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Introductory Chapter: Analytical and Numerical Approaches in Engineering Elasticity

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1. Introduction

In this section, the historical development of the “elasticity theory” was presented briefly, and recent studies performed about the elasticity concept were categorized and listed according to their basic engineering problem groups. The mentioned literature survey has been performed by searching the keywords “elasticity,” “analytic,” and “solution” between the years 2014 and 2018. The most important general aspects of the “elasticity theory” were described in four groups as “the unknowns,” “the used equations,” “the modeling procedures,” and “solution methods.” In the future, in the consideration of these explained theoretical, numerical, and experimental properties, the researchers can be concentrating on the origin of the problem and new solution methods in deciding the exact nature of the material.

The elasticity concept of solid materials is the deformation with the external force application and recovery to its original shape after the forces removed. In the strength measurement of the material, stress (force per area) and strain (deformation per unit length) criteria have been used. The elasticity theory was presented in order to explain the basic theoretical concepts and their analytical solution methods, the deformations that were assumed to be very small and corresponding stress distributions. The classical elasticity theory was explained by theorems of “uniqueness of solution” and “existence of solution” as they have been declared in the basic mathematical concepts. The “uniqueness of solution” theorem was restricted to a single solution space by satisfying the related boundary or the initial conditions. If there were no any boundary or initial conditions, the solution space would have to be infinity. The “existence of solution” theorem was created by explaining the default displacement functions, checking the equilibrium equations for stress definitions, and satisfying the partial differential equations with the infrastructure of the default solutions. The purpose of the elasticity theory was the determination of this unique and exact solution in elastic region of the material. In linear elastic region, superposition method and combined loading applications are widely used in engineering.

2. Historical development in elasticity

The historical development of the concept of “elasticity” by considering mathematics, physics, and engineering mechanics was summarized in **Figure 1** [1, 2].

