Chapter

Fuzzy Forecast Based on Fuzzy Time Series

Ming-Tao Chou

Abstract

This chapter mainly uses fuzzy time series for interval prediction and long-term significance level analysis. In this study, the Taiwan Shipping and Transportation Index (Taiwan STI) is used to illustrate the prediction process. Nine steps have been used to establish the interval prediction of the Taiwan Shipping and Transportation Index (Taiwan STI), and ΔS is called a long-term significance level (up/down/stable) is used to illustrate the long-term prediction significance level. By means of interval prediction and long-term prediction significance level, the future trends for this index and more internal messages related to this index can be provided to relevant researchers.

Keywords: fuzzy time series, interval prediction, long-term prediction significance level

1. Introduction

In 1965, Zadeh [1] proposed the concept of fuzzy sets as a tool to test the unknown degree of membership. Many fuzzy studies then attempted to use this method as a theoretical framework, which is widely used in the research fields of natural sciences and social sciences, obtaining good study achievements [2–22]. The fuzzy time series is also an analysis method derived from the concept of fuzzy sets. In 1993, Song and Chissom [18–21] successfully combined the concept of fuzzy sets with the time series model and began studies on fuzzy time series. Chen [3] proposed the simplified and easy-to-calculate method for Song and Chissom's model [18–21], so that the computation complexity of fuzzy time series is dramatically reduced. Lee and Chou [14] also proposed that rational settings of the lower and upper boundary in intervals of the universal set for fuzzy time series have improved their accuracy and reliability. Liaw [17] proposed a simple test method for whether a fuzzy time series has a fuzzy trend, in which the method is used to determine whether the data for analysis is in a steady state. Chou [12] added to Chen and Hsieh's defuzzification method [2] in the fuzzy time series, so that the long-term level for the series can be obtained, and the model originally used for single-point prediction can be applied to long-term prediction and interval prediction. This article mainly uses the algorithmic method of Chou's [12] research process for illustrating the fuzzy time series, taking the Taiwan Shipping and Transportation Index (STI) [23] as an example.

The remainder of this chapter is organized as follows. Section 2 presents the definition of fuzzy time series and Section 3 defines the long-term predictive significance level process. A numerical example of STI is shown in Section 4, and concluding remarks are mentioned in conclusion.

2. Definition of fuzzy time series

Fuzzy sets, presented by Zadeh [1], have numerous presentations, such as in fuzzy sets, fuzzy decision analysis, and fuzzy time series. The concept is also widely applied in social science article and applications [2–22]. Fuzzy time series is developed rapidly since their introduction by Song and Chissom [18–21]. Current fuzzy time series methods have benefited from both theoretical developments as well as relevant applications in research [2–22], which has led to more diverse uses. This trend indicates that the development of fuzzy time series has markedly improved. The definitions of the fuzzy time series used in this article are described as follows.

Definition 1 [18–21]. A fuzzy number on the real line $\mathfrak{R}$ is a fuzzy subset of $\mathfrak{R}$ that is normal and convex.

Definition 2 [18–21]. Let $Y(t)(t = \dots, 0, 1, 2, \dots)$, a subset of $\mathfrak{R}$, be the universe of discourse on which the fuzzy sets $f_i(t)(t = 1, 2, \dots)$ are defined, and let $F(t)$ be the collection of $f_i(t)(t = 1, 2, \dots)$. Then, $F(t)$ is called fuzzy time series on $Y(t)(t = \dots, 0, 1, 2, \dots)$.

Definition 3 [18–21]. Let I and J be the index sets for $F(t-1)$ and $F(t)$, respectively. If for any $f_j(t) \in F(t)$, where $j \in J$, there then exists $f_i(t-1) \in F(t-1)$, where $i \in I$, such that there exists a fuzzy relation $R_{ij}(t, t-1)$ and $f_j(t) = f_i(t-1) \circ R_{ij}(t, t-1)$, where '$\circ$' is the max–min composition. Then, $F(t)$ is said to be caused by only $F(t-1)$. Denote this as $f_i(t-1) \to f_j(t)$, or equivalently, $F(t-1) \to F(t)$.

