Chapter

Using Gray-Markov Model and Time Series Model to Predict Foreign Direct Investment Trend for Supporting China's Economic Development

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Abstract

Foreign direct investment (FDI) is one of the important factors affecting China's economic development, the prediction of which is the basis of its development and decision-making. Based on elaborating the significant role in China's economic growth and the status quo of utilizing foreign investment over the period between 2000 and 2016, this chapter attempts to construct Gray-Markov model (GMM) and time series model (TSM) to forecast the trend of China's utilization of FDI and then compares the precision of two different prediction models to obtain a better one. Results indicate that although it is qualified, traditional Gray model needs to be optimized; GMM is built to help modify the result, improve Gray-related degrees, and narrow the gap with real value. Comparing the accuracy of GMM with that of TSM, we can conclude that the fitting effect of GMM is better. To increase the credibility of these results, this chapter is based on the data of Beijing and Chongqing from 1990 till 2016, also verifying that the fitting effect of GMM is superior to that of the TSM. Then, we can safely draw a conclusion that the prediction model of GMM is more credible, which has a certain referencing value for the utilization of FDI.

Keywords: foreign direct investment (FDI), Gray-Markov model (GMM), time series model (TSM)

1. Introduction

In the light of the definition of the International Monetary Fund (IMF) and the Organization for Economic Cooperation and Development, foreign direct investment (FDI) is an investment in the form of a controlling ownership in a business in one country by an entity based in another country. The primary purpose of the host country in attracting FDI is to promote the country's economic development and industrial upgrading. This will facilitate domestic enterprises to improve their technology and quality, gradually supporting the development of foreign enterprises to enter the global value chain [1]. Influencing the supply chain system, FDI has significantly promoted the sound and rapid development of the national economy.

Therefore, it is necessary to focus on the future tendency of FDI in the supply chain system when we investigate the transformation and innovation of Chinese economy.

Since the late 1970s, FDI attracted by China has been steadily increasing, regardless of the changes and fluctuation of the international economic environment and the total flow of FDI globally. Statistically, over the period from 1979 to 2010, China's actual use of FDI amounted to $1048.31 billion [2], and FDI keeps a rapid growth. According to the data of Ministry of Commerce of the People's Republic of China (PRC) (**Figure 1**), the FDI in China presented a rising trend over the period from 1990 to 2016. The vital roles in the economic development of China are as follows. Firstly, the proportion of basic industries in China declines generally, and the proportion of agricultural output drops by 18% over the period between 1978 and 2011 [3]. Secondly, for a long time, FDI mainly concentrates in secondary and tertiary industries, accelerating the restructuring and upgrading of China's industries [4]. Finally, FDI provides investment capital and promotes the rapid development of China's import and export trade, improving China's status in international trade.

Due to the remarkable role of FDI, a multitude of scholars began to track and study the FDI in developing countries, build analytical framework, and launch a new field of research of FDI in developing countries. The statistics shows that China has become an emerging market for FDI. Dees indicates that FDI has positive effects on the GDP, technological progress, and the improvement of management system [5]. Nourzad considers that FDI promotes economy development through technology transfer [6], while Mah argues that the latter one promotes the former one [7]. Taking the reform policy (implemented in July 2005) as the boundary, Pan and Song explore the impact of the effective exchange rate of RMB on FDI [8]. Research shows that they are in a long-term equilibrium relationship before implementing reform policy. After the policy, the exchange rate of RMB has the Granger causality for FDI, and the appreciation of RMB can promote the flow of FDI. Additionally, De Mello shows that FDI can increase the added value associated with it [9]. Based on the data from 1971 to 2012, Dreher et al. conclude that the membership in international organizations is an essential and decisive factor of FDI

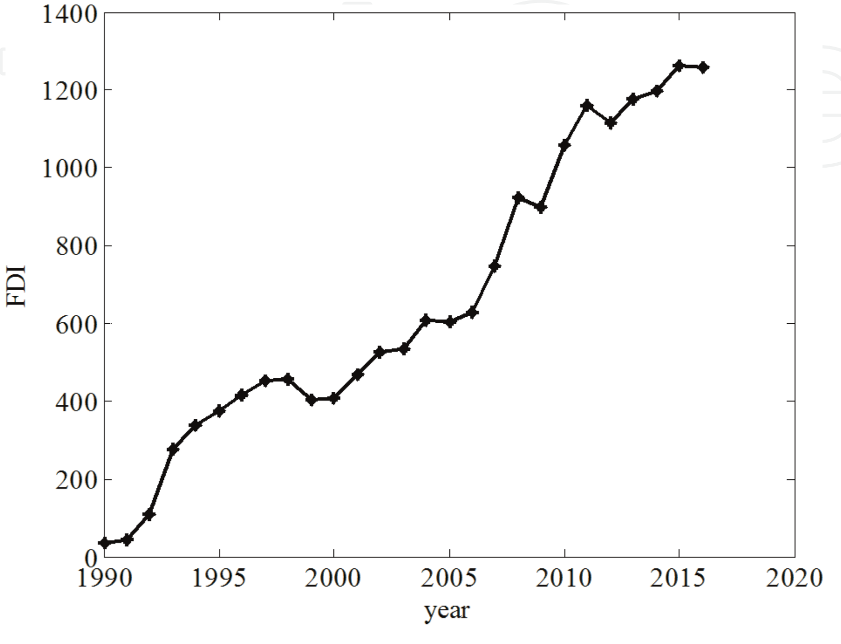

Figure 1.
The horizontal curve of FDI in China.

liquidity and has a promoting effect on FDI mobility [10]. Badr and Ayed do a quantitative study of the relationship between FDI and economic development in South American countries, and they find that FDI can be determined by some economic factors, having no important effect on economic development [11]. Kathuria et al. apply panel data to examining the effectiveness of public policy in attracting FDI [12]. Lin et al. divide the FDI company into five strategies [13]; Brülhat and Schmidheiny estimate the rivalness of state-level inward FDI [14].