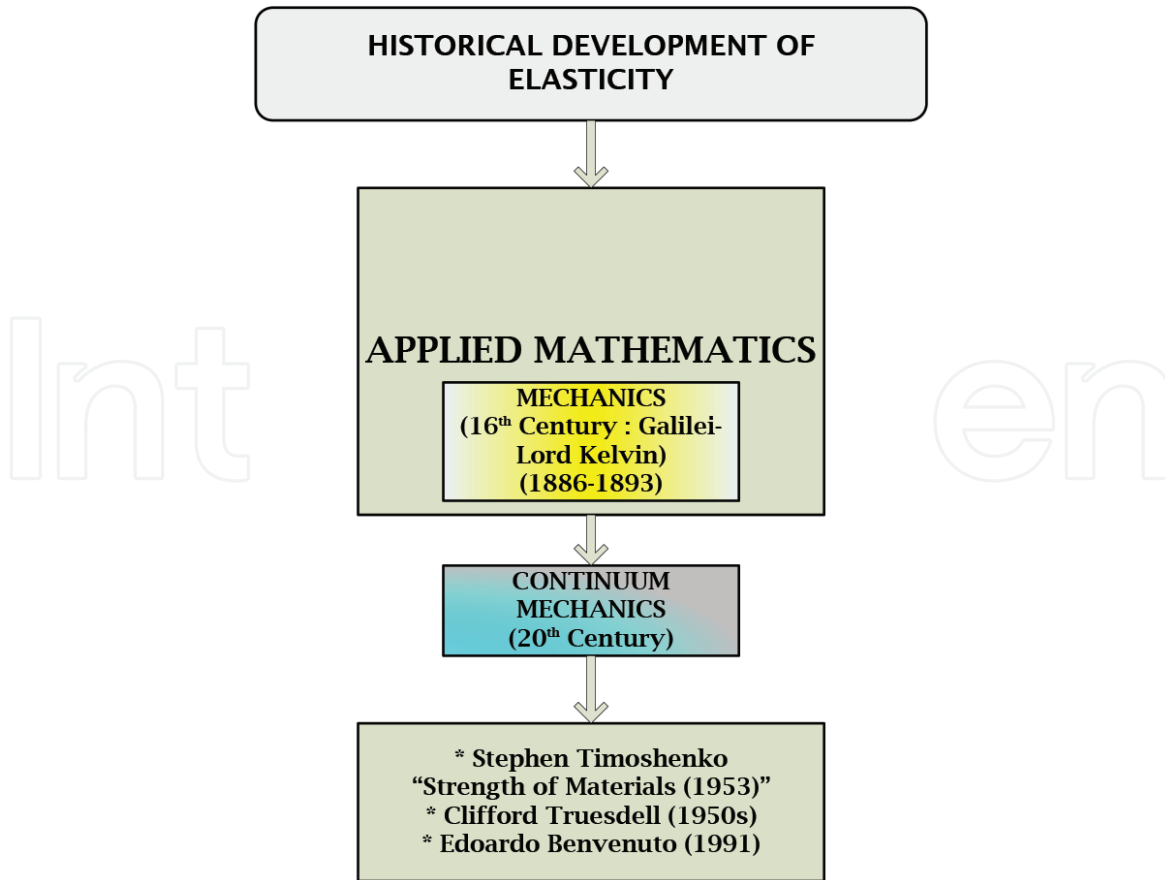


Figure 1.
Development of elasticity between the sixteenth and twentieth centuries.

The scientific studies performed on engineering problems have been grouped as analytical, numerical, and experimental. The main solution techniques listed below form the first step aspects in performing the experiments and obtaining the numerical solutions by considering innovations: (i) characteristics of the solution methods, (ii) learning the mathematical theories, (iii) the physics of the problem, and (iv) learning the problem-solving methodologies. The second step aspects have been listed as “solving problems by mathematical techniques” and “obtaining new formulas.” The scientific progress has been continued thanks to the studies done since the sixteenth century. The development in scientific area occurred in the elasticity concept has been summarized and visualized in the consideration of the scientists who have lived between the sixteenth and twentieth centuries and their studies [1, 2]. These famous scientists were Galilei (1564–1642), Mariotte (1620–1684), Hooke (1635–1703), Leibniz (1646–1716), Bernoulli (1700–1782), Baumgarten (1706–1757), Euler (1707–1783), Coulomb (1736–1806), Young (1773–1829), Poisson (1781–1840), Navier (1785–1836), Cauchy (1789–1857), Saint-Venant (1797–1886), Borchardt (1817–1880), Rankine (1820–1872), Kirchhoff (1824–1887), Maxwell (1831–1879), Clebsch (1833–1872), Kohlrausch (1840–1910), Amagat (1841–1915), Voigt (1850–1919), Mallock (1851–1933), Lamme (1864–1924), Röntgen (1872,1919), Synge (1897–1995), and Everett (1930–1982) (**Figure 1**).

3. Classification of engineering problems in the context of elasticity

In this section, the results of the literature review on elasticity were evaluated by referring to the articles (total number of articles, 157) between 2014 and 2018. Important information has gained from the literature survey about the elasticity

theory and its related recent engineering solutions, as well as information about the theoretical, numerical, and experimental scientific researches and scientific innovations. The brief classification of the main engineering problems was summarized in **Figure 2**.

