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# Criteria for Adequacy Estimation of Mathematical Descriptions of Physical Processes

Yuri Menshikov

## Abstract

In this chapter, adequacy estimation criteria for mathematical descriptions in the form of ordinary differential equations were proposed. Adequate mathematical descriptions can increase the objectivity of the results of mathematical modeling for future use. These descriptions make it possibly reasonable to use the results of mathematical modeling to optimize and predict the behavior of physical processes. Interrelations between criteria are considered. The proposed criteria are easily transferred on mathematical descriptions in algebraic form.

**Keywords:** mathematical simulation, adequate descriptions, criteria of adequacy, applications

## 1. Introduction

Mathematical modeling (simulation) of physical processes is an important tool for the study of the environment.

Mathematical modeling is a means of studying the real objects, processes, or systems by replacing the real objects on the mathematical models, which are more comfortable to study with the aid of computers.

The mathematical model is an approximate representation of real-world objects, processes, and systems, expressed in mathematical terms. In this case, a significant feature of original are saved from researcher's point of view.

First of all, some definitions and some concepts are given for the convenience of exposition.

A *mathematical model* of a real object will be called as mathematical dependencies and connections between the elements of a mathematical model. These elements are chosen on the basis of the interests of the researcher himself and the ultimate goals of study of the object. Usually, dependencies and relationships have forms of differential equations, integral equations, algebraic connections, etc.

The functions of external influences and external loads that are present in the mathematical model of the object in the form of symbols will be called as models of *external loads*.

The initial conditions, boundary conditions, and other conditions for the mathematical model will be called as *additional conditions*.

The totality of the mathematical model of the object, models of external influences, and additional conditions will be called as a *mathematical description* of the object.

The study of the behavior of the mathematical model of an object under the influence of models of external loads and additional conditions will be called as *mathematical modeling or mathematical simulation*.

The practical significance of the results of mathematical modeling or simulation of physical processes depends on the degree of coincidence of the results of mathematical modeling of the selected mathematical description of the real process with experimental data [1]. Such property of a mathematical description of a physical process is usually called *adequacy*.

### 1.1 Preliminary definition

A mathematical description will be called as an *adequate mathematical description* (AMD) of the process under study if the results of mathematical modeling (simulation) using this description coincide with experimental data with the accuracy of experimental measurements.

The definition of adequacy will be clarified later for some types of mathematical models.

If the coincidence of the results of mathematical modeling with experiment is bad, then further use of these mathematical descriptions is problematic.

It is note that authors of works on mathematical modeling concern seldom questions of adequacy of the constructed mathematical description of process to real measurements [2–5]. Sometimes, such adequacy is proved by the real facts; sometimes, authors refer to results of other authors; and sometimes, there have not been any arguments.

The considered situation requires formation of some uniform approach to this problem, common methodological approach, general algorithms, and common criteria of estimation of adequacy degree.

- Currently, there are two main approaches to the problem of constructing an adequate mathematical description [1, 6–8]: for a mathematical model with a priori chosen structure and inaccurate parameters, a model of external influence is determined, which together with the mathematical model of the process provide the adequacy condition (coincidence with experiment);
- a model of external loads is given a priori and then parameters of mathematical model or of its structure are selected, in such a way that results of mathematical simulation match up with experiment.

Having a comparison of the results of mathematical modeling with experimental data in definition of adequate mathematical description ensures the objectivity of the results of the synthesis of a mathematical description. In the literature, this approach is called as an identification method: estimation of the parameters of an adequate mathematical description based on the results of measurements of the characteristics of the state of the physical process [9, 10].

Mathematical models of physical processes can be presented as systems of ordinary differential equations, systems of partial differential equations, algebraic relations, integral equations, etc.

It should be noted that in many works, the accuracy of the results of mathematical modeling is several times lower than the accuracy of experimental data.

In the given work, the mathematical models of physical processes described only by the system of the ordinary differential equations will be examined [2, 3]. Such idealization of real processes or dynamic systems is widely used in various areas for the description of control systems [11], as well as of mechanical systems with the

concentrated parameters [5, 12], economic processes [13], biological processes [14], ecological processes [15], etc. In some works with the help of such systems, human emotions are simulated [16].

Many problems investigated in the given work, have place for other types of mathematical models of physical processes, for example, for mathematical models in the form of the partial differential equations.

The chapter proposes several criteria for checking the adequacy of the constructed mathematical descriptions for cases when the mathematical model of the physical process is represented by a system of differential equations.

The author hopes that the offered criteria of adequacy will be useful in a construction of the adequate mathematical descriptions of real physical processes.

## 2. Criteria of adequacy mathematical description of quantitative type

Consider the specified criteria for mathematical descriptions in the form of a system of differential equations.

For simplicity, we select the physical processes with mathematical models in the form of linear system of ordinary differential equations [17–19]:

$$\dot{x}(t) = Cx(t) + Dz(t), \quad (1)$$

where  $C, D$ —matrices with constant coefficients, which are given approximately,  $x = (x_1, x_2, \dots, x_n)^T$ —vector-function variables, characterized the state of process ( $(\cdot)^T$ —a mark of transposition),  $z(t) = (z_1(t), z_2(t), \dots, z_m(t))^T$ —vector-function of external load; and  $x \in X, z \in Z, X, Z$  are normalized functional spaces.

We assume that state variables  $x_i(t), 1 \leq i \leq n$  of system (1) correspond to some real characteristics of process which is under investigation  $\tilde{x}_i(t), 1 \leq i \leq n$ .

By *mathematical description* of the physical process, we mean the set of the system of Eq. (1), the vector of the external loads functions  $z(t) = (z_1(t), z_2(t), \dots, z_m(t))^T$  and the initial conditions  $x(t_0) = x^0$ . In other words, a *mathematical description* is a collection of mathematical models, models of external influences, and initial conditions.

The process of solving the system of differential Eq. (1) under the influence of selected models of external loads  $z(t) = (z_1(t), z_2(t), \dots, z_m(t))^T$ , taking into account, the initial conditions  $x(t_0) = x^0$ , is usually called *mathematical modeling* or *mathematical simulation*.

An *adequate mathematical description* of a physical process of such type with respect to all variables  $x_1(t), x_2(t), \dots, x_n(t)$  of quantitative type will be called as the mathematical description for which the results of mathematical simulation of variables  $x_1(t), x_2(t), \dots, x_n(t)$  coincide with the results of experimental measurements  $x_1^{ex}(t), x_2^{ex}(t), \dots, x_n^{ex}(t)$  of the characteristic  $x_1(t), x_2(t), \dots, x_n(t)$  with the accuracy of the experiments  $\delta_1, \delta_2, \dots, \delta_n$ :

$$\|x_i(t) - x_i^{ex}(t)\|_X \leq \delta_i, 1 \leq i \leq n. \quad (2)$$

In practice, the measurement of the characteristics of state variables is limited to only one or two components. We formulate a refined definition of the adequacy of a mathematical description for the case of a single variable.

