Chapter

The Black Hole Binary Gravitons and Related Problems

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Abstract

The energy spectrum of graviton emitted by the black hole binary is calculated in the first part of the chapter. Then, the total quantum loss of energy is calculated in the Schwinger theory of gravity. In the next part, we determine the electromagnetic shift of energy levels of H-atom electrons by calculating an electron coupling to the black hole thermal bath. The energy shift of electrons in H-atom is determined in the framework of nonrelativistic quantum mechanics. In the last section, we determine the velocity of sound in the black hole atmosphere, which is here considered as the black hole photon sea. Derivation is based on the thermodynamic theory of the black hole photon gas.

Keywords: graviton, Schwinger source theory, spectrum of H-atom, Coulomb potential, black hole spectrum, energy shift, sound

1. The graviton spectrum of the black hole binary

In 1916, Schwarzschild published the solution of the Einstein field equations [1] that were later understood to describe a black hole [2, 3], and in 1963, Kerr generalized the solution to rotating black holes [4]. The year 1970 was the starting point of the theoretical work leading to the understanding of black hole quasinormal modes [5–7], and in the 1990s, higher-order post-Newtonian calculations [8] were performed and later the extensive analytical studies of relativistic two-body dynamics were realized [9, 10]. These advances, together with numerical relativity breaks through in the past decade [11–13]. Numerous black hole candidates have now been identified through electromagnetic observations [14–16]. The black hole binary and their rotation and mergers are open problem of the astrophysics, and it is the integral part of the binary black hole physics.

The binary pulsar system PSR B**1913+16** (also known as PSR J1915+1606) discovered by Hulse and Taylor [17] and subsequent observations of its energy loss by Taylor and Weisberg [18] demonstrated the existence of gravitational waves [19].

By the early 2000s, a set of initial detectors was completed, including TAMA 300 in Japan, GEO600 in Germany, the Laser Interferometer Gravitational-Wave Observatory (LIGO) in the United States, and Virgo in Italy. In 2015, Advanced LIGO became the first of a significantly more sensitive network of advanced detectors (a second-generation interferometric gravitational wave detector) to begin observations [20].

Taylor and Hulse, working at the Arecibo Radiotelescope, discovered the radio pulsar PSR B**1913+16** in a binary, in 1974, and this is now considered as the best general relativistic laboratory [21].

Pulsar PSR B**1913+16** is the massive body of the binary system where each of the rotating pairs is 1.4 times the mass of the Sun. These neutron stars rotate around each other in an orbit not much larger than the Sun's diameter, with a period of 7.8 h. Every 59 ms, the pulsar emits a short signal that is so clear that the arrival time of a 5 min string of a set of such signals can be resolved within 15 μs.

A pulsar model based on strongly magnetized, rapidly spinning neutron stars was soon established as consistent with most of the known facts [22]; its electrodynamical properties were studied theoretically [23] and shown to be plausibly capable of generating broadband radio noise detectable over interstellar distances. The binary pulsar PSR B**1913+16** is now recognized as the harbinger of a new class of unusually short-period pulsars, with numerous important applications.

Because the velocities and gravitational energies in a high-mass binary pulsar system can be significantly relativistic, strong-field and radiative effects come into play. The binary pulsar PSR B**1913+16** provides significant tests of gravitation beyond the weak-field, slow-motion limit [24, 25].

We do not repeat here the derivation of the Einstein quadrupole formula in the Schwinger gravity theory [26]. We show that just in the framework of the Schwinger gravity theory, it is easy to determine the spectral formula for emitted gravitons and the quantum energy loss formula of the binary system. The energy loss formula is general, including black hole binary, and it involves arbitrarily strong gravity.

Since the measurement of the motion of the black hole binaries goes on, we hope that sooner or later the confirmation of our formula will be established.

1.1 The Schwinger approach for the problem

Source methods by Schwinger are adequate for the solution of the calculation of the spectral formula of gravitons and energy loss of binary. Source theory [27, 28] was initially constructed to describe the particle physics situations occurring in high-energy physics experiments. However, it was found that the original formulation simplifies the calculations in the electrodynamics and gravity, where the interactions are mediated by photon and graviton, respectively. The source theory of gravity forms the analogue of quantum electrodynamics because, while in QED the interaction is mediated by the photon, the gravitational interaction is mediated by the graviton [29]. The basic formula in the source theory is the vacuum-to-vacuum amplitude [30]:

$$\langle 0_+ | 0_- \rangle = e^{\frac{i}{\hbar} W(S)}, \tag{1}$$

where the minus and plus symbols refer to any time before and after the region of space–time with action of sources. The exponential form is postulated to express the physically independent experimental arrangements, with result that the associated probability amplitudes multiply and the corresponding W expressions add [27, 28].