The studies evaluated in the literature review were listed below in 10 main headings. The distribution of articles corresponding to research concepts is presented in **Figure 3**. These are (1) historical development, (2) analytical and experimental studies related to the finite element method (FEM), (3) experimental studies, (4) analytical studies and finite element analysis (FEA), (5) analytical studies,

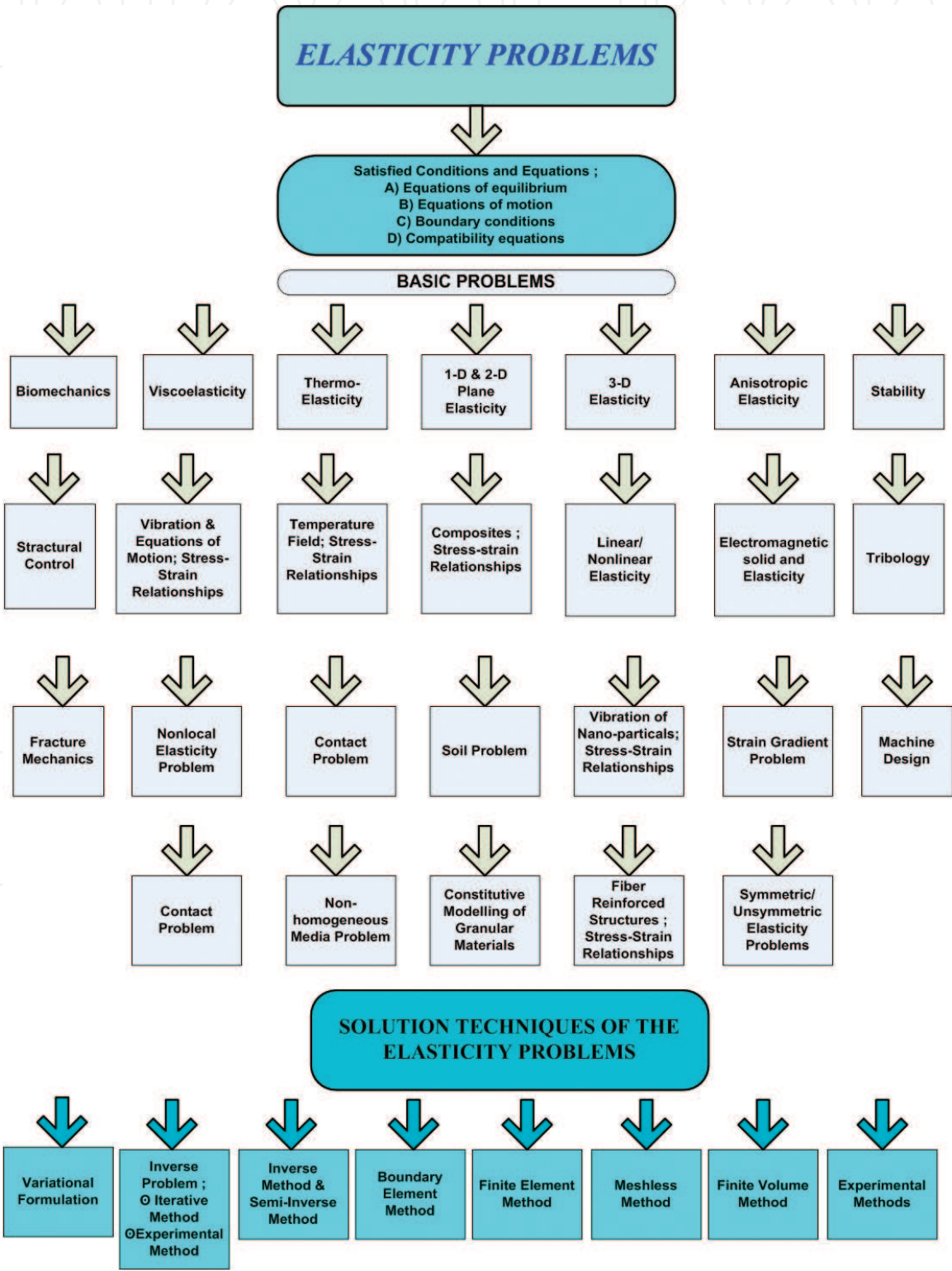


Figure 2.
Classification of the basic elasticity problems and their solution techniques.

