

# We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900

Open access books available

186,000

International authors and editors

200M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index  
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?  
Contact [book.department@intechopen.com](mailto:book.department@intechopen.com)

Numbers displayed above are based on latest data collected.  
For more information visit [www.intechopen.com](http://www.intechopen.com)



# Vibration Characteristics of Single-Walled Carbon Nanotubes Based on Nonlocal Elasticity Theory Using Wave Propagation Approach (WPA) Including Chirality

*Muzamal Hussain and Muhammad Nawaz Naeem*

## Abstract

This chapter deals with the vibrational properties of single-walled carbon nanotubes (SWCNTs), based on nonlocalized theory of elasticity (NLT). The nanotube pilot control with nonlinear parameters was derived from Euler's beam theory. The wave propagation (WPA) approach was used to derive the frequency equation describing the natural frequencies of vibration in SWCNTs. Complex exponentials depend on the boundary conditions given at the edges of the carbon nanotubes used. Vibration frequency spectra were obtained and evaluated for different physical parameters such as diameter ratio for single chiral carbon nanotubes and flexural strength for chiral SWCNT. The results show that the natural frequencies are significantly reduced by increasing the nonlocal parameters, but by increasing the ratio of the diameter length (aspect ratio), the natural frequency increases. The frequency of SWCNTs is calculated with the help of MATLAB computer software. These results are compared to previously known numerical simulations.

**Keywords:** nonlocal, wave propagation approach, vibration, MATLAB

## 1. Introduction

Vibrational properties of CNTs play important critical roles in controlling the performance of various scientific and engineering fields and stability of CNT-based devices, superconductivity, and material strength analysis. New technologies and innovative improvements such as nano-probe, wood mirror, nano-electronic devices, chemical release, and drug release have been proposed. The important application of the current investigation of rotating FG-CNT is in nano-engineering structure as nano-components like sensors and actuators. The use of carbon nanotubes (CNTs) has been practiced in a variety of fields such as field emission, construction, electronics, and fashion [1]. CNT's free vibration surveys have been

tested in relation to their physical properties and behavior. Much of the work was done with high rate of elasticity and characterization, a very effective Young modulus [2], and the bond strength between carbon atoms [3]. In the past 15 years, researchers have used different models such as the ring [4], the beam [5], the shell [6], and other continuous models [2, 7] to capture the object anew. Due to their attractive applications, dynamic features such as buckling, stability, and vibration are explored in a theoretical way and avoid potential risks for future use. Therefore, a new model is needed to capture the nanoscale structure. Researchers [8–12] have conducted investigations of higher-order elasticity theories. Other nonclassical theories of elasticity have attracted the attention of researchers such as the theory of stress [13, 14], theories of stress [15, 16], and nonlocal theory [17, 18].

In NLT, the pressure applied at a certain point depends on the stress at all points, which is quite different from conventional theory. Wang et al. [19] and Yang et al. [20] presented a survey of SWCNT based on nonlocal Timoshenko beam theory (TBM). CNT analysis has been explained by some researchers [21–25]. Bocko and Lengvarský studied the vibration frequencies of CNTs for different termination conditions and mode shapes using nonlocal elastic theory. Chawis et al. [26] analyzes vibrational behavior of SWCNTs with small-scale effects using nonlocal theory. Recently, some researchers have investigated the vibrational behavior of SWCNTs [27–29].

In addition, many researchers investigated the vibrational behavior of the above structures using different types of theories. The method chosen for studying nanoscale systems is the NLT (WPA) wave propagation method, which allows the study of fundamental frequencies of SWCNTs through combinations of different parameters. The beam model (BM) [30] is used to calculate the associated frequencies and shapes of MWCNT. Bocko and Lengvarský [31] investigate the bending free vibration of SWCNTs with four different boundary conditions. A continuum approach is used for the computation of natural frequency based on nonlocal theory of bending beam. The natural frequencies are given for several nano-parameters with two different diameters of nanotubes and continuously changed length. It is concluded that when tube length increases, the frequency decreases the nonlocal parameter and diameter. Chawis et al. [26] reported SWCNT vibration based on nonlocal theory to access the scale length. With the addition of nonlocal parameter in Euler beam theory, the governing equation is derived.

NLT-based are another option of robust research techniques of CNTs within the acceptable error range compared to previously used BMs and the subsequent other approaches [2–5, 21, 25, 32–34]. For solving ordinary differential equations (ODEs), Fourier variable separation method is used. According to the current model, the basic natural frequencies of chiral SWCNTs are calculated and obtained for various physical parameters such as the aspect ratio (length-to-diameter ratios) of SWCNTs with different nonlocal parameters and the effect of bending stiffness (rigidity) on the vibration frequencies of SWCNT. This is also our motivation for doing the present work.

## **2. Governing equation of motion**

In classical theory, the physical mass acts as a local action. In conventional theory, the stress produced at a point is influenced by stress at the point. According to Eringen [35], in nonlocal theory, the stress applied at a given point depends on the stress at all points [1], which is quite different from conventional theory. Under this assumption, the nonlocal relationship based on homogeneous isotropic beams can be expressed as

$$\varepsilon_{xx} - \tilde{\omega}^2 \kappa_j^2 \frac{\partial^2 \varepsilon_{xx}}{\partial x^2} = E v_{xx} \quad (1)$$

The factor  $p = \tilde{\omega}^2 \kappa_j^2$  is termed as the small-scale effect, where  $\tilde{\omega}$  and  $\kappa_j$  are defined as the material constant and lattice spacing length or internal characteristic length.