Definition 4 [18–21]. If, for any $f_j(t-1) \in F(t)$, where $j \in J$, there exists $f_i(t-1) \in F(t-1)$, where $i \in I$, and a fuzzy relation $R_{ij}(t, t-1)$, such that $f_j(t) = f_i(t-1) \circ R_{ij}(t, t-1)$. Let $R(t, t-1) = \cup_{ij} R_{ij}(t, t-1)$, where $\cup$ is the union operator. Then, $R(t, t-1)$ is called the fuzzy relation between $F(t)$ and $F(t-1)$. Thus, we define this as the following fuzzy relational equation:

$$F(t) = F(t-1) \circ R(t, t-1).$$

Definition 5 [18–21]. Suppose that $R_1(t, t-1) = \cup_{ij} R^1{}_{ij}(t, t-1)$ and $R_2(t, t-1) = \cup_{ij} R^2{}_{ij}(t, t-1)$ are two fuzzy relations between $F(t)$ and $F(t-1)$. If, for any $f_j(t) \in F(t)$, where $j \in J$, there exists $f_i(t-1) \in F(t-1)$, where $i \in I$, and fuzzy relations $R^1{}_{ij}(t, t-1)$ and $R^2{}_{ij}(t, t-1)$ such that $f_i(t) = f_i(t-1) \circ R^1_{ij}(t, t-1)$ and $f_i(t) = f_i(t-1) \circ R^2_{ij}(t, t-1)$, then define $R_1(t, t-1) = R_2(t, t-1)$.

Definition 6 [18–21]. Suppose that $F(t)$ is only caused by $F(t-1)$, $F(t-2)$, ..., or $F(t-m)\ \ (m > 0)$. This relation can be expressed as the following fuzzy relational equation: $F(t) = F(t-1) \circ R_0(t, t-m)$, which is called the first-order model of $F(t)$.

Definition 7 [18–21]. Suppose that $F(t)$ is simultaneously caused by $F(t-1)$, $F(t-2)$, ..., and $F(t-m)\ \ (m > 0)$. This relation can be expressed as the following fuzzy relational equation: $F(t) = (F(t-1) \times F(t-2) \times \dots \times F(t-m)) \circ R_a(t, t-m)$, which is called the m^{th}-order model of $F(t)$.

Definition 8 [3]. $F(t)$ is fuzzy time series if $F(t)$ is a fuzzy set. The transition is denoted as $F(t-1) \to F(t)$.

Definition 9 [7]. Let $d(t)$ be a set of real numbers: $d(t) \subseteq R$. We define an exponential function where

1. $y = \exp d(t) \Leftrightarrow \ln y = d(t)$ and

2. $\exp(\ln d(t)) = d(t), \ln(\exp x) = d(t)$.

Definition 10 [14]. The universe of discourse $U = [D_L, D_U]$ is defined such that $D_L = D_{\min} - st_\alpha(n)/\sqrt{n}$ and $D_U = D_{\max} + st_\alpha(n)/\sqrt{n}$ when $n \le 30$ or $D_L = D_{\min} - \sigma Z_\alpha/\sqrt{n}$ and $D_U = D_{\max} + \sigma Z_\alpha/\sqrt{n}$ when $n > 30$, where $t_\alpha(n)$ is the $100(1-\alpha)$ percentile of the t distribution with n degrees of freedom. z_α is the $100(1-\alpha)$ percentile of the standard normal distribution. Briefly, if Z is an $N(0, 1)$ distribution, then $P(Z \ge z_\alpha) = \alpha$.

Definition 11 [14]. Assuming that there are m linguistic values under consideration, let A_i be the fuzzy number that represents the i^{th} linguistic value of the linguistic variable, where $1 \le i \le m$. The support of A_i is defined as follows:

$$\begin{cases} D_L + (i-1)\dfrac{D_U - D_L}{m}, \quad D_L + \dfrac{i(D_U - D_L)}{m}, 1 \le i \le m-1 \\ D_L + (i-1)\dfrac{D_U - D_L}{m}, \quad D_L + \dfrac{i(D_U - D_L)}{m}, i = m. \end{cases}$$

Definition 12 [17]. For a test H_0 : *nonfuzzy trend* against H_1 : *fuzzy trend*, where the critical region $C^* = \left\{C \middle| C_2^k + C_2^{n-k} > C_\lambda = C_2^n \times (1-\lambda)\right\}$, the initial value of the significance level α is 0.2.