An *adequate mathematical description* of a physical process of such type with respect to the variable  $x_k(t), 1 \leq k \leq n$  ( $ALMD_{x_k}$ ) of quantitative type will be called as the mathematical description for which the results of mathematical simulation of

a variable  $x_k(t)$  coincide with the results of experimental measurements  $x_k^{ex}(t)$  of the characteristic  $x_k(t)$  with the accuracy of the experiment  $\delta_k$ :

$$\|x_k(t) - x_k^{ex}(t)\|_X \leq \delta_k. \quad (3)$$

For the rest of the variables, coincidence with experiment is not determined. Adequate mathematical descriptions are similarly determined in the case of several measurements of state variables. The metrics of comparison in this case is determined by the objectives of specific studies.

In [12, 20], which were considered before, coincidence with experiment is 10 times below the accuracy of experiment.

The criteria of mathematical description adequacy of quantitative type, which are offered in the given chapter, can be used for other types of mathematical descriptions of physical processes, for example, for mathematical descriptions in the form of the partial differential equations [21]. They have many common features.

It can be shown that there are an infinite set adequate mathematical descriptions for the same physical experiment.

In addition, qualitatively, different physical processes can have adequate mathematical descriptions for the same experiment.

There exist two approaches to problem of construction of adequate mathematical description of quantitative type [22, 23]:

1. Mathematical model of process of type (1) is given a priori with inexact parameters and then the models of external loads were determined for which the results of simulation coincide with experiment [22, 23];
2. Some models of external loads are given a priori and then mathematical model of process of type (1) is chosen for which the results of simulation coincide with experiment [6–8].

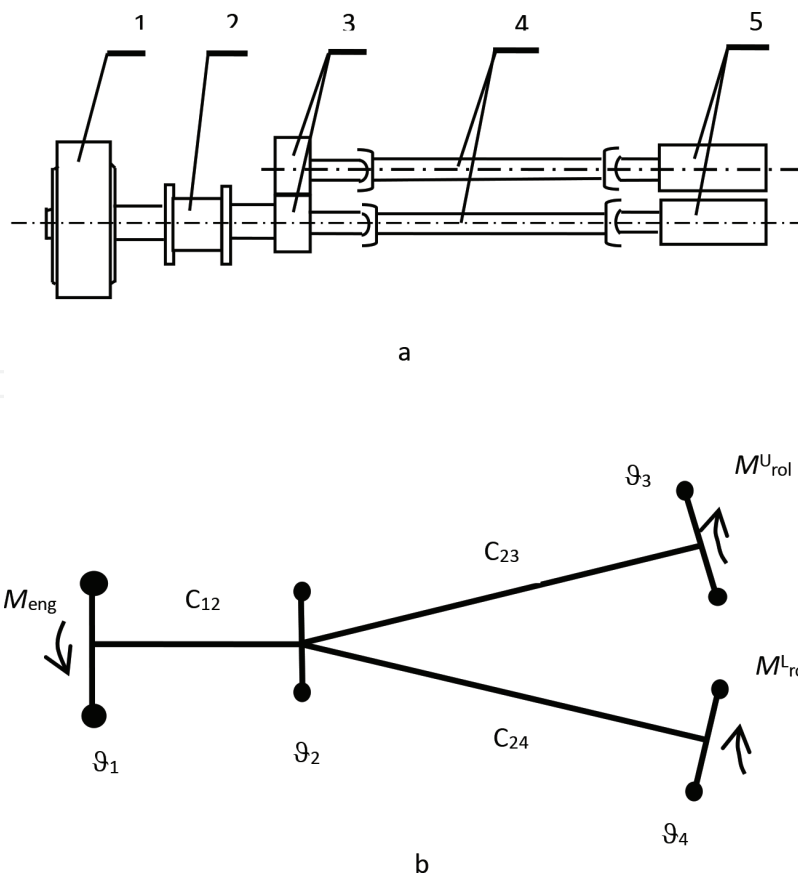
Now, we will consider the synthesis of adequate mathematical description of quantitative type in the frame of first approach analyzing the process with the concentrated parameters, for which the motion is described by ordinary differential equations of  $n$ -order (1).

We assume that some functions of state  $x_1(t), x_2(t), \dots, x_r(t), r \leq n$  in system (1) are obtained from experiment and presented by graphs. Besides, we suppose that some functions of external loads, for example,  $z_1(t), z_2(t), \dots, z_l(t), l \leq m$  are unknown. According to first approach, it is necessary to develop the construction of such model of external load component, which is characterized by the functions of state  $x_1(t), x_2(t), \dots, x_r(t)$  of mathematical model (1), and will coincide with experimental measurements  $\tilde{x}_1(t), \tilde{x}_2(t), \dots, \tilde{x}_r(t)$  with inaccuracy of initial data. Such mathematical model of process behavior together with obtained model of external load can be considered as *adequate mathematical description of quantitative type* of process.

Such method of obtaining of mathematical models of external loads (functions  $z_1(t), z_2(t), \dots, z_l(t), l \leq m$ ) is determined in literature as a method of identification [9, 10]. By the way, physical reasons of occurrence of such external loads are not being taken into account. They are only functions, which in combination with mathematical model (1) provide results of modeling, which coincide with experiment with the given accuracy.

Consider an example of a mathematical description that satisfies the criterion of the adequacy of a quantitative type for all variables  $x_1(t), x_2(t), \dots, x_n(t)$ .





**Figure 1.**  
Kinematic scheme of the main mechanical line of rolling mill.

## 2.1 Vibrations in the main mechanical line of the rolling mill

Now, we consider in detail, the problem in which the dynamics of the main mechanical lines of rolling mills is investigated [24, 25]. One variant of the kinematic scheme of it is presented in **Figure 1**. (a) where the engine is marked by label (1), the coupling is marked by label (2), gears is marked by label (3), driving shafts is marked by label (4), operational barrels is marked by label (5).

