We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900
Open access books available

185,000
International authors and editors

200M
Downloads

Our authors are among the

154
Countries delivered to

TOP 1%
most cited scientists

12.2%
Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com



Chapter

On the Deviation of the Lunar Center of Mass to the East: Two Possible Mechanisms Based on Evolution of the Orbit and Rounding Off the Shape of the Moon

Boris P. Kondratyev

Abstract

It is known that the Moon's center of mass (COM) does not coincide with the geometric center of figure (COF) and the line "COF/COM" is not directed to the center of the Earth, but deviates from it to the South-East. Here, we discuss two mechanisms to explain the deviation of the lunar COM to the East from the mean direction to Earth. The first mechanism considers the secular evolution of the Moon's orbit, using the effect of the preferred orientation of the satellite with synchronous rotation to the second (empty) orbital focus. It is established that only the scenario with an increase in the orbital eccentricity e leads to the required displacement of the lunar COM to the East. It is important that high-precision calculations confirm an increase e in our era. In order to fully explain the shift of the lunar COM to the East, a second mechanism was developed that takes into account the influence of tidal changes in the shape of the Moon at its gradual removal from the Earth. The second mechanism predicts that the elongation of the lunar figure in the early era was significant. As a result, it was found that the Moon could have been formed in the annular zone at a distance of 3–4 radii of the modern Earth.

Keywords: Moon, displacement of center of mass, formation and evolution, gravitation

1. Introduction

At the dawn of modern astronomy, Hevelius and Galileo established that the optical libration of the Moon in longitude leads to a small (5^0–8^0) seeming (for terrestrial observer) oscillations of the figure of our satellite in the East-West direction with a period in the anomalistic month. These oscillations disappear when the Moon is at perigee and apogee. Oscillations of a different kind—optical oscillations in latitude—occur with amplitude $6^0 40'$ and a period of one draconic month with the disappearance of the deviation, when the Moon is at the nodes of the orbit.

IntechOpen

If the Moon was absolutely spherically symmetric, these optical librations would not have resulted in additional rotational oscillations of its body. But since due to the interaction with the Earth, the lunar body has tidal bulges, this leads to the appearance of moments of force from external celestial bodies. Newton [1] predicted that deviations of an elongated body of the Moon from the direction to the Earth must lead to real small rotational librations of the satellite relative to the inertial reference system. These small oscillations are called the physical libration of the Moon.

It is necessary to understand that when moving along the orbit of the Moon, its main axis is not directed at the center of mass of the Earth-Moon system and, on the average, at the second (empty) focus of the lunar orbit [2, 3]. The latter will play an important role in our theory.

Due to the proximity of the Moon in our time, the movement of our satellite is studied with such high accuracy that even a small asymmetry of its internal structure must be taken into account. This asymmetry is manifested in that the center of the Moon's mass COM is offset relative to the geometric center of the lunar figure *COF*.

This effect of shift is briefly mentioned in [4, 5]. Using astrometric data, an approximate numerical evaluation of the offset was given in [6] and in a more accurate version in [7]. A new approach based on the analysis of data obtained from the Lunar Laser Ranging experiment allowed in [8] clarifies the parameters of the shift of the Moon's center of mass.

Note that the definition of *COF* depends on the adopted model (sphere, ellipsoid, etc.), so that results of different researchers may be slightly different. However, according to many sources, it is reasonably safe to suggest that two points of the centers on the Moon really do not coincide.

To consider the internal asymmetry of the mass distribution in the lunar body, we introduce a coordinate system with the origin at the center of mass of the Moon, where the X-axis is directed (approximately) to the Earth, the Y-axis to the left (if viewed from the Earth), and the Z-axis—downward. Then, according to the United Lunar Control Network (ULCN), which takes into account the findings of many studies, including information from spacecraft [9], the displacement of the center of the figure relative to the center of mass "COM/COF" is equal to [10]

$$\Delta x \approx -1.71 \ km, \quad \Delta y \approx -0.73 \ km, \quad \Delta z \approx -0.26 \ km. \tag{1}$$

Based on the results of a study of the topography of the lunar surface using laser altimetry from a satellite, the displacement of the "COM/COF" was determined more accurately [11]:

$$\Delta x \approx -1.7752 \ km, \quad \Delta y \approx -0.7311 \ km, \quad \Delta z \approx -0.2399 \ km. \tag{2}$$

As follows from the analysis of observational data (1) or (2), the effect of displacement of the center of the figure relative to the Moon's center of mass includes not only the shift of the center of mass toward the Earth $0.001 \cdot R$ ($R = 1737.10$ km— the average radius of the Moon) but also the spatial deviation of the line "COM/COF" to the North-West. Note that in the literature it often also speaks of the displacement of the center of mass of the Moon relative to the center of its figure; for the observer from the Earth, this shift of the center of mass occurs down (to the South) and to the left (to the East). Then, all the signs in (1) and (2) are reversed. According to (2), the total displacement of the lunar COM is equal to $\Delta \approx 1.935 \ km$.

Besides, the shifts (1) and (2) of the center of the Moon's mass are global in nature, and, ultimately, they already include many different factors (see, e.g., [12]). Therefore, in particular, it is impossible to interpret the displacement of the center of mass only as a displacement of the lunar core alone.

Despite the seemingly geometric simplicity of the problem, the offset of the center of the Moon's mass remains an unexplored problem in the lunar science. The importance of this problem is that the Moon is close enough to the Earth and the accuracy of observations of its spin-orbital motion by the method LLR is now so much high that for correct interpretation of these movements it is necessary to take into account many celestial mechanical disturbances, including the indicated internal asymmetry of the Moon's body.

Here, we study the problem of the shift of the Moon's center of mass to the East. To do this, we consider two geometric mechanisms that allow us to explain this important feature of the internal structure of the Moon and shed light on some of the currently controversial features of its evolution and origin (see also [13–15]).

2. Optical libration of the Moon for the observer from the second focus

Instead of the term "the direction of the Moon's surface" often used in references, it is more accurate to speak of the direction of the main lunar axis of inertia, which only in two cases—at the position of the Moon at apogee and perigee—is directed to the center of mass of the Earth-Moon system. To do this, we first consider the optical libration of the Moon in longitude and place the observer in the point of the second (empty) focus of the orbit [2].