The trend of FDI in the future is an important reference for China's economic development. However, much literature focuses on the development of FDI itself and its influencing factors, and there is little research on the future development. This is what we do in this chapter. Currently, the predictive analysis model for economic and trade development can be divided into linear prediction method and nonlinear prediction one. The linear prediction method mainly includes historical average level prediction method, time series prediction method, and Kalman filter prediction method, to name just a few. The nonlinear prediction methods concern Gray theory, Markov chain, support vector machine, and boom prediction method. The historical average prediction algorithm is simple and easy to understand and the parameters can be estimated by using the least squares method. However, it is too simple to accurately reflect the randomness and nonlinearity, and therefore it cannot be applied to unexpected events. The Kalman filter uses the flexible recursive state space model, with the advantages of linear, unbiased, and minimum mean variance. Nevertheless, because the Kalman filter prediction model belongs to the linear model, its performance becomes worse in the nonlinearity and uncertainty [15]. The time series model is simple in modeling, with high prediction accuracy in the case of full historical data. The Gray model can be modeled with less information, handling data easily and having higher accuracy, which can be extensively used in several fields [15–18]. However, Gray model becomes less attractive for time series with large stochastic fluctuation. Markov stochastic process predicts the development and changes of dynamic system according to the transfer probability of different states, and the transfer probability reflects the influence degrees of various stochastic factors and the internal law of the transition states. Therefore, it is more suitable to predict the problems with large stochastic fluctuation. What cannot be ignored is that Markov model requires data to meet the characteristics of no effect. Consequently, when using a simple model, it is very difficult to obtain a better prediction result, and the combination method becomes a popular method.

Through the vector autoregressive moving average (VARMA), Bhattacharya et al. compare and analyze the consumer price index sequence (CPI) and improve the forecasting accuracy [16]. The Gray model (proposed by a Chinese scholar, Professor Deng) and the Markov model (proposed by a Russian mathematician, Markov) have been combined very early, which is called Gray-Markov model (GMM). Based on the Gray prediction model, GMM is used to solve the inaccurate problems resulting from the large random fluctuation of the data and widely promoted in the fields of financial economy, agricultural economy, and resource and energy [17–20]. On the basis of GM(1,1), Li et al. propose an improved GM(2,1) model [21]. Based on the model of GM(1,1) and Markov stochastic process and combining Taylor formula approximation method, Li et al. construct a model of T-MC-RGM(1,1) and verify its validity by the example of thermal power station in Japan [22].

The level of FDI in China is influenced by many factors such as fixed investment, laws and regulations, corporate culture, innovation ability, and financial market stability, among others. To clearly recognize and describe the role of FDI, the foreign investment system is abstracted as a Gray system with no physical prototype and incomplete information, which can be predicted with GM(1,1) model. Meanwhile, the FDI level in the previous year has no direct influence on that in the next year, in line with the no-effect characteristic of Markov stochastic process. On the basis of the previous study of Gray-Markov model, it is used to predict the tendency of FDI in China, addressing the shortcomings of the Gray model for the low precision of the data sample with large fluctuation and compensating for the limitation that the Markov model requires the data to have a smooth process. As a comparison, the time series prediction model is introduced to evaluate FDI. Then, the fitting results are compared to decide the optimal prediction model.

2. Gray-Markov model

Gray-Markov model is a forecasting method integrating the Gray theory with the Markov theory [17–25]. Firstly, GM(1,1) is constructed to obtain the predicted residual value. Then, the error state can be divided according to the residual values, and the error state can be obtained in light of the Markov prediction model. Then, based on the error state and transition matrix, the predicted sequence from GM(1,1) can be adjusted to obtain more precise predicting internals. The traditional GM(1,1) has its advantage in short-term prediction, while it has a poor fitting effect in forecasting the long-range and fluctuating data series. And the benefit of Markov stochastic process is the prediction of the large data series with random volatility. GMM has been proposed by He to predict the yield of cocoon and oil tea in Zhejiang Province. Subsequently, this model is widely used in the prediction of transportation, air accidents, and rainfall. Accordingly, we use GMM to predict FDI of China [26–28].

2.1 Gray model

The Gray system theory, founded and developed by Chinese scholar Deng, extends the viewpoints and methods of general system theory, information theory, and cybernetics to the abstract system of society, economy, and ecology, incorporating the development of mathematical methods to develop the theory and method of Gray system. The modeling process is as follows.

(1) Raw series are

$$X^{(0)} = \left\{ x^{(0)}(1), x^{(0)}(2), \ldots\ldots, x^{(0)}(m) \right\} \tag{1}$$

(2) To weaken the randomness of the original data, the accumulated generating series is derived:

$$X^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i). \tag{2}$$

(3) Based on the sequence of $X_{(t)}^{(1)}$, a new sequence $Z_{(t)}^{(1)}$ is derived as follows:

$$Z^{(1)}(k) = \frac{1}{2}x^{(1)}(k) + \frac{1}{2}x^{(1)}(k-1) \tag{3}$$

(4) Then, whitened differential equation is obtained:

$$x^{(0)}(k) + aZ^{(1)}(k) = b \tag{4}$$

In Eq. (4) a is development coefficient, b is the parameter of Gray action, and Φ is identification parameter vector. Then, the least squares estimation of parameters satisfies the following equation:

$$\hat{\Phi} = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \left(B^T B\right)^{-1} B^T Y \tag{5}$$

and

$$B = \begin{pmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -Z^{(1)}(m) & 1 \end{pmatrix}, Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(m) \end{pmatrix} \tag{6}$$

By differentiating $x^{(1)}(k)$, a whitened differential equation can be written as $\frac{dx^{(1)}}{dt} + ax^{(1)}(k) = b$

(5) The whitened time response is as follows:

$$\hat{x}^{(1)}(k+1) = \left(x^{(1)}(1) - \frac{\hat{b}}{\hat{a}}\right)e^{(-\hat{a}k)} + \frac{\hat{b}}{\hat{a}} \tag{7}$$

Reducing the sequence of $\hat{x}^{(1)}(k+1)$ $(k = 1, 2, \ldots, m-1)$, the following sequence is obtained:

$$\hat{X}^{(0)} = \left\{\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(0)}(m)\right\} \tag{8}$$

(6) Model testing

Model test is divided into residual test and Gray-relating test. Residual test is to obtain the difference between predicting value and the actual value. Firstly, the absolute residuals and relative residuals about $X^{(0)}$ and $\hat{X}^{(0)}$ are calculated:

$$\varDelta^{(0)}(i) = \hat{x}^{(0)}(i) - x^{(0)}(i)(i = 1, 2, \ldots, n) \tag{9}$$

$$\phi(i) = \frac{\varDelta^{(0)}(i)}{\hat{x}^{(0)}(i)}(i = 1, 2, \ldots, n) \tag{10}$$

Then, below is the average value of relative residuals:

$$\varPhi = \frac{1}{n}\sum_{i=1}^{n}\phi_i \tag{11}$$

Given the value of α, it is called residual qualification model when $\varPhi < \alpha$. The value of α can be 0.01, 0.05, or 0.10, and the corresponding model is perfect, qualified, and barely qualified.