The four-mass model with weightless elastic connections is chosen as mathematical model of dynamic system of the main mechanical line of the rolling mill [24, 25]. The system of vibrations equations is obtained from the Lagrang's equations of second kind:

$$\begin{aligned} \ddot{M}_{12} + \omega_{12}^2 M_{12} - \frac{c_{12}}{\vartheta_2} M_{23} - \frac{c_{12}}{\vartheta_2} M_{24} &= \frac{c_{12}}{\vartheta_1} M_{eng}(t); \\ \ddot{M}_{23} + \omega_{23}^2 M_{23} - \frac{c_{23}}{\vartheta_2} M_{12} + \frac{c_{23}}{\vartheta_2} M_{24} &= \frac{c_{23}}{\vartheta_3} M^U_{rol}(t); \\ \ddot{M}_{24} + \omega_{24}^2 M_{24} - \frac{c_{24}}{\vartheta_2} M_{12} + \frac{c_{24}}{\vartheta_4} M_{23} &= \frac{c_{24}}{\vartheta_4} M^L_{rol}(t). \end{aligned} \quad (4)$$

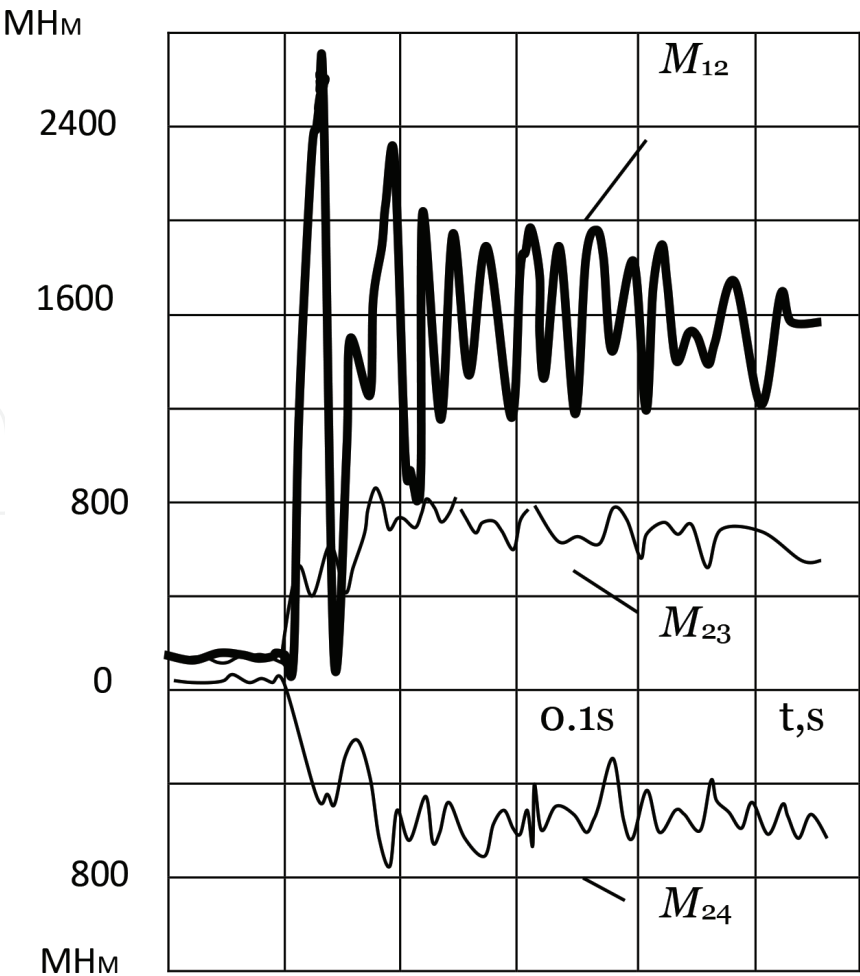
Here, the following designations were accepted:  $M_{eng}$ —moment of engine,  $\vartheta_i$ —moments of inertia of the concentrated weights,  $c_{ik}$ —rigidity of the appropriate elastic connection,  $M^U_{rol}$ ,  $M^L_{rol}$ —moments of technological resistance put to the upper and lower operational barrels accordingly, and  $M_{ik}$ —moments of elasticity forces, which are applied to shafts between mass  $\vartheta_i$  and  $\vartheta_k$ ;  $\omega_{ik}^2 = (\vartheta_i \vartheta_k)^{-1} c_{ik} (\vartheta_i + \vartheta_k)$ .

Actually, the constructed mathematical model may correspond to real process and may not. It is necessary to check up correctness of the constructed mathematical model. For this purpose, the data of experiment are used. If the results of mathematical modeling coincide with results of experiment (with accuracy of measurements), then mathematical description of process is considered as adequate to a reality in the quantitative sense. In other words, the mathematical description corresponds to real process.

The information related to the real motion of the main mechanical line of rolling mill was obtained by an experimental way [23, 24, 26]. Such information is being understood as availability of functions  $M_{12}(t)$ ,  $M_{23}(t)$ ,  $M_{24}(t)$ . The records of functions  $M_{12}(t)$ ,  $M_{23}(t)$ ,  $M_{24}(t)$  of a given process are shown in **Figure 2**.

It is obvious, that the results of mathematical modeling of system (4) depend directly on character of change of external loads, which is applied to operational barrels of the rolling mill and external impact of the engine  $M_{eng}$ ,  $M_{rol}^U$ ,  $M_{rol}^L$ . Sometimes, it is possible to pick up such loadings  $M_{eng}$ ,  $M_{rol}^U$ ,  $M_{rol}^L$  in which the results of mathematical modeling  $M_{12}(t)$ ,  $M_{23}(t)$ ,  $M_{24}(t)$  coincide with experiment (**Figure 2**).

If such choice is possible, then mathematical model (4) combined with the found loads  $M_{eng}$ ,  $M_{rol}^U$ ,  $M_{rol}^L$  will give adequate mathematical description of real process. It is necessary to note that in many papers analyzing the problem of mathematical modeling with the use of system of a differential Eq. (4) together with functions  $M_{eng}$ ,  $M_{rol}^U$ ,  $M_{rol}^L$  are determined as mathematical model of process. Coincidence is understood as coincidence with the accuracy of experimental measurements.



**Figure 2.**  
The records of functions  $M_{12}(t)$ ,  $M_{23}(t)$ ,  $M_{24}(t)$ .

According to this approach, it is necessary to construct such models of external loads  $M_{rol}^U, M_{rol}^L$  on system (4), for which the functions  $M_{12}(t), M_{23}(t), M_{24}(t)$  of elastic moments in the links of the model (solution of the system(4)), coincide with the corresponding experimental functions of the moments of elastic forces in the links of the main line of the rolling mill (**Figure 2**).

