Chapter

Two Systems of Maxwell's Equations and Two Corresponding Systems of Wave Equations in a Rotating Dielectric Medium

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Abstract

In this chapter, on the base of two basic systems of Maxwell's equations for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ in a uniformly rotating dielectric medium, the two corresponding systems of wave equations have been derived (to the first order in an angular velocity Ω). From their comparative analysis, it can be seen that the structure of the wave equations for electromagnetic field vectors in the first system is asymmetrical with respect to Ω, while the structure of such equations in the second one is symmetrical. On this basis, it can be concluded that if the principle of symmetry is accepted as a criterion for selection, then second system of wave equations (and, therefore, corresponding the second set of Maxwell's equations) for vectors $\vec{E}$ and $\vec{B}$ may be preferred.

Keywords: Maxwell's equations, wave equations, Sagnac effect, fiber-optic gyro, ring laser gyro

1. Introduction

In order to develop the theory of fiber-optic gyro (or, e.g., ring laser gyro with resonator containing a dielectric medium with index of refraction n), one needs to have a system of Maxwell's equations and corresponding system of wave equations for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ which are written in a frame of reference which uniformly rotates in an inertial frame with angular velocity Ω ($\Omega = |\vec{\Omega}|$). Since the module v of vector $\vec{v} = \vec{\Omega} \times \vec{r}$ of a tangential velocity of such rotating device is much smaller than the speed of light, it is sufficient for these systems to be linear in v or, equivalently, in Ω.

From the literature, it can be seen that there are mainly two basic systems of Maxwell's equations for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ derived from the first principles and written for the case of a uniformly rotating dielectric medium. These two systems of equations are in good agreement with the experiments conducted for ring optical interferometers, fiber-optic gyros, and ring laser gyros containing in their arms the gas discharge tubes with Brewster's windows. Both systems are based on the Galilean description of rotation: $x = x^0 \cos\Omega t^0 + y^0 \sin\Omega t^0, y = -x^0 \sin\Omega t^0 + y^0 \cos\Omega t^0, z = z^0, t = t^0$ (superscript $'0'$ refers to the inertial frame).

In the absence of free charges and currents, the first system, which was first obtained in work [1] from the formalism of the theory of general relativity, may be written in the form (we keep the terms only up to first order in Ω):

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0, \tag{1}$$

$$\vec{\nabla} \cdot \vec{B} = 0, \tag{2}$$

$$\vec{\nabla} \times \left(\vec{B} - \frac{1}{c^2}\ \vec{v} \times \vec{E}\right) - \frac{1}{c^2}\frac{\partial}{\partial t}\left(n^2\ \vec{E} - \vec{v} \times \vec{B}\right) = 0, \tag{3}$$

$$\vec{\nabla} \cdot \left(\vec{E} - \frac{1}{n^2}\ \vec{v} \times \vec{B}\right) = 0, \tag{4}$$

and the second one, which was first obtained in work [2] on the base of the use of the tetrad method in this theory, may be presented as

$$\vec{\nabla} \times \vec{E} + \frac{\partial}{\partial t}\left(\vec{B} + \frac{1}{c^2}\ \vec{v} \times \vec{E}\right) = 0, \tag{5}$$

$$\vec{\nabla} \cdot \left(\vec{B} + \frac{1}{c^2}\ \vec{v} \times \vec{E}\right) = 0, \tag{6}$$

$$\vec{\nabla} \times \vec{B} - \frac{1}{c^2}\frac{\partial}{\partial t}\left(n^2\ \vec{E} - \vec{v} \times \vec{B}\right) = 0, \tag{7}$$

$$\vec{\nabla} \cdot \left(\vec{E} - \frac{1}{n^2}\ \vec{v} \times \vec{B}\right) = 0. \tag{8}$$

Systems of Maxwell's Eqs. (1–4) and (5–8) are written here in units of the SI. They contain only two electromagnetic field vectors $\vec{E}$ and $\vec{B}$. In the named original works [1] (see expressions (5), (14b) therein) and [2] (see relations (6.11), (6.12) therein), these systems contain, besides vectors $\vec{E}$ and $\vec{B}$, two more electromagnetic field vectors $\vec{D}$ and $\vec{H}$ (which must be excluded). Systems (1–4) and (5–8) can be also found, for example, in subsequent works [3–8] and [9], respectively. In the case of vacuum (when $n = 1$), sets (1–4) and (5–8) are transformed into the ones known correspondingly from works [10] and [11].

In the above systems, all the quantities are specified by the formulas

$$\begin{aligned}
&\vec{\nabla} = \hat{x}(\partial/\partial x) + \hat{y}(\partial/\partial y) + \hat{z}(\partial/\partial z),\ \vec{E} = E_x\,\hat{x} + E_y\,\hat{y} + E_z\,\hat{z},\ \vec{B} = B_x\,\hat{x} + B_y\,\hat{y} + B_z\,\hat{z},\\
&\vec{\Omega} = \Omega_x\,\hat{x} + \Omega_y\,\hat{y} + \Omega_z\,\hat{z},\ \vec{r} = x\hat{x} + y\hat{y} + z\hat{z},\ \vec{v} = \vec{\Omega} \times \vec{r} = v_x\,\hat{x} + v_y\,\hat{y} + v_z\,\hat{z},\\
&v_x = \Omega_y z - \Omega_z y, v_y = \Omega_z x - \Omega_x z, v_z = \Omega_x y - \Omega_y x
\end{aligned} \tag{9}$$