Definition 13 [8]. Let $d(t)$ be a set of real numbers $d(t) \subseteq R$. An upper interval for $d(t)$ is a number b such that $x \le b$ for all $x \in d(t)$. The set $d(t)$ is said to be an interval higher if $d(t)$ has an upper interval. A number, max, is the maximum of $d(t)$ if max is an upper interval for $d(t)$ and $\max \in d(t)$.

Definition 14 [8]. Let $d(t) \subseteq R$. The least upper interval of $d(t)$ is a number $\overrightarrow{\max}$ satisfying:

1. $\overrightarrow{\max}$ is an upper interval for $d(t)$ such that $x \le \overrightarrow{\max}$ for all $x \in d(t)$ and

2. $\overrightarrow{\max}$ is the least upper interval for $d(t)$, that is, $x \le b$ for all $x \in d(t) \Rightarrow \overrightarrow{\max} \le b$.

Definition 15 [8]. Let $d(t)$ be a set of real numbers $d(t) \subseteq R$. A lower interval for $d(t)$ is a number b such that $x \ge b$ for all $x \in d(t)$. The set $d(t)$ is said to be an interval below if $d(t)$ has a lower interval. A number, min, is the minimum of $d(t)$ if min is a lower interval for $d(t)$ and $\min \in d(t)$.

Definition 16 [8]. Let $d(t) \subseteq R$. The least lower interval of $d(t)$ is a number $\overleftarrow{\min}$ satisfying:

1. $\overleftarrow{\min}$ is a lower interval for $d(t)$ such that $x \ge \overleftarrow{\min}$ for all $x \in d(t)$ and.

2. $\overleftarrow{\min}$ is the least lower interval for $d(t)$, that is, $x \ge b$ for all $x \in d(t) \Rightarrow \overleftarrow{\min} \le b$.

Definition 17 [8]. The long-term predictive value interval $(\overleftarrow{\min}, \overrightarrow{\max})$ is called the static long-term predictive value interval.

Definition 18 [2]. Let $A_i = (\alpha_i, \ \beta_i, \ \gamma_i), i = 1, 2, \ldots, n,$ be n triangular fuzzy numbers. By using the graded mean integration representation (GMIR) method, the GMIR value $P(A_i)$ of A_i is $P(A_i) = (\alpha_i + 4\beta_i + \gamma_i)/6$. $P(A_i)$ and $P(A_j)$ are the GMIR values of the triangular fuzzy numbers A_i and A_j, respectively.

Definition 19 [12]. Set up new triangular fuzzy numbers by S = ($\overleftarrow{\min}$, $\hat{d}(t)$, $\overrightarrow{\max}$). After GMIR transformation, S becomes a real number ΔS. This is called the long-term significance level with fuzzy time series. The ΔS is a real number satisfying the following:

1. ΔS is called a long-term significance level up, only if: ΔS > $\hat{d}(t)$;

2. ΔS is called a long-term significance level down, only if: ΔS < $\hat{d}(t)$; and

3. ΔS is called a long-term significance level stable, only if: ΔS = $\hat{d}(t)$.

3. Procedure of fuzzy time series forecasting

This section proposes a method to forecast the long-term predictive significance level by Chou. The stepwise procedure of the proposed method consists the following steps [8], illustrated as a flowchart in **Figure 1** [5–12].

Step 1. Let $d(t)$ be the data under consideration and let $F(t)$ be fuzzy time series. Following Definition 11, a difference test is performed to determine whether stability of the information. Recursion is performed until the information is in a stable state, where the critical region is $C^* = \left\{C \middle| C_2^k + C_2^{n-k} > C_\lambda = C_2^n \times (1-\lambda)\right\}$.

Step 2. Determine the universe of discourse $U = [D_L, D_U]$.