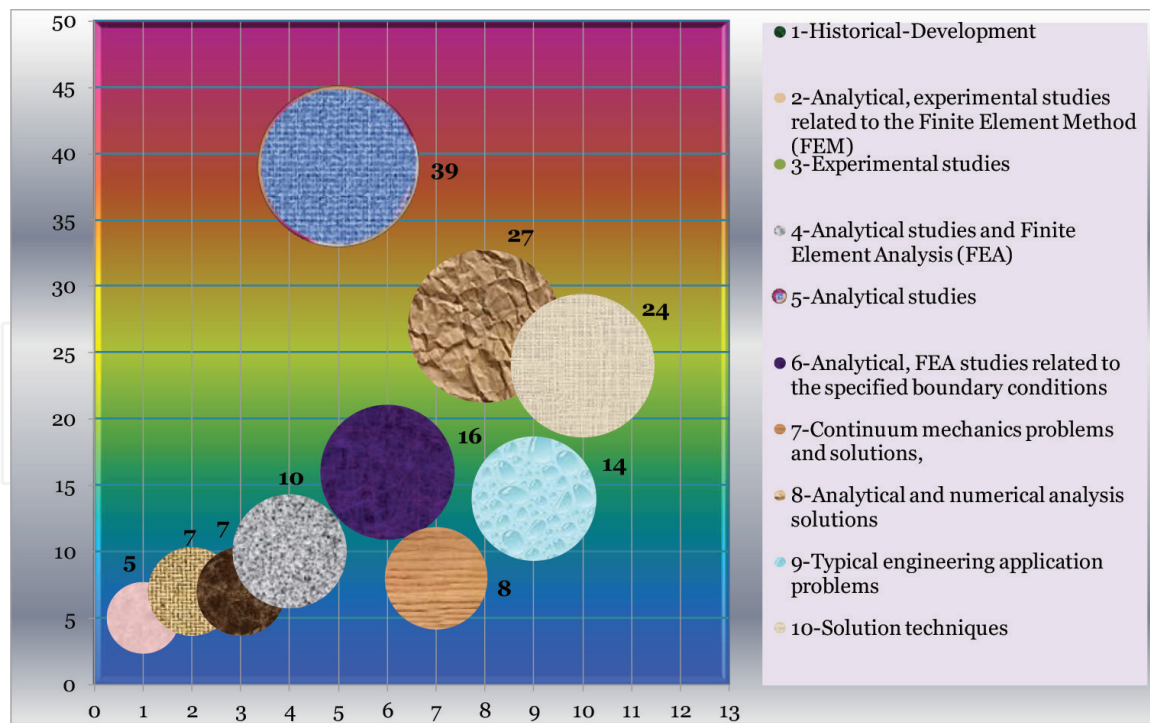


Figure 3.

The results of the literature review on elasticity were evaluated by referring to the 157 articles between 2014 and 2018.

(6) analytical and FEA studies related to the specified boundary conditions, (7) continuum mechanics problems and solutions, (8) analytical and numerical analysis solutions, (9) typical engineering application problems, and (10) solution techniques. The types of elasticity problems have been grouped according to the science innovations and related industrial applications. The numerical problems have been solved in three basic steps. The first step was to check the basic differential equations in terms of satisfaction with the placement of the estimated displacement functions. The second step was to check the “initial values” or the “boundary conditions” of the problem [3–5]. Values were substituted into the differential equations in order to satisfy the conditions at these defined coordinates or at time domains. The boundary conditions have been classified in two groups as “the essential” (displacement) and “the natural” (force) boundary conditions. The initial conditions were the first-stage variations named initiative and time-dependent variables. The third step was the satisfaction of the continuity conditions on the compatibility equations by means of assumed displacement functions. The basic elasticity problems were grouped into 26 subtitles as described in **Figure 2**. In this figure, the number of generally used proposed solution techniques analytically and numerically was equal to eight.