where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are the unit vectors which form an orthogonal coordinate basis $\{\hat{x}\,\hat{y}\,\hat{z}\}$ of a rotating frame; E_x, E_y, and E_z and B_x, B_y, and B_z are the components of vectors $\vec{E}$ and $\vec{B}$ in this basis; $\vec{\Omega}$ is the vector of angular velocity with which the basis $\{\hat{x}\,\hat{y}\,\hat{z}\}$ rotates in the inertial frame; Ω_x, Ω_y, and Ω_z are the components of vector $\vec{\Omega}$; $\vec{r}$ is the radius vector of the given observation point in basis $\{\hat{x}\,\hat{y}\,\hat{z}\}$; x, y, and z

are the components of vector $\vec{r}$; $\vec{v}$ is the vector of linear tangential velocity of the observation point calculated in the inertial frame; v_x,v_y, and v_z are the components of vector $\vec{v}$; $n = (\varepsilon_r \mu_r)^{1/2}$ is the index of refraction of a dielectric medium; and ε_r and μ_r are the relative permittivity and permeability of a medium, respectively.

Remark about systems (1–4) and (5–8)

⊲ 1. According to the textbook [12], in a uniformly rotating frame of reference with spatial rectangular coordinates x, y, z and time coordinate t, the quadratic form ds^2 may be presented as $ds^2 = g_{ik}\,dx^i\,dx^k$ where $x^0 = ct$, $x^1 = x$, $x^2 = y$, and $x^3 = z$. In general, when $\vec{\Omega} = \Omega_x \hat{x} + \Omega_y \hat{y} + \Omega_z \hat{z}$, the nonzero components of a space–time metric tensor (with determinant $g = -1$) of such rotating frame may be written as $g_{00} = 1 - v^2/c^2, g_{11} = g_{22} = g_{33} = -1, g_{01} = g_{10} = -v_x/c, g_{02} = g_{20} = -v_y/c$, and $g_{03} = g_{30} = -v_z/c$. Then, components $\kappa_{\alpha\beta} = -g_{\alpha\beta} + g_{0\alpha}\,g_{0\beta}/g_{00}$ of a spatial metric tensor [with determinant $\kappa = (1 - v^2/c^2)^{-1}$] of such frame will be $\kappa_{11} = 1 + \kappa v_x^2/c^2$, $\kappa_{22} = 1 + \kappa v_y^2/c^2$, $\kappa_{33} = 1 + \kappa v_z^2/c^2$, $\kappa_{12} = \kappa_{21} = \kappa v_x v_y/c^2$, $\kappa_{13} = \kappa_{31} = \kappa v_x v_z/c^2$, and $\kappa_{23} = \kappa_{32} = \kappa v_y v_z/c^2$. From these formulas for $\kappa_{\alpha\beta}$ and κ, it follows that the spatial metric tensor of a rotating frame of reference, in a linear with respect to Ω approximation, has a diagonal form with nonzero elements $\kappa_{11} = \kappa_{22} = \kappa_{33} = 1$ and its determinant $\kappa = 1$. Therefore, in such approximation, geometry of space in a rotating frame of reference remains Euclidean (flat space). So, spatial rectangular coordinates x, y, and z (or, e.g., cylindrical coordinates ρ, ϕ, z) of the given observation point in this frame have their usual sense, and the operator $\vec{\nabla}$ in systems (1–4) and (5–8) may be used in usual way. ⊳.

As one can see, the above two systems of Maxwell's Eqs. (1–4) and (5–8) for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ are not identical: system (1–4) has asymmetrical structure with respect to Ω in a sense that rotation manifests itself only in the third and fourth equations but not in the first and second ones; system (5–8) has symmetrical structure with respect to Ω because rotation manifests itself in all four equations. The reason of such difference between these two systems is that they were obtained in works [1, 2] with the help of two qualitatively different theoretical approaches.

In this situation, one may ask the following questions: (1) what will the form of the two corresponding systems of wave equations for the named vectors in first and second cases be? (2) Which system of such wave equations (first or second) is preferred? The answers to these questions are not given in the literature.

So, the task of this research is to derive the wave equations for vectors $\vec{E}$ and $\vec{B}$—at first on the base of system of Maxwell's Eqs. (1–4) and then on the base of system (5–8). The results obtained in both cases must be compared. All calculations must be performed with accuracy approximated to the first order in v ($v = |\vec{v}|$) or, equivalently, Ω ($\Omega = |\vec{\Omega}|$).

2. Auxiliary relations

In this section, we are going to present some useful formulas for the quantities $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{G}\right)$, $\vec{\nabla} \times \left(\vec{v} \times \vec{G}\right)$, $\vec{\nabla} \cdot \left(\vec{v} \times \vec{G}\right)$, $\vec{\nabla}\left(\vec{v} \cdot \vec{G}\right)$, and $\vec{\nabla}^2 \left(\vec{v} \times \vec{G}\right)$ which involve vectors $\vec{v} = \vec{\Omega} \times \vec{r}$ and $\vec{G}$ ($\vec{G} = \vec{E}, \vec{B}$).