Consider the construction of an adequate mathematical description within the framework of the first approach. To construct, for example, a model  $M_{rol}^u$ , which corresponds to the moment of the external load to the upper work roll, consider the second equation in the system (4). The solution of this equation has the form

$$M_{23}(t) = M_{23}(0) \cos \omega_{23}t + \dot{M}_{23}(0) \omega_{23}^{-1} \sin \omega_{23}t + \frac{c_{23}}{\vartheta_3 \omega_{23}} \int_0^t M_{rol}^u(\tau) \sin \omega_{23}(t - \tau) d\tau \\ + c_{23} (\vartheta_2 \omega_{23})^{-1} \int_0^t [M_{12}(\tau) - M_{24}(\tau)] \sin \omega_{23}(t - \tau) d\tau$$

or

$$\int_0^t \sin \omega_{23}(t - \tau) M_{rol}^u(\tau) d\tau = F(t), \quad (5)$$

where

$$F(t) = \vartheta_3 \omega_{23} (c_{23})^{-1} \{ M_{23}(t) - [M_{23}(0) \cos \omega_{23}t + \dot{M}_{23}(0) \omega_{23}^{-1} \sin \omega_{23}t] \} \\ - \vartheta_3 (\vartheta_2)^{-1} \int_0^t [M_{12}(\tau) - M_{24}(\tau)] \sin \omega_{23}(t - \tau) d\tau.$$

We will assume that function  $F(t)$  in (5) belongs to the normalized space  $L_2[0, T]$  ( $[0, T]$  is the period of time at which the function  $M_{rol}^u$  is studied) and the solution  $M_{rol}^u$  of Eq. (5) belongs to the normalized functional space  $C[0, T]$ .

Let us rewrite the equation (5) in the more compact form

$$A_p z = u_\delta, \quad (6)$$

where  $z$  is the searched element,  $u_\delta$  is the given element, which belong, respectively, to the functional spaces  $C[0, T]$  and  $L_2[0, T]$ ,  $A_p$  is the integral operator. In this case,  $z = M_{rol}^u(t), u_\delta = F(t)$ .

Since the right-hand side  $F(t)$  of the integral Eq. (5) is determined from the experiment, it is natural to assume that instead of the exact right-hand side  $u_{ex} = F_{ex}(t)$  of the Eq. (6), some approximation of it is given  $u_\delta = F(t)$ :

$$\|u_\delta - u_T\|_{L_2[0, T]} = \|F(t) - F_T(t)\|_{L_2[0, T]} \leq \delta, \delta \text{ is given.}$$

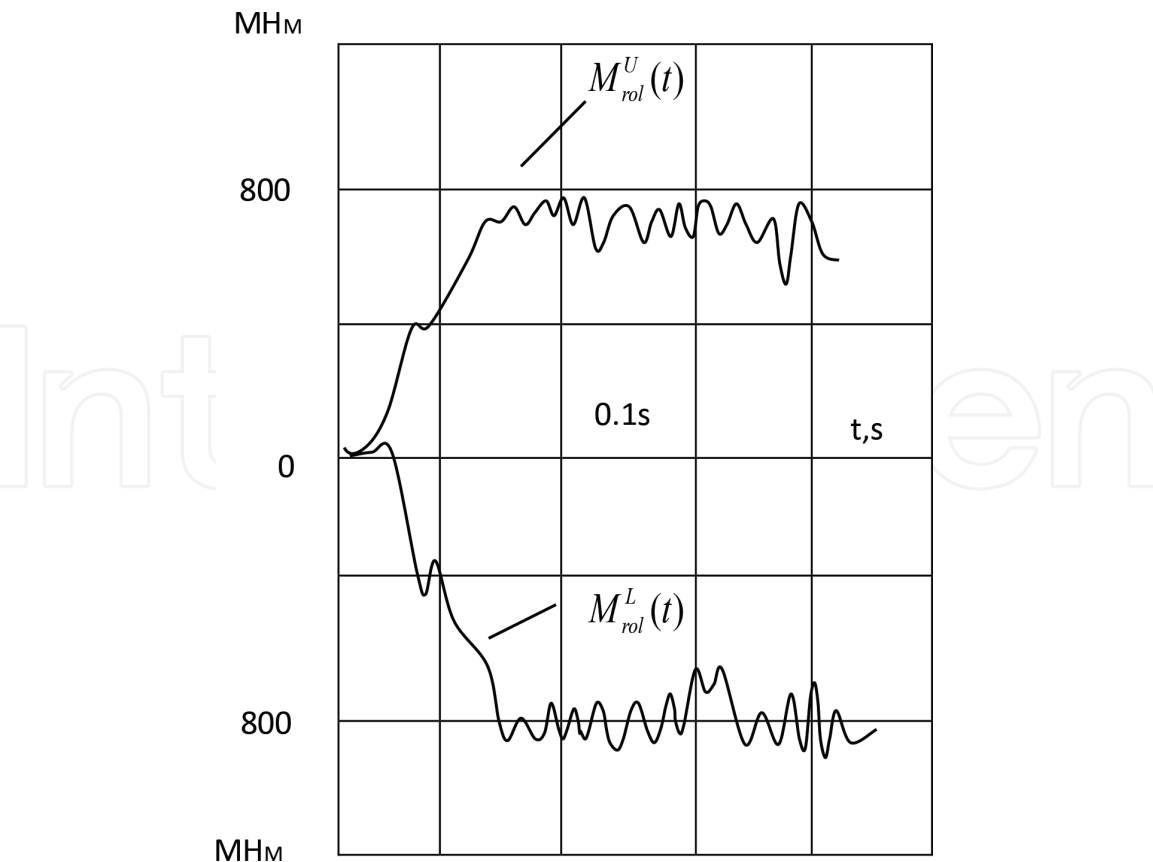
The set of possible solutions  $Q_\delta \subset C[0, T]$  of Eq. (6) consists of elements that correspond to the equation with given accuracy:

$$Q_\delta = \{z : \|A_p z - u_\delta\|_{L_2[0, T]} \leq \delta\}.$$

Each function in the set  $Q_\delta$  together with the given mathematical model (6) provides an adequate mathematical description of the physical process.

In this case, the problem of identifying model of external load in the rolling mill is considered as the inverse of the synthesis problem [17].





**Figure 3.**  
The results of identification (functions  $M^U_{rol}(t)$ ,  $M^L_{rol}(t)$ ) for case of rolling that were shown on **Figure 2**.

In this chapter, oscillograms of the moments of the forces of elasticity in the links of the main line of rolling mill 1150, obtained in [24, 25], are used. A copy of this oscillogram is shown in **Figure 2**. The value of  $T$  is chosen equal to 0.48 s.

When synthesizing the model of external load on the lower work roll of the state, it is necessary to use the last differential equation in the system (4).

In **Figure 3**, the graphs of the models of external loads on the upper and lower working rolls for rolling case shown on **Figure 2**.

Thus, models of external loads  $M^U_{rol}(t)$ ,  $M^L_{rol}(t)$  were obtained that together with the mathematical model (4) and the initial conditions yield simulation results that coincide with the experimental measurements with the accuracy of the experiment. In other words, the mathematical model (4), models of external loads and initial conditions give an *adequate mathematical description of quantitative type* for all variables of physical process.

Now, we consider another example of astrodynamical processes mathematical description, which has the property of adequacy of quantitative type in only one variable.