In the flat space-time, the field of gravitons is described by the amplitude (1) with the action ($c = 1$ in the following text) [31]

$$W(T) = 4\pi G \int (dx)(dx') \left[T^{\mu\nu}(x) D_+(x - x') T_{\mu\nu}(x') - \frac{1}{2} T(x) D_+(x - x') T(x') \right], \tag{2}$$

where the dimensionality of $W(T)$ has the dimension of the Planck constant $\hbar$ and $T_{\mu\nu}$ is the momentum and energy tensor that, for a particle trajectory $\mathbf{x} = \mathbf{x}(t)$, is defined by the equation [32]

$$T^{\mu\nu}(x) = \frac{p^{\mu}p^{\nu}}{E}\delta(\ \mathbf{x} - \mathbf{x}(t)), \tag{3}$$

where p^{μ} is the relativistic four-momentum of a particle with a rest mass m and

$$p^{\mu} = (E,\ \mathbf{p}) \tag{4}$$

$$p^{\mu}p_{\mu} = -m^2, \tag{5}$$

and the relativistic energy is defined by the known relation

$$E = \frac{m}{\sqrt{1 - \mathbf{v}^2}}, \tag{6}$$

where $\mathbf{v}$ is the three-velocity of the moving particle.

Symbol $T(x)$ in Eq. (2) is defined as $T = g_{\mu\nu}T^{\mu\nu}$, and $D_+(x - x')$ is the graviton propagator whose explicit form will be determined later.

1.2 The power spectral formula in general

It may be easy to show that the probability of the persistence of vacuum is given by the following formula [27]:

$$|\langle 0_+|0_-\rangle|^2 = \exp\left\{-\frac{2}{\hbar}\ \mathrm{Im}\,\mathrm{W}\right\} \stackrel{d}{=} \exp\left\{-\int dt d\omega \frac{1}{\hbar\omega}P(\omega, t)\right\}, \tag{7}$$

where the so-called power spectral function $P(\omega, t)$ has been introduced [27]. For the extraction of the spectral function from Im W, it is necessary to know the explicit form of the graviton propagator $D_+(x - x')$. This propagator involves the graviton property of spreading with velocity c. It means that its mathematical form is identical with the photon propagator form. With regard to Schwinger et al. [33], the x-representation of $D(k)$ in Eq. (2) is as follows:

$$D_+(x - x') = \int \frac{(dk)}{(2\pi)^4} e^{ik(x - x')} D(k), \tag{8}$$

where

$$D(k) = \frac{1}{|\mathbf{k}^2| - \left(k^0\right)^2 - i\epsilon}, \tag{9}$$

which gives

$$D_+(x - x') = \frac{i}{4\pi^2}\int_0^{\infty} d\omega \frac{\sin \omega|\mathbf{x} - \mathbf{x}^{'}|}{|\mathbf{x} - \mathbf{x}^{'}|} e^{-i\omega|t - t'|}. \tag{10}$$

Now, using Eqs. (2), (7), and (10), we get the power spectral formula in the following form:

$$P(\omega,t)=4\pi G\omega\int(d\mathbf{x})(d\mathbf{x}')dt'\frac{\sin\omega|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\cos\omega(t-t')$$
$$\times\left[T^{\mu\nu}(\mathbf{x},t)T_{\mu\nu}(\mathbf{x}',t')-\frac{1}{2}g_{\mu\nu}T^{\mu\nu}(\mathbf{x},t)g_{\alpha\beta}T^{\alpha\beta}(\mathbf{x}',t')\right].\tag{11}$$

1.3 The power spectral formula for the binary system

In the case of the binary system with masses m_1 and m_2, we suppose that they move in a uniform circular motion around their centre of gravity in the xy plane, with corresponding kinematical coordinates:

$$\mathbf{x}_1(t)=r_1\big(\mathbf{i}\cos(\omega_0 t)+\mathbf{j}\sin(\omega_0 t)\big)\tag{12}$$

$$\mathbf{x}_2(t)=r_2\big(\mathbf{i}\cos(\omega_0 t+\pi)+\mathbf{j}\sin(\omega_0 t+\pi)\big)\tag{13}$$

with

$$\mathbf{v}_i(t)=d\mathbf{x}_i/dt,\quad \omega_0=v_i/r_i,\quad v_i=|\mathbf{v}_i|\quad (i=1,2).\tag{14}$$

For the tensor of energy and momentum of the binary, we have

$$T^{\mu\nu}(x)=\frac{p_1^\mu p_1^\nu}{E_1}\delta(\mathbf{x}-\mathbf{x}_1(t))+\frac{p_2^\mu p_2^\nu}{E_2}\delta(\mathbf{x}-\mathbf{x}_2(t)),\tag{15}$$

where we have omitted the tensor $t_{\mu\nu}^G$, which is associated with the massless, gravitational field distributed all over space and proportional to the gravitational constant G [32].