A. Consider the term $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{G}\right)$. It is known (see, e.g., handbook [13]) that

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{G}\right) = -\vec{\nabla}^2 \vec{G} + \vec{\nabla} \left(\vec{\nabla} \cdot \vec{G}\right). \tag{10}$$

B. Consider the identity $\vec{\nabla} \times \left(\vec{v} \times \vec{G}\right) = \left(\vec{G} \cdot \vec{\nabla}\right) \vec{v} - \left(\vec{v} \cdot \vec{\nabla}\right) \vec{G} + \vec{v} \left(\vec{\nabla} \cdot \vec{G}\right) - \vec{G} \left(\vec{\nabla} \cdot \vec{v}\right)$. In the case $\vec{v} = \vec{\Omega} \times \vec{r}$, we have $\left(\vec{G} \cdot \vec{\nabla}\right) \vec{v} = \vec{\Omega} \times \vec{G}$, $\vec{\nabla} \cdot \vec{v} = 0$, and, to the first order in Ω, $\vec{v} \left(\vec{\nabla} \cdot \vec{G}\right) = 0$. Therefore

$$\vec{\nabla} \times \left(\vec{v} \times \vec{G}\right) = -\left(\vec{v} \cdot \vec{\nabla}\right) \vec{G} + \vec{\Omega} \times \vec{G}. \tag{11}$$

C. Consider the identity $\vec{\nabla} \cdot \left(\vec{v} \times \vec{G}\right) = \vec{G} \cdot \left(\vec{\nabla} \times \vec{v}\right) - \vec{v} \cdot \left(\vec{\nabla} \times \vec{G}\right)$. Since $\vec{\nabla} \times \vec{v} = 2\,\vec{\Omega}$,

$$\vec{\nabla} \cdot \left(\vec{v} \times \vec{G}\right) = 2\,\vec{\Omega} \cdot \vec{G} - \vec{v} \cdot \left(\vec{\nabla} \times \vec{G}\right). \tag{12}$$

D. Consider the identity $\vec{\nabla} \left(\vec{v} \cdot \vec{G}\right) = \left(\vec{v} \cdot \vec{\nabla}\right) \vec{G} + \left(\vec{G} \cdot \vec{\nabla}\right) \vec{v} + \vec{v} \times \left(\vec{\nabla} \times \vec{G}\right) + \vec{G} \times \left(\vec{\nabla} \times \vec{v}\right)$. Since $\left(\vec{G} \cdot \vec{\nabla}\right) \vec{v} = \vec{\Omega} \times \vec{G}$, $\vec{\nabla} \times \vec{v} = 2\,\vec{\Omega}$, and $\vec{G} \times \left(\vec{\nabla} \times \vec{v}\right) = -2\left(\vec{\Omega} \times \vec{G}\right)$,

$$\vec{\nabla} \left(\vec{v} \cdot \vec{G}\right) = \left(\vec{v} \cdot \vec{\nabla}\right) \vec{G} - \vec{\Omega} \times \vec{G} + \vec{v} \times \left(\vec{\nabla} \times \vec{G}\right). \tag{13}$$

E. Consider the vector $\vec{\nabla}^2 \left(\vec{v} \times \vec{G}\right)$. There is the formula

$$\vec{\nabla}^2 \left(\vec{v} \times \vec{G}\right) = \vec{\nabla}^2 \left(\vec{v} \times \vec{G}\right)_x \hat{x} + \vec{\nabla}^2 \left(\vec{v} \times \vec{G}\right)_y \hat{y} + \vec{\nabla}^2 \left(\vec{v} \times \vec{G}\right)_z \hat{z}. \tag{14}$$

Let us first calculate the projection $\vec{\nabla}^2 \left(\vec{v} \times \vec{G}\right)_x$ of this vector onto the axis $\hat{x}$. Taking into account (9), we have

$$\vec{\nabla}^2 \left(\vec{v} \times \vec{G}\right)_x = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) (v_y G_z - v_z G_y), \tag{15}$$

or

$$\vec{\nabla}^2 \left(\vec{v} \times \vec{G}\right)_x = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \left[(\Omega_z x - \Omega_x z) G_z - (\Omega_x y - \Omega_y x) G_y\right]. \tag{16}$$

After the calculation, we get

$$\begin{aligned}\vec{\nabla}^2 \left(\vec{v} \times \vec{G}\right)_x = {} & v_y \left(\frac{\partial^2 G_z}{\partial x^2} + \frac{\partial^2 G_z}{\partial y^2} + \frac{\partial^2 G_z}{\partial z^2}\right) - v_z \left(\frac{\partial^2 G_y}{\partial x^2} + \frac{\partial^2 G_y}{\partial y^2} + \frac{\partial^2 G_y}{\partial z^2}\right) \\ & - 2\Omega_x \left(\frac{\partial G_y}{\partial y} + \frac{\partial G_z}{\partial z}\right) + 2\left(\Omega_y \frac{\partial G_y}{\partial x} + \Omega_z \frac{\partial G_z}{\partial x}\right).\end{aligned} \tag{17}$$

Let us add to the right-hand side of (17) the following terms: $-2\Omega_x (\partial G_x / \partial x)$ and $+2\Omega_x (\partial G_x / \partial x)$. Then

$$\vec{\nabla}^2\left(\vec{v}\times\vec{G}\right)_x = v_y\left(\frac{\partial^2 G_z}{\partial x^2}+\frac{\partial^2 G_z}{\partial y^2}+\frac{\partial^2 G_z}{\partial z^2}\right) - v_z\left(\frac{\partial^2 G_y}{\partial x^2}+\frac{\partial^2 G_y}{\partial y^2}+\frac{\partial^2 G_y}{\partial z^2}\right)$$
$$-2\Omega_x\left(\frac{\partial G_x}{\partial x}+\frac{\partial G_y}{\partial y}+\frac{\partial G_z}{\partial z}\right)+2\left(\Omega_x\frac{\partial G_x}{\partial x}+\Omega_y\frac{\partial G_y}{\partial x}+\Omega_z\frac{\partial G_z}{\partial x}\right), \tag{18}$$

or

$$\vec{\nabla}^2\left(\vec{v}\times\vec{G}\right)_x = \left[\vec{v}\times\left(\vec{\nabla}^2\vec{G}\right)\right]_x - 2\Omega_x\left(\vec{\nabla}\cdot\vec{G}\right)+2\frac{\partial}{\partial x}\left(\vec{\Omega}\cdot\vec{G}\right). \tag{19}$$