Equation (1) can be written as

$$\varepsilon_{xx} - p \frac{\partial^2 \varepsilon_{xx}}{\partial x^2} = E v_{xx} \quad (2)$$

where  $\varepsilon_{xx}$ ,  $v_{xx}$ , and  $E$  are, respectively, the normal pressure, normal stress, and modulus of the child. In general, the parameter is called a nonlocal parameter, and in classical theory, this parameter is used to investigate the vibration, pinching, and bending problems of the beam [36, 37]. According to Euler's theory [36] equation can be written as

$$\alpha(x) p \frac{\partial^2 s}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left[ EJ(x) \frac{\partial^2 s}{\partial t^2} \right] = 0 \quad (3)$$

where  $\alpha$  and  $J$  are the mass per unit length and moment of inertia of CNT, respectively. By using the Fourier method of variable separation, the two leading differential equation systems are normalized (ODE). In this system, an equation involving the space variable  $x$  and other equations is connected to the time variable

$$S(x, t) = \beta(x) T(t) \quad (4)$$

$$\alpha p \frac{\partial^2}{\partial t^2} \beta(x) T(t) + \frac{\partial^2}{\partial x^2} \left[ EJ \frac{\partial^2}{\partial x^2} \beta(x) T(t) \right] = 0 \quad (5)$$

$$\alpha p \beta(x) \frac{d^2 T}{dt^2} + EJ T(t) \frac{d^4 \beta}{dx^4} = 0 \quad (6)$$

$$EJ T(t) \frac{d^4 \beta}{dx^4} = -\alpha p \beta(x) \frac{d^2 T}{dt^2} \quad (7)$$

For harmonic response

$$T(t) = \cos \omega t \text{ or } \sin \omega t \text{ or } e^{i\omega t} \quad (8)$$

$$\alpha p \beta(-\omega^2 \cos \omega t) + EJ \cos \omega t \frac{d^4 \beta}{dx^4} = 0 \quad (9)$$

$$\frac{d^4 \beta}{dx^4} - \frac{\alpha p \omega^2}{EJ} \beta(x) = 0 \quad (10)$$

$$\frac{d^4 \beta}{dx^4} - \mu^4 \beta(x) = 0 \quad (11)$$

Here  $\beta(x)$  denotes the mode shape (eigenshape).

For parameter  $\mu$

$$\mu^4 = \frac{\alpha p \omega^2}{EJ} \quad (12)$$

where the general solution of fourth-order ODEs is

$$\beta(x) = \gamma_1 \sin \mu x + \gamma_2 \cos \mu x + \gamma_3 \sin \mu x + \gamma_4 \cosh \mu x \quad (13)$$

where  $\gamma_1, \gamma_2, \gamma_3$  and  $\gamma_4$  are the unknown constants. Equation (11) becomes

$$\beta^{iv}(x) - \mu^4 \beta(x) = 0 \quad (14)$$

### 3. Application of WPA

For the solution of CNT problem, an analytical technique wave propagation approach is evoked. A simple approach called the propagation waveform (WPA) was developed by Zhang et al. [38]. Thus, a simple and effective technique is applied as the wave propagation method [34, 39, 40] used for problem-solving in the form of differential equations. Prior to this, many techniques have been sequentially used to study the vibration of CNTs [41–43]. Previously, the current approach was used continuously to study the vibration of carbon nanotubes [27, 29, 44–47].

$$\beta(x) = e^{-iq_m} \quad (15)$$

where  $q_m$  denoted the wave number in axial direction and used for support conditions of SWCNTs [29].  $\omega = 2\pi f$  is the angular frequency.

