally probable and phase errors are distributed normally and symmetrically around the standard positions, then, in accordance with the maximum likelihood rule, it is advisable to use equidistant decision-making boundaries:

$$G_i = i \frac{360}{m}, \quad i = 0, \dots, m \quad (40)$$

Then the rule for forming a bit stream at the output of the investigated example of a digital demodulator when using, for example, four phase angles on  $i$ -th modulation interval for one carrier frequency can be written as a set of conditions for forming a pair of binary symbols:

$$s_{2i}, s_{2i+1} = \begin{cases} 1, 0; & \text{if } G_0 \leq \Psi_i < G_1; \\ 0, 0; & \text{if } G_1 \leq \Psi_i < G_2; \\ 0, 1; & \text{if } G_2 \leq \Psi_i < G_3; \\ 1, 1; & \text{if } G_3 \leq \Psi_i < G_4; \end{cases} \quad i = 0, 1, 2, \dots \quad (41)$$

Parameter		Value		
		Samples	Seconds	Hertz
Stage 1	Period of orthogonality	73	0.009	—
	Subcarriers distance	—	—	110
	Duration of the OFDM character	107.083	0.013	—
	Sign of the prefixes correlation—negative			
	Starting number of the first full OFDM character	107	—	—
	Time of analysis	—	0.5	—
	Sample size of measurements	4001	—	—
Stage 2	The possible frequency grid $i \in 0...31$	—	—	$55 + i \cdot 110$
	Numbers of carriers—17			
	Nominals of carriers $j \in 0...16$			$825 + j \cdot 110$
	Number of phase angles PSK—4			
	Number and nominal value of carrier of synchronization channel =0	—	—	825
	Numbers and nominal values of information carriers $k \in 1...16$	—	—	$825 + k \cdot 110$

**Table 1.**  
*Analysis experiment results.*

The full stream of transmitted characters is formed with this rule. The result of the work of the considered method of automatic technical radio monitoring of the source of OFDM signals is represented as received by binary sequences, for example,

$X_0 =$ 

	0	1	2	3	4	5	6	7	8	9	10	11
0	1	0	0	0	1	0	0	1	1	0	0	...

$X_1 =$ 

	0	1	2	3	4	5	6	7	8	9	10	11
0	1	1	1	1	0	0	1	0	1	1	0	...

and so on.

The correctness of the considered method for analyzing and demodulating the OFDM signals under conditions of partial uncertainty has been tested using both real and simulated digital measurements of signals.

Based on the considered algebraic algorithms for automatic analysis, recognition, and demodulation of signals, a software package was developed. **Table 1** presents an example of the main results of the use of the software package for the analysis and demodulation of a complex signal of the standard MIL188-110A. The moment of the beginning of a signal observation was accidental. The test signal was sampled in the low-frequency band 0.3÷3.4 kHz after spectrum transfer by decimation of measurements. In this example, the signal contained 16 informational subcarrier frequencies and 1 synchronization subchannel at a modulation rate of 75 OFDM characters/s and relative PSK-4. The sampling rate was 8000 sample/s.

From the data in the table, it follows that it took 0.5 seconds of observation to identify all the parameters of the signal and start demodulation. Note that signal processing can be done in real time or based on the results of previously stored digital samples.

## 5. Conclusions

The considered statistical method for analyzing the structure and demodulation of OFDM signals under conditions of a priori uncertainty of solving radio monitoring tasks has been practically tested. It has demonstrated the high accuracy of parameter identification. The relatively low computational complexity of correlation and algebraic analysis makes it possible to identify the structure and the parameters of signals practically in seconds.

The noise immunity of the analysis is achieved by solving a SLAE with rectangular overdetermined matrixes of coefficients. To eliminate phase errors due to the asynchrony of the sample relative to the clock modulation intervals, a method for calculating phase corrections is proposed. The method uses the known parameters of the time-frequency structure of the signal. The application of the phase correction method provides ideal conditions for identifying the modulation type of subcarrier oscillations. Mathematical formalization of solving the problem of determining the modulation multiplicity, based on generating the multimodal reference functions and sequential calculating the degree of mutual correlation, allows us to completely automate the process of identifying the secondary parameters which are necessary for demodulating the signals of subcarrier frequencies. The further research can be focused on the generalization of the method for any structures of mono and poly frequency signals including those with a linear frequency modulation.


### Author details

Sergey G. Rassomakhin

Information Systems and Technologies Security Department, V.N. Karazin Kharkiv National University, Kharkiv, Ukraine

\*Address all correspondence to: [rassomakhin@karazin.ua](mailto:rassomakhin@karazin.ua)

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