Similarly, we may obtain

$$\vec{\nabla}^2\left(\vec{v}\times\vec{G}\right)_y = \left[\vec{v}\times\left(\vec{\nabla}^2\vec{G}\right)\right]_y - 2\Omega_y\left(\vec{\nabla}\cdot\vec{G}\right)+2\frac{\partial}{\partial y}\left(\vec{\Omega}\cdot\vec{G}\right), \tag{20}$$

$$\vec{\nabla}^2\left(\vec{v}\times\vec{G}\right)_z = \left[\vec{v}\times\left(\vec{\nabla}^2\vec{G}\right)\right]_z - 2\Omega_z\left(\vec{\nabla}\cdot\vec{G}\right)+2\frac{\partial}{\partial z}\left(\vec{\Omega}\cdot\vec{G}\right). \tag{21}$$

Therefore

$$\vec{\nabla}^2\left(\vec{v}\times\vec{G}\right) = \vec{v}\times\left(\vec{\nabla}^2\vec{G}\right) - 2\,\vec{\Omega}\,\left(\vec{\nabla}\cdot\vec{G}\right)+2\,\vec{\nabla}\,\left(\vec{\Omega}\cdot\vec{G}\right). \tag{22}$$

Finally, taking into account that, to the first order in Ω, $\vec{\Omega}\,\left(\vec{\nabla}\cdot\vec{G}\right)=0$, we get

$$\vec{\nabla}^2\left(\vec{v}\times\vec{G}\right) = \vec{v}\times\left(\vec{\nabla}^2\vec{G}\right)+2\,\vec{\nabla}\,\left(\vec{\Omega}\cdot\vec{G}\right). \tag{23}$$

Formulas (10)–(13) and (23) will be used in the next sections.

3. First system of wave equations for vectors $\vec{E}$ and $\vec{B}$ in a rotating medium

In this section, we are going to derive the first system of wave equations for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ which will correspond to the original system of Maxwell's Eqs. (1–4).

3.1 Equation for vector $\vec{E}$

To derive the wave equation for vector $\vec{E}$, we apply the operator $\vec{\nabla}\times$ to expression (1):

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right)+\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{B}\right)=0. \tag{24}$$

Taking into account (10) and (3), we rewrite (24) in the form

$$\vec{\nabla}^2\,\vec{E} - \frac{n^2}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}+\frac{1}{c^2}\frac{\partial}{\partial t}\left(\vec{v}\times\frac{\partial\vec{B}}{\partial t}\right)-\frac{1}{c^2}\frac{\partial}{\partial t}\left[\vec{\nabla}\times\left(\vec{v}\times\vec{E}\right)\right]-\vec{\nabla}\,\left(\vec{\nabla}\cdot\vec{E}\right)=0, \tag{25}$$

or, using (1),

$$\vec{\nabla}^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} - \frac{1}{c^2} \frac{\partial}{\partial t} \left[\vec{\nabla} \times \left(\vec{v} \times \vec{E} \right) + \vec{v} \times \left(\vec{\nabla} \times \vec{E} \right) \right] - \vec{\nabla} \left(\vec{\nabla} \cdot \vec{E} \right) = 0. \quad (26)$$

With the help of (11), we get

$$\vec{\nabla}^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \frac{1}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{E} - \vec{\Omega} \times \vec{E} - \vec{v} \times \left(\vec{\nabla} \times \vec{E} \right) \right] - \vec{\nabla} \left(\vec{\nabla} \cdot \vec{E} \right) = 0. \quad (27)$$

Consider the term $\vec{\nabla} \left(\vec{\nabla} \cdot \vec{E} \right)$ in (27). According to (4), $\vec{\nabla} \cdot \vec{E} = \frac{1}{n^2} \vec{\nabla} \cdot \left(\vec{v} \times \vec{B} \right)$. Taking into account (12) and (3), we have $\vec{\nabla} \cdot \left(\vec{v} \times \vec{B} \right) = 2 \vec{\Omega} \cdot \vec{B} - (n^2/c^2)(\partial/\partial t) \left(\vec{v} \cdot \vec{E} \right)$, so

$$\vec{\nabla} \left(\vec{\nabla} \cdot \vec{E} \right) = \frac{2}{n^2} \vec{\nabla} \left(\vec{\Omega} \cdot \vec{B} \right) - \frac{1}{c^2} \frac{\partial}{\partial t} \vec{\nabla} \left(\vec{v} \cdot \vec{E} \right). \quad (28)$$

Consider the last term in (28). In accordance with (13),

$$\vec{\nabla} \left(\vec{v} \cdot \vec{E} \right) = \left(\vec{v} \cdot \vec{\nabla} \right) \vec{E} - \vec{\Omega} \times \vec{E} + \vec{v} \times \left(\vec{\nabla} \times \vec{E} \right), \quad (29)$$

therefore

$$\vec{\nabla} \left(\vec{\nabla} \cdot \vec{E} \right) = \frac{2}{n^2} \vec{\nabla} \left(\vec{\Omega} \cdot \vec{B} \right) - \frac{1}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{E} - \vec{\Omega} \times \vec{E} + \vec{v} \times \left(\vec{\nabla} \times \vec{E} \right) \right]. \quad (30)$$