$$\beta^{iv}(x) = q_m^4 e^{-iq_m x} \quad (16)$$

After putting these values in Eq. (14), we get

$$q_m^4 e^{-iq_m x} - \mu^4 e^{-iq_m x} = 0 \quad (17)$$

$$\mu^4 = q_m^4 \quad (18)$$

By using Eq. (12), we can write as

$$\frac{\alpha p \omega^2}{EJ} = q_m^4 \quad (19)$$

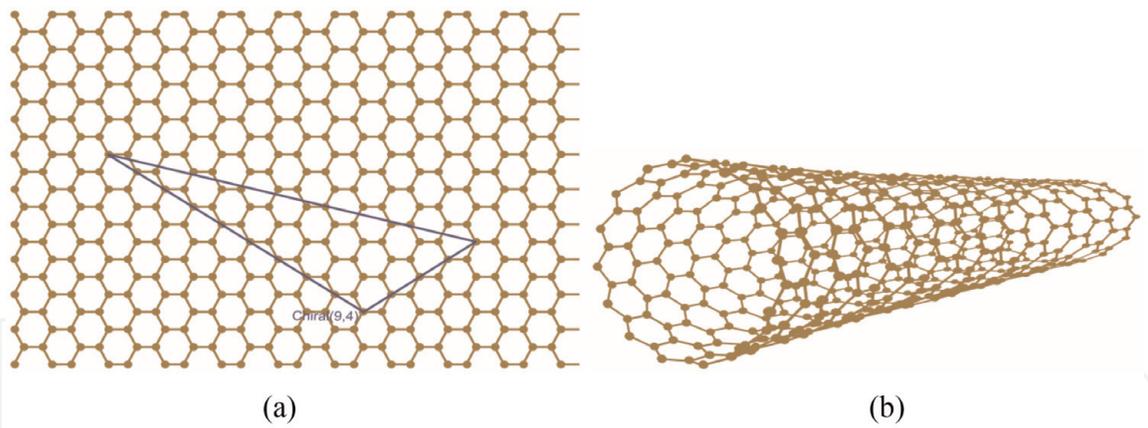
### 4. Nonlocal boundary conditions

In the article, the vibration of chiral SWCNTs with CC boundary conditions was investigated. In addition, the network interpretive directives ( $m, n$ ) for chiral CNTs can be expressed as ( $m, n$ ) for  $n$   $m$  correspondingly as shown in **Figure 1**. General boundary conditions C—C and C—F are considered for the system; then it is used in order to find the frequency equation of SWCNTs and eigenfrequencies of different indices for chiral SWCNTs such as (12, 5), (22, 7), and (25, 10).

Support conditions in the form of frequency.

From Eq. (19)

$$\text{For clamped – clamped, } \frac{\alpha p \omega^2}{EJ} = \left( \frac{(2n+1)\pi}{2L} \right)^4 \quad (20)$$



**Figure 1.**  
 (a) Graphene sheet. (b) SWCNT.

where  $q_m = \frac{(2n+1)\pi}{2L}$  (CC boundary condition).

$$\text{For clamped - free } \frac{\alpha p \omega^2}{EJ} = \left( \frac{(2n-1)\pi}{2L} \right)^4 \quad (21)$$

where  $q_m = \frac{(2n-1)\pi}{2L}$  (CF boundary condition).