As shown in Eq. (12), Gray correlation degree measures the correlating coefficient between the original sequence and the reference sequence:

$$\varepsilon_i(k) = \frac{\min_i \min_k |x(k) - x_i(k)| + \rho \max_i \max_k |x(k) - x_i(k)|}{|x(k) - x_i(k)| + \rho \max_i \max_k |x(k) - x_i(k)|} \tag{12}$$

i denotes the ith group of fitting data, and k denotes the kth one in a certain group. ρ denotes the distinguish coefficient varying from 0 to 1, which is always set as 0.5. However, the correlation coefficient varies with moments, which results in disperse information. Combining the correlation coefficient in different moments together, we can obtain the correlation degree between the original curve and the fitting curve:

$$r_i = \frac{1}{n}\sum_{k=1}^{n} \varepsilon_i(k) \tag{13}$$

2.2 Markov model

Markov chain is proposed by Andrey Markov (1856–1922), and it is a discrete time stochastic process with Markov property in mathematics. Given the current knowledge and information, historical information has no impact on the future. To improve prediction accuracy, Markov model is used to handle the data obtained by GM(1,1). It is critical to divide state and build transition matrix.

2.2.1 Dividing states

To divide states, four rules are suggested to follow. Firstly, the partition state must have at least one true value in each state. Secondly, elements in a one-step transition matrix cannot be the same. Thirdly, the actual values must fall into one state. Finally, the state must pass Markov test. The numbers vary according to the original data. In this chapter, the overall level of FDI in China is on the rise while fluctuating in detail. Therefore, the level of FDI is a non-stable stochastic process. Taking the curve of $\hat{Y}(k) = \hat{x}^{(0)}(k+1)$ as reference, the sequence can be divided into n states. The intervals can be denoted as $Q_i = [Q_{1i}, Q_{2i}]$ and $i = 1, 2, \ldots, n$, in which $Q_{1i} = \hat{Y}(k) + E_{1i}$ and $Q_{2i} = \hat{Y}(k) + E_{2i}$.

2.2.2 Transition matrix

Assuming that there are n states denoting as $E_1, E_2, \ldots, E_n$, the transition probability amounts to frequency approximately in general, namely, $P_{ij} = \frac{M_{ij}^{(l)}}{M_i}$. Then, we can get the lth step transition matrix $P(l) = \left(P_{ij}^{(l)}\right)_{n\times n}$. $M_{ij}^{(l)}$ is the data of raw series transferring l step from the state Q_i to the state Q_j.

2.2.3 The forecasting value

The eventual forecast is in the center of the Gray zone, which is denoted as $Y'(k) = \frac{1}{2}(Q_{1i} + Q_{2i}) = \hat{Y}(k) = \frac{1}{2}(E_{1i} + E_{2i})$. Eventually, the forecasting sequence is obtained as $Y'(k) = \{Y'(1), Y'(2), \ldots, Y'(m)\}$.

3. Time series model (TSM)

Burg suggests that recursive algorithm estimated by the AR(P) model is the most practical one [29], while Hannan proposes time series with multidimensional linear stationary RMA(p, q). The times series model mainly includes the autoregressive model and the moving average model [30–32], and generally the modeling steps are as follows.

3.1 Preliminary analysis of data and modeling identification

Time series prediction is a statistical method processing dynamic data, which is a random sequence arranged in chronological order or a set of ordered random variables defined in probabilistic space $\{X_t, t = 1, 2, \ldots, n\}$, in which the parameter t represents time. In the TSM, if the samples' autocorrelation function $\{\hat{\rho}_k\}$ decreases to zero based on the negative exponential function, then it can be preliminarily judged that this sequence is a stationary autoregressive moving average model (ARMA). If the absolute value of the sample autocorrelation function in the q-step delay $\hat{\rho}_k(k \le q)$ is greater than twice of the standard deviation and the value of $\hat{\rho}_k(k > q)$ is less than twice of the standard deviation, then the sequence is q-step moving average model (MA(q)). In a similar vein, we can judge p-step autoregressive moving average model (AR(p)) according to the truncation situation of partial autocorrelation function $\{\hat{\varphi}_{kk}\}$.

3.2 Parameter estimation

In order to fit the TSM, we need to estimate the autoregressive coefficient φ_i, the moving average coefficient θ_i, the mean μ, and the variance σ_ε^2 of the white noise sequence in the ARMA model.

3.3 Diagnostic test

The purpose of diagnostic test is to check and test the rationality of the model, including residual test, autocorrelation function of residual error and partial autocorrelation function test, and the significance test of parameters in the model.

3.4 Optimal model selection

Model recognition is only a preliminary selection of TSM. Considering the actual observed errors and statistical errors, several models are taken as candidate models. And the most common methods of selecting optimal models include F-test method, criterion function method (AIC criterion, BIC criterion, SBC criterion).

4. Comparison of GMM and TSM

4.1 GMM predicting FDI of China

Take the FDI value of China over the period from 1990 to 2016 as the original data (unit, \$100 million; data source, Ministry of Commerce of the PRC):

$$X^{(0)} = \{34.87,\ 43.66,\ 110.08,\ \ldots,\ 1260\}$$

Based on Eq. (5) and using the software MATLAB, the least squares estimation (LSE) of FDI is as follows:

$$\Phi = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = \begin{pmatrix} -0.0697 \\ 243.795 \end{pmatrix}$$

Based on Eq. (7), time-response function can be written as $\hat{x}(k+1) = 3530.59e^{0.0697k} - 3495.72$. Residual values can be obtained according to relative error based on the prediction value of GM(1,1) model. To improve the predicting accuracy, the relative error can be divided into five states (E1, E2, E3, E4, E5) between 1990 and 2010. The relative error status can be seen in **Tables 2**.

According to the original FDI value over a period from 1990 to 2010 and the relative error of prediction value in GM(1,1), the transition matrixes of different steps $P_1^{(i)}(i = 1, 2, 3, 4, 5)$ are shown as follows:

$$P_1^{(1)} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} & 0 & \frac{1}{5} & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}, P_1^{(2)} = \begin{pmatrix} \frac{3}{5} & 0 & 0 & \frac{2}{5} & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{6} & 0 & \frac{1}{3} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{pmatrix}, P_1^{(3)} = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix},$$

$$P_1^{(4)} = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 \\ 1 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}, P_1^{(5)} = \begin{pmatrix} 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ \frac{2}{5} & 0 & \frac{1}{5} & \frac{2}{5} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Residual State	E1	E2	E3	E4	E5
Meaning	Extremely underestimated	Underestimated	Reasonable	Overestimated	Extremely overestimated
Range	[−0.17, −0.10]	[−0.10, 0.02]	[0.02, 0.07]	[0.07, 0.12]	[0.12, 0.83]

Table 1.
Relative error status of FDI level in China.