Finally, substituting (30) into (27), we obtain the desired wave equation for vector $\vec{E}$:

$$\vec{\nabla}^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \frac{2}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{E} - \vec{\Omega} \times \vec{E} \right] - \frac{2}{n^2} \vec{\nabla} \left(\vec{\Omega} \cdot \vec{B} \right) = 0. \quad (31)$$

3.2 Equation for vector $\vec{B}$

In order to derive the wave equation for vector $\vec{B}$, we apply the operator $\vec{\nabla} \times$ to expression (3):

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{B} \right) - \frac{1}{c^2} \vec{\nabla} \times \left[\vec{\nabla} \times \left(\vec{v} \times E \right) \right] - \frac{n^2}{c^2} \frac{\partial}{\partial t} \left(\vec{\nabla} \times \vec{E} \right) + \frac{1}{c^2} \frac{\partial}{\partial t} \left[\vec{\nabla} \times \left(\vec{v} \times \vec{B} \right) \right] = 0. \quad (32)$$

Taking into account (10), (1), and (2), we rewrite (32) as

$$\vec{\nabla}^2 \vec{B} - \frac{n^2}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} - \frac{1}{c^2} \frac{\partial}{\partial t} \left[\vec{\nabla} \times \left(\vec{v} \times \vec{B} \right) \right] + \frac{1}{c^2} \vec{\nabla} \times \left[\vec{\nabla} \times \left(\vec{v} \times \vec{E} \right) \right] = 0. \quad (33)$$

Since $\vec{\nabla} \times \left[\vec{\nabla} \times \left(\vec{v} \times \vec{E} \right) \right] = -\vec{\nabla}^2 \left(\vec{v} \times \vec{E} \right) + \vec{\nabla} \left[\vec{\nabla} \cdot \left(\vec{v} \times \vec{E} \right) \right]$,

$$\vec{\nabla}^2 \vec{B} - \frac{n^2}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} - \frac{1}{c^2} \frac{\partial}{\partial t} \left[\vec{\nabla} \times \left(\vec{v} \times \vec{B} \right) \right] - \frac{1}{c^2} \vec{\nabla}^2 \left(\vec{v} \times \vec{E} \right) + \frac{1}{c^2} \vec{\nabla} \left[\vec{\nabla} \cdot \left(\vec{v} \times \vec{E} \right) \right] = 0. \tag{34}$$

Using (11), we have

$$\vec{\nabla}^2 \vec{B} - \frac{n^2}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} + \frac{1}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{B} - \vec{\Omega} \times \vec{B} \right] - \frac{1}{c^2} \vec{\nabla}^2 \left(\vec{v} \times \vec{E} \right) + \frac{1}{c^2} \vec{\nabla} \left[\vec{\nabla} \cdot \left(\vec{v} \times \vec{E} \right) \right] = 0. \tag{35}$$

Consider the term $\vec{\nabla} \left[\vec{\nabla} \cdot \left(\vec{v} \times \vec{E} \right) \right]$ in (35). In accordance with (12), $\vec{\nabla} \cdot \left(\vec{v} \times \vec{E} \right) = 2\, \vec{\Omega} \cdot \vec{E} - \vec{v} \cdot \left(\vec{\nabla} \times \vec{E} \right)$, or, with (1), $\vec{\nabla} \cdot \left(\vec{v} \times \vec{E} \right) = 2\, \vec{\Omega} \cdot \vec{E} + (\partial/\partial t) \left(\vec{v} \cdot \vec{B} \right)$. Then

$$\vec{\nabla} \left[\vec{\nabla} \cdot \left(\vec{v} \times \vec{E} \right) \right] = 2\, \vec{\nabla} \left(\vec{\Omega} \cdot \vec{E} \right) + \frac{\partial}{\partial t} \vec{\nabla} \left(\vec{v} \cdot \vec{B} \right), \tag{36}$$

or, taking into account (13),

$$\vec{\nabla} \left[\vec{\nabla} \cdot \left(\vec{v} \times \vec{E} \right) \right] = 2\, \vec{\nabla} \left(\vec{\Omega} \cdot \vec{E} \right) + \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{B} - \vec{\Omega} \times \vec{B} \right] + \frac{\partial}{\partial t} \left[\vec{v} \times \left(\vec{\nabla} \times \vec{B} \right) \right]. \tag{37}$$

With the help of (3), we find

$$\vec{\nabla} \left[\vec{\nabla} \cdot \left(\vec{v} \times \vec{E} \right) \right] = 2\, \vec{\nabla} \left(\vec{\Omega} \cdot \vec{E} \right) + \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{B} - \vec{\Omega} \times \vec{B} \right] + \vec{v} \times \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}. \tag{38}$$

Substituting (38) into (35), we obtain

$$\vec{\nabla}^2 \vec{B} - \frac{n^2}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} + \frac{2}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{B} - \vec{\Omega} \times \vec{B} \right] + \frac{2}{c^2} \vec{\nabla} \left(\vec{\Omega} \cdot \vec{E} \right) - \frac{1}{c^2} \left[\vec{\nabla}^2 \left(\vec{v} \times \vec{E} \right) - \vec{v} \times \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \right] = 0. \tag{39}$$