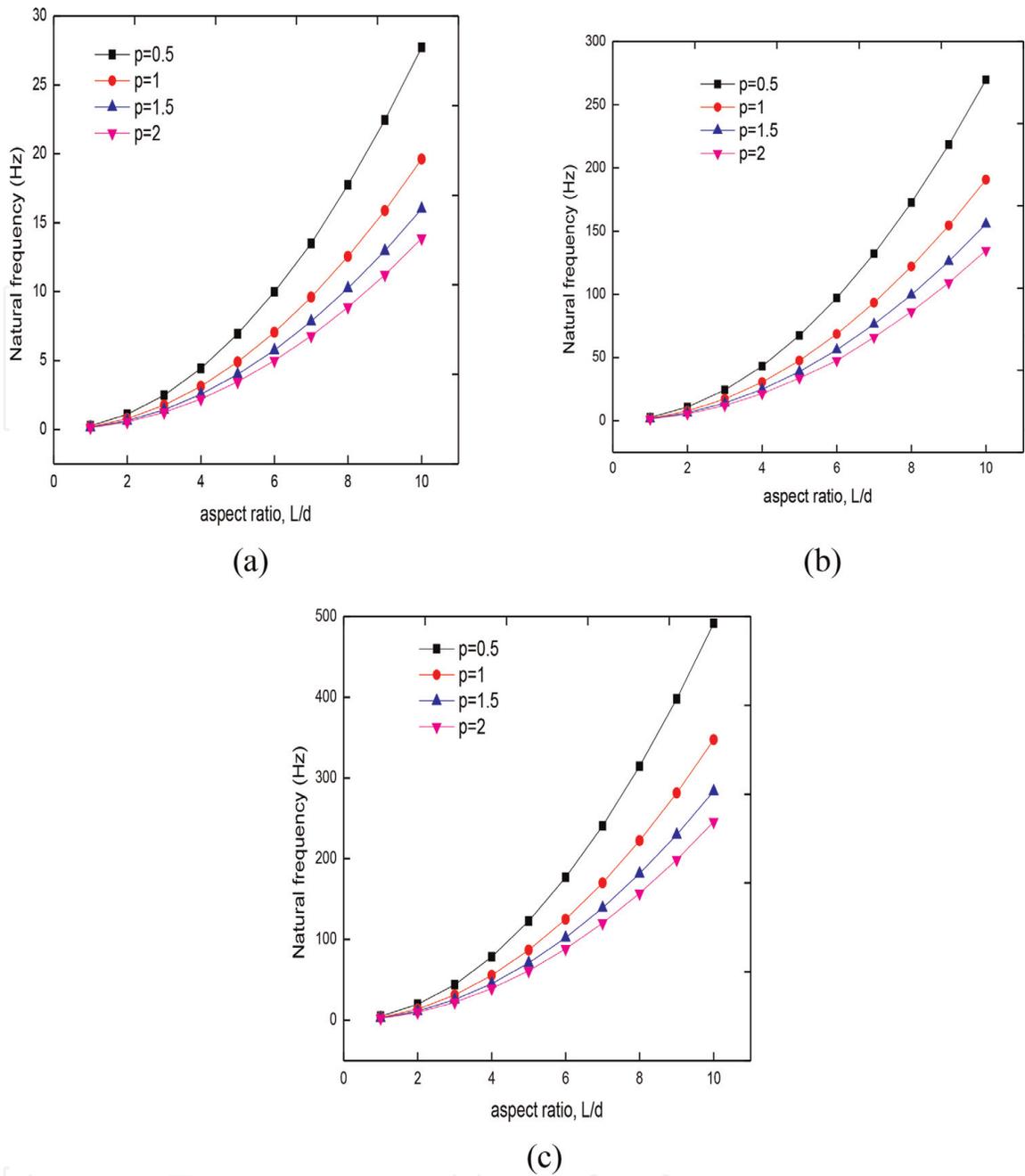
## 5. Results and discussion

The fundamental natural frequencies (FNF)  $f$  (Hz) of SWCNTs obtained from nonlocal theory (NLT) based on wave propagation approach (WPA) with C-C and C-F boundary conditions are presented. A comparison of nondimensionalized natural frequencies  $\Delta = \omega R \sqrt{\rho/E}$  of SWCNT is presented in **Table 1**. It is noted that from **Table 1**, the frequency value of present model have the small values as the values followed by the Alibeigloo and Shaban [48] shows a frequency difference between these studies. It can be seen that the error percentage is negligible, hence showing high rate of convergence. The results of nondimensional frequency are computed for two different values of  $n = 1, 2$  with circumferential wave number ( $m = 0, 1, 2, 3, 4, 5$ ) as shown in **Table 1**.

In order to analyze the effect of nonlocal parameter and bending rigidity on the vibration of chiral SWCNTs for different scales, the effects of nonlocal parameters on natural frequencies are illustrated in **Figures 2–5**. The natural frequencies are reduced by increasing nonlocal parameters ( $p = 0.5, 1, 1.5, 2$ ). The results for the

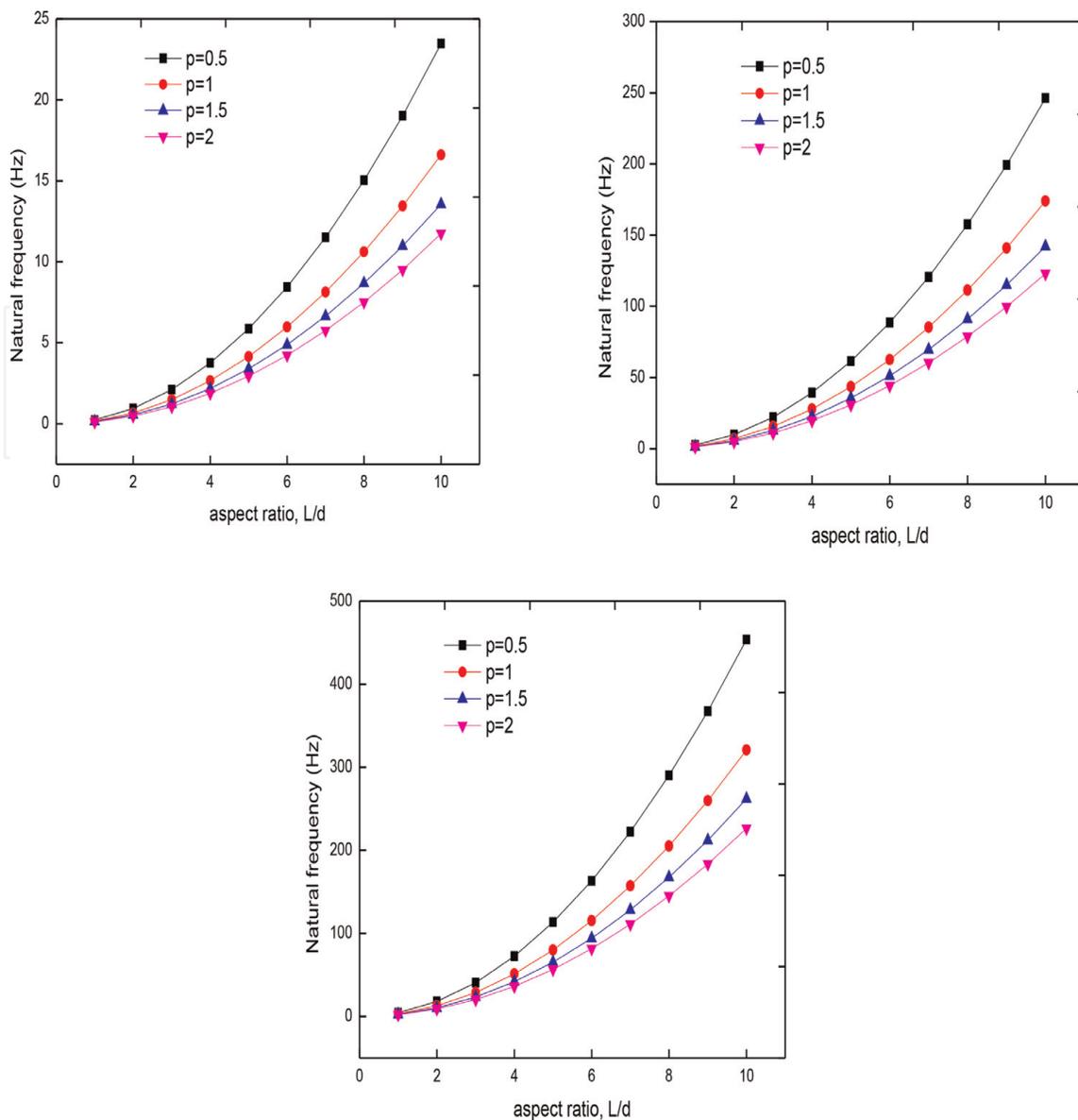
$m$	$n = 1$		$n = 2$	
	Alibeigloo and Shaban [48]	Present	Alibeigloo and Shaban [48]	Present
0	0.97087	0.97063	0.99351	0.99289
1	0.59721	0.59698	0.88357	0.88301
2	0.34025	0.34019	0.68072	0.68013
3	0.20145	0.20099	0.50059	0.5003
4	0.12886	0.12872	0.36918	0.36897
5	0.09105	0.9087	0.27671	0.27662

**Table 1.**  
 Comparison of nondimensional frequencies  $\Delta = \omega R \sqrt{\rho/E}$  ( $L/R = 1, n = 1$ ).



**Figure 2.** FNFs versus aspect ratio for CC chiral SWCNTs (a) (12, 5), (b) (22, 7), and (c) (25, 10) with different nonlocal parameter  $p$ .