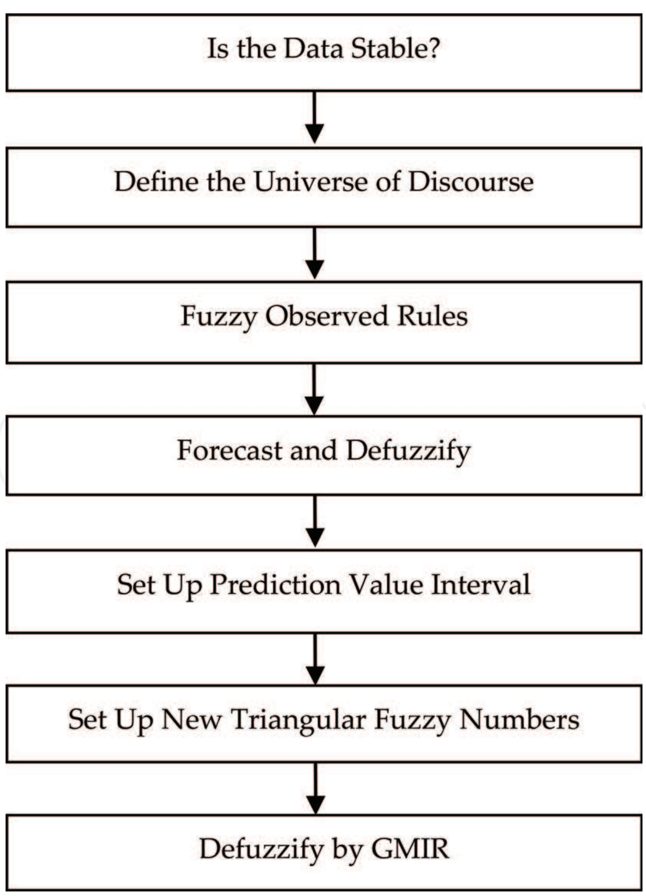

Figure 1.
Procedure of the proposed model.

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Step 3. Define A_i by letting its membership function be as follows:

$$u_{A_i}(x) = \begin{cases} 1 \ \text{ for } x \in \left[D_L + (i-1)\dfrac{D_U - D_L}{m}, D_L + \dfrac{i(D_U - D_L)}{m}\right) \\ \text{where } 1 \le i \le m-1; \\ 1 \ \text{ for } x \in \left[D_L + (i-1)\dfrac{D_U - D_L}{m}, D_L + \dfrac{i(D_U - D_L)}{m}\right] \\ \text{where } i = m; \\ 0 \ \text{ otherwise.} \end{cases}$$

Step 4. Then, $F(t) = A_i$ if $d(t) \in \mathrm{supp}(A_i)$, where $\mathrm{supp}(\cdot)$ denotes the support.

Step 5. Derive the transition rule from period $t-1$ to t and denote it as $F(t-1) \to F(t)$. Aggregate all transition rules. Let the set of rules be $R = \{r_i | r_i : P_i \to Q_i\}$.

Step 6. The value of $d(t)$ can be predicted using the fuzzy time series $F(t)$ as follows. Let $T(t) = \{r_j | d(t) \in \mathrm{supp}(P_j), \textit{where } r_j \in R\}$ be the set of rules fired by $d(t)$, where $\mathrm{supp}(P_j)$ is the support of P_j. Let $\overline{\mathrm{supp}(P_j)}$ be the median of $\mathrm{supp}(P_j)$. The predicted value of $d(t)$ is $\sum_{r_j \in T(t-1)} \frac{\overline{\mathrm{supp}(Q_j)}}{|T(t-1)|}$.

Step 7. The long-term predictive value interval for $d(t)$ is given as $(\overleftarrow{\min}, \overrightarrow{\max})$.

Step 8. Set up new triangular fuzzy numbers by $\Delta S = (\overleftarrow{\min}, \hat{d}(t) \overrightarrow{\max})$.

Step 9. Defuzzify S to be ΔS.

4. Numerical example of Shipping and Transportation Index in Taiwan

In this study, the Shipping and Transportation Index (STI) in Taiwan is used for a numerical example. The STI reflects the spot rates of the Taiwan Stock Exchange Corporation. The STI data are sourced from the Taiwan Stock Exchange Corporation [23], the historical data for which is defined here as the STI, and month-averaged data for the period between January, 2015, and June, 2018, was collected.

Over these 42 data points, the analysis produces an average of 4.226, with a standard deviation of 0.172, maximum value of 4.571, and minimum value of 4.067. These descriptive statistics show that the STI has largely remained at the 1124.70 level. As shown in **Figure 2**, its current rate of return is negative.