Year	Original	Relative error of GM	State	Year	Original	Relative error of GM	State
1990	34.87	0	E3	2001	468.78	0.0848	E4
1991	43.66	0.8288	E5	2002	527.43	0.0397	E3
1992	110.08	0.5974	E5	2003	535.05	0.0914	E4
1993	275.15	0.0615	E3	2004	606.30	0.0398	E3
1994	337.67	−0.0741	E2	2005	603.25	0.109	E4
1995	375.21	−0.1131	E1	2006	630.21	0.1318	E4
1996	417.26	−0.1545	E1	2007	747.68	0.0394	E3
1997	452.57	−0.1679	E1	2008	923.95	−0.1071	E1
1998	454.63	−0.0941	E1	2009	900.33	−0.0061	E2
1999	403.19	0.095	E4	2010	1057.35	−0.102	E1
2000	407.15	0.1477	E4	—	—	—	—

Data source: China Statistical Yearbook over the period from 2000 to 2006, Ministry of Commerce of the PRC.

Table 2.
Comparison of GM(1,1) prediction value and original value of FDI of China.

Based on the transition matrix, we can obtain the error state over a period from 2011 to 2016 (see **Table 3**). Taking the middle value of the error state to modify the prediction value of GM(1,1) model, then the modified value can be seen in **Table 3**. And $x^{(0)}(k)$, $\hat{x}^{(0)}(k)$, and $\phi(i)$ represent the original value, predicting value and relative error of GM(1,1). $\hat{x'}^{(0)}(k)$ and $\phi'(i)$ represent the modified value and relative error of GMM.

In the light of Eqs. (9)–(11), the relative residual error of GM(1,1) and GMM is $0.0584 < \alpha = 0.1$ and $0.0458 < \alpha = 0.05$, respectively. Therefore, the GM(1,1) model is barely qualified, and the modified GMM model is qualified. Gray correlation degrees of the two models are 67 and 79.9%, respectively. In summary, the

Year	$x^{(0)}(k)$	$\hat{x}^{(0)}(k)$	$\phi(i)$	$\hat{x'}^{(0)}(k)$	$\phi'(i)$
1990	34.87	0.0349	0	33.4752	−0.0471
1991	43.66	0.2550	0.8288	127.5082	0.6739
1992	110.08	0.2734	0.5974	136.7182	0.2332
1993	275.15	0.2932	0.0615	281.4594	0.0173
1994	337.67	0.3144	−0.0741	326.9385	−0.0328
1995	375.21	0.3371	−0.1131	379.20437	0.0193
1996	417.26	0.3614	−0.1545	406.5945	−0.0172
1997	452.57	0.3875	−0.1679	435.9630	−0.0289
1998	454.63	0.4155	−0.0941	467.4528	0.0360
1999	403.19	0.4455	0.0950	392.0632	0.0000
2000	407.15	0.4777	0.1477	420.3821	0.0582
2001	468.78	0.5122	0.0848	450.7465	−0.0113
2002	527.43	0.5492	0.0397	527.2409	−0.0056
2003	535.05	0.5889	0.0914	518.2134	−0.0040
2004	606.30	0.6314	0.0398	606.1574	−0.0055
2005	603.25	0.6770	0.1090	595.7787	0.0154
2006	630.21	0.7259	0.1318	638.8121	0.0407
2007	747.68	0.7784	0.0394	747.2223	−0.0059
2008	923.95	0.8346	−0.1071	938.8998	0.0246
2009	900.33	0.8949	−0.0061	930.6540	0.0326
2010	1057.35	0.9595	−0.1020	1079.4327	0.0291
2011	1160.11	1.0288	−0.1276	1157.4006	0.0065
2012	1117.20	1.1031	−0.0128	1241.0002	0.1077
2013	1175.90	1.1828	0.0058	1330.6383	0.1241
2014	1195.60	1.2682	0.0573	1116.0363	−0.0417
2015	1262.70	1.3598	0.0714	1196.648	−0.0260
2016	1260.00	1.4580	0.1358	1283.0826	0.0451

Annotations: The unit of $x^{(0)}(k)$ and $\hat{x'}^{(0)}(k)$ is 1 billion dollars. The unit of $\hat{x}^{(0)}(k)$ is 10^3 billion dollars. Data source: China Statistical Yearbook.

Table 3.
Residual checklist of Markov model and GM(1,1).

prediction accuracy of GMM has been improved, and its fitting effect exceeds the model of GM(1,1).

4.2 TSM predicting FDI of China

Now we will build a TSM based on the FDI value of China over the period from 1990 to 2016, obtain the predicting data, compare the difference between the predicted data and the original date, and evaluate the accuracy of this model.

Figure 1 shows the changing tendency of FDI in China over the period between 1990 and 2016. The raw data series show the seasonal change and overall growth, but the data series are not stable. Through the seasonal difference method to process the data, the seasonal difference order of three was selected. After the differential processing, the data sequence has been stabilized, eliminating the growing trend (**Figure 2**).

We determine the order of TSM based on sample autocorrelation function and partial autocorrelation function. After the one-step delay, the sample autocorrelation function falls to a standard error of twice times and has the property of truncation. After the two-step delay, the sample partial autocorrelation function falls to a standard error of twice times and has the property of truncation.

In the light of the calculation of SAS software, now we compare the model of ARMA(2,1), AR(2), and MA(1) (see **Tables 4** and **5**).

Comparing the AIC and SBC values for ARMA(2,1), AR(2), and MA(1) models (see **Table 4**), we find the model MA(1) to be the most inferior. Considering the AIC and SBC criterion values of ARMA(2,1) and AR(2) and the significance of parameters, it is found that fitting effect of the AR(2) model is the best.

As shown in **Table 5**, the P-value (Pr > ChiSq) for self-correlation test of the residual sequence with the 6-step delay, the 12-step delay and the 18-step delay are greater than that of the significant level $\alpha = 0.1$. Therefore, we cannot reject the hypothesis that residuals are non-autocorrelated. That is to say, the residual is regarded as a white noise sequence. This illustrates that the AR(2) model has extracted sufficient information from the raw series and it is a rational model:

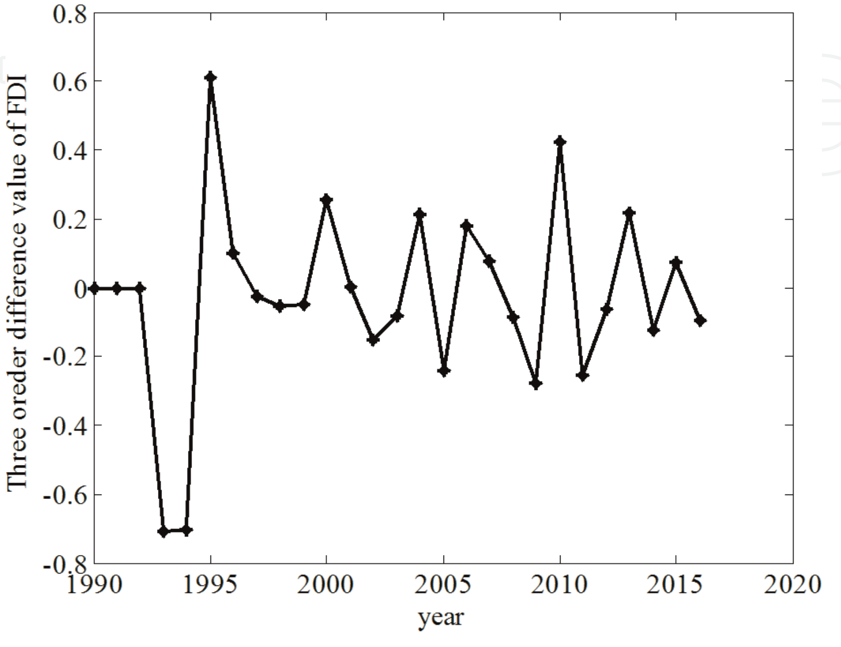

Figure 2.
The curve about time after differential.

Model	Parameter	Estimate	P-value	AIC	SBC
AR(2)	MU	2.0711	<0.0001***	1.0603	4.5944
	AR1,1	1.5357	<0.0001***		
	AR1,2	−0.5392	0.0102**		
MA(1)	MU	0.4676	0.0038***	28.7297	31.08558
	MA1,1	−0.7099	0.0001***		
ARMA(2,1)	MU	1.9380	<0.0001***	0.7062	5.4184
	MA1,1	−0.4940	0.1192		
	AR1,1	1.2352	0.0015***		
	AR1,2	−0.2358	0.5028		

*Annotations: ***, **, and * indicate a significant level of 0.01, 0.05, and 0.1, respectively.*

Table 4.
Prediction results of TSM.

To lag	6	12	18
Chi-square	3.91	5.44	8.02
Pr > ChiSq	0.4187	0.8599	0.9481

Table 5.
Self-correlation test of AR(2) model.

$$(1 - 1.53571B + 0.53921B^2)(1 - B^3)X_t = \varepsilon_t$$

where $X = num - 2.0711$, $t = year$, and num represents the FDI value of the corresponding year.

4.3 Comparison of prediction results of two models

4.3.1 Accuracy assessment

Regarding how to select the appropriate accuracy evaluation criteria, Yokuma and Armstrong [33] have done a survey of expert opinions. They think that accuracy, clear physical meaning, and being easy to implement can be the critical evaluation criteria [33]. Accordingly, three criteria are used to evaluate the accuracy of the prediction model.

Criterion	Mean squared error	Mean absolute error	Mean absolute percentage error
Index	$MSE = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{x}_i)^2$	$MAE = \frac{1}{n}\sum_{i=1}^{n}\lvert x_i - \hat{x}_i\rvert$	$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left\lvert\frac{x_i - \hat{x}_i}{x_i}\right\rvert$
GM(1,1)	7.3991e+03	66.0812	0.3117
GMM	2.4731e+03	29.5558	0.1181
Time series	1.6644e+04	85.6101	0.1448

Annotations: $\hat{x}_i$ is the predicting value, x_i is the original value, and n is the predicting number.

Table 6.
Three criteria to evaluate the accuracy of models.

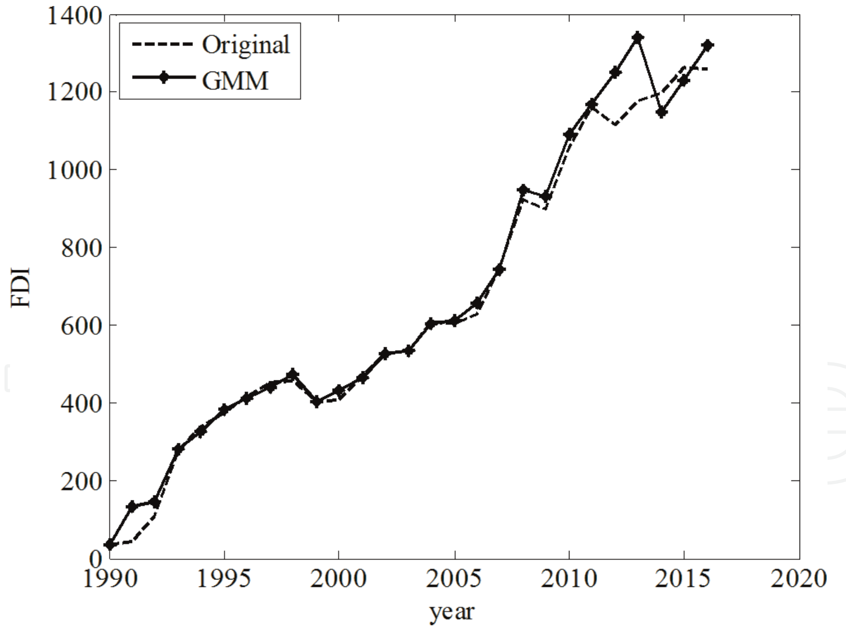

Figure 3.
Original value and that in GMM.

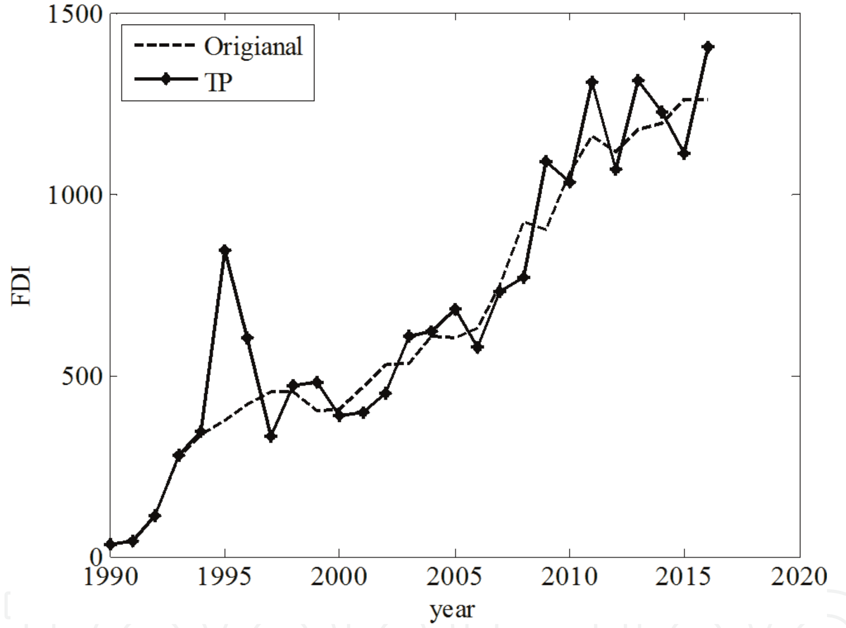

Figure 4.
Original value and that in TSM.