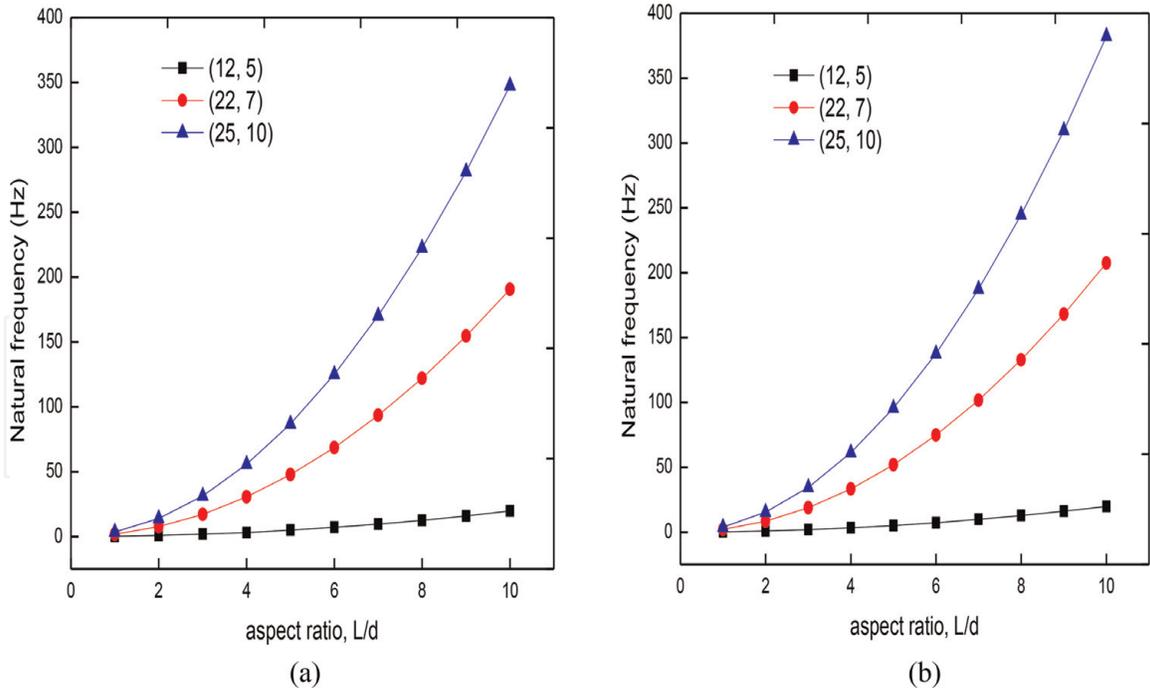
chiral clamped SWCNT with the indices (12, 5), (22, 7), and (25, 10) are shown in **Figure 2**. For the effect of nonlocal parameters with chiral index  $C=C(=12, 5)$  at  $L/d(=1\sim 10)$ , the first 10 frequencies at  $p = 0.5$  are 0.2773, 1.1094, 2.4961, 4.4376, 6.9337, 9.9846, 13.501, 17.7503, 22.4653, and 27.7349. When  $p = 2$ , then the frequency peaks are 0.1387, 0.5547, 1.2481, 2.2188, 3.4669, 4.9923, 6.7951, 8.8752, 11.2326, and 13.8675. Now, for  $C=C(=22, 7)$ , with the same parameters, the first 10 frequencies at  $p = 0.5$  and 2 are 2.6958, 10.7833, 26.2624, 43.1331, 67.3955, 97.0496, 132.0953, 172.5326, 218.3616, and 269.5822 and 1.3472, 5.3886, 12.1244, 21.5566, 33.6788, 48.4975, 66.0105, 86.2178, 109.1808, and 134.7152, respectively. Now, for  $C=C(=25, 10)$ , with the same parameters, the first 10 frequencies at  $p = 0.5$  and 2 are 4.9142, 19.6569, 44.2280, 78.6276, 122.8556, 176.9120, 240.7969, 314.5103, 398.0521, and 491.4223 and 2.4571, 9.8284, 22.1140, 39.3138, 61.4278, 88.4560, 120.3985, 15.2551, 199.0260, and 245.7112, respectively. It can be seen that FNF is reduced by increasing nonlocal parameters ( $p = 0.5, 1, 1.5, 2$ ). To illustrate the effect