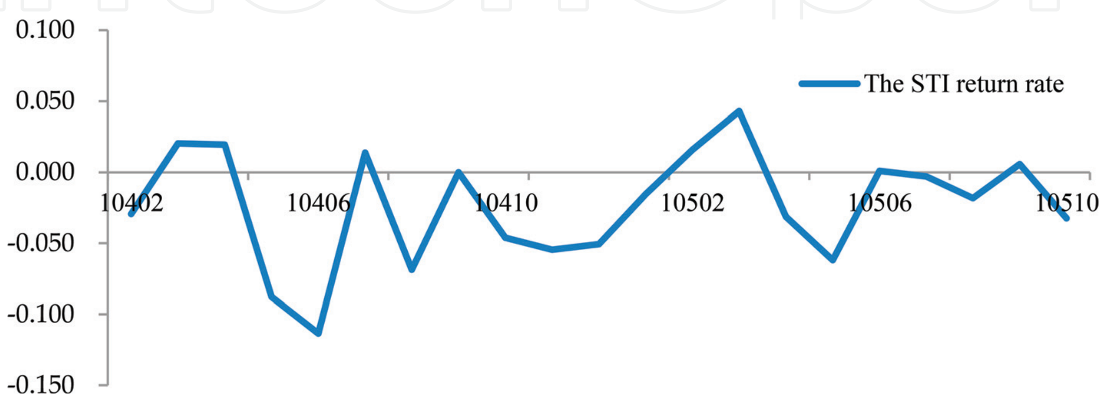

Figure 2.
Rate of return of the STI.

The following steps in the procedure are performed when using fuzzy time series to analyze STI.

Step 1. First, we take the logarithm of the STI data to reduce variation and improve the forecast accuracy, letting $S\mathrm{TI}(\tilde{t}) = \ln S\mathrm{TI}(t)$.

Step 2. Maintaining stationary data while forecasting helps to improve the forecast quality; therefore, we conduct a stationary test on the STI data. For fuzzy time series, a fuzzy trend test can measure whether the STI's fuzzy trend moves upward or downward. Using this fuzzy trend test, the STI data can be converted into a stationary series. If the original STI data exhibited a fuzzy trend, it can be eliminated by taking the difference. We then repeat the test after taking the first difference to measure if the STI data exhibits a fuzzy trend. If a fuzzy trend is again observed, then we take the second difference, and so on.

Letting $\mathrm{STI}(t)$ be the historical data under consideration and fuzzy time series, a difference test is used (following Definition 11) to understand whether the stability of the information. Recursion is performed until the information is determined to be stable. Once the region $C'' = \{C | C = C_2^{22} + C_2^{42-22}\} = 432 < \{C | C_2^{42} \times (1 - 0.2)\} = 688.8$, the STI data are considered in a stable state and are not rejected.

Step 3. According to the interval setting of the STI data, we define the upper and lower bounds, which facilitate dividing the linguistic value intervals later. From Definition 10, the discourse $U = [D_L, D_U]$. From **Table 1**, $D_{\min} = 4.067$, $D_{\max} = 4.571$, $s = 0.172$, and $n = 42$ can be obtained. Letting$\alpha = 0.05$, since n is large than 30, a standard normal Z was used. Thus, $Z_{0.05} = 1.645$, $D_L = D_{\min} - st_\alpha/\sqrt{n} \approx 3.627$, and $D_U = D_{\max} + st_\alpha/\sqrt{n} \approx 5.011$. That is, $U = [3.627, 5.011\,]$.

Step 4. After defining the upper and lower bounds of the STI data in Step 3, we can define the SCFI range by determining the membership function as well as the linguistic values. We can also define the range of the subinterval for each linguistic value, assuming that the following linguistic values are under consideration: extremely few, very few, few, some, many, very many, and extremely many. According to Definition 11, the supports of fuzzy numbers that represent these linguistic values are given as follows:

$$u_{A_i}(x) = \begin{cases} 1 \ \text{ for } x \in [3.627 + (i-1)(0.129), 3.627 + i(0.198)) \\ \text{where } 1 \le i \le m - 1; \\ 1 \ \text{ for } x \in [3.627 + (i-1)(0.129), 3.627 + i(0.198)] \\ \text{where } i = m; \\ 0 \ \text{ otherwise.} \end{cases}$$

where A_1 = "extremely few," A_2 = "very few," A_3 = "few," A_4 = "some," A_5 = "many," A_6 = "very many," and A_7 = "extremely many." Thus, the supports are $\mathrm{supp}(A_1) = [3.627, 3.825)$, $\mathrm{supp}(A_2) = [3.825, 4.023)$, $\mathrm{supp}(A_3) = [4.023, 4.221)$, $\mathrm{supp}(A_4) = [4.221, 4.419)$, $\mathrm{supp}(A_5) = [4.419, 4.617)$, $\mathrm{supp}(A_6) = [4.617, 4.815)$, and $\mathrm{supp}(A_7) = [4.815, 5.011]$.