4.3.2 Comparing predicted values with actual values

As shown in **Table 6**, the prediction accuracy of GMM has been improved manifestly compared with that in GM(1,1) model. Therefore, the forecasting value in GMM is closer to the actual level of China's FDI. Then, from **Figures 3** and **4**, we can clearly see that GMM model has a better fitting effect than that in TSM.

5. Empirical analysis of FDI in Chongqing and Beijing

From discussions above, it is found that GMM has higher prediction accuracy and better fitting effects than those of TSM of Chinese FDI level. To further verify the credibility of this result, we construct GMM and TSM based on the FDI level of

Beijing (1990–2016) and Chongqing (1990–2015). The divided states involved in the GMM are shown in **Table 7**, and the transition matrixes of GMM associated with Beijing and Chongqing are denoted as P_2 and P_3. For simplification, we only list the form of transition matrix P_2. The comparison of GM(1,1) and GMM can be seen in **Tables 8** and **9**. The average relative errors of GM(1,1) and GMM of Beijing (Chongqing) are 0.0312 (0.5285) and −0.0029 (−0.1051), respectively. The Gray

Area	Error state	E1	E2	E3	E4	E5
Beijing	Range	[−0.47, −0.2]	[−0.2, −0.1]	[−0.1, 0.1]	[0.1, 0.28]	[0.28, 0.65]
Chongqing	Range	[0, 0.28]	[0.28, 0.55]	[0.55, 0.75]	[0.75, 0.81]	[0.81,0.97]

Table 7.
Residual states of FDI in Beijing and Chongqing.

Year	Original value	GM(1,1)	GMM	TSM	To state	GR	MR
1990	27,696	0.0277	0.0277	0.0277	E3	0	0
1991	24,482	0.0693	0.0371	0.0245	E5	0.6466	0.3395
1992	34,984	0.0780	0.0417	0.0364	E5	0.5514	0.1614
1993	66,693	0.0878	0.0711	0.0311	E4	0.2401	0.0618
1994	144,460	0.0988	0.1121	0.0863	E1	−0.4625	−0.2885
1995	140,277	0.1112	0.1262	0.1342	E1	−0.2617	−0.1117
1996	155,290	0.1251	0.1420	0.1969	E1	−0.2410	−0.0934
1997	159,286	0.1408	0.1620	0.1514	E2	−0.1310	0.0165
1998	216,800	0.1585	0.1799	0.2127	E1	−0.3677	−0.2050
1999	197,525	0.1784	0.2052	0.2126	E2	−0.1071	0.0373
2000	168,368	0.2008	0.1626	0.2680	E4	0.1615	−0.0352
2001	176,818	0.2260	0.1831	0.1767	E4	0.2176	0.0341
2002	172,464	0.2544	0.1361	0.2213	E5	0.3220	−0.2673
2003	219,126	0.2863	0.2319	0.1890	E4	0.2346	0.0551
2004	255,974	0.3222	0.2610	0.2560	E4	0.2056	0.0193
2005	352,834	0.3627	0.3627	0.2878	E3	0.0271	0.0271
2006	455,191	0.4082	0.4694	0.3979	E2	−0.1151	0.0303
2007	506,572	0.4594	0.5283	0.5179	E2	−0.1026	0.0412
2008	608,172	0.5171	0.5947	0.5870	E2	−0.1761	−0.0227
2009	612,094	0.5820	0.5820	0.6851	E3	−0.0517	−0.0517
2010	636,358	0.6550	0.6550	0.7277	E3	0.0285	0.0285
2011	705,447	0.7373	0.8368	0.7195	E5	0.0431	0.1570
2012	804,160	0.8298	0.6721	0.8222	E4	0.0309	−0.1964
2013	852,418	0.9339	0.7565	0.9097	E4	0.0873	−0.1268
2014	904,085	1.0512	1.0512	1.0009	E3	0.1399	0.1399
2015	1,299,635	1.1831	1.3428	1.0292	E1	−0.0985	0.0322
2016	1,302,858	1.3316	1.5114	1.4400	E1	0.0216	0.1380

*Annotations: In **Table 8**, the unit of original value is 1 million dollars. GM, GMM, and TSM represent the predicted value of Gray model, Gray-Markov model, and time series model, and their unit is 10^6 million dollars. GR and MR represent the residuals of Gray model and Gray-Markov model. Data source: Beijing Statistical Yearbook (1990–2017), Beijing Municipal Bureau of Statistics.*

Table 8.
Comparison of predicted errors of GMM and GM(1,1) of Beijing FDI level.

relational degrees of GM(1,1) and GMM of Beijing (Chongqing) are 64.62% (75.26%) and 79.39% (86.82%), respectively. Therefore, the errors of GM(1,1) and GMM are barely qualified or qualified, and hence GMM is superior to GM(1,1):

$$P_1^{(1)} = \begin{pmatrix} \frac{2}{4} & \frac{4}{4} & 0 & 0 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{5} & 0 & \frac{1}{5} & \frac{2}{5} & \frac{1}{5} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}, P_1^{(2)} = \begin{pmatrix} \frac{2}{4} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{2}{5} & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \end{pmatrix}, P_1^{(3)} = \begin{pmatrix} \frac{1}{4} & \frac{2}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{2}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{5} & \frac{2}{5} & 0 & \frac{2}{5} & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \end{pmatrix},$$

$$P_1^{(4)} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 \end{pmatrix}, P_1^{(5)} = \begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{5} & \frac{2}{5} & \frac{2}{5} & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 \end{pmatrix}$$