After the insertion of Eq. (15) into Eq. (11), we get [33]

$$P_{total}(\omega,t)=P_1(\omega,t)+P_{12}(\omega,t)+P_2(\omega,t),\tag{16}$$

where $(t'-t=\tau)$

$$P_1(\omega,t)=\frac{G\omega}{r_1\pi}\int_{-\infty}^{\infty}d\tau\frac{\sin[2\omega r_1\sin(\omega_0\tau/2)]}{\sin(\omega_0\tau/2)}\cos\omega\tau$$
$$\times\left(E_1^2(\omega_0^2r_1^2\cos\omega_0\tau-1)^2-\frac{m_1^4}{2E_1^2}\right),\tag{17}$$

$$P_2(\omega,t)=\frac{G\omega}{r_2\pi}\int_{-\infty}^{\infty}d\tau\frac{\sin[2\omega r_2\sin(\omega_0\tau/2)]}{\sin(\omega_0\tau/2)}\cos\omega\tau$$
$$\times\left(E_2^2(\omega_0^2r_2^2\cos\omega_0\tau-1)^2-\frac{m_2^4}{2E_2^2}\right),\tag{18}$$

$$P_{12}(\omega,t)=\frac{4G\omega}{\pi}\int_{-\infty}^{\infty}d\tau\frac{\sin\omega\left[r_1^2+r_2^2+2r_1r_2\cos(\omega_0\tau)\right]^{1/2}}{\left[r_1^2+r_2^2+2r_1r_2\cos(\omega_0\tau)\right]^{1/2}}\cos\omega\tau$$
$$\times\left(E_1E_2(\omega_0^2r_1r_2\cos\omega_0\tau+1)^2-\frac{m_1^2m_2^2}{2E_1E_2}\right).\tag{19}$$

1.4 The quantum energy loss of the binary

Using the following relations

$$\omega_0\tau=\varphi+2\pi l,\quad \varphi\in(-\pi,\pi),\quad l=0,\ \pm1,\ \pm2,\ldots\tag{20}$$

$$\sum_{l=-\infty}^{l=\infty} \cos 2\pi l \frac{\omega}{\omega_0} = \sum_{l=-\infty}^{\infty} \omega_0 \delta(\omega - \omega_0 l), \tag{21}$$

we get for $P_i(\omega, t)$, with ω being restricted to positive:

$$P_i(\omega, t) = \sum_{l=1}^{\infty} \delta(\omega - \omega_0 l) P_{il}(\omega, t). \tag{22}$$

Using the definition of the Bessel function $J_{2l}(z)$

$$J_{2l}(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi \cos\left(z \sin\frac{\varphi}{2}\right) \cos l\varphi, \tag{23}$$

from which the derivatives and their integrals follow, we get for P_{1l} and P_{2l} the following formulae:

$$\begin{aligned} P_{il} = \frac{2G\omega}{r_i} \left(\left(E_i^2\left(v_i^2 - 1\right) - \frac{m_i^4}{2E_i^2}\right) \int_0^{2v_i l} dx J_{2l}(x) \right. \\ \left. + 4E_i^2\left(v_i^2 - 1\right) v_i^2 J'_{2l}(2v_i l) + 4E_i^2 v_i^4 J''_{2l}(2v_i l) \right), \quad i = 1, 2. \end{aligned} \tag{24}$$

Using $r_2 = r_1 + \epsilon$, where ϵ is supposed to be small in comparison with radii r_1 and r_2, we obtain

$$\left[r_1^2 + r_2^2 + 2r_1 r_2 \cos\varphi\right]^{1/2} \approx 2a \cos\left(\frac{\varphi}{2}\right), \tag{25}$$

with

$$a = r_1\left(1 + \frac{\epsilon}{2r_1}\right). \tag{26}$$

So, instead of Eq. (19), we get

$$\begin{aligned} P_{12}(\omega, t) = \frac{2G\omega}{a\pi} \int_{-\infty}^{\infty} d\tau \frac{\sin\left[2\omega a \cos\left(\omega_0 \tau/2\right)\right]}{\cos\left(\omega_0 \tau\right)/2]} \cos\omega\tau \\ \times \left(E_1 E_2 \left(\omega_0^2 r_1 r_2 \cos\omega_0 \tau + 1\right)^2 - \frac{m_1^2 m_2^2}{2E_1 E_2} \right). \end{aligned} \tag{27}$$

Now, we can approach the evaluation of the energy loss formula for the binary from the power spectral of Eqs. (24) and (27). The energy loss is defined by the relation

$$\begin{aligned} -\frac{dU}{dt} = \int P(\omega) d\omega = \\ \int d\omega \sum_{i,l} \delta(\omega - \omega_0 l) P_{il} + \int P_{12}(\omega) d\omega = -\frac{d}{dt}\left(U_1 + U_2 + U_{12}\right). \end{aligned} \tag{28}$$

From [34] we have Kapteyn's formula:

$$\sum_{l=1}^{\infty} \frac{J_{2l}(2lv)}{l^2} = \frac{v^2}{2}. \tag{29}$$

After differentiating the last relation with respect to v, we have

$$\sum_{l=1}^{\infty} lJ'''_{2l}(2lv) = 0. \tag{30}$$

From [34] we learn other Kapteyn's formulae:

$$\sum_{l=1}^{\infty} 2lJ'_{2l}(2lv) = \frac{v}{(1-v^2)^2}, \tag{31}$$

and

$$\sum_{l=1}^{\infty} l\int_0^{2lv} J_{2l}(x)dx = \frac{v^3}{3(1-v^2)^3}. \tag{32}$$