4. General principles in the elasticity theory

Elasticity concept is explainable by the natural elastic behavior of the materials. In elastic region, material deformed in a nonpermanent form up to the elastic limit was reached. The relationship between stress (σ) and strain (ε) under loading and unloading cases was explained by the linear and nonlinear equations. The slopes of the linear curves developed in linear elastic region were known as Young’s modulus E , and shear modulus G , of the materials under tensile/compression and torsion tests. During these tests, total calculated area under the linear curves was defined as the total potential energy stored in the material. Proportionally, stress development

and strains occurred in the structure according to the applied load. Principally, application of the stress distributions should be very slowly; on the other hand, at each incremental loading step, the equilibrium state and its equilibrium equations of the specimen should be satisfied. This controlled operation and action-reaction principle have worked under the control mechanism of the testing machine. The total work done by incremental external forces “ dW ” was equal to total potential energy stored incrementally “ dU ” in the structure of linear elastic region. Using this principle, the governing equations were satisfied by $dW - dU = 0$. Otherwise, in the case which used high strain rates $\dot{\epsilon}$, the material behavior would have been examined in the material nonlinearity concept. In the nonlinear elastic material experimental tests, the resulting stress-strain curves represented the combination of the behavior of nonlinear continuous or multiple nonlinear continuous forms. In nonlinear curves, the stored potential energy “ U ” developed in the elastic limit range was calculated in the consideration of two areas: the first area under the $\sigma - \epsilon$ nonlinear curve described as the stored potential energy by strain increments $\epsilon + d\epsilon$ and the second area above the curve, known as the complementary potential energy by stress increments $\sigma + d\sigma$ stored in the material. Both linear and nonlinear elasticity equations were derived according to the assumption that during loading and unloading stages of the experiments, the material stores its potential energy within the molecules and there was no loss of energy. As known in the molecular concept, the binding energy keeps the molecules together at any instant of time, and in the lack of energy loss such as heat or light, there will be no loss in the total mass of the molecular system. This phenomenon shows us that the system, which has no energy loss, does not combine (no binding status) with another solid object or with atoms that oscillates at short distances. Otherwise, in the case of the material decreases in amount as losing its mass as energy in the form of heat or light during the binding process, the removed energy corresponding to the removed mass can be explained by Einstein’s equation $E = mc^2$. Here, E is the binding energy, m is the mass change in the system, and c is the speed of light, respectively. The elasticity solutions were grouped in terms of a variety of the material, geometry, and loading types. Generally, the used geometries were selected as bar-, beam-, plate-, and shell-type isotropic or composite-type structures. In order to obtain analytical and numerical solutions, the three-dimensional elasticity problems can be reduced into two-dimensional problems in the consideration of the plane stress and plane strain concepts of the elasticity. By these methods the total number of unknowns will be equal to total numbers of equations. Otherwise, some unknown values will stay in unsolvable or undefined forms. Geometrical, material, and loading symmetries reduce problem-solving difficulties in the analytical and numerical models. On the other hand, continuity conditions in geometries automatically satisfies the continuity conditions in the analytical and numerical solutions of elasticity. For example, the existence of the fourth-order partial derivatives of the assumed solution approximation functions is checking the continuity and compatibility equations. Singularity problems may be discarded by omitting the very small holes, empty spaces, gaps in macroscale, or dislocations and beside these the distances between small particles in microscale. In the case of a three-dimensional problem in elasticity, 15 unknowns were defined as mentioned below. These were six stress components, six strain components, and three displacement components. These unknown values were to be calculated by using 15 elasticity equations, three equilibrium equations, six stress-strain relationships, and six strain-displacement relationships. Continuity conditions were satisfied by considering the six compatibility equations which were derived from 15 elasticity equations in three-dimensional problems. Boundary conditions and the initial conditions were both defined on the boundaries and at the starting time domains, respectively, in order to obtain the solutions under the limitation of

approximate and true percentage minimum error calculations. In the case of three-dimensional elasticity problem, 15 unknown values have to be solved by 15 governing equations (the list of the unknowns were six stress components