Recall that in the first approximation the Moon moves on ellipse (now the eccentricity of the orbit is $e = 0.0549$), and this motion is synchronous, since there is the resonance 1:1 of periods of axial rotation and revolution of the Moon around the Earth. According to the Kepler's first law, the motion is described by the formula

$$r = \frac{p}{1 + e\cos\upsilon}, \quad p = a_1(1 - e^2). \tag{3}$$

Here, a_1 is the main semiaxis, and e is the eccentricity of an ellipse. The angle of the true anomaly υ is associated with the angle of the eccentric anomaly E

$$\cos\upsilon = \frac{\cos E - e}{1 - e\cos E}. \tag{4}$$

The time that has elapsed since the Moon was at perigee ($E = 0, \upsilon = 0$), until the moment when the angles are equal (E, υ), is equal to

$$t = \frac{(E - e\sin E)}{2\pi}T, \tag{5}$$

where T is the period of revolution on the ellipse. Since the lunar axial angular velocity Ω must be equal the mean motion $n = \frac{2\pi}{T}$, the rotation angle δ of the major axis of inertia of the Moon (see **Figure 1**) in the time t will be

$$\delta = t \cdot \Omega = t \cdot n = \frac{Tn}{2\pi}(E - e\sin E) = E - e\sin E. \tag{6}$$

From the triangle f_1MC (**Figure 1**) follows that

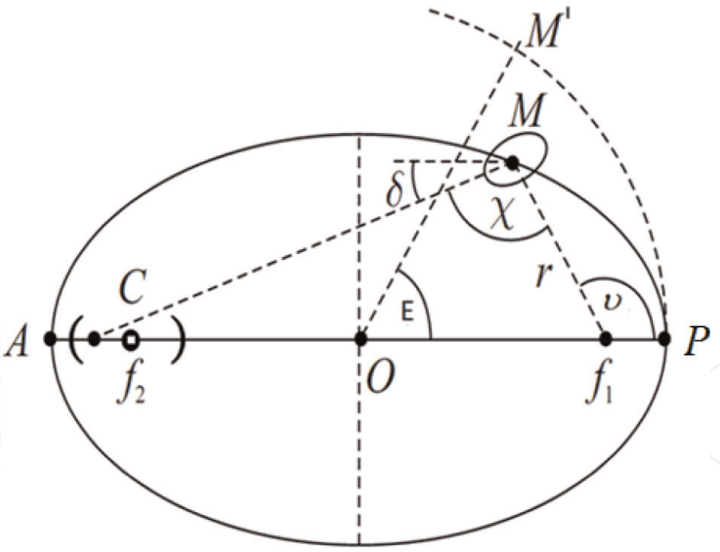


Figure 1.
The large ellipse is the orbit of the Moon M (for clarity, the ellipticity is exaggerated), and P and A are the points of perigee and apogee. The point of active focus f_1 is the center of mass of the Earth-Moon system, and f_2 is the point of the second (passive) focus.

$$\frac{d_{Cf_1}}{\sin\chi} = \frac{r}{\sin(\upsilon-\chi)}, \quad \delta+\chi=\upsilon, \tag{7}$$

so

$$d_{Cf_1} = r\cdot\frac{\sin\chi}{\sin(\upsilon-\chi)} = r\frac{\sin\chi}{\sin\delta}.$$

Then, the distance $\Delta = d_{Cf_2} = 2a_1e - d_{Cf_1}$ is

$$\frac{\Delta}{a_1} = 2e - \frac{1-e^2}{1+e\cos\upsilon}\frac{\sin\chi}{\sin\delta} = 2e - \frac{1-e^2}{1+e\cos\upsilon}\{\sin\upsilon\ \operatorname{ctg}\delta - \cos\upsilon\}. \tag{8}$$

Here, $\operatorname{ctg}\delta$ is the function of the angle E (or true anomaly υ)

$$\operatorname{ctg}\delta = \operatorname{ctg}(E - e\sin E) = \frac{1+\frac{\sqrt{1-e^2}\sin\upsilon}{e+\cos\upsilon}\cdot\operatorname{tg}\left[\frac{e\sqrt{1-e^2}\sin\upsilon}{1+e\cos\upsilon}\right]}{\frac{\sqrt{1-e^2}\sin\upsilon}{e+\cos\upsilon} - \operatorname{tg}\left[\frac{e\sqrt{1-e^2}\sin\upsilon}{1+e\cos\upsilon}\right]}. \tag{9}$$

Therefore, the required distance $\frac{\Delta}{a_1}$ from the point f_2, which is a continuation of the lunar major inertia axis that crosses the apsidal line, is not, generally speaking, zero and equal to

$$\frac{\Delta}{a_1} = e + \cos E - \operatorname{ctg}\delta\sqrt{1-e^2}\sin E \tag{10}$$

Expanding in powers of a small eccentricity gives

$$\begin{aligned}
\frac{\Delta}{a_1} &= -\frac{\cos\upsilon}{2}e^2 - \frac{1}{3}\left(1+\frac{\cos^2\upsilon}{2}\right)e^3 - \frac{\cos\upsilon}{8}(7-4\cos^2\upsilon)e^4 + \ldots; \quad \Delta\le 0;\\
\frac{\Delta}{a_1} &= -\frac{\cos E}{2}e^2 + \frac{1}{3}\left(\frac{1}{2}-2\cos^2 E\right)e^3 - \frac{\cos E}{24}(1+8\cos^2 E)e^4 + \ldots; \quad \Delta\ge 0.
\end{aligned} \tag{11}$$

The results of calculations using formula (10) are shown in **Figure 2**.

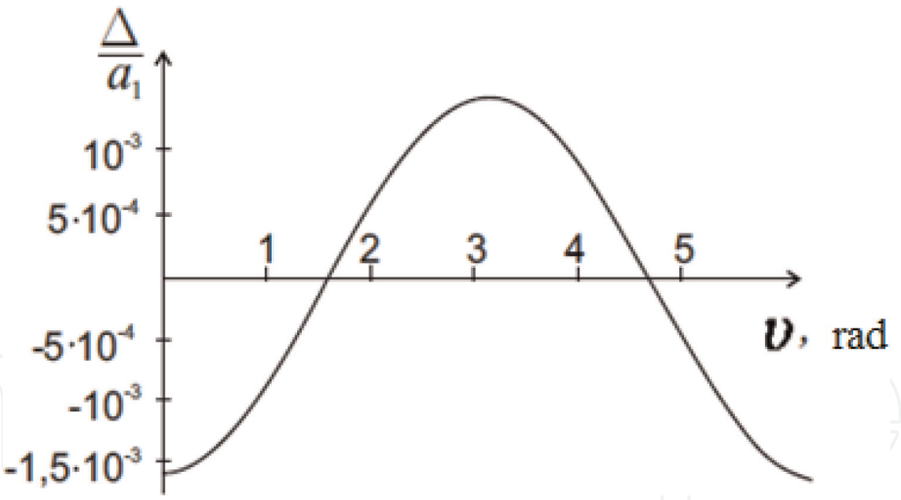


Figure 2.
Graph of deviation $\frac{\Delta}{a_1}$ as a function of the true anomaly υ.

It is important to emphasize that, according to formula (11), the effect of the deviation $\frac{\Delta}{a_1}$ is already in the first approximation proportional to the square of the eccentricity of the Moon's orbit.