So, after the application of Eqs. (30), (31) and (32) to Eqs. (24) and (28), we get

$$-\frac{dU_i}{dt} = \frac{Gm_i^2 v_i^3 \omega_0}{3r_i\left(1-v_i^2\right)^3}\left[13v_i^2 - 15\right]. \tag{33}$$

Instead of using Kapteyn's formulae for the interference term, we will perform a direct evaluation of the energy loss of the interference term by the ω-integration in (27) [35]. So, after some elementary modification in the ω-integral, we get

$$\begin{gathered} -\frac{dU_{12}}{dt} = \int_0^\infty P(\omega)d\omega = \\ A\int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\omega\omega e^{-i\omega\tau}\sin\left[2\omega a\cos\omega_0\tau\right]\left[\frac{B(C\cos\omega_0\tau+1)^2 - D}{\cos\left(\omega_0\tau/2\right)}\right], \end{gathered} \tag{34}$$

with

$$A = \frac{G}{a\pi}, \quad B = E_1E_2, \quad C = v_1v_2, \quad D = \frac{m_1^2 m_2^2}{2E_1E_2}. \tag{35}$$

Using the definition of the δ-function and its derivative, we have, instead of Eq. (34), with $v = a\omega_0$

$$\begin{gathered} -\frac{dU_{12}}{dt} = A\omega_0\pi\int_{-\infty}^{\infty} dx\,\frac{\left[B(C\cos x+1)^2 - D\right]}{\cos\left(x/2\right)} \\ \times\left[\delta'(x - 2v\cos\left(x/2\right)) - \delta'(x + 2v\cos\left(x/2\right))\right]. \end{gathered} \tag{36}$$

According to the Schwinger article [36], we express the delta function as follows:

$$\delta(x \pm 2v\cos\left(x/2\right)) = \sum_{n=0}^{\infty}\frac{(\pm 2v\cos\left(x/2\right))^n}{n!}\left(\frac{d}{dx}\right)^n\delta(x). \tag{37}$$

Then

$$\delta'(x \pm 2v\cos\left(x/2\right)) = \sum_{n=0}^{\infty}\frac{(\pm 2v\cos\left(x/2\right))^n}{n!}\left(\frac{d}{dx}\right)^{n+1}\delta(x) = \tag{38}$$

and it means that

$$\frac{[\delta'(x+2v\cos(x/2)) - \delta'(x-2v\cos(x/2))]}{\cos(x/2)} = (-2)\sum_{n=1}^{\infty}\frac{(2v)^{2n-1}(\cos(x/2))^{2(n-1)}}{(2n-1)!}\left(\frac{d}{dx}\right)^{2n}\delta(x) \tag{39}$$

Now, we can write Eq. (36) in the following form after some elementary operations:

$$-\frac{dU_{12}}{dt} = A\omega_0\pi\int_{-\infty}^{\infty}dx\left(B(C\cos x+1)^2 - D\right)\times(-2)\sum_{n=1}^{\infty}\frac{(2v)^{2n-1}(\cos(x/2))^{2(n-1)}}{(2n-1)!}\left(\frac{d}{dx}\right)^{2n}\delta(x), \tag{40}$$

where $\left(B(C\cos x+1)^2 - D\right)$ can be written as follows:

$$\left(B(C\cos x+1)^2 - D\right) = 4BC^2(\cos^4(x/2) + \left[4CB - 4BC^2\right](\cos^2(x/2) + \left[BC^2 - 2CB + B - D\right]. \tag{41}$$

After the application of the per partes method, we get from Eq. (40) the following mathematical object:

$$\begin{aligned}-\frac{dU_{12}}{dt} &= (-2)A\left[4BC^2\right]\omega_0\pi\int_{-\infty}^{\infty}dx\,\delta(x)\sum_{n=1}^{\infty}\left(\frac{d}{dx}\right)^{2n}(2v)^{2n-1}\frac{(\cos(x/2))^{2n+2}}{(2n-1)!}\\ &- 2A\left[4CB - 4BC^2\right]\omega_0\pi\int_{-\infty}^{\infty}dx\,\delta(x)\sum_{n=1}^{\infty}\left(\frac{d}{dx}\right)^{2n}(2v)^{2n-1}\frac{(\cos(x/2))^{2n}}{(2n-1)!}\\ &- 2A\left[BC^2 - 2CB + B - D\right]\omega_0\pi\int_{-\infty}^{\infty}dx\,\delta(x)\sum_{n=1}^{\infty}\left(\frac{d}{dx}\right)^{2n}(2v)^{2n-1}\frac{(\cos(x/2))^{2(n-1)}}{(2n-1)!}.\end{aligned} \tag{42}$$