Year	Original value	GM(1,1)	GMM	TSM	To state	GR	MR
1990	332	0.0033	0.0029	0.0003	E1	0	−0.1628
1991	977	0.3159	0.0347	0.0010	E5	0.9691	0.7188
1992	10,247	0.3647	0.1276	0.0029	E3	0.7190	0.1972
1993	25,915	0.4210	0.2463	0.0846	E2	0.3844	−0.0523
1994	44,953	0.4860	0.4180	0.0686	E1	0.0751	−0.0755
1995	37,926	0.5611	0.3282	0.0842	E2	0.3241	−0.1554
1996	21,878	0.6478	0.2267	0.0406	E3	0.6623	0.0350
1997	38,466	0.7478	0.4375	0.0165	E2	0.4856	0.1207
1998	43,107	0.8633	0.5050	0.0755	E2	0.5007	0.1465
1999	23,893	0.9967	0.2193	0.0563	E4	0.7603	−0.0897
2000	24,436	1.1506	0.2531	0.0181	E4	0.7876	0.0347
2001	25,649	1.3284	0.2922	0.0296	E4	0.8069	0.1223
2002	28,089	1.5335	0.1687	0.0329	E5	0.8168	−0.6651
2003	31,112	1.7704	0.1947	0.0360	E5	0.8243	−0.5976
2004	40,508	2.0439	0.4497	0.0417	E4	0.8018	0.0991
2005	51,575	2.3596	0.5191	0.0599	E4	0.7814	0.0065
2006	69,595	2.7241	0.9534	0.0776	E3	0.7445	0.2700
2007	108,534	3.1448	1.1007	0.1059	E3	0.6549	0.0139
2008	272,913	3.6306	3.1223	0.1930	E1	0.2483	0.1259

Year	Original value	GM(1,1)	GMM	TSM	To state	GR	MR
2009	401,643	4.1914	3.6046	0.6941	E1	0.0417	−0.1143
2010	304,264	4.8388	2.8307	0.6809	E2	0.3712	−0.0749
2011	582,575	5.5862	3.2679	0.2878	E2	−0.0429	−0.7827
2012	352,418	6.4490	3.7727	1.2269	E2	0.4535	0.0659
2013	414,353	7.4452	4.3554	0.2769	E2	0.4435	0.0487
2014	423,348	8.5952	1.8909	0.5837	E4	0.5075	−1.2388
2015	377,183	9.9228	2.1830	0.5124	E4	0.6199	−0.7278
2016	332	0.0033	0.0029	0.0003	E1	0	−0.1628

Annotations: In ***Table 9****, the unit of original value is 1 million dollars. GM, GMM, and TSM represent the predicted value of Gray model, Gray-Markov model, and time series model, and their unit is* 10^5 *million dollars. GR and MR represent the residuals of Gray Model and Gray-Markov model. Data source: Chongqing Statistical Yearbook (1990–2016), Beijing Municipal Bureau of Statistics.*

Table 9.
Comparison of predicted errors of GMM and GM(1,1) of Chongqing FDI level.

Similar to Section 4.2, TSM of Beijing FDI can be modeled as MA (1):

$$(1 + 0.82673\mathrm{B})\left(1 - \mathrm{B}^2\right)\mathrm{X}_t = \varepsilon_t$$

where $X = num - 0.27264$ and $t = year$.
TSM of Chongqing FDI can be modeled as ARMA(1,2,1):

$$(1 - 0.82442\mathrm{B})(1 + \mathrm{B})\left(1 - \mathrm{B}^2\right)\mathrm{X}_t = \varepsilon_t$$

where $X = num - 2.178452$ and $t = year$.
Figure 5 (**Figure 6**) shows the difference between the original value and the predicting value in Gray-Markov model (time series model) of foreign direct investment in Beijing. It is apparent that the fitting effect of GMM is better than

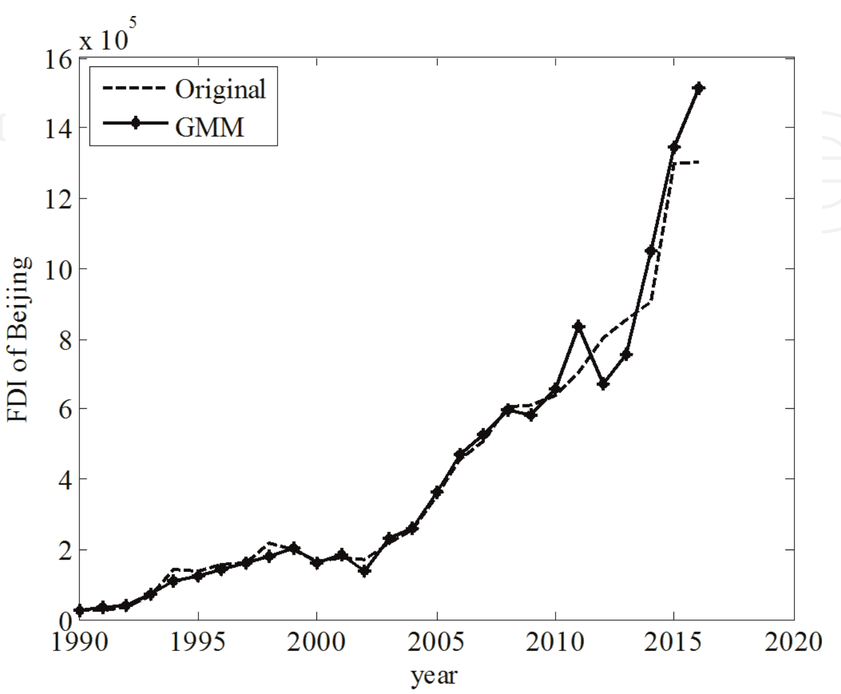

Figure 5.
Original value and that in GMM of BJ. Annotations: BJ denotes the city of Beijing.

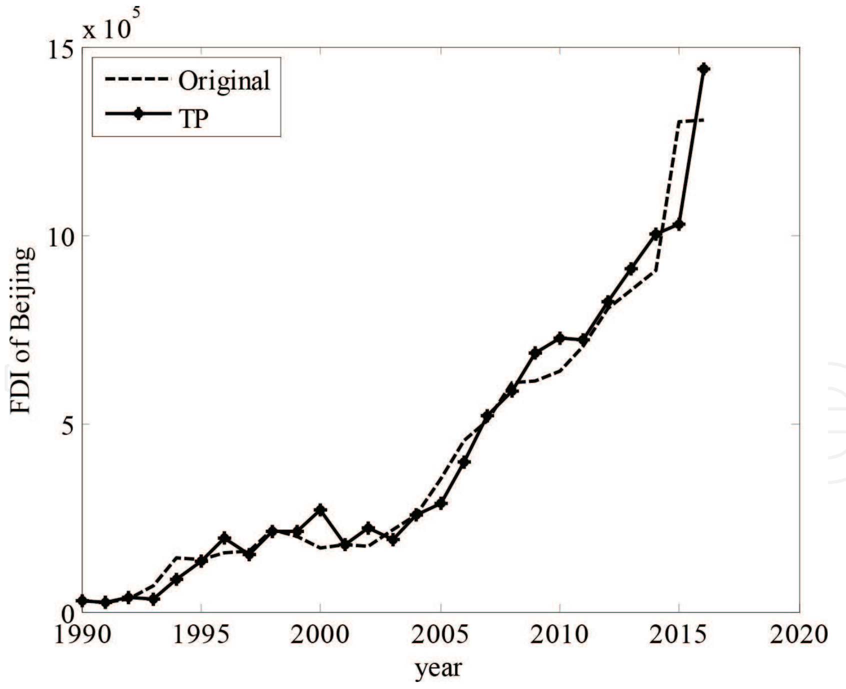

Figure 6.
Original value and that in TSM of BJ. Annotations: BJ denotes the city of Beijing.

that of TSM. The similar conclusion can be drawn from **Figures** 7 and **8**.
Tables 9 and **10** show the predicting effect of GMM is better than that of TSM from the point of predicting errors and accuracy. There is no doubt that it is a good thing to predict accurately the foreign direct investment of the forthcoming 5 or 10 years for the domain specialists. Because if the predicting results is lower or higher than they expected, they could pay attention to seeking the critical factors and policy which have impacts on FDI and adjust them in advance.