Thus, when the Moon is moving on the ellipse around the Earth, the end of the major axis of inertia will be approximately directed to the point of the second focus. Strictly speaking, this end of the axis will perform (without taking into account the very small physical libration of the Moon in longitude) oscillatory motions in the vicinity f_2 in the interval

$$-1.5933 \cdot 10^{-3} \leq \frac{\Delta}{a_1} \leq 1.4275 \cdot 10^{-3}. \tag{12}$$

In our era, in a linear measure, this is approximately

$$-612 km \leq \Delta \leq 548 km. \tag{13}$$

The results of calculations (12) and (13) show a small asymmetry oscillations ($\sim$ 11%) relative to the right and left sides of the point f_2. Emphasize that the physical libration of the Moon in longitude has a very small amplitude and with a large reserve of fits in the interval (13).

3. Resolution alternatives to choose between two options for the lunar orbit evolution

Since Darwin [16], many efforts were made to examine the secular evolution of the Moon's orbit,

but so far it has not been established whether the orbit of the Moon in the past more or less oblate than now. In the literature, this issue is still under discussion. In this regard, the study of the shift of the Moon's center of mass to the East may shed some light on this important issue.

Many researchers agree that gravitational differentiation of the Moon occurred in the early era (see, e.g., [17]), with the result that the Moon's center of mass is slightly ($\sim 0.001 \cdot R$) shifted toward the Earth. We shall not discuss here the question of the gravitational differentiation of the Moon and just to note that one of the reasons for the displacement of the Moon's center of mass to the Earth can be some asymmetry of tidal forces from the Earth into two hemispheres of the Moon

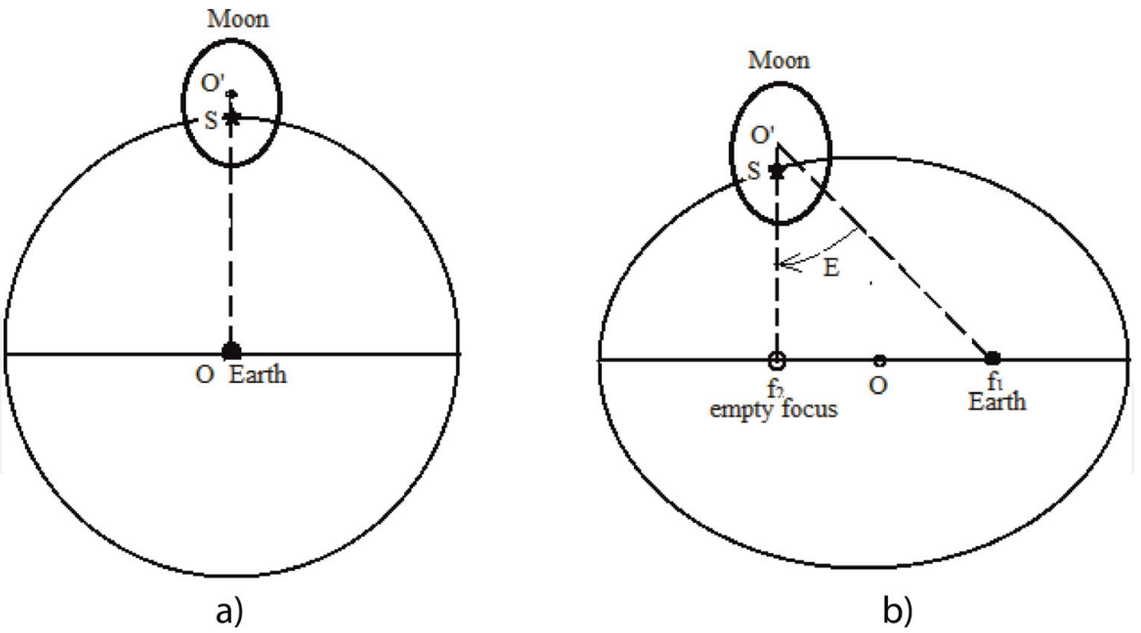


Figure 3.
(a) Orientation of the displaced center of mass S *of the young Moon after the differentiation of the substance of its body. A large circle is the orbit of the Moon in the early epoch, and a small ellipse is the cross section of the Moon. Since the orbit is circular, the focuses* f_1 *and* f_2 *coincide with the center* O*. Relative sizes are not respected. The line* $O'S$ *is directed straight to the Earth; therefore, the Earth's observer would see both points coinciding with each other from Section 2, the motion on the ellipse the line passing through the center of the Moon's figure and its center of mass be directed to the second (empty) focus of the orbit. Therefore, in our era, when the eccentricity of the lunar orbit has increased to its current value* $e = 0.0549$*, we will observe the picture as in b. (b) The orientation of the lunar center of mass* S *in our era in the first version of the evolution of the Moon's orbit. The large ellipse is the orbit of the Moon, and the small ellipse is the cross section of the Moon. The Earth is in the first focus* f_1 *of the lunar orbit. The angle* E *characterizes the orientation of the Moon COM* S *relative to the direction to Earth.*

(Sect. 4.1). One of the manifestations of the offset center of mass can be a different thickness of crust in the near side and the far side of the Moon [18].

Thus, the core of the Moon was formed during the gravitational differentiation, and then under the influence of a small asymmetry of tidal forces, the process of displacement of the lunar center of mass toward the Earth began to occur. This offset COM for the Earth observer can be characterized by the orientation angle E between the line $\ddot{}COF/COM\ddot{}$ and the direction to the center of the Earth (**Figure 3b**).

3.1 On the difference on tidal forces from the Earth in near and far lunar hemispheres

Assuming that the differentiation of the Moon occurred (according to cosmogonic times) rather quickly, it is necessary to require that the shift of the lunar center of mass toward the Earth occurred even before the Moon hardened.

The real cause of the displacement of the Moon's center of mass to the Earth could be some asymmetry of tidal forces. Let us perform the required calculations. After the capture of the Moon in resonance 1:1, it was possible to talk about near and far of its hemispheres. It is clear that the forces in the nearest and farthest points are, respectively, equal to

$$F_1 \approx \frac{2GM_\oplus}{R_0^2} x\left(1+\frac{3}{2}x\right),$$
$$F_2 \approx \frac{2GM_\oplus}{R_0^2} x\left(1-\frac{3}{2}x\right), \quad x = \frac{R}{R_0}, \tag{14}$$

where R_0 is the distance between the centers of the Earth-Moon and R is the distance from the center of the Moon to the near (far) points of its surface. The difference of these forces will be

$$\Delta F_{\oplus} = F_1 - F_2 \approx \frac{6GM_{\oplus}}{R_0^2} x^2. \tag{15}$$

In the era of its formation, the Moon could be much closer to Earth than in our era (see, e.g., [16, 18, 19]). Due to the proximity to the Earth of the young Moon, the difference in tidal forces (15) in both lunar hemispheres was much more in the early era than it is now. In the era of the differentiation of the Moon, it was this difference in tidal forces (15) that caused the displacement of the center of mass of the Moon toward the Earth. Based on these provisions, we note that the very solution to the question of the displacement of the Moon's *COM* to the East is closely related to the further secular evolution of its form and orbit. In particular, to find out how the lunar *COM* would be located relative to the Earth's observer in the modern era, when its orbit evolved and eccentricity acquired modern significance, consider two possible options with the initial eccentricity of the young Moon orbit.