We get after some elementary operations $\int\delta f(x) = f(0)$

$$J_1 = \sum_{n=1}^{\infty}\left(\frac{d}{dx}\right)^{2n}(2v)^{2n-1}\frac{(\cos(x/2))^{2n+2}}{(2n-1)!}\Bigg|_{x=0} = \sum_{n=0}^{\infty}f(n)v^{2n} = F(v^2), \tag{43}$$

$$J_2 = \sum_{n=1}^{\infty}\left(\frac{d}{dx}\right)^{2n}(2v)^{2n-1}\frac{(\cos(x/2))^{2n}}{(2n-1)!}\Bigg|_{x=0} = \sum_{n=0}^{\infty}g(n)v^{2n} = G(v^2) \tag{44}$$

and

$$J_3 = \sum_{n=1}^{\infty}\left(\frac{d}{dx}\right)^{2n}(2v)^{2n-1}\frac{(\cos(x/2))^{2(n-1)}}{(2n-1)!}\Bigg|_{x=0} = \sum_{n=0}^{\infty}h(n)v^{2n} = H(v^2) \tag{45}$$

where f, g, h, F, G, H are functions which must be determined.
So we get instead of Eq. (41) the following final form:

$$-\frac{dU_{12}}{dt} = (-2)A\left[4BC^2\right]\omega_0\pi G(v^2) - 2A\left[4CB - 4BC^2\right]\omega_0\pi F(v^2) - 2A\left[-2CB + BC^2 + B - D\right]\omega_0\pi H(v^2) \tag{46}$$

Let us remark that we can use simple approximation in Eq. (41) as follows: $(\cos(x/2))^{2n+2} \approx (\cos(x/2))^2$, $(\cos(x/2))^{2n} \approx (\cos(x/2))^2$, $(\cos(x/2))^{2(n-1)} \approx (\cos(x/2))^2$. Then, after using the well-known formula

$$\left(\frac{d}{dx}\right)^{2n} \cos^2(x/2) = \frac{1}{2}\cos(x + \pi n) \tag{47}$$

and

$$\frac{1}{2}\cos(x + \pi n)\bigg|_{x=0} = \frac{1}{2}(-1)^n. \tag{48}$$

So, instead of Eq. (46), we have

$$-\frac{dU_{12}}{dt} = A\omega_0\pi\left\{2BC + BC^2 + B - D\right\} \sum_{n=1}^{\infty} \frac{(2v)^{2n-1}(-1)^n}{(2n-1)!}. \tag{49}$$

2. Energy shift of H-atom electrons due to the black hole thermal bath

We here determine the electromagnetic shift of energy levels of H-atom electrons by calculating an electron coupling to the black hole thermal bath. The energy shift of electrons in H-atom is determined in the framework of nonrelativistic quantum mechanics.

The Gibbons-Hawking effect is the statement that a temperature can be associated to each solution of the Einstein field equations that contain a causal horizon.

Schwarzschild space-time involves an event horizon associated with temperature T of a black hole of mass M. We consider here the influence of the heat bath of the Gibbons-Hawking photons on the energy shift of H-atom electrons.

The analogical problems are solved in the scientific respected journals. There is a general conviction of an analogy between the black hole and the hydrogen atom. Corda [37] used the model where Hawking radiation is a tunneling process. In his article the emission is expressed in terms of the black hole quantum levels. So, the Hawking radiation and black hole quasinormal modes by Corda [38] are analogical to hydrogen atom by Bohr.

In this model [39] the corresponding wave function is written in terms of a unitary evolution matrix. So, the final state is a pure quantum state with no information loss. Black hole is defined as the quantum systems, with discrete quantum spectra, with Hooft's assumption that Schrödinger equations are universal for all universe dynamics.

Thermal photons by Gibbons and Hawking are blackbody photons, with the Planck photon distribution law [40–42], derived from the statistics of the oscillators inside of the blackbody. Later Einstein [43] derived the Planck formula from the Bohr model of atom where photons and electrons have the discrete energies related with the Bohr formula $\hbar\omega = E_i - E_f$, E_i, E_f being the initial and final energies of electrons.

Now, we determine the modification of the Coulomb potential due to blackbody photons. At the start, the energy shift in the H-atom is the potential $V_0(\mathbf{x})$, generated by nucleus of the H-atom. The potential at point $V_0(\mathbf{x} + \delta\mathbf{x})$ is [44, 45]

$$V_0(\mathbf{x} + \delta\mathbf{x}) = \left\{1 + \delta\mathbf{x}\nabla + \frac{1}{2}(\delta\mathbf{x}\nabla)^2 + \dots\right\}V_0(\mathbf{x}). \tag{50}$$

The average of the last equation in space enables the elimination of the so-called the effective potential:

$$V(\mathbf{x}) = \left\{1 + \frac{1}{6}(\delta\mathbf{x})^2_T\Delta + \ldots\right\}V_0(\mathbf{x}), \tag{51}$$

where $(\delta\mathbf{x})^2_T$ is the average value of the square coordinate shift caused by the thermal photons. The potential shift follows from Eq. (51):

$$\delta V(\mathbf{x}) = \frac{1}{6}(\delta\mathbf{x})^2_T\Delta V_0(\mathbf{x}). \tag{52}$$