Consider the last term in (39). In accordance with (23), $\vec{\nabla}^2 \left(\vec{v} \times \vec{E} \right) = \vec{v} \times \left(\vec{\nabla}^2 \vec{E} \right) + 2\, \vec{\nabla} \left(\vec{\Omega} \cdot \vec{E} \right)$; hence

$$\vec{\nabla}^2 \left(\vec{v} \times \vec{E} \right) - \vec{v} \times \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{v} \times \left(\vec{\nabla}^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \right) + 2\, \vec{\nabla} \left(\vec{\Omega} \cdot \vec{E} \right). \tag{40}$$

It is clear that, to the first order in Ω,

$$\vec{v} \times \left(\vec{\nabla}^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \right) = 0. \tag{41}$$

So, expression (40) may be rewritten as

$$\vec{\nabla}^2 \left(\vec{v} \times \vec{E} \right) - \vec{v} \times \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 2\, \vec{\nabla} \left(\vec{\Omega} \cdot \vec{E} \right). \tag{42}$$

After inserting (42) into (39), we obtain the desired wave equation for vector $\vec{B}$:

$$\vec{\nabla}^2 \vec{B} - \frac{n^2}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}\left[\left(\vec{v}\cdot\vec{\nabla}\right)\vec{B} - \vec{\Omega}\times\vec{B}\right] = 0. \tag{43}$$

3.3 Result of Section 3

According to (31) and (43), the first system of wave equations for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ in a rotating medium has the following form:

$$\begin{aligned}
&\vec{\nabla}^2 \vec{E} - \frac{n^2}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}\left[\left(\vec{v}\cdot\vec{\nabla}\right)\vec{E} - \vec{\Omega}\times\vec{E}\right] - \frac{2}{n^2}\,\vec{\nabla}\left(\vec{\Omega}\cdot\vec{B}\right) = 0,\\
&\vec{\nabla}^2 \vec{B} - \frac{n^2}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}\left[\left(\vec{v}\cdot\vec{\nabla}\right)\vec{B} - \vec{\Omega}\times\vec{B}\right] = 0.
\end{aligned} \tag{44}$$

4. Second system of wave equations for vectors $\vec{E}$ and $\vec{B}$ in a rotating medium

In this section, we are going to obtain the second system of wave equations for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ which will correspond to the original system of Maxwell's Eqs. (5–8).

4.1 Equation for vector $\vec{E}$

To derive the wave equation for vector $\vec{E}$, we apply the operator $\vec{\nabla}\times$ to expression (5):

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right) + \frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{B}\right) + \frac{1}{c^2}\frac{\partial}{\partial t}\left[\vec{\nabla}\times\left(\vec{v}\times\vec{E}\right)\right] = 0. \tag{45}$$

Taking into consideration (7) and (10), we rewrite (45) in the form

$$\vec{\nabla}^2 \vec{E} - \frac{n^2}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} + \frac{1}{c^2}\frac{\partial}{\partial t}\left(\vec{v}\times\frac{\partial \vec{B}}{\partial t}\right) - \frac{1}{c^2}\frac{\partial}{\partial t}\left[\vec{\nabla}\times\left(\vec{v}\times\vec{E}\right)\right] - \vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right) = 0, \tag{46}$$

or, using (5),

$$\vec{\nabla}^2 \vec{E} - \frac{n^2}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{1}{c^2}\frac{\partial}{\partial t}\left[\vec{\nabla}\times\left(\vec{v}\times\vec{E}\right) + \vec{v}\times\left(\vec{\nabla}\times\vec{E}\right)\right] - \vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right) = 0. \tag{47}$$

Taking into account (11), we have

$$\vec{\nabla}^2 \vec{E} - \frac{n^2}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} + \frac{1}{c^2}\frac{\partial}{\partial t}\left[\left(\vec{v}\cdot\vec{\nabla}\right)\vec{E} - \vec{\Omega}\times\vec{E} - \vec{v}\times\left(\vec{\nabla}\times\vec{E}\right)\right] - \vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right) = 0. \tag{48}$$

Consider the term $\vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right)$ in (48). According to (8), $\vec{\nabla}\cdot\vec{E}=\frac{1}{n^2}\ \vec{\nabla}\cdot\left(\vec{v}\times\vec{B}\right)$, where, taking into account (12),

$$\vec{\nabla}\cdot\left(\vec{v}\times\vec{B}\right)=2\ \vec{\Omega}\cdot\vec{B}-\vec{v}\cdot\left(\vec{\nabla}\times\vec{B}\right). \tag{49}$$

With the help of (7), we find $\vec{\nabla}\cdot\left(\vec{v}\times\vec{B}\right)=2\ \vec{\Omega}\cdot\vec{B}-(n^2/c^2)\,(\partial/\partial t)\left(\vec{v}\cdot\vec{E}\right)$; hence

$$\vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right)=\frac{2}{n^2}\ \vec{\nabla}\left(\vec{\Omega}\cdot\vec{B}\right)-\frac{1}{c^2}\frac{\partial}{\partial t}\ \vec{\nabla}\left(\vec{v}\cdot\vec{E}\right). \tag{50}$$

Consider the term $\vec{\nabla}\left(\vec{v}\cdot\vec{E}\right)$ in (50). In accordance with (13),

$$\vec{\nabla}\left(\vec{v}\cdot\vec{E}\right)=\left(\vec{v}\cdot\vec{\nabla}\right)\vec{E}-\vec{\Omega}\times\vec{E}+\vec{v}\times\left(\vec{\nabla}\times\vec{E}\right). \tag{51}$$