3.2 The first version: the evolution of the lunar orbit with increase in its eccentricity

First, suppose that in the early epoch the orbit of the Moon was more circular than in our epoch. Consequently, during the secular evolution, the Moon's orbit became more and more eccentric, up to its modern value of eccentricity $e = 0.0549$.

Recall now that the Moon's COM, already shifted toward the Earth, after the solidification of the lunar body will be fixed relative to its main axes of inertia. Since in the early epoch the orbit of the Moon was almost circular, the line connecting the geometrical center of the figure of the Moon and its center of mass was directed exactly to the Earth (**Figure 3a**).

However, since in this version of the secular evolution the orbit of the Moon becomes more eccentric, two foci appear (**Figure 3b**). In accordance with the laws of celestial mechanics, as we know

From **Figure 3b**, it can be seen that, for the observer from the Earth (point f_1), the center of mass S will now be located on the left (to the East) from the direction to the center of the Moon (see also **Figure 5**). *Thus, in the first variant of the evolution of the Moon's orbit, the modern Earth's observer, in accordance with* **Figure 3b**, *will see the Moon's center of mass displaced to the left (to the East) from the direction to the center of the figure. It is this location of the center of mass of the Moon relative to the center of its figure that we observe in our era.*

The contribution of this mechanism to the displacement of the Moon's center of mass to the East will be made in Section 4.

3.3 The second version of the evolution: from more flattened to less flattened lunar orbit

If we assume that the orbit of the young Moon was more eccentric in the early era than it is now, that is, during the secular evolution, the Moon's orbit was rounded; then in our era, when the orbital eccentricity decreased to the current value $e = 0.0549$, instead of **Figure 3b**, we will see the location of the center of mass of the Moon, as shown in **Figure 4**.

Thus, **Figure 4** shows that in the second version of the evolution of the orbit a modern observer from the Earth would see that the center of mass of the Moon is

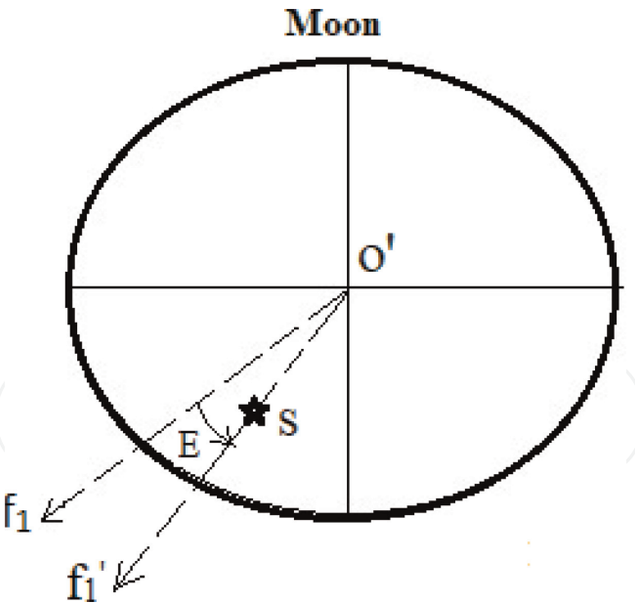


Figure 4.
The final configuration of the location of the lunar COMS in the second version of the evolution of its orbit. The line $O'f_1$ sets the direction (for the Earth observer) to the center of mass of the Moon in the early era, and the line $O'f_1'$ sets the direction to the center of the figure of the Moon for the observer from the Earth in our time. In this version, the observer would see that the center of mass of the Moon S is shifted to the right (to the West, as indicated by the arrow) from the average direction to the center O' of the figure of the Moon.

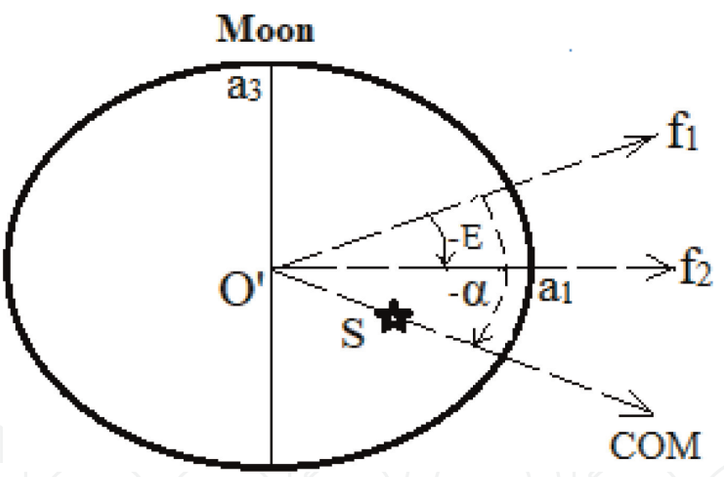


Figure 5.
Elongated figure of the Moon in the early era (its cross section of the plane $x_2 = 0$ is shown by the ellipse with the semiaxes $a_1 > a_3$). The arrows represent the directions from the center O' of the Moon to both foci f_1 and f_2 of its orbit around the Earth (the Earth in focus f_1), as well as to the center of the mass S of the Moon. The angle α between the line on S and the line to f_1 measures orientation of the Moon's center of mass, and the angle E between the directions to the first focus f_1 and second focus f_2 measures the deflection of an ellipse from a circle.

shifted to the right (to the West) from the direction to the center of the figure. *However, this is contrary to observations, so the second version of the evolution must be discarded.*

4. Correction factor to mechanism of orbit evolution

Let us consider again (**Figure 1**) the motion of a satellite in an elliptical orbit around a body of greater mass. The equation of an ellipse is given by formula (3). From the triangle $O'f_1f_2$ by the sine theorem, we find the relation

$$\sin E = \frac{2e \sin \upsilon (1 + e \cos \upsilon)}{1 + e^2 + 2e \cos \upsilon}. \tag{16}$$

Then, the average angle E is given by the integral

$$\langle E \rangle = \frac{1}{\pi} \int_0^{\pi} \arcsin \left[\frac{2e \sin \upsilon (1 + e \cos \upsilon)}{1 + e^2 + 2e \cos \upsilon} \right]. \tag{17}$$

In particular, for the Moon's orbit, the current value of eccentricity is equal $e \approx 0.0549$, and formula (17) gives

$$\langle E \rangle \approx 0.0700. \tag{18}$$

Taking into account (18), in the framework of the first variant of the evolution mechanism of the lunar orbit from the circle to the ellipse with the modern value of eccentricity, we find that the ratio of the average angle $\langle E \rangle$ to the angle $\operatorname{arctg} \frac{\Delta y}{\Delta x}$ will be

$$\kappa = \frac{\langle E \rangle}{\operatorname{arctg}(0.7311/1.7752)} \approx 0.18. \tag{19}$$

Therefore, the first orbital evolution mechanism helps to explain approximately 18% of the observed current Moon's offset *COM* to the East. In the linear measure, it is

$$|\Delta y| \approx 0.132 \ \ km. \tag{20}$$

We emphasize that the conclusion of the theory that evolution of the orbit of the Moon occurred with increasing eccentricity is consistent with the fact that at the present time the eccentricity of the orbit of the Moon is really growing and, therefore, in the past it was less than today [20, 21] (see also [22–25]).