The shift of the energy levels is given by the standard quantum formula [44]:

$$\delta E_n = \frac{1}{6}(\delta\mathbf{x})^2_T(\psi_n\Delta V_0\psi_n). \tag{53}$$

In case of the Coulomb potential, which is the case of the H-atom, we have

$$V_0 = -\frac{e^2}{4\pi|\mathbf{x}|}. \tag{54}$$

Then for the H-atom we can write

$$\delta E_n = \frac{2\pi}{3}(\delta\mathbf{x})^2_T\frac{e^2}{4\pi}|\psi_n(0)|^2, \tag{55}$$

where we used the following equation for the Coulomb potential

$$\Delta\frac{1}{|\mathbf{x}|} = -4\pi\delta(\mathbf{x}). \tag{56}$$

The motion of electron in the electric field is evidently described by elementary equation:

$$\delta\ddot{\mathbf{x}} = \frac{e}{m}\mathbf{E}_T, \tag{57}$$

which can be transformed by the Fourier transformation into the following equation

$$|\delta\mathbf{x}_{T\omega}|^2 = \frac{1}{2}\left(\frac{e^2}{m^2\omega^4}\right)\mathbf{E}^2_{T\omega}, \tag{58}$$

where the index ω concerns the Fourier component of the above functions.

Using Bethe idea [46] of the influence of vacuum fluctuations on the energy shift of electron, the following elementary relations were applied by Welton [45], Akhiezer et al. [44] and Berestetzkii et al. [47]:

$$\frac{1}{2}\mathbf{E}^2_\omega = \frac{\hbar\omega}{2}, \tag{59}$$

and in case of the thermal bath of the blackbody, the last equation is of the following form [48]:

$$\mathbf{E}^2_{T\omega} = \varrho(\omega) = \left(\frac{\hbar\omega^3}{\pi^2c^3}\right)\frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}, \tag{60}$$

because the Planck law in (60) was written as

$$\varrho(\omega) = G(\omega) < E_\omega > = \left(\frac{\omega^2}{\pi^2 c^3}\right) \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1}, \tag{61}$$

where the term

$$< E_\omega > = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1} \tag{62}$$

is the average energy of photons in the blackbody and

$$G(\omega) = \frac{\omega^2}{\pi^2 c^3} \tag{63}$$

is the number of electromagnetic modes in the interval ω, $\omega + d\omega$.
Then,

$$(\delta \mathbf{x}_{T\omega})^2 = \frac{1}{2}\left(\frac{e^2}{m^2\omega^4}\right)\left(\frac{\hbar\omega^3}{\pi^2 c^3}\right)\frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}, \tag{64}$$

where $(\delta \mathbf{x}_{T\omega})^2$ involves the number of frequencies in the interval $(\omega, \omega + d\omega)$.
So, after some integration, we get

$$(\delta \mathbf{x})_T^2 = \int_{\omega_1}^{\omega_2} \frac{1}{2}\left(\frac{e^2}{m^2\omega^4}\right)\left(\frac{\hbar\omega^3}{\pi^2 c^3}\right)\frac{d\omega}{e^{\frac{\hbar\omega}{kT}} - 1} = \frac{1}{2}\left(\frac{e^2}{m^2}\right)\left(\frac{\hbar}{\pi^2 c^3}\right) F(\omega_2 - \omega_1), \tag{65}$$

where $F(\omega)$ is the primitive function of the omega-integral with

$$\frac{1}{\omega}\frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}, \tag{66}$$

which is not elementary, and it is not in the tables of integrals.

Frequencies ω_1 and ω_2 can be determined from the field of thermal photons. It was performed for the Lamb shift [44, 47] caused by the interaction of the Coulombic atom with the field fluctuations. The Bethe-Welton method is valid here too and so we take Bethe-Welton frequencies. It means an electron does not respond to the fluctuating field if the frequency is much less than the atom binding energy given by the Rydberg constant [49] $E_{Rydberg} = \alpha^2 mc^2/2$. So, the lower frequency limit is

$$\omega_1 = E_{Rydberg}/\hbar = \frac{\alpha^2 mc^2}{2\hbar}, \tag{67}$$

where $\alpha \approx 1/137$ is so-called the fine structure constant.