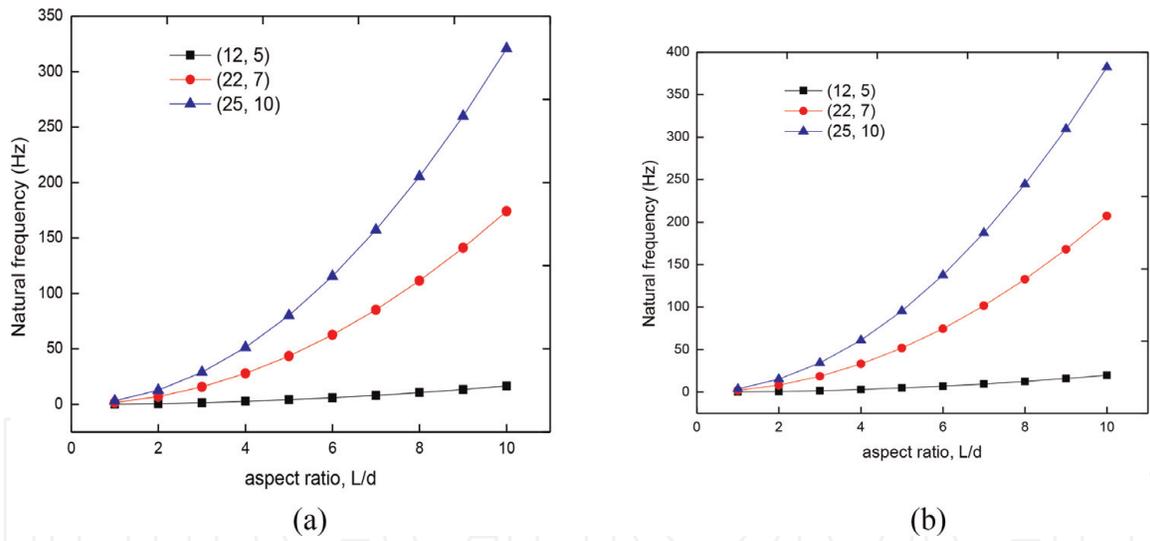
**Figure 3.** FNFs versus aspect ratio for CF chiral SWCNTs (a) (12, 5), (b) (22, 7), and (c) (25, 10) with different nonlocal parameter  $p$ .

of different nonlocal parameters on natural frequencies for chiral SWCNTs with indices (12, 5), (22, 7), and (25, 10) based on NLT as shown in **Figures 2 and 3**. It is remarkable that from **Figures 2–5**, the FNF values of the chiral CC tubes are certainly higher than the chiral CF values of the SWCNTs.

**Figures 4 and 5** show the FNFs against aspect ratio with varying bending strength index ( $EI$ ). They refer to instances when  $EI$  changes from  $5.1122e^{-9}$  to  $7.2629e^{-9}$  nm with nonlocal parameters  $p = 1$ . These figures show the natural frequency behavior of the calculated SWCNT system under bending rigidity ( $EI$ ) parameters. It is considered that with the increase of the bending rigidity ( $EI = 5.1122e^{-9}$  to  $7.2617e^{-9}$  nm), the fundamental natural frequencies increase, and with the increase of aspect ratio, the frequencies also increase as C—C = (12, 5)  $f$  (Hz): 0.1961–0.2337 [C—F (12, 5)  $f$  (Hz): 0.1660–0.1978] and C—C = (22, 7)  $f$  (Hz): 1.9062–2.2719 [CF (22, 7)  $f$  (Hz): 1.7406–2.0745] and C—C = (25, 10)  $f$  (Hz): 3.4749–4.1415 [C—F (25, 10)  $f$  (Hz): 3.2077–3.8230] at  $L/d = 1$ . The fundamental natural frequencies at  $L/d = 10$  are as C—C = (12, 5)  $f$  (Hz): 27.7349–23.3737 [C—F (12, 5)  $f$  (Hz): 16.5992–19.7835] and CC = (22, 7)  $f$  (Hz): 190.6234–227.1912 [C—F (22, 7)  $f$  (Hz): 174.0556–207.4452] and C—C = (25, 10)  $f$  (Hz): 347.4881–414.1476



**Figure 4.** FNFs with aspect ratio for CC chiral SWCNTs (12, 5), (22, 7), and (25, 10) with nonlocal parameter  $p = 1$  and bending rigidity (a)  $EI = 5.1122e^{-9}nm$  and (b)  $EI = 7.2629e^{-9}nm$ .



**Figure 5.** FNFs with aspect ratio for CF chiral SWCNTs (12, 5), (22, 7), and (25, 10) with nonlocal parameter  $p = 1$  and bending rigidity (a)  $EI = 5.1122e^{-9}nm$  and (b)  $EI = 7.2629e^{-9}nm$ .

[C—F (25, 10)  $f$  (Hz): 320.7685–382.3023]. These relate to the case where  $EI$  varies from  $5.1122e^{-9}$  to  $7.2617e^{-9}$  nm and the nonlocal parameter  $p = 1$ . A trend of increasing frequencies of indices with bending rigidity is as  $(25, 10) > (22, 7) > (12, 5)$ .

The tendency to increase the frequency of indices with bending stiffness is  $(25, 10) > (22, 7) > (12, 5)$ . **Figure 5** shows that FNF, calculated by NLT, is based on WPA, with (12, 5), (22, 7), and (25, 10) CF chiral SWCNT, respectively. It was observed that FNF increased with increasing  $EI$  (hardness) and its value increased with increasing  $L/d$ . From our results, we can easily conclude that the climbing frequencies for bending the hardness of the curves (12, 5), (22, 7), and (25, 10) are as follows:  $(12, 5) < (22, 7) < (25, 10)$ .