Based on theoretical analysis of mathematical vortex model of planetary systems, the analytical expression for planetary distances in the prevailing planetary systems was obtained. These distances are functions of the coordinates of the centers of vortical rings of primary planetary vortex. Comparison of theoretical and real distances planets of the Solar system show their good agreement.

Known in the cosmogonic theories of the solar system, the law of Titsius-Bode (1772) of planetary distances  $r_n$

$$r_n = 0,4 + 0,3 \cdot 2^n \text{ a.o.} \tag{7}$$

is a successful empirical approximation of the real sequence of distances  $r_n$  of planets with number  $n$  from to Sun. In this case, the first planet (Mercury)

corresponds to the value  $n \rightarrow -\infty$ , Venus— $n = 0$ , the Earth— $n = 1$ , etc., and the conditional unformed planet between Mars and Jupiter must be attributed to the value of  $n = 3$ . Despite the excellent conformance of this law to the average number of planets, the law (7) for the first and distant planets of Neptune and Pluto is not fulfilled [27].

In the twentieth century, some attempts were made [28, 29] to theoretically obtain the law of planetary distances, but in the basis of these theories, the authors had to impose new arbitrary hypotheses. Schmidt [28] introduces a hypothetical function of the distribution of kinetic moments in the masses of the primary nebula, and for the simplest functions it receives a quadratic law, a geometric progression, and others. Kuiper [29] deduces his law on the basis of the theory of tidal stability using the concept of “critical density of Rosh.” However, the law it received give the distance between planets, which differ on several orders from the real distance.

Below, based on the mathematical vortical model of the formation of planetary systems [30, 31], the analytical law of planetary distances for any planetary systems was obtained, which gives a good agreement with real distances in the solar system, which has another form compared with (7).

## 2.2 The theory of planetary vortex

The general picture and the basic relations in the primary vortex explosion, which creates stars and their planetary systems, is constructed in [30] on the basis of a separate exact solution of the Euler hydrodynamic equations for spherical eddy currents [30]. The main physical feature of this axisymmetric spatial flow, called the planetary vortex [30], is the presence of a vortex dipole in the center of the vortex dipole, which flows through a moving, twisted stream of outer space, and the interaction of these motions generates vortical flow of a planetary vortex [30].

Using the method of integrating the complete nonlinear system of Euler's hydrodynamic equations was introduced and flow functions  $\Psi(y, \theta)$  constructed. The function of flow in spherical coordinates  $(r, \theta, \varphi)$  [30] is constructed.

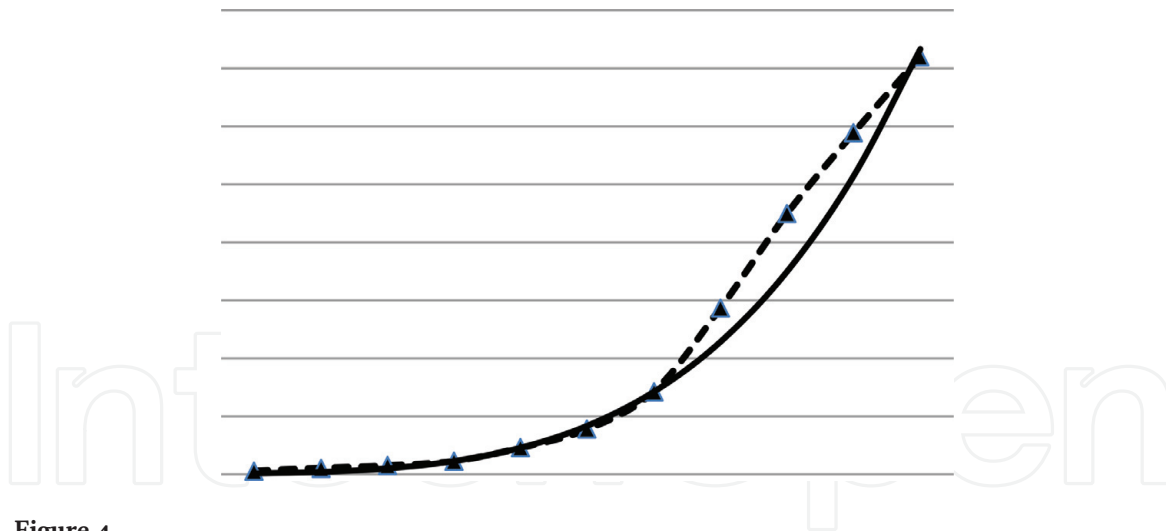
The planetary vortex described above as a complicated vortex flow is the initial stage of the formation of a star planetary system from the primary nebula that has fallen into the vortex region. Further prolonged evolution of this vortex to the state of the planetary system is characterized by a variety of complex physical processes such as: collision, accretion, accumulation of massive bodies, and their gravitation; the formation of a massive star and its light and gravitational action; mutual gravitational and resonant influence of system structures, etc. [27].

Since the forces of gravitation, collision, and others acting between parts of a single vortex are internal, they do not change its integral physical invariants.

In [30, 31], the modern planetary distances were calculated and they are shown in the graphs (**Figure 4**). As we can see, the theoretical curve in the entire range of distances is almost equidistant from the curve of real distributions of distances in the solar system with deviations in both directions of the order of 20%.

Finally, the technique developed here for the calculation of the primary vortex parameters can be applied in the reverse direction to determine according to the data of several open planets of the main parameter of the planetary system and the establishment of the method of work [38] of the complete structure of new exoplanetary systems: the number of vortex planets, their distances from the star, angular velocities, etc. This will give astronomers-observers reasoned data for the search for new, yet open exoplanets in stellar planetary systems, which have already opened 2–6 planets [32, 33].

Thus, the mathematical model of the process of formation of planetary systems, which describes interplanetary distances well, is constructed.



**Figure 4.**  
*Planetary distances in the solar system as functions of planetary numbers.*

The development of this mathematical model does not take into account important physical factors that have a significant effect on the behavior of interplanetary matter. To such factors, it is necessary to attribute, first of all, gravitational interaction and heat flows. Because of this, one should not expect a good coincidence of the real characteristics of the physical process with the results of mathematical modeling. This situation occurs when the criterion of adequacy of the qualitative type is not met (see Section 2). Consequently, the coincidence of the results of mathematical modeling (with the adequate mathematical description constructed only one or two of the system's variables) with all the main characteristics of the physical process is an exception, as a rule, and it can take place only if the adequacy of a quality type is performed.

### 3. Criteria of adequacy of mathematical description of qualitative type

We will consider what prospects of adequate descriptions are valid for further use and what goals should be selected as the creation of adequate mathematical descriptions.

It will be useful to address to classical works in this area. In work [34], the following statement was done: "...the imitation modeling is the creation of experimental and applied methodology which aimed at the use of it for a prediction of the future behavior of system."