Besides, the following should be noted. As is well known, due to perturbations, all elements of the lunar orbit are subject to periodic perturbations [20, 26]. Thus, for several thousand years, the eccentricity of the Moon's orbit changes due to solar perturbations in the range from 0.0255 to 0.0775. However, here we do not consider the periodic perturbations: throughout in this chapter, we are talking about tidal secular change in the average eccentricity of the Moon's orbit, which is now equal $e \approx 0.0549$.

5. Second mechanism of displacement of the Moon's center of mass to the East

Because of proximity of the Moon to Earth during an early era, which is offered by many researchers, the main factor of formation for the Moon is a tidal force from our planet. In the tidal field of the Earth, the figure of the early Moon stretched out, which was also facilitated by its capture in spin-orbit resonance 1:1. Therefore, for our approximate calculations, we can simulate the figure of the Moon using the elongated (toward the Earth) spheroid with the semiaxes $a_1 > a_2 = a_3$. The equation of the surface of this spheroid in Cartesian coordinates $Ox_1x_2x_3$ is

$$\frac{x_1^2}{a_1^2} + \frac{x_2^2 + x_3^2}{a_3^2} = 1. \tag{21}$$

The main symmetry semiaxis a_1 of this spheroid was initially directed exactly to the Earth.

Let us consider **Figure 5**. Due to the small orbit eccentricity, the angle E between the main axis of the Moon's figure and the direction to f_1 was also initially small. However, in the evolution of the Moon's orbit from the less eccentric to the more eccentric, as was shown in the first mechanism, the angle E will increase monotonically. This factor changes the orientation of the figure of the Moon relative to the observer on the Earth, and the angle α will also increase. From a geometrical point of view, during the evolution of the lunar orbit, the angle E can change only in the interval of values $0 \le E \le 2e \approx 0.11$. Moreover, taking into account the averaging performed above (see form. (18)), the right part of the interval will be adjusted

$$0 \le E \le 0.070. \tag{22}$$

In addition, although the angle α can vary from zero (in the early era of lunar evolution) up to the current value $\alpha_0 = \arctan\left(\frac{0.7311}{1.7752}\right) \approx 0.39$, but also taking into account the action of the first mechanism, the interval will be changed:

$$0 \le \alpha \le \alpha_0, \tag{23}$$

where

$$\alpha_0 = \arctan\left(\frac{0.7311 - 0.1243}{1.7752}\right) \approx 0.329. \tag{24}$$

We emphasize that because of inequalities (22) and (23), the center of mass of the Moon will have that arrangement which is shown in **Figures 3b** and **5**.

The problem consists in studying dependence between the angle α and the changing form of the Moon during the secular evolution in the gravitational tidal field of the Earth.

6. Differential equation for evolution of the angle α

As you know (see, e.g., [27]), a change in the shape of an ellipsoidal body can be described by a linear velocity field. In particular, the evolution of the prolate spheroid (21) in the moving frame of reference, whose axes coincide with the main axes of this body at any time, can be represented by the velocity field:

$$u_1 = \frac{\dot{a}_1}{a_1}x_1, \quad u_2 = \frac{\dot{a}_2}{a_2}x_2, \quad u_3 = \frac{\dot{a}_3}{a_3}x_3. \tag{25}$$

Here, the point above denotes the time derivative $\frac{d}{dt}$. Since for incompressible figures the condition of volume preservation should be fulfilled (in this case—for the volume of the prolate spheroid (21)), we have the additional ratio

$$\text{div } \mathbf{u} = \frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_3}{a_3} = 0. \tag{26}$$

In the velocity field (25), the Moon's shape will always remain a second-order surface, and the streamlines will be represented by pieces of hyperboles (**Figure 6**).

Owing to symmetry, the elongation of the spheroid (21) is described by the only polar oblateness $\varepsilon = 1 - \frac{a_3}{a_1}$. Consider changing ε for the Moon's shape. In this case

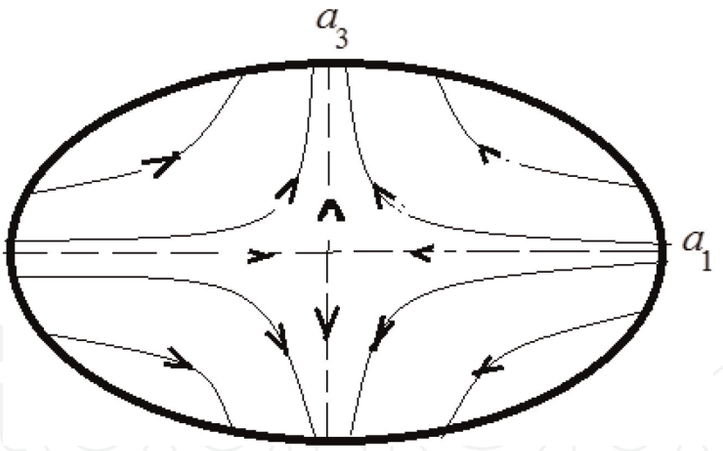


Figure 6.
Streamlines at deformation of the Moon's shape (the section is shown by ellipse). Arrows depict the direction of deformation at the stage of rounding the figure in the early era of evolution.

two components (first and third) of the velocity field in (25) taking into account a condition of incompressibility (26) will take the form

$$u_1 = \gamma x_1, \quad u_3 = -\frac{1}{2}\gamma x_3; \quad \gamma = \frac{1}{a_1}\frac{da_1}{dt}. \tag{27}$$

In the plane Ox_1x_3, the condition $x_2 = 0$ is satisfied, and expressions for angles E and α, (see **Figure 5**) will be equal:

$$E = -\text{arctg}\frac{x_3}{x_1}, \quad E - \alpha = -\text{arctg}\frac{x_3'}{x_1'}. \tag{28}$$

Here, (x_1, x_3) and (x_1', x_3') are the coordinates of the points of intersection of the Moon's surface by the rays $O'f_1$ and $O'S$, respectively. Therefore,

$$\alpha = \text{arctg}\frac{x_3'}{x_1'} - \text{arctg}\frac{x_3}{x_1}. \tag{29}$$

Differentiating expression (29) with respect to time t, we find

$$\dot{\alpha} = \frac{x_1'\dot{x}_3' - x_3'\dot{x}_1'}{x_1'^2 + x_3'^2} - \frac{x_1\dot{x}_3 - x_3\dot{x}_1}{x_1^2 + x_3^2}; \tag{30}$$

By substituting in (30) the components of the velocity field (27), we obtain

$$\dot{E} = -\frac{3}{4}\gamma \sin 2E; \quad \dot{E} - \dot{\alpha} = -\frac{3}{4}\gamma \sin 2(E - \alpha). \tag{31}$$