## 6. Conclusion

In this study, the influence of boundary conditions on the vibration of single-walled carbon nanotubes was analyzed in chiral fashion with indices (12, 5), (22, 7), and (25, 10), respectively. An attempt of nonlocal elasticity theory models has been employed to study the vibration characteristics of SWCNTs analytically, and the WPA is exploited to develop the ODE of the vibrations of the SWCNTs. The influences of different boundary conditions and bending rigidity of chiral SWCNTs against aspect ratio have investigated. As can be seen from these, by increasing the aspect ratio of the carbon nanotube, fundamental natural frequency increases. In addition, as can be seen, increasing the bending rigidity results in the increase of the fundamental frequencies. The frequencies of CC end condition are higher than CF end condition for all computations in this chapter.

IntechOpen

### Author details

Muzamal Hussain\* and Muhammad Nawaz Naeem  
Department of Mathematics, Government College University Faisalabad,  
Faisalabad, Pakistan

\*Address all correspondence to: [muzamal45@gmail.com](mailto:muzamal45@gmail.com)

### IntechOpen

© 2019 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/3.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. 

## References

- [1] Iijima S. Helical microtubules of graphitic carbon. *Nature*. 1991;**345**: 56-58
- [2] Li C, Chou TW. A structural mechanics approach for the analysis of carbon nanotubes. *International Journal of Solids and Structures*. 2003;**40**(10): 2487-2499
- [3] Sakhaee-Pour A, Ahmadian MT, Vafai A. Vibrational analysis of single-walled carbon nanotubes using beam element. *Thin-Walled Structures*. 2009; **47**(6):646-652
- [4] Vodenitcharova T, Zhang LC. Effective wall thickness of a single-walled carbon nanotube. *Physical Review B*. 2003;**68**(16):165401
- [5] Hsu JC, Chang RP, Chang WJ. Resonance frequency of chiral single-walled carbon nanotubes using Timoshenko beam theory. *Physics Letters A*. 2008;**372**(16):2757-2759
- [6] Yakobson BI, Brabec CJ, Bernholc J. Nanomechanics of carbon tubes: Instabilities beyond linear response. *Physical Review Letters*. 1996;**76**(14): 2511-2514
- [7] Cao G, Chen X, Kysar JW. Strain sensing of carbon nanotubes: Numerical analysis of the vibrational frequency of deformed single-wall carbon nanotubes. *Physical Review B*. 2005;**72**(19):195412
- [8] Murmu T, Pradhan SC. Thermo-mechanical vibration of a single-walled carbon nanotube embedded in an elastic medium based on nonlocal elasticity theory. *Computational Materials Science*. 2009;**46**:854-859
- [9] Demir Ç, Civalek O. Nonlocal finite element formulation for vibration. *International Journal of Engineering and Applied Sciences (IJEAS)*. 2016;**8**: 109-117
- [10] Civalek O, Demir C, Akgoz B. Static analysis of single walled carbon nanotubes (SWCNT) based on Eringen's nonlocal elasticity theory. *International Journal of Engineering and Applied Sciences (IJEAS)*. 2009;**2**:47-56
- [11] Narendar S, Gopalakrishnan S. Critical buckling temperature of single-walled carbon nanotubes embedded in a one-parameter elastic medium based on nonlocal continuum mechanics. *Physica E: Low-dimensional Systems and Nanostructures*. 2011;**43**:1185-1191
- [12] Yayli MO. Torsion of nonlocal bars with equilateral triangle cross sections. *Journal of Computational and Theoretical Nanoscience*. 2013;**10**: 376-379
- [13] Toupin RA. Theory of elasticity with couple stresses. *Archive for Rational Mechanics and Analysis*. 1964;**17**:85-112
- [14] Mindlin RD, Tiersten HF. Effects of couple-stresses in linear elasticity. *Archive for Rational Mechanics and Analysis*. 1962;**11**:415-448
- [15] Fleck NA, Hutchinson JW. A phenomenological theory for strain gradient effects in plasticity. *Journal of the Mechanics and Physics of Solids*. 1993;**41**:1825-1857
- [16] Fleck NA, Hutchinson JW. A reformulation of strain gradient plasticity. *Journal of the Mechanics and Physics of Solids*. 2001;**49**:2245-2271
- [17] Eringen AC. On differential equations of nonlocal elasticity and solutions of screw dislocation and surface-waves. *Journal of Applied Physics*. 1983;**54**:4703-4710
- [18] Eringen AC, Edelen DGB. On nonlocal elasticity. *International Journal of Engineering Science*. 1972;**10**:233-248