So, the adequate mathematical descriptions are intended for the forecast of behavior of real process at first. It is possible with the aid of adequate mathematical modeling to predict behavior of real process in new conditions of operation. For example, it is possible to test more intensive mode of operations of the real machine without risk of its destruction. Such tool (adequate mathematical description) allows determining the optimum parameters of real process.

Let us now consider the conditions under which it is possible to further use adequate mathematical descriptions for "...a prediction of the future behavior of system."

Obviously, the structure of system (1), its parameters, and the specific type of external influences are determined by the properties of a real physical process.

Let the selected structure of the mathematical model of the physical process include parameters  $p = (p_1, p_2, \dots, p_k)^T$  (e.g., the mass of the elements, the stiffness of the elastic elements, etc.), which are reflecting the actual physical characteristics.

The structure of the mathematical model also includes dependencies that reflected real physical patterns and dependencies of the process under study.

For the purpose of further substantiated use of mathematical descriptions, it is necessary to require that there is a one-to-one correspondence between the components of the vector parameter  $p$  of mathematical description and the actual physical elements. In addition, it is necessary to require that the interconnections between the parameters of a mathematical model comply with the physical laws of the process being studied, and the main external loads had been included. This important correspondence will be called the *main correspondence* (MC). The execution of the MC believed the fulfillment of the criterion of *adequacy of the qualitative type*. In other words, a mathematical description of a physical process satisfies the criterion of adequacy of a qualitative type if the main correspondence is fulfilled.

An additional requirement for the implementation of the MC is explained firstly by the fact that the quantitative agreement of the results of mathematical modeling with a specific experiment is possible for mathematical descriptions of qualitatively different physical processes due to the selection of parameters of mathematical descriptions.

The implementation of the MC leads to the fact that the models of external influences obtained by the method of identification will correspond to the real external influences on the physical process. At least, these models will not contradict the physical meaning. If we return to the example of the synthesis of an adequate mathematical description of the process of mechanical oscillations in the main line of the rolling mill, then it can be argued that the MC is being executed. By virtue of this, the obtained models of external influences have a reasonable physical interpretation (do not contradict the physical meaning). The external load smoothly increases from zero to a steady-state value (see **Figure 3**).

When fulfilling the adequacy of a mathematical description of a qualitative type, it becomes possible to argue that a mathematical description that satisfies two criteria of adequacy will retain its useful properties for other experiments in the future under small changes in the conditions of the physical process. In other words, this description can be used for “a prediction of the future behavior of system.” An example of such a successful application could be further mathematical modeling using an adequate mathematical description for the rolling mill [23].

In the second example, given in Section 1, the main correspondence is not fulfilled, and therefore, the application of the obtained results in the new conditions will not be justified.

The algorithm for constructing an adequate mathematical description of a qualitative type cannot be formalized, as in the case of the adequacy of a mathematical description of a quantitative type. The process of constructing such a description mainly depends on subjective factors, such as the scientific tasks of studying the physical process using mathematical modeling methods.

In some cases, it is impossible to perform a check of mathematical description adequacy of a quantitative type due to lack of experimental data in principle. Let us give an example of a mathematical description that satisfies only the criterion of adequacy of a qualitative type.

An important and relatively new field of applications of methods of mathematical modeling is a tectonic processes study [35]. This work presents a complete algorithm required for the successful application of mathematical modeling methods of operations, which does not include the verification operation. Consider this algorithm in more detail.

It is widely known that earthquakes predicting is challenging and unsolved still (but it can be happen in future). Even where earthquakes have unambiguously



occurred within the parameters of a prediction, statistical analysis has generally shown these to be not better than lucky guesses. Now, there are hundreds of well-known earthquakes precursors and a number of theories to explain their origin. However, the problem of earthquake prediction in many of its aspects still remains open.

Utsu studied theoretically the relation between the size of aftershock activity and the magnitude of the main shock [36]. Independence of the occurrence of main shocks has been assumed in many models, some chapters discuss the migration of large earthquakes and casual relationship between seismic activities in different geophysical regions. Trigger models assume a series of primary events (main shocks) distributed completely random in time. Each of these primary events may generate secondary series of events. Epidemic-type model can be considered as birth and death process.

Utsu proposes a new model that takes into account the influence of strain solitary waves as a “trigger” of some shocks and appropriate methods of forecasting. Authors analyze the 2011 Japan earthquake. These studies show that solitary waves can be generated as aftershocks hypocenters at the Moho surface.

Significant amount of works have been devoted to research of solitons in solids. For example, considering structural-phenomenological approach one distinguishes damaged environment with microstructure, Kosser’s continuum with limited traffic, Leru’s continuum, environment with deformities [37], grainy environment, which has the soliton solutions of motion equations. It is known that the pulse perturbations in rocks are different from seismic waves of harmonic type.

Some researchers [36–38] have considered generalizations of the traveling wave solutions. The necessary and sufficient conditions for the existence of appropriate solutions were obtained.

Let us consider an anisotropic elastic medium. By [39], it follows that relations between stress and strain in Hooke’s law contain 21 free coefficients. The system of motion equations in this case has the form:

$$\rho \frac{\partial^2 u}{\partial t^2} = ((c_{11}, c_{66}, c_{55}, 2c_{16}, 2c_{15}, 2c_{56}), \Theta u) + ((c_{16}, c_{26}, c_{45}, c_{12} + c_{66}, c_{14} + c_6, c_{46} + c_{25}), \Theta v) + ((c_{15}, c_{46}, c_{35}, c_{14} + c_{56}, c_{13} + c_{55}, c_{36} + c_{45}), \Theta w), \quad (8)$$

$$\rho \frac{\partial^2 v}{\partial t^2} = ((c_{16}, c_{26}, c_{45}, c_{66} + c_{12}, c_{56} + c_{14}, c_{25} + c_{46}), \Theta u) + ((c_{66}, c_{22}, c_{44}, 2c_{26}, 2c_{46}, 2c_{24}), \Theta v) + ((c_{56}, c_{24}, c_{34}, c_{46} + c_{25}, c_{36} + c_{45}, c_{23} + c_{44}), \Theta w), \quad (9)$$

$$\rho \frac{\partial^2 w}{\partial t^2} = ((c_{15}, c_{46}, c_{35}, c_{14} + c_{56}, c_{13} + c_{55}, c_{45} + c_{36}), \Theta u) + ((c_{56}, c_{24}, c_{34}, c_{46} + c_{25}, c_{45} + c_{36}, c_{23} + c_{44}), \Theta v) + ((c_{55}, c_{44}, c_{33}, 2c_{45}, 2c_{35}, 2c_{34}), \Theta w), \quad (10)$$

where  $u$ ,  $v$ , and  $w$  are the displacements along the respective axes of a Cartesian coordinate system,  $\rho(x_1, x_2, x_3)$  is the density,

$\Theta = \left( \frac{\partial^2}{\partial x_1^2}, \frac{\partial^2}{\partial x_2^2}, \frac{\partial^2}{\partial x_3^2}, \frac{\partial^2}{\partial x_2 \partial x_1}, \frac{\partial^2}{\partial x_3 \partial x_1}, \frac{\partial^2}{\partial x_2 \partial x_3} \right)$ ,  $C = \|c_{ij}\|_{i,j=1,6}$  is matrix of elastic constants.