Step 5. According to the subinterval setting of each linguistic value, we classified each historical dataset of the STI into its corresponding interval to measure the value corresponding to the linguistic value for each interval. The fuzzy time series $F(t)$ was given by $F(t) = A_i$ when $d(t) \in \mathrm{supp}(A_i)$. Therefore, $F(201501) = A_5$, $F(201502) = A_6$, $F(201503) = A_5$, $F(201504) = A_6$, ..., and $F(201806) = A_3$. **Table 1** shows the comparison between the actual SCFI data and the fuzzy enrollment data.

Step 6. We apply fuzzy theory to define the corresponding value for the intervals of the STI data, arrange the corresponding method for the STI data, and

Year	Actual	ln(Actual)	Fuzzified	The forecast value
201501	95.611	4.560	A_5	4.518
201502	92.839	4.531	A_5	4.518
201503	94.750	4.551	A_5	4.518
201504	96.622	4.571	A_6	4.617
201505	88.503	4.483	A_5	4.518
201506	79.003	4.369	A_4	4.221
201507	80.103	4.383	A_4	4.221
201508	74.787	4.315	A_4	4.221
201509	69.560	4.242	A_4	4.221
201510	71.416	4.269	A_4	4.221
201511	67.625	4.214	A_3	4.221
201512	64.282	4.163	A_3	4.221
201601	63.301	4.148	A_3	4.221
201602	64.315	4.164	A_3	4.221
201603	67.143	4.207	A_3	4.221
201604	65.073	4.176	A_3	4.221
201605	61.163	4.114	A_3	4.221
201606	61.221	4.114	A_3	4.221
201607	61.043	4.112	A_3	4.221
201608	59.942	4.093	A_3	4.221
201609	60.293	4.099	A_3	4.221
201610	58.372	4.067	A_3	4.221
201611	58.736	4.073	A_3	4.221
201612	57.892	4.059	A_3	4.221
201701	59.278	4.082	A_3	4.221
201702	62.746	4.139	A_3	4.221
201703	65.467	4.182	A_3	4.221
201704	62.142	4.129	A_3	4.221
201705	76.626	4.339	A_4	4.221
201706	63.029	4.144	A_3	4.221
201707	64.728	4.170	A_3	4.221
201708	68.464	4.226	A_4	4.221
201709	70.555	4.256	A_4	4.221
201710	67.830	4.217	A_4	4.221
201711	66.696	4.200	A_3	4.221
201712	67.640	4.214	A3	4.221
201801	69.769	4.245	A_4	4.221
201802	65.206	4.178	A_3	4.221
201803	64.669	4.169	A_3	4.221
201804	64.671	4.169	A_3	4.221

Year	Actual	ln(Actual)	Fuzzified	The forecast value
201805	63.759	4.155	A_3	4.221
201806	62.712	4.139	A_3	4.221

Table 1.
Fuzzy historical STI data and the forecasted results.

$r_1 : A_3 \to A_3$	$r_5 : A_5 \to A_4$
$r_2 : A_3 \to A_4$	$r_6 : A_5 \to A_5$
$r_3 : A_4 \to A_4$	$r_7 : A_5 \to A_6$
$r_4 : A_4 \to A_3$	$r_8 : A_6 \to A_5$