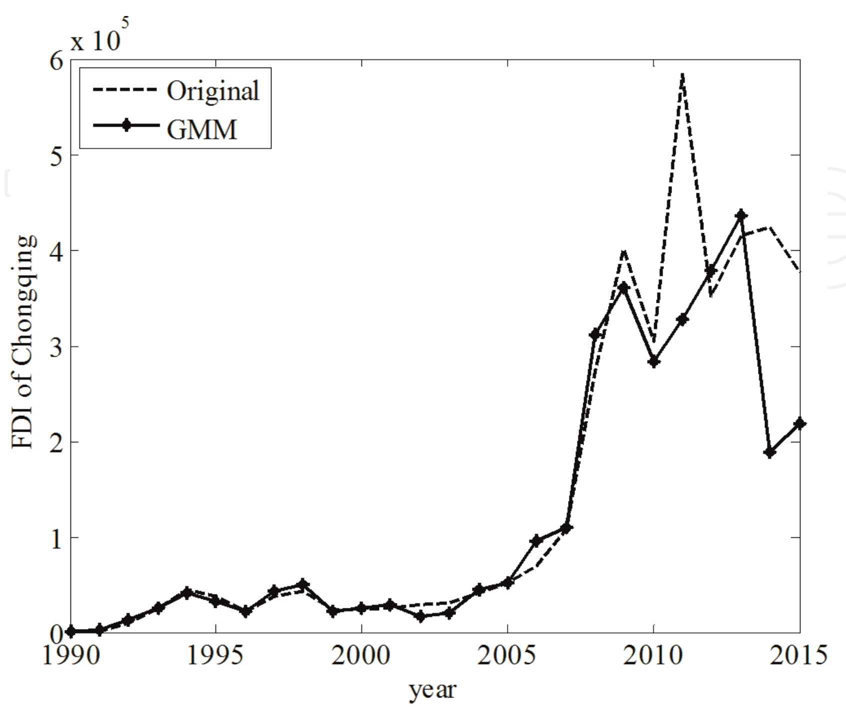

Figure 7.
Original value and that in GMM of CQ. Annotations: CQ denotes the city of Chongqing.

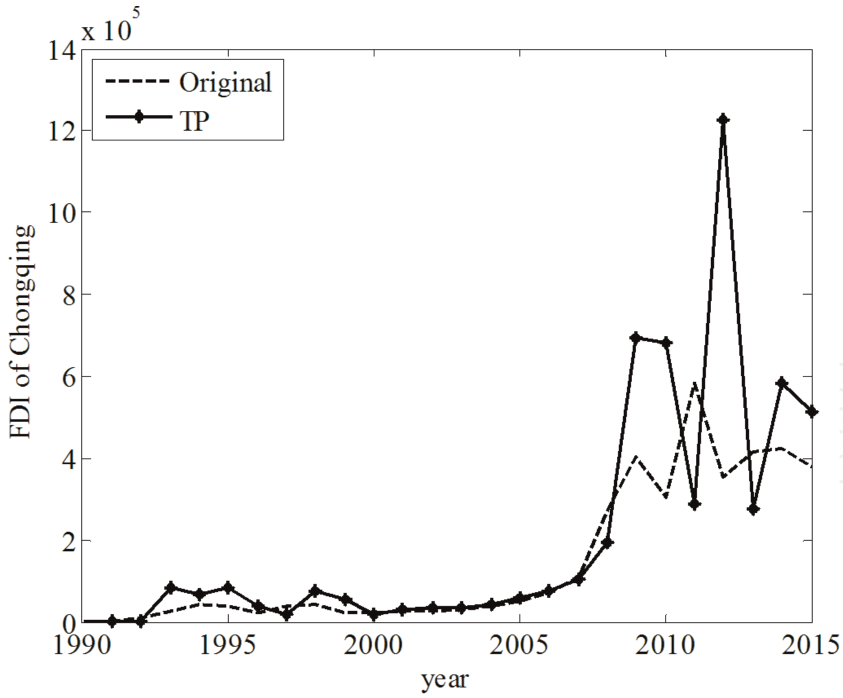

Figure 8.
Original value and that in TSM of CQ. Annotations: CQ denotes the city of Chongqing.

Area	Index	$MSE = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{x}_i)^2$	$MAE = \frac{1}{n}\sum_{i=1}^{n}\lvert x_i - \hat{x}_i\rvert$	$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left\lvert\frac{x_i - \hat{x}_i}{x_i}\right\rvert$
Beijing	GM(1,1)	3.3244e+09	4.7473e+04	0.2587
	GMM	4.3599e+09	3.9369e+04	0.1032
	TSM	5.4378e+09	4.6731e+04	0.1335
Chongqing	GM(1,1)	3.9173e+10	1.3478e+05	2.9584
	GMM	5.8230e+09	3.4080e+04	0.2663
	TSM	4.4524e+10	1.0126e+05	0.6018

Annotations: MSE, MAE, and MAPE denote mean squared error, mean absolute error, and mean absolute percentage error.

Table 10.
Comparison of predicting accuracy of two models.

6. Conclusions and future work

Our contributions are threefold. Firstly, comparing the predicting results of the Gray-Markov model and the time series model and the original value, respectively, we can find that the fitting effect of the former (GMM) is better than the latter (TSM) and so does its scientific and practical importance. Secondly, the predicting results of GMM show that the level of foreign investment in China has been increasing by years. Thirdly, in order to further enhance Chinese international status and attract more foreign investment, the government should play a role at a macro level to reduce excessive market administrative intervention, establish a service-oriented government, and reduce the relevant approval procedures for international investment.

In the future work, the Gray-Markov model and time series model can be combined with other predicting model (e.g., support vector machine and dynamic Bayesian) to improve the accuracy. Also these models have the potential to be applied in the other areas such as finance (e.g., stocks, funds, and security), risk (e.g., financial risk and operational risk), and business (e.g., consumer price index and incomes).

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