Thus, the derivative of the angle α will be equal to

$$\dot{\alpha} = -\frac{3}{4}\gamma[\sin 2E - \sin 2(E - \alpha)]. \tag{32}$$

More convenient than (32), below will be the next form of differential equation:

$$\frac{d\alpha}{dt} = -\frac{3}{2}\gamma \sin\alpha \cdot \cos(2E - \alpha). \tag{33}$$

7. Solution of Eq. (33)

Let us turn to the analysis of the differential equation (33) and transform the derivative $\frac{d\alpha}{dt}$:

$$\frac{d\alpha}{dt} = \frac{d\alpha}{d\varepsilon}\frac{d\varepsilon}{dt}. \tag{34}$$

As

$$\frac{d\varepsilon}{dt} = \frac{d}{dt}\left(1 - \frac{a_3}{a_1}\right) = \frac{a_3\dot{a}_1 - a_1\dot{a}_3}{a_1^2}, \tag{35}$$

therefore, in agreement with (34),

$$\frac{d\varepsilon}{dt} = \frac{3}{2}\gamma(1-\varepsilon). \tag{36}$$

Substituting (36) in (34) and then the result in (33), we have

$$\dot{\alpha} = \frac{3}{2}\gamma(1-\varepsilon)\frac{d\alpha}{d\varepsilon} = -\frac{3}{2}\gamma\sin\alpha\cos(2E-\alpha). \tag{37}$$

As a result, the differential equation for the angle α takes the form

$$\frac{d\alpha}{d\varepsilon} = -\frac{\sin\alpha\cos(2E-\alpha)}{1-\varepsilon}. \tag{38}$$

Separating the variables in (38) and integrating and taking into account the auxiliary formula

$$\int \frac{d\alpha}{\sin\alpha\cos(2E-\alpha)} = \frac{1}{\cos 2E}\ln\frac{\sin\alpha}{\cos(2E-\alpha)}, \tag{39}$$

we obtain a solution for equation (38) in the form

$$\frac{1}{\cos 2E}\ln\frac{\sin\alpha}{\cos(2E-\alpha)} = C + \ln(1-\varepsilon), \tag{40}$$

where C is the integration constant. Potentiating expression (40), we find the solution in the form

$$\varepsilon(\alpha, E) = 1 - C\cdot\exp\left\{\left[\frac{\sin\alpha}{\cos(2E-\alpha)}\right]^{\frac{1}{\cos 2E}}\right\}. \tag{41}$$

8. Analysis of the solution (41) and estimation of the elongation of the lunar figure in early era

In formula (41), the constant integration C is defined by the known observational data. As in the modern epoch of tidal evolution of the Moon the supplemented relations

$$\varepsilon \approx 0.0125, \quad E = 0.07, \quad \alpha = \arctan\frac{0.7311 - 0.1243}{1.7752} \approx 0.32937, \tag{42}$$

then the formula (41) gives

$$C \approx 0.713. \tag{43}$$

Thus, the solution of equation (41) will get in the form

$$\varepsilon(\alpha, E) = 1 - 0.713 \cdot \exp\left(\left[\frac{\sin\alpha}{\cos(2E - \alpha)}\right]^{\frac{1}{\cos 2E}}\right). \tag{44}$$

Formula (44) represents the solution of the problem: it describes the change in the Moon's oblateness ε during the tidal evolution and establishes the dependence between ε and the angle α. Recall that α is the angle between the directions (from the center of the Moon) to the first focus of the orbit and the Moon's *COM*. As we already know, in the course of evolution, the angle α varied (in radians) within the limits given in (23).

The graphic image of the function of two variables from (44) is shown in **Figure 7**.

Graphs for the two extreme values of the angle E are shown in **Figure 8**. As seen in **Figure 8**, the oblateness ε of the figure is very little depending on the angle E. Moreover, in the initial era, ε for all E has the same value and could not exceed the value

$$\varepsilon = 1 - \frac{a_3}{a_1} \approx 0.285. \tag{45}$$

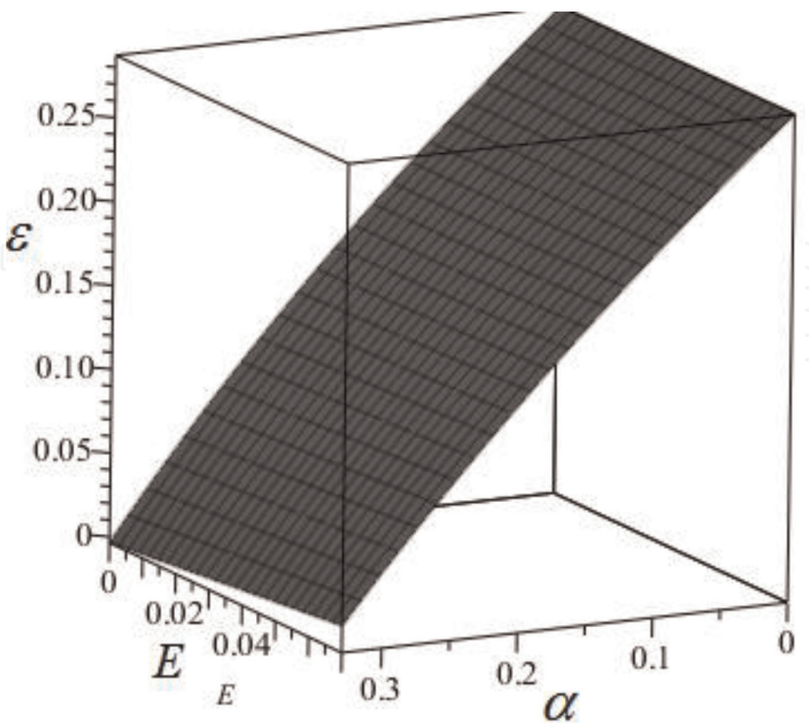


Figure 7.
3D image of the function $\varepsilon(\alpha, E)$. The angle α is set in radians. The oblateness of shape of the Moon $\varepsilon(\alpha, E)$ in the early era very little depends on the value of the parameter E, and its value does not exceed $\varepsilon \approx 0.285$.