- [19] Wang B, Deng ZC, Zhang K. Nonlinear vibration of embedded single-walled carbon nanotube with geometrical imperfection under harmonic load based on nonlocal Timoshenko beam theory. *Journal of Applied Mathematics and Mechanics*. 2013;**34**:269-280
- [20] Yang J, Ke LL, Kitipornchai S. Nonlinear free vibration of single-walled carbon nanotubes using nonlocal Timoshenko beam theory. *Physica E: Low-Dimensional System and Nanotechnology*. 2010;**42**:1727-1735
- [21] Azrar A, Azrar L, Aljinaidi AA. Length scale effect analysis on vibration behavior of single-walled carbon nanotubes with arbitrary boundary conditions. *Revue de Mécanique Appliqué et Théorique*. 2011;**2**:475-484
- [22] Azrar A, Azrar L, Aljinaidi AA, Hamadiche M. Dynamics instability analysis of multi-walled carbon nanotubes conveying fluid. *Advanced Materials Research*. 2013;**682**:153-160
- [23] Ansari R, Rouhi H, Sahmani S. Calibration of the analytical nonlocal shell model for vibrations of double-walled carbon nanotubes with arbitrary boundary conditions using molecular dynamics. *International Journal of Mechanical Sciences*. 2011;**53**:786-792
- [24] Ansari R, Sahmani S, Rouhi H. Rayleigh-Ritz axial buckling analysis of single-walled carbon nanotubes with different boundary conditions. *Physics Letters A*. 2011;**375**:1255-1263
- [25] Swain A, Roy T, Nanda BK. Vibration behavior of single-walled carbon nanotube using finite element. *International Journal on Theoretical and Applied Research in Mechanical Engineering*. 2013;**2**:129-133
- [26] Chawis T, Somchai C, Li T. Non-local theory for free vibration of single-walled carbon nanotubes. *Advanced Materials Research*. 2013;**747**:257-260
- [27] Hussain M, Naeem MN, Shahzad A, He M. Vibrational behavior of single-walled carbon nanotubes based on cylindrical shell model using wave propagation approach. *AIP Advances*. 2017;**7**(4):045114
- [28] Rouhi H, Bazdid-Vahdati M, Ansari R. Rayleigh-Ritz vibrational analysis of multi-walled carbon nanotubes based on the non-local Flügge shell theory. *Journal of Composites*. 2015:750392. <http://dx.doi.org/10.1155/2015/750392>
- [29] Hussain M, Naeem MN. Vibration analysis of single-walled carbon nanotubes using wave propagation approach. *Mechanical Sciences*. 2017;**8**(1):155-164
- [30] Yoon J, Ru CQ, Mioduchowski A. Noncoaxial resonance of an isolated multiwall carbon nanotube. *Physical Review B*. 2002;**66**(23):233402
- [31] Bocko J, Lengvarský P. Vibration of single-walled carbon nanotubes by using nonlocal theory. *American Journal of Mechanical Engineering*. 2014;**2**:195-198
- [32] Duan WH, Wang CM, Zhang YY. Calibration of nonlocal scaling effect parameter for free vibration of carbon nanotubes by molecular dynamics. *Journal of Applied Physics*. 2007;**101**(2):024305
- [33] Elishakoff I, Pentaras D. Fundamental natural frequencies of double-walled carbon nanotubes. *Journal of Sound and Vibration*. 2009;**322**(4):652-664
- [34] Natsuki T, Endo M, Tsuda H. Vibration analysis of embedded carbon nanotubes using wave propagation approach. *Journal of Applied Physics*. 2006;**99**(3):034311
- [35] Eringen AC. *Nonlocal Continuum Field Theories*. Springer Science & Business Media; 2002

- [36] Thongyothee C, Chucheepsakul S. Finite element modeling of van der Waals interaction for elastic stability of multi-walled carbon nanotubes. *Advanced Materials Research*. 2008;**55**: 525-528
- [37] Reddy JN, Pang SD. Nonlocal continuum theories of beams for the analysis of carbon nanotubes. *Journal of Applied Physics*. 2008;**103**(2):023511
- [38] Zhang XM, Liu GR, Lam KY. Vibration analysis of thin cylindrical shells using wave propagation approach. *Journal of Sound and Vibration*. 2001; **239**(3):397-403
- [39] Wang L, Guo W, Hu H. Group velocity of wave propagation in carbon nanotubes. *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*. 2008;**464**(2094):1423-1438
- [40] Liew KM, Wang Q. Analysis of wave propagation in carbon nanotubes via elastic shell theories. *International Journal of Engineering Science*. 2007; **45**(2):227-241
- [41] Hussain M, Naeem M, Shahzad A, He M. Vibration characteristics of fluid-filled functionally graded cylindrical material with ring supports. In: *Computational Fluid Dynamics*. Rijeka, Croatia: InTech Open; 2018. DOI: 10.5772/intechopen.72172. ISBN: 978-953-51-5706-9
- [42] Vibration characteristics of zigzag and chiral FGM rotating carbon nanotubes sandwich with ring supports. *Journal of Mechanical Engineering Science, Part C*. 2019
- [43] Hussain M, Naeem MN. Effects of ring supports on vibration of armchair and zigzag FGM rotating carbon nanotubes using Galerkin's method. *Composites Part B Engineering*. 2019;**163**:548-561. DOI: 10.1016/j.compositesb.2018.12.144
- [44] Hussain M, Naeem M. Effect of various edge conditions on free vibration characteristics of rectangular plates. In: *Advance Testing and Engineering*. Rijeka, Croatia: InTech Open; 2018. ISBN: 978-953-51-6706-8
- [45] Hussain M, Naeem M. Vibration of single-walled carbon nanotubes based on Donnell shell theory using wave propagation approach. In: *Novel Nanomaterials—Synthesis and Applications*. Rijeka, Croatia: InTech Open; 2018. DOI: 10.5772/intechopen.73503. ISBN: 978-953-51-5896-7
- [46] Hussain M, Naeem MN, Isvandzibaei MR. Effect of Winkler and Pasternak elastic foundation on the vibration of rotating functionally graded material cylindrical shell. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*. 2018;**232**(24): 4564-4577. DOI: 0954406217753459
- [47] Hussain M, Naeem MN, Shahzad A, He M, Habib S. Vibrations of rotating cylindrical shells with FGM using wave propagation approach. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*. 2018;**232**(23): 4342-4356
- [48] Alibeigloo A, Shaban M. Free vibration analysis of carbon nanotubes by using three-dimensional theory of elasticity. *Acta Mechanica*. 2013;**224**(7): 1415-1427