There, the following concept was introduced. Authors found the solutions of (8)–(10) as follows:

$$(u, v, w)^T = (\psi_u(t), \psi_v(t), \psi_w(t))^T W(x, y, z, t), \quad (11)$$

where  $W(\cdot) = \exp\left(-\frac{g(x-\tilde{x}(t))}{\varepsilon_1} - \frac{g(y-\tilde{y}(t))}{\varepsilon_2} - \frac{g(z-\tilde{z}(t))}{\varepsilon_3}\right)$ ,  $g(\cdot) \in G$  is the class of positive-definite, unimodal, twice continuously differentiated functions with a minimum at zero, and for which the second derivative is not a constant,  $\psi_u(t)$ ,  $\psi_v(t)$ ,  $\psi_w(t)$  are the functions which determined the amplitude of the relevant perturbations,  $\varepsilon_1, \varepsilon_2, \varepsilon_3$  are constants which determined the localization perturbations,  $\tilde{x}(t), \tilde{y}(t), \tilde{z}(t)$  are the functions which determined the trajectory of solitary waves.

In [39], authors got the necessary and sufficient conditions of existent solutions of the system (8)–(10) in the form (11). In [40], the various crystal systems for the existence of the appropriate type of motion equations solutions are studied.

The main hypothesis of method of earthquakes forecast is that a single shock causes the appearance of one or more solitary waves that move from the hypocenter of the earthquake. Each wave passing through the zone of accumulation of seismic energy, causing a new earthquake can in turn generate new solitons. The method of prediction involves the separation from the total population of earthquake subsequences caused by the same soliton and the construction of a hypothetical trajectory of the solitons. Knowing the distance between individual impulses along the trajectory of the soliton can estimate its speed. Knowing some point of its trajectory, it is possible to make an assessment of the trajectory. With the rate and trajectory of each soliton, one can estimate its position at any time. Having information about the position of the soliton at some time can determine the “soliton component” of shock probability at this time.

As initial data, we consider a sequence of the form:  $(x_1, t_1, \mu_1), (x_2, t_2, \mu_2), \dots, (x_k, t_k, \mu_k)$ , where  $x_1, x_2, \dots, x_k$  are earthquake hypocenters,  $\mu_1, \mu_2, \dots, \mu_k$  are magnitudes, and  $t_1, t_2, \dots, t_k$  are times of shocks.

Let a trajectory of the soliton be described parametrically:  $x(t) = (x_1(t), x_2(t))$ ,  $v(t)$  is speed. Then, we have:

$$v(t) = \frac{dx(t)}{dt} = \sqrt{(x_1'(t))^2 + (x_2'(t))^2},$$

$$\int_{t_i}^{t_{i+1}} v(t) dt = \int_{t_i}^{t_{i+1}} \sqrt{(x_1'(t))^2 + (x_2'(t))^2} dt = l(x(t_i), x(t_{i+1})),$$

$$v_i = \frac{l(x(t_i), x(t_{i+1}))}{t_{i+1} - t_i} = \int_{t_i}^{t_{i+1}} v(t) dt / (t_{i+1} - t_i),$$

where  $l(x(t_i), x(t_{i+1}))$  is the length of the curve corresponding to time interval  $(t_i, t_{i+1})$ .

Assume that the speed of the soliton is monotone—decreasing function. If the motion occur in the region of constant density, the ratio will be implemented:

$v_1 = v_2 = \dots = v_{k-1}$ . In the field of variable density:  $v_1 > v_2 > \dots > v_{k-1}$ .

Then, we consider the approximate speed  $\tilde{v}_1 = \frac{x_2 - x_1}{t_2 - t_1}$ ,  $\tilde{v}_2 = \frac{x_3 - x_2}{t_3 - t_2}$ , ...,  $\tilde{v}_{k-1} = \frac{x_k - x_{k-1}}{t_k - t_{k-1}}$ .

If a density is constant, then  $\tilde{v}_1 + \varepsilon_1 = \tilde{v}_2 + \varepsilon_2 = \dots = \tilde{v}_{k-1} + \varepsilon_{k-1}$ , where  $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_{k-1}$  is the curvature parameters. If a density is variable, then:

$\tilde{v}_1 > \tilde{v}_2 > \dots > \tilde{v}_{k-1}$ . This is criteria for the identification a subsequence of individual solitons trajectory.

In **Figure 5**, the results of seismic process analysis that occurred on the Japanese islands for 3 days before the earthquake of magnitude 8.9 (occurred on March 11, 2011) are shown. Here, the numbers from 0 to 12 mark epicenters of the foreshock, the epicenter of the main shock indicates maximum circle radius (it is near to the epicenter of the foreshock number 1). Curves and straight lines marked the soliton trajectory, which are calculated using a special software. The calculations take into account the hypothetical rate of the solitary waves and their possible reflection from the areas with a high density of rocks.