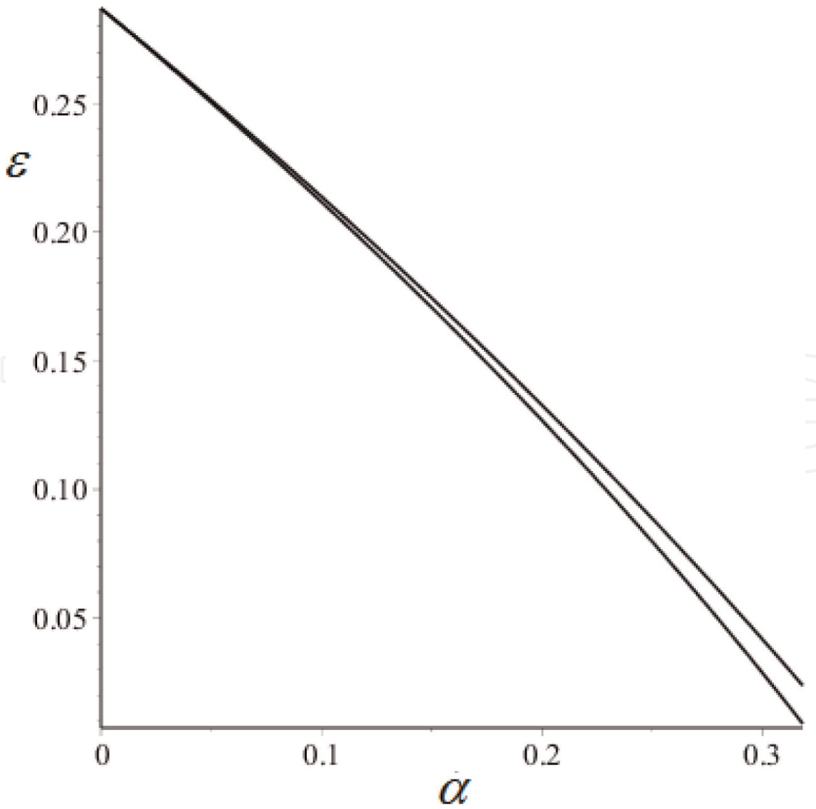


Figure 8.
The dependence of the oblateness ε of the Moon shape from the angle α between the line ¨COM/COF¨ of the Moon and the mean direction to the Earth. The graph shows the change ε during the tidal evolution. Two extreme angle values $E = 0.0$ (upper curve) and $E = 0.07$ are taken for comparison. The beginning and the end of the evolutionary process correspond to the values $\alpha \approx 0$ and $\alpha \approx 0.329$.

Thus, the second mechanism explains both the displacements of the center of mass of the Moon to the East and predicts that the oblateness of the Moon in the early era could not exceed the value $\varepsilon \approx 0.285$.

9. Some consequences: how close to the earth could the Moon be formed

Above we established that on the known shift of the Moon's center of mass to the East, we can find the oblateness (45), which the Moon could have in the epoch of its formation. The corresponding spheroid eccentricity will be equal to

$$e \approx 0.70. \tag{46}$$

Proceeding from (46) and using the theory of tidal equilibrium figures, it is possible to estimate how close to each other might be the Earth and the Moon in the early era. For this purpose, without loss of generality, we assume that the satellite is uniform (at the Moon, as we know, and now concentration of substance very small), and its mass in comparison with the mass of the Earth can be neglected. Then, in the tidal approach for the potential of the Earth, the equation of hydrostatic equilibrium of the satellite with synchronous rotation has the first integral [28]:

$$\frac{p}{\rho} + const = \varphi + \frac{1}{2}\Omega^2\left(3x_1^2 - x_3^2\right); \qquad \Omega^2 = \frac{GM_\oplus}{R_\oplus^3}. \tag{47}$$

Here, p is the pressure, ρ is the density, φ is the quadratic internal gravitational potential of the satellite, Ω is the angular velocity rotation of the satellite, and $R_\oplus$ is the distance between the centers of the Earth and the Moon. For satellite with the form of the prolate spheroid (21), we have [27]

$$\begin{aligned}\varphi &= \pi G\rho\left[I - A_1 x_1^2 - A_3\left(x_2^2 + x_3^2\right)\right];\\ A_1 &= \frac{1-e^2}{e^3}\ln\frac{1+e}{1-e} - 2\frac{1-e^2}{e^2};\\ A_3 &= \frac{1}{e^2} - \frac{1-e^2}{2e^3}\ln\frac{1+e}{1-e}.\end{aligned} \tag{48}$$

The internal pressure of the equilibrium figure should also be a quadratic function from the coordinates

$$p = p_0\left(1 - \frac{x_1^2}{a_1^2} - \frac{x_2^2 + x_3^2}{a_3^2}\right). \tag{49}$$

From the first integral (47) is possible to find a square of angular velocity rotation of satellite

$$\frac{\Omega^2}{\pi G\rho} = 2\frac{A_1 - (1-e^2)A_3}{4-e^2}. \tag{50}$$

Since

$$\frac{\Omega^2}{2\pi G\rho} = \frac{M_\oplus}{2\pi\rho R_\oplus^3} = \frac{\kappa}{x^3}, \tag{51}$$

where we have identified the following characters

$$x = \frac{R_\oplus}{R_\oplus}; \quad \kappa = \frac{2}{3}\frac{\rho_\oplus}{\rho} \approx 1.09875, \tag{52}$$

the ratio (51) can be represented as

$$\frac{\kappa}{x^3} = \frac{A_1 - (1-e^2)A_3}{4-e^2}. \tag{53}$$

Substituting the value e from (46) into the right-hand side (53), we obtain the cubic equation

$$\frac{\kappa}{x^3} = 0.0324, \tag{54}$$

from which we find the required distance

$$x \approx 3.24. \tag{55}$$

Thus, the Moon with oblateness (45) could form at a very close distance from the Earth: at a distance of only three and a quarter of the mean radii of the modern Earth. This result slightly corrects the one we received earlier [15].

Note that the prolate spheroid with meridional eccentricity (45) is a stable figure of equilibrium. In fact, the instability of this type of figure occurs only when $e \geq 0.883$ (see, e.g., [28]).

10. Discussion and conclusions

Here, it is necessary to add the following. As is well known, in the problem of secular perturbations, the perturbation function is replaced by its secular part. The influence of the Sun leads only to periodic perturbations of the eccentricity of the lunar orbit, which we do not take into account here. In this chapter, we ignore periodic oscillations and consider only tidal secular changes in the average eccentricity of the lunar orbit.

As for the tidal influence of the Sun on the figure of the Moon, it turns out to be insignificant compared to the influence of the Earth. Indeed, the ratio of force $\Delta F_\odot$ to force $\Delta F_\oplus$ from (15) is equal to

$$\frac{\Delta F_\odot}{\Delta F_\oplus} = \frac{M_\odot}{M_\oplus}\left(\frac{R_\oplus}{R_\odot}\right)^4 \approx 10^{-5}.$$

Therefore, to solve the posed problem within the framework of our model, the influence of the Sun can be neglected.

In the theory of the tidal evolution of the Moon's orbit and its form, we encounter problems that are difficult to give exact answers. Above, we examined some of the conclusions from those observational facts that the center of mass of the Moon is slightly shifted to the East. Two geometrical mechanisms have been developed to explain this shift.

The first mechanism considers the secular evolution of the Moon's orbit, using the effect of the preferred orientation of the satellite with synchronous rotation to the second orbital focus. According to this mechanism, *only the scenario of secular evolution of the orbit with the increase of eccentricity leads to the desired offset of the center of the Moon's mass to the East*. It is important to note that this conclusion that the evolution of the Moon's orbit occurred with an increase e is consistent with the fact that at present the eccentricity of the lunar orbit is indeed increasingly, and therefore in the past, it was less than today [20, 21] (see also [22–25]).