$[\sigma_x \ \sigma_y \ \sigma_z \ \tau_{xy} \ \tau_{yz} \ \tau_{xz}]$ and six strain components $[\varepsilon_x \ \varepsilon_y \ \varepsilon_z \ \gamma_{xy} \ \gamma_{yz} \ \gamma_{xz}]$, and additionally the three displacement components $[u \ v \ w]$) [3, 4]. In solid mechanics and elasticity theory, the governing partial differential equations, the constitutive and kinematics equations, and the initial and boundary conditions have been all defined. However, if at least one of the above conditions has remained partially or entirely unknown, then one has a so-called inverse problem (**Figure 2**) [5]. On the other hand, the elasticity “inverse problem” has been defined for the problems in which they consist of recovering the missing displacements to the solution space corresponding to the applied force data by using the iterative calculation steps. Obviously, lost or uncalculated data developing on one part of a whole domain boundary have directly affected the final configuration of the stress-strain and displacement components and their resultant solution spaces at the other part of this boundary. The proposed solutions were both numerical and analytical (**Figure 2**). Inverse problem of elasticity in other words Cauchy problem (Cauchy-Navier equations of elasticity) has been defined on the accessible outer boundary of the structure. The Cauchy stress tensor components were related with the infinitesimal (incremental calculations) strain tensor components which have been identified in deformed configuration with successive iterations.

The stress-strain relationship in terms of indicial notation is given below:

$$\sigma_{ij} = 2 \mu \varepsilon_{ij} + \lambda \delta_{ij} \varepsilon_{kk} \quad (1)$$

Here, μ, λ are the Lamé constants. The Cauchy strain components represent the geometrical nonlinearity of the material according to the deformed configuration.

The inverse problem solution depends on the stepwise calculated and so updated Cauchy stress and strain distributions, over the whole boundary of the geometry. Experimentally, tractions and displacements have been measured by nondestructive tests. In isotropic, fiber, and particulate composite material concepts, the stress-strain distributions $\sigma - \varepsilon$ have been examined according to the defined total number of elastic constants in stiffness $[C]$ matrix. The inverse of the stiffness matrix named as the compliance matrix $[S] = [C]^{-1}$ includes the elastic constants in $\varepsilon - \sigma$ strain versus stress equations. In the generalized Hook's law, anisotropic crystalline materials have been defined with 36 constants. Strain energy function has to be used to show that the number of independent material constants can be reduced from 36 to 21. The solution techniques as iterative methods, inverse method, semi-inverse method, variational formulation, finite element method, finite volume method, and meshless method have been listed in **Figure 2**. The experimental solution techniques have been explained by tensile, compression, torsion, impact, and bending mechanical tests. Nondestructive tests (NDT) have been used to obtain informational data from the surfaces of the materials (nanoindentation-hardness testing).

5. Conclusion

In this introduction chapter, the historical development of the elasticity concept and its engineering properties were presented briefly. According to Newton's action and reaction principle, the materials behave linear or nonlinear elastically under typical loading. Elasticity theory provides necessarily required equations and solution techniques. The action-response principle defined between the work done by

the forces and the potential energy stored has been explained by the material elastic constants. The mechanical response of a homogeneous isotropic linearly elastic material can be explained by two physical constants, Young's modulus and Poisson's ratio. The elastic properties of particle composites, consisting in a dispersion of nonlinear (spherical or cylindrical) nonhomogeneities into a linear solid matrix, were explained by homogenization procedure. The linear elastic constants of fiber composite materials have been defined according to their three principle directions [6]. These principle directions coincided with the fiber orientations located in each layer. By contrast, the physical-mechanical properties of nonlinear elastic materials have generally been described by parameters which have formations as the scalar functions of the deformation, and their material properties have been determined by selecting the suitable solution techniques.

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