Then

$$\vec{\nabla}\left(\vec{\nabla}\cdot\vec{E}\right)=\frac{2}{n^2}\ \vec{\nabla}\left(\vec{\Omega}\cdot\vec{B}\right)-\frac{1}{c^2}\frac{\partial}{\partial t}\left[\left(\vec{v}\cdot\vec{\nabla}\right)\vec{E}-\vec{\Omega}\times\vec{E}+\vec{v}\times\left(\vec{\nabla}\times\vec{E}\right)\right]. \tag{52}$$

Substituting (52) into (48), we obtain the desired wave equation for vector $\vec{E}$:

$$\vec{\nabla}^2\,\vec{E}-\frac{n^2}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}+\frac{2}{c^2}\frac{\partial}{\partial t}\left[\left(\vec{v}\cdot\vec{\nabla}\right)\vec{E}-\vec{\Omega}\times\vec{E}\right]-\frac{2}{n^2}\ \vec{\nabla}\left(\vec{\Omega}\cdot\vec{B}\right)=0. \tag{53}$$

4.2 Equation for vector $\vec{B}$

In order to derive the wave equation for vector $\vec{B}$, we apply the operator $\vec{\nabla}\times$ to expression (7):

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{B}\right)-\frac{n^2}{c^2}\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{E}\right)+\frac{1}{c^2}\frac{\partial}{\partial t}\left[\vec{\nabla}\times\left(\vec{v}\times\vec{B}\right)\right]=0. \tag{54}$$

Taking into account (5) and (10), we rewrite (54) in the form

$$\vec{\nabla}^2\,\vec{B}-\frac{n^2}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}-\frac{1}{c^2}\frac{\partial}{\partial t}\left(\vec{v}\times\frac{n^2}{c^2}\frac{\partial\vec{E}}{\partial t}\right)-\frac{1}{c^2}\frac{\partial}{\partial t}\left[\vec{\nabla}\times\left(\vec{v}\times\vec{B}\right)\right]-\vec{\nabla}\left(\vec{\nabla}\cdot\vec{B}\right)=0, \tag{55}$$

or, using (7),

$$\vec{\nabla}^2\,\vec{B}-\frac{n^2}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}-\frac{1}{c^2}\frac{\partial}{\partial t}\left[\vec{\nabla}\times\left(\vec{v}\times\vec{B}\right)+\vec{v}\times\left(\vec{\nabla}\times\vec{B}\right)\right]-\vec{\nabla}\left(\vec{\nabla}\cdot\vec{B}\right)=0. \tag{56}$$

Consider the term $\vec{\nabla}\times\left(\vec{v}\times\vec{B}\right)$ in (56). According to (11), $\vec{\nabla}\times\left(\vec{v}\times\vec{B}\right)=-\left(\vec{v}\cdot\vec{\nabla}\right)\vec{B}+\vec{\Omega}\times\vec{B}$. Thus,

$$\vec{\nabla}^2 \vec{B} - \frac{n^2}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} + \frac{1}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{B} - \vec{\Omega} \times \vec{B} - \vec{v} \times \left(\vec{\nabla} \times \vec{B} \right) \right] - \vec{\nabla} \left(\vec{\nabla} \cdot \vec{B} \right) = 0. \tag{57}$$

Consider the term $\vec{\nabla} \left(\vec{\nabla} \cdot \vec{B} \right)$ in (57). In accordance with (6),

$$\vec{\nabla} \cdot \vec{B} = -\left(1/c^2\right) \vec{\nabla} \cdot \left(\vec{v} \times \vec{E} \right). \tag{58}$$

Taking into account (12), we have

$$\vec{\nabla} \cdot \left(\vec{v} \times \vec{E} \right) = 2 \vec{\Omega} \cdot \vec{E} - \vec{v} \cdot \left(\vec{\nabla} \times \vec{E} \right), \tag{59}$$

or, using (5),

$$\vec{\nabla} \cdot \left(\vec{v} \times \vec{E} \right) = 2 \vec{\Omega} \cdot \vec{E} + \frac{\partial}{\partial t} \left(\vec{v} \cdot \vec{B} \right). \tag{60}$$

So

$$\vec{\nabla} \cdot \vec{B} = -\frac{2}{c^2} \vec{\Omega} \cdot \vec{E} - \frac{1}{c^2} \frac{\partial}{\partial t} \left(\vec{v} \cdot \vec{B} \right). \tag{61}$$

Hence

$$\vec{\nabla} \left(\vec{\nabla} \cdot \vec{B} \right) = -\frac{2}{c^2} \vec{\nabla} \left(\vec{\Omega} \cdot \vec{E} \right) - \frac{1}{c^2} \frac{\partial}{\partial t} \vec{\nabla} \left(\vec{v} \cdot \vec{B} \right). \tag{62}$$

Consider the term $\vec{\nabla} \left(\vec{v} \cdot \vec{B} \right)$ in (62). In accordance with (13),

$$\vec{\nabla} \left(\vec{v} \cdot \vec{B} \right) = \left(\vec{v} \cdot \vec{\nabla} \right) \vec{B} - \vec{\Omega} \times \vec{B} + \vec{v} \times \left(\vec{\nabla} \times \vec{B} \right). \tag{63}$$

Then

$$\vec{\nabla} \left(\vec{\nabla} \cdot \vec{B} \right) = -\frac{2}{c^2} \vec{\nabla} \left(\vec{\Omega} \cdot \vec{E} \right) - \frac{1}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{B} - \vec{\Omega} \times \vec{B} + \vec{v} \times \left(\vec{\nabla} \times \vec{B} \right) \right]. \tag{64}$$