The second frequency follows from the cutoff, determined by the neglection of the relativistic effect in our theory. So, we write

$$\omega_2 = \frac{mc^2}{\hbar}. \tag{68}$$

If we express the thermal function in the form of the geometric series

$$\frac{1}{e^{\frac{\hbar\omega}{kT}} - 1} = q\left(1 + q^2 + q^3 + \ldots..\right); \quad q = e^{-\frac{\hbar\omega}{kT}}, \tag{69}$$

$$\int_{\omega_1}^{\omega_2} q\left(1+q^2+q^3+\ldots..\right)\frac{1}{\omega}d\omega = \ln|\omega| + \sum_{k=1}^{\infty}\frac{\left(-\frac{\hbar\omega}{kT}\right)^k}{k!k} + \ldots.; \quad q = e^{-\frac{\hbar\omega}{kT}} \tag{70}$$

and the first thermal contribution is

$$Thermal\ \ contribution = \ln\frac{\omega_2}{\omega_1} - \frac{\hbar}{kT}(\omega_2 - \omega_1), \tag{71}$$

then, with Eq. (55)

$$\delta E_n \approx \frac{2\pi}{3}\left(\frac{e^2}{m^2}\right)\left(\frac{\hbar}{\pi^2c^3}\right)\left(\ln\frac{\omega_2}{\omega_1} - \frac{\hbar}{kT}(\omega_2-\omega_1)\right)|\psi_n(0)|^2, \tag{72}$$

where according to Sokolov et al. [50]

$$|\psi_n(0)|^2 = \frac{1}{\pi n^2 a_0^2} \tag{73}$$

with

$$a_0 = \frac{\hbar^2}{me^2}. \tag{74}$$

Let us only remark that the numerical form of Eq. (72) has deep experimental astrophysical meaning.

Haroche [51] and his group performed experiments with the Rydberg atoms in a cavity. We used here Gibbons-Hawking black hole for the determination of the energy shift of H-atom electrons in the black hole gas.

3. Velocity of sound in the black hole photon gas

We have seen that the black hole can be modeled by the blackbody, and it means that there is the velocity of sound in the Gibbons-Hawking black hole thermal bath. So, let us derive the sound velocity from the thermodynamics of photon gas and energy mass relation.

In order to be pedagogically clear, we start with the derivation of the speed of sound in the real elastic rod.

Let A be the cross-section of the element Adx of a rod on the axis x. Let $\varphi(x,t)$ be the deflection of Adx at point x at time t. The shift of the Adx at point $x+dx$ is evidently

$$\varphi + \frac{\partial\varphi}{\partial x}dx. \tag{75}$$

Now, we suppose that the force tension $F(x,t)$ acting on the Adx of the rod is given by Hooke's law:

$$F(x,t) = EA\frac{\partial\varphi}{\partial x}, \tag{76}$$

where E is Young's modulus of elasticity. We easily derive that

$$F(x+dx) - F(x) \approx EA\frac{\partial^2\varphi}{\partial x^2}dx. \tag{77}$$

The mass of Adx is ϱAdx, where ϱ is the mass density of the rod and the dynamical equilibrium is expressed by Newton's law of force:

$$\varrho Adx\varphi_{tt} = EA\varphi_{xx}dx \tag{78}$$

or

$$\varphi_{tt} - v^2\varphi_{xx} = 0, \tag{79}$$

where

$$v = \left(\frac{E}{\varrho}\right)^{1/2} \tag{80}$$

is the velocity of sound in the rod.

The complete solution of Eq. (79) includes the initial and boundary conditions. We suppose that Eq. (80) is of the universal validity also for gas in the cylinder tube. If $(\Delta L/L)$ is the relative prolongation of a rod, then an analogue for the tube of gas is $\Delta V/V$, $F \to \Delta p$, where V is the volume of a gas and p is gas pressure. Then, the modulus of elasticity as the analogue of Eq. (76) is

$$E = -\frac{dp}{dV}V. \tag{81}$$

The sound in ideal gas is the adiabatic thermodynamic process with no heat exchange. This is the model of the sound spreading in the gas of blackbody photons. Such process is described by the thermodynamic equation:

$$pV^{\kappa} = const, \tag{82}$$

where κ is the Poisson constant defined as $\kappa = c_p/c_v$, with c_p, c_v being the specific heat under constant pressure and under constant volume.

After differentiation of Eq. (82), we get the following equation:

$$dpV^{\kappa} + \kappa V^{\kappa-1}dV = 0, \tag{83}$$

or

$$\frac{dp}{dV} = -\kappa\frac{p}{V}. \tag{84}$$

After inserting Eq. (84) into Eq. (81), we get from Eq. (80) the so-called Newton-Laplace formula:

$$v = \sqrt{\kappa\frac{p}{\varrho}}, \tag{85}$$

with ϱ being the gas mass density.

The equilibrium radiation density has the Stefan-Boltzmann form:

$$u = aT^4; \quad a = 7,5657.10^{-16}\frac{\mathrm{J}}{\mathrm{K}^4\mathrm{m}^3}. \tag{86}$$

Then, with regard to the thermodynamic definition of the specific heat,

$$c_v = \left(\frac{\partial u}{\partial T}\right)_V = 4aT^3. \tag{87}$$

Similarly, with regard to the general thermodynamic theory,

$$c_p = c_v + \left[\left(\frac{\partial u}{\partial V} \right)_T + p \right] \left(\frac{\partial V}{\partial T} \right)_p = c_v, \tag{88}$$

because $\left(\frac{\partial V}{\partial T}\right)_T = 0$ for photon gas, and in such a way, $\kappa = 1$ for photon gas. According to the theory of relativity, there is a relation for mass and energy, namely, $m = E/c^2$. At the same time, the pressure and the internal energy of the blackbody gas are related as $p = u/3$. So, in our case

$$\varrho = u/c^2 = \frac{aT^4}{c^2}; \quad p = \frac{u}{3}. \tag{89}$$

So, after the insertion of formulae in Eq. (88) into Eq. (85), the final formula for the sound velocity in photon blackbody sea is the following:

$$v = c\sqrt{\frac{\kappa}{3}} = \frac{c}{3}\sqrt{3}, \tag{90}$$

which was derived by Partovi [52] using the QED theory of the photon gas. We correctly derived $v/c < 1$.