To fully explain the displacement of the center of the Moon's mass to the East, a second mechanism was developed, which takes into account the influence of tidal changes in the shape of the Moon as it gradually moves away from the Earth. The essence of the second mechanism is fully consistent with the fact that the distance between Earth and Moon is now really increasing and the Earth's spin is slowing in reaction.

In addition, the second mechanism predicts that the Moon's figure flattening in the early era was very significant and reached the value of $\varepsilon \approx 0.285$. In turn, based on the theory of tidal equilibrium figures, it allowed us to estimate how close to Earth could the Moon be formed as an astronomical body. According to formula (55), the Moon was formed in the ring zone at a distance of 3–4 medium radii of the present Earth. This result seems to be consistent with the modern view that the Moon was formed as a result of a gigantic impact in the immediate vicinity of the proto-Earth.

Since the formation of the Moon as a celestial body and so far the Earth-Moon system has been and remains a binary planet, the physical laws of its development have always been the same. In the early era, however, the tidal forces between the Earth and the Moon were much more important. Indeed, now the tidal force has very little effect on the Moon, because of which it is removed from the Earth for only 3.8 cm per year. However, studying the evolution of the moon still requires a great effort of researchers.

In summary, we can say that the method presented here really allows to take into account additional observational facts in the structure of the Moon. We have

shown that from the hidden fact that in our era there is a slightly shift of the center of the Moon's mass to the East, and not to the West, you can get valuable information about the evolution of the orbit of the Moon and its shape. This finding supports the scenario [29] that the Moon could be formed about 4.5 billion in the surrounding "donut" from the hot gas that appeared after the collision of Theia with proto-Earth.

IntechOpen

Author details

Boris P. Kondratyev[1,2,3]

1 Sternberg Astronomical Institute, M.V. Lomonosov Moscow State University, Russia

2 Faculty of Physics of the M.V. Lomonosov Moscow State University, Russia

3 Central Astronomical Observatory at Pulkovo, Russia

*Address all correspondence to: work@boris-kondratyev.ru

IntechOpen

© 2019 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/3.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. CC BY

References

[1] Newton I. Mathematical principles of natural philosophy. In: Bernard Cohen I, Whitman A, editors. A Guide to Newton's Principia. University of California Press; 1999

[2] Kondratyev BP. On one inaccuracy of Isaac Newton. Kvant. 2009;**5**:38

[3] Murray K, Dermott S. Dynamics of the Solar System. M.: FIZMATLIT, 2010; p. 588 (trans. from English).

[4] Yakovkin AA. The radius and shape of the moon. Bulletin AOE. 1934;**4**

[5] Bohme S. Bearbeitung der Aufnahmen von F. Hayn zur Ortsbestimmung des Mondes. Astronomische Nachrichten. 1953;**256**: 356

[6] Shakirov KS. The influence of the internal structure of the moon on its rotation. Izvestia AOE. 1963;**34**

[7] Lipsky YN, Nikonov VA. The position of the center of the figure of the moon. Astronomicheskii Zhurnal. 1971; **48**:445

[8] Calame O. Free librations of the Moon determined by an analysis of laser range measurements. Moon. 1976;**15**:343

[9] Archinal BA, Rosiek MR, Kirk RL, Redding BL. Completion of the Unified Lunar Control Network 2005 and Topographic Model. Virginia: US Geological Survey. 2006

[10] Iz H, Ding XL, Dai CL, Shum CK. Polyaxial figures of the Moon. Journal of Geodesy. 2011;**1**(4):348

[11] Barker MK, Mazarico E, Neumann GA, Zuber MT, Kharuyama J, Smith DE. A new lunar digital elevation model from the Lunar Orbiter Laser Altimeter and SELENE Terrain Camera. Icarus. 2016;**273**:346

[12] Lemoine FG, Goossens S, Sabaka TJ, Nicholas JB, Mazarico E, Rowlands DD, et al. GRGM900C: A degree-900 lunar gravity model from GRAIL primary and extended mission data. GeoRL. 2014;**41**: 3382. DOI: 10.1002/2014GL060027

[13] Kondratyev BP. The deviation of the lunar center of mass to the east of the direction toward the earth. A Mechanism Based on Orbital Evolution. Astronomy Reports. 2018;**62**(8):542. DOI: 10.1134/S106377291808005X

[14] Kondratyev BP. The deviation of the lunar center of mass to the east of the direction toward the earth. A Mechanism Based on Rounding of the Figure of the Moon. Astronomy Reports. 2018; **62**(10):705. DOI: 10.1134/S106377 2918100062

[15] Kondratyev BP. On the deviation of the lunar center of mass to the East. Two possible mechanisms based on evolution of the orbit and rounding off the shape of the Moon. Astrophysics and Space Science. 2018;**186**(186)

[16] Darwin GH. Tidal Friction in Cosmogony, Scientific Papers 2. Cambridge University Press; 1908

[17] Urey HC. Chemical evidence relative to the origin of the solar system. MNRAS. 1966;**131**:212

[18] Wieczorek MA, Neumann GA, Nimmo F, Kiefer WS, Taylor GJ, Melosh HJ, et al. The crust of the Moon as seen by GRAIL. Science. 2013;**339**(6120):671. DOI: 10.1126/science.1231530

[19] Zhong S. Origin and Evolution of the Moon. 2014. 2014IAUS, 298, 457Z. DOI: 10.1017/S1743921313007229

[20] Macdonald GJF. Tidal friction. Reviews of Geophysics. 1964;**2**:467. DOI: 10.1029/RG002i003p00467

[21] Goldreich P. History of the lunar orbit. Reviews of Geophysics and Space Physics. 1966;**4**:411

[22] Simon JL, Bretagnon P, Chapront J, Chapront-Touze M, Francou G, Laskar J. Numerical expressions for precession formulae and mean elements for the moon and planets. A&A. 1994; **282**:663

[23] Chapront J, Chapront-Touzé M, Francou G. A new determination of lunar orbital parameters, precession constant and tidal acceleration from LLR measurements. A&A. 2002;**387**:700

[24] Laskar J, Fienga A, Gastineau M, Manche H. La2010: A new orbital solution for the long-termmotion of the Earth. A&A. 2011;**532**(**A89**):15

[25] Folkner WM, Williams JG, Boggs DH, Park RS, Kuchynka P. The Planetary and Lunar Ephemerides DE430 and DE431. The Interplanetary Network Progress Report. 42-196. 2014. pp. 1-81

[26] Deprit A. The movement of the moon in space. In: Kopal Z, editor. Physics and Astronomy of the Moon. New York and London: Academic Press; 1971

[27] Kondratyev BP. Dinamika ellipsoidal'nykh gravitiruiushchikh figure. Moscow: Nauka; 1989

[28] Chandrasekhar S. Ellipsoidal Equilibrium Figures. New Haven and London: Yale University Press; 1969

[29] Simon SJ, Stewart ST, Petaev MI, Leinhardt ZM, Mace MT, Jacobsen SB. The Origin of the Moon Within a Terrestrial Synestia. JGR. 2018. DOI: 10.1002/2017JE005333