Substituting (64) into (57), we obtain the desired wave equation for vector $\vec{B}$:

$$\vec{\nabla}^2 \vec{B} - \frac{n^2}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} + \frac{2}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{B} - \vec{\Omega} \times \vec{B} \right] + \frac{2}{c^2} \vec{\nabla} \left(\vec{\Omega} \cdot \vec{E} \right) = 0. \tag{65}$$

4.3 Result of Section 4

According to (53) and (65), the second system of wave equations for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ in a rotating medium has the following form:

$$\begin{aligned} &\vec{\nabla}^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \frac{2}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{E} - \vec{\Omega} \times \vec{E} \right] - \frac{2}{n^2} \vec{\nabla} \left(\vec{\Omega} \cdot \vec{B} \right) = 0, \\ &\vec{\nabla}^2 \vec{B} - \frac{n^2}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} + \frac{2}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{B} - \vec{\Omega} \times \vec{B} \right] + \frac{2}{c^2} \vec{\nabla} \left(\vec{\Omega} \cdot \vec{E} \right) = 0. \end{aligned} \tag{66}$$

5. Two systems of wave equations for vectors $\vec{E}$ and $\vec{B}$ in a rotating medium: a comparative analysis

The first system of wave equations for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ [which was derived from the corresponding original system of Maxwell's Eqs. (1–4)] has the form

$$\vec{\nabla}^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \frac{2}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{E} - \vec{\Omega} \times \vec{E} \right] - \frac{2}{n^2} \vec{\nabla} \left(\vec{\Omega} \cdot \vec{B} \right) = 0, \tag{67}$$

$$\vec{\nabla}^2 \vec{B} - \frac{n^2}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} + \frac{2}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{B} - \vec{\Omega} \times \vec{B} \right] = 0. \tag{68}$$

The second system of wave equations [which was obtained from the corresponding original system of Maxwell's Eqs. (5–8)] is

$$\vec{\nabla}^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \frac{2}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{E} - \vec{\Omega} \times \vec{E} \right] - \frac{2}{n^2} \vec{\nabla} \left(\vec{\Omega} \cdot \vec{B} \right) = 0, \tag{69}$$

$$\vec{\nabla}^2 \vec{B} - \frac{n^2}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} + \frac{2}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{B} - \vec{\Omega} \times \vec{B} \right] + \frac{2}{c^2} \vec{\nabla} \left(\vec{\Omega} \cdot \vec{E} \right) = 0. \tag{70}$$

Systems (67–70) have been derived in preprint [14] on the base of the procedure developed (for the case of vacuum) in work [15]. It should be noted that Eqs. (67) and (69) for vector $\vec{E}$ in both systems, after dropping the terms $\vec{\Omega} \times \vec{E}$ and $\vec{\nabla} \left(\vec{\Omega} \cdot \vec{B} \right)$, take the approximate form

$$\vec{\nabla}^2 \vec{E} - \frac{n^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \frac{2}{c^2} \frac{\partial}{\partial t} \left[\left(\vec{v} \cdot \vec{\nabla} \right) \vec{E} \right] = 0, \tag{71}$$

known, for example, from work [8] (see relation (33) therein).

Expressions (67, 68) and (69, 70) represent two qualitatively different systems of wave equations for vectors $\vec{E}$ and $\vec{B}$ in a uniformly rotating dielectric medium. From a comparative analysis of these systems, it follows:

1. The structure of equations for vectors $\vec{E}$ and $\vec{B}$ in the first system (67, 68) is asymmetrical with respect to Ω. Therefore, the propagation of $\vec{E}$- and $\vec{B}$-components of electromagnetic waves in a rotating medium in general case $\vec{\Omega} = \Omega_x \hat{x} + \Omega_y \hat{y} + \Omega_z \hat{z}$ will be governed by qualitatively different laws.

2. The structure of the wave equations in the second system (69, 70) is symmetrical with respect to Ω. Hence, the propagation of the named field components in such case will be governed by similar laws.

Systems of wave Eqs. (67, 68) and (69, 70) [in conjunction with corresponding original sets of Maxwell's Eqs. (1–4) and (5–8)] may serve as a theoretical basis for a detailed study of the process of electromagnetic wave propagation in a rotating dielectric medium. But before the beginning of such study, the researcher must first solve the problem of choosing between them (because the final results will be different).

[In the author's opinion, from the position of the principle of symmetry, the system of wave Eqs. (69, 70) is preferable to system (67, 68), and the corresponding original system of Maxwell's Eqs. (5–8) is preferable to the original system (1–4).]

6. Conclusion

In this chapter, on the base of two basic systems of Maxwell's Eqs. (1–4) and (5–8) for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ in a uniformly rotating dielectric medium, the two corresponding systems of wave Eqs. (67, 68) and (69, 70) have been derived (to the first order in Ω).

From their comparative analysis, it can be seen that the structure of the wave equations for electromagnetic field vectors in system (67, 68) is asymmetrical with respect to Ω, while the structure of such equations for these vectors in system (69, 70) is symmetrical.

On this basis, it can be concluded that if the principle of symmetry is accepted as a criterion for selection, then system of wave Eqs. (69, 70) for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ may be preferred. The same conclusion may be also made about the original system of Maxwell's Eqs. (5–8) on the base of which set of wave Eqs. (69, 70) has been obtained.

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