Table 2.
*Fuzzy transitions derived from **Table 1**.*

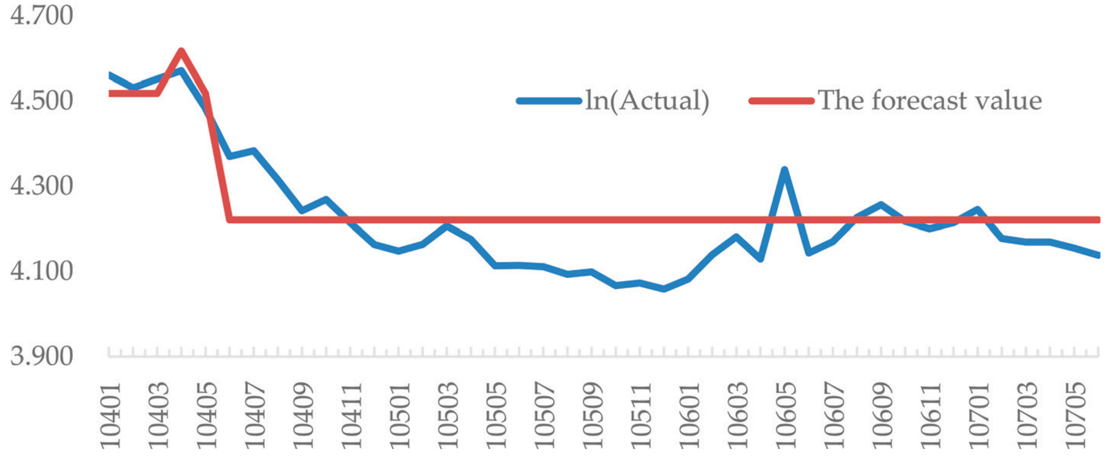

Figure 3.
Forecast STI and actual STI.

integrate the changes from all the rules to determine the rules for the STI. The transition rules are derived from **Table 1**. For example, $F(201501) \to F(201502)$ is $A_5 \to A_5$. **Table 2** shows all transition rules obtained from **Table 1**.

Step 7. We calculate each rule by determining all the rules of the STI, and the calculation results can be used to forecast future values. **Table 1** shows the forecasting results from 201001 to 201806.

Step 8. The calculated STI rules can define the intervals of the STI data; using these intervals, we can determine the variation in future long-term intervals. The long-term predictive value interval for the STI is given as (3.726, 4.913). Thus, the long-term predictive interval for the STI is given as (41.506, 136.022). Therefore, the current long-term S STI is bounded by this interval. According to Step 8, the fuzzy STI of 201501 shown in **Table 1** is A_5, and from **Table 2**, we can see that the rules are the fuzzy logical relationships in Rule 8 of **Table 2**, in which the current state of fuzzy logical relationships is A_3. Thus, the 201806 STI predictive value is 41.506.

Step 9. Letting defuzzified S be ΔS, the STI 201806 forecast value based on our investigation is 68.090, and its trading range is between 41.506 and 136.002. Thus, the new triangular fuzzy numbers by S = (41.506, 68.090, 136.002). Thus, the defuzzified S is ΔS = 74.981, and ΔS = 74.981 > $\hat{d}(t)$ = 68.090. ΔS is called a long-term significance level up.

The result shows that based on the long-term significance level, the STI is currently oversold. This result and the risk–reward ratio are both related within the group. We used **Table 1** data in our analysis according to the root mean square percentage error (R.M.S.P.E.) method, with an average prediction error of 1.708%. **Figure 3** shows the forecast visitor arrivals determined through fuzzy time series analysis and the actual STI values. Based on the fuzzy time series results, the average STI is estimated to be 68.090 in 201806 (**Figure 3**).

5. Conclusions and future work

In this article, a long-term predictive value interval model is developed for forecasting the STI. This model facilitates minimizing the uncertainties associated with fuzzy numbers. The method is examined by forecasting the STI by using data from which $\Delta S = 74.981$ and $\Delta S > \hat{d}(t)$ is obtained. For index returns, the current rate of return is negative and its volatility is increasing. The long-term predictive significance level of the STI is at the ΔS level; the STI should thus exhibit extreme volatility.

The current model for the STI 201806 forecast level deviates insignificantly from the actual values for an average of 68.090 and is within the group; the prediction error does not exceed 1.708% of the significance level. By constructing a fuzzy time series forecasting model for the STI with an error of less than 1.708%, with the traditional fuzzy time excluded from the single-point forecast comparison, this model provides a long-term predictive significance level.

Furthermore, the proposed method can be computerized. Thus, by improving fuzzy linguistic assessments as well as the evaluation of fuzzy time series, decision makers can automatically obtain the final long-term predictive significance level.

The STI used in this chapter is used as a forecasting example. If you predict that the future will rise, you can use the buying strategy. For example, if the index returns in the future, you can use the selling strategy.

The four functions of management are mainly four functions: planning, organization, leadership and control. The fuzzy time series mode used in this chapter can be applied to controlled projects to compare and correct whether the re-executed work meets expectations. If you meet expectations, re-plan the original settings.

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