So, we have performed the derivation of the velocity of sound in the relic photon sea. It is not excluded that the relic sound can be detected by the special microphones of Bell Laboratories. If we use van der Waals equation of state or the Kamerlingh Onnes virial equation, the obtained results will be modified with regard to the basic results.

Our derivation of the light velocity in the blackbody photon gas was based on the classical thermodynamic model with the adiabatic process ($\delta Q = 0$), controlling the spreading of sound in the gas. Partovi [52] derived additional radiation corrections to the Planck distribution formula and the additional correction to the speed of sound in the relic photon sea. His formula is of the form

$$v_{sound} = \left[1 - \frac{88\pi^2\alpha^2}{2025} \left(\frac{T}{T_e} \right)^4 \right] \frac{c}{\sqrt{3}}, \tag{91}$$

where α is the fine structure constant and $T_e = 5.9$ G Kelvin. We see that our formula is the first approximation in the Partovi expression.

There is the Boltzmann statistical theory of transport of sound energy in a gas [53]. After the application of this theory to the photon gas or relic photon gas, we can obtain results involving the cross-section of the photon-photon interaction [47]:

$$\sigma_{\gamma\gamma} = 4,7\alpha^4 \left(\frac{c}{\omega} \right)^2; \quad \hbar\omega \ll mc^2, \tag{92}$$

and

$$\sigma_{\gamma\gamma} = \frac{973}{10125\pi} \alpha^2 r_e^2 \left(\frac{\hbar\omega}{mc^2} \right)^6; \quad \hbar\omega \gg mc^2, \tag{93}$$

where $r_e = e^2/mc^2 = 2,818 \times 10^{-13}$ cm is the classical radius of electron and $\alpha = e^2/\hbar c$ is the fine structure constant with numerical value $1/\alpha = 137,04$.

4. Discussion and summary

We have derived the spectral density of gravitons and the total quantum loss of energy of the black hole binary. The energy loss is caused by the emission of gravitons during the motion of the two black hole binaries around each other under their gravitational interaction. The energy loss formulae of the production of gravitons are derived here by the Schwinger method. Because the general relativity and theory of gravity do not necessarily contain the last valid words to be written about the nature of gravity and it is not, of course, a quantum theory [21], they cannot give the answer on the production of gravitons and the quantum energy loss, respectively. So, this article is the original text that discusses the quantum energy loss caused by the production of gravitons by the black hole binary system. It is evident that the production of gravitons by the binary system forms a specific physical situation, where a general relativity can be seriously confronted with the source theory of gravity.

This article is an extended version of an older article by the present author [33], in which only the spectral formulae were derived. Here we have derived the quantum energy loss formulae, with no specific assumption concerning the strength of the gravitational field. We hope that future astrophysical observations will confirm the quantum version of the energy loss of the binary black hole.

In the next part of the chapter, the electromagnetic shift of energy levels of H-atom electrons was determined by calculating an electron coupling to the Gibbons-Hawking electromagnetic field thermal bath of the black hole. The energy shift of electrons in H-atom is determined in the framework of nonrelativistic quantum mechanics.

In the last section, we have determined the velocity of sound in the blackbody gas of photons inside of the black hole. Derivation was based on the thermodynamic theory of the photon gas and the Einstein relation between energy and mass. The spectral form for the n-dimensional blackbody was not here considered. The text is based mainly on the author articles published in the international journals of physics [33, 54, 55].

There is the fundamental problem concerning the maximal mass of the black hole. The theory of the space–time with maximal acceleration constant was derived by authors [56, 57]. In this theory the maximal acceleration constant is the analogue of the maximal velocity in special theory of relativity. Maximal acceleration determines the maximal black hole mass where the mass of the black hole is restricted by maximal acceleration of a body falling in the gravity field of the black hole.

Another question is what is the relation of our formulae to the results obtained by LIGO (Laser Interferometer Gravitational-Wave Observatory)? LIGO is the largest and most sensitive interferometer facility ever built. It has been periodically upgraded to increase its sensitivity. The most recent upgrade, Advanced LIGO (2015), detected for the first time the gravitational wave, with sensitivity far above the background noise. The event with number GW150914 was identified with the result of a merger of two black holes at a distance of about 400 Mpc from Earth [58]. Two additional significant detections, GW151226 and GW170104, were reported later. We can say that at this time it is not clear if the LIGO results involve information on the spectrum of gravitons calculated in this chapter.

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