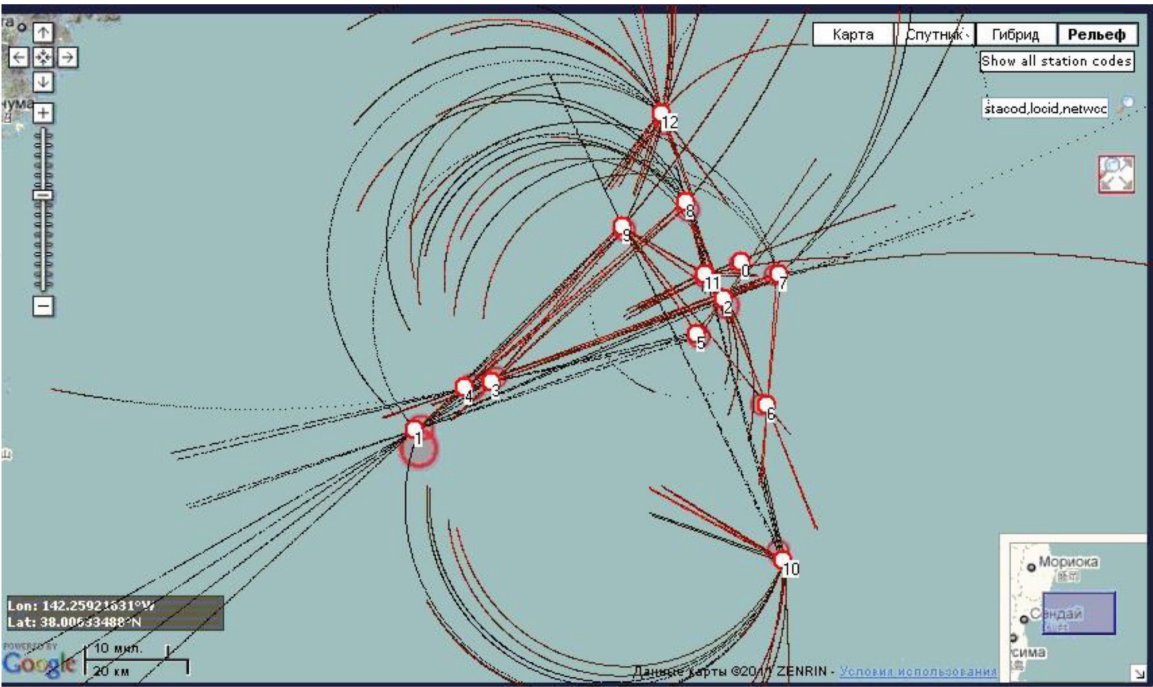
As you can see, the foreshock is arranged so that a large number of possible waves pass through the region, where there was a maximum magnitude shock. Clearly, traced kind of a soliton with a focusing effect is at the point where there was the main shock.

Thus, here it proposed the mathematical model of the process of earthquake sequences, taking into account the impact of slow solitary wave soliton type as a “trigger” to some shocks. The proposed theory allows us to construct forecasts when geophysics seismic process is similar to that which occurred on Japanese islands in 2011.

The considered example of a mathematical description cannot be checked for the adequacy of a quantitative type due to objective reasons. But this description meets the criterion of adequacy of the qualitative type and so the results of mathematical modeling do not contradict the physical meaning.

Further, consider an example of a mathematical description, the adequacy of which cannot be fundamentally assessed.

Methods of mathematical modeling penetrate recently into many nontraditional areas of human activity such as the study and modeling of emotions [16]. Let us consider in more detail the peculiarities of the application of methods of mathematical modeling in this field.



**Figure 5.**  
*Results of seismic process analysis that occurred on the Japanese islands on March 11, 2011 for 3 days before the earthquake of magnitude 8.9.*

The mathematician Rinaldi investigated as first Petrarch's emotional cycle and established an ODE model, starting point for the investigations in two directions: mapping the mathematical model to a suitable modeling concept, and trying to extend the model for love dynamics in modern times.

A control-oriented approach observes emotions and inspiration as states fading over time—behaving like a transfer function approaching a steady state. This observation suggests a modeling approach by transfer functions. Both model approaches allow an easy extension to modern times.

In literature, two special contributions can be found:

- Love affairs and differential equations by Strogatz [41], —harmonic oscillators making reference to Romeo and Juliet;
- Laura and Petrarch: an Intriguing Case of Cyclical Love Dynamics by Rinaldi [42]—presenting a nonlinear ODE with cyclic solutions.

Both contributions start directly with nonlinear oscillations, observing a certain historic emotional behavior of prominent couples. Laura group at Vienna University of Technology tries to consider general modeling concepts for emotional relations, which cover or coincide with Petrarch's emotional cycle, in case of appropriate parameterization.

Following a suggestion of Strogatz [41] here examines a sequence of dynamical models involving coupled ordinary differential equations describing the time-variation of the love or hate displayed by individuals in a romantic relationship. The models start with a linear system of two individuals and advance to love triangles, and finally to include the effect of nonlinearities, which are shown to produce chaos.

An obvious difficulty in any model of love is defining what is meant by love and quantifying it in some meaningful way.

### 3.1 Simple linear model

Strogatz [40] considers a love affair between Romeo and Juliet, where  $R(t)$  is Romeo's love (or hate if negative) for Juliet at time  $t$  and  $J(t)$  is Juliet's love for Romeo.

The simplest model is linear with

$$\begin{aligned}\frac{dR(t)}{dt} &= aR(t) + bJ(t), \\ \frac{dJ(t)}{dt} &= cR(t) + dJ(t),\end{aligned}\tag{12}$$

where  $a$  and  $b$  specify Romeo's "romantic style," and  $c$  and  $d$  specify Juliet's style. The parameter  $a$  describes the extent to which Romeo is encouraged by his own feelings, and  $b$  is the extent to which he is encouraged by Juliet's feelings. The resulting dynamics are two-dimensional, governed by the initial conditions and the four parameters, which may be positive or negative.

A similar linear model has been proposed by Rinaldi [42] in which a constant term is added to each of the derivatives in (12) to account for the appeal (or repulsion if negative) that each partner presents to the other in the absence of other feelings. Such a model is more realistic since it allows feelings to grow from a state of indifference and provides an equilibrium not characterized by complete apathy. However, it does so at the expense of introducing two additional parameters. While the existence of a non-apathetic equilibrium may be very important to the

individuals involved, it does not alter the dynamics other than to move the origin of the RJ state space.

Romeo can exhibit one of four romantic styles depending on the signs of  $a$  and  $b$ , with names adapted from those suggested by Strogatz [40] and his students:

1. Eager beaver:  $a > 0, b > 0$  (Romeo is encouraged by his own feelings as well as Juliet's).
2. Narcissistic nerd:  $a > 0, b < 0$  (Romeo wants more of what he feels but retreats from Juliet's feelings).
3. Cautious (or secure) lover:  $a < 0, b > 0$  (Romeo retreats from his own feelings but is encouraged by Juliet's).
4. Hermit:  $a < 0, b < 0$  (Romeo retreats from his own feelings as well as Juliet's).

Note that for the mathematical description (12), the criterion of the adequacy of the quantitative type and the criterion of the adequacy of the qualitative type cannot be checked due to the specificity of the process under study. Therefore, the further use of the results of mathematical modeling is unreasonable. However, there are no obstacles to nontraditional interpretations of the results of mathematical modeling of emotional processes.

## 4. Conclusion

The proposed adequacy criteria for mathematical descriptions in the form of ordinary differential equations make it possible to reasonably use the results of mathematical modeling to optimize and predict the behavior of physical processes.

The proposed criteria are easily transferred on mathematical descriptions in algebraic form [43].


Criteria for the adequacy of mathematical descriptions in the form of partial differential equations are currently missing in the literature. However, some criteria for the adequacy of mathematical descriptions can be transferred to the specified descriptions. For example, the criterion of adequacy of the qualitative type can be transferred almost unchanged.

## Author details

Yuri Menshikov  
University of Dnipro, Dnipro, Ukraine

\*Address all correspondence to: menshikov2003@list.ru

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