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Electrostatic Waves in Magnetized Electron-Positron Plasmas

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Abstract

The behavior of arbitrary amplitude linear and nonlinear electrostatic waves that propagate in a magnetized four component, two-temperature, electron-positron plasma is presented. The characteristics of the dispersive properties of the associated linear modes using both fluid and kinetic theory are examined. The fluid theory analysis of the electrostatic linear waves shows the existence of electron acoustic, upper hybrid, electron plasma and electron cyclotron branches. A kinetic theory analysis is then used to study the acoustic mode, in particular the effect of Landau damping, which for the parameter regime considered is due to the cooler species. Consequently, it is found that a large enough drift velocity is required to produce wave growth. Nonlinear electrostatic solitary waves (ESWs), similar to those found in the broadband electrostatic noise observed in various regions of the earth's magnetosphere is further investigated. A set of nonlinear differential equations for the ESWs, which propagate obliquely to an external magnetic field is derived and numerically solved. The effect of various plasma parameters on the waves is explored and shows that as the electric driving force is increased, the electric field structure evolves from a sinusoidal wave to a spiky bipolar form. The results are relevant to both astrophysical environments and related laser-induced laboratory experiments.

Keywords: electrons, positrons, electrostatic waves, nonlinear waves

1. Introduction

Electron-positron plasmas play a significant role in the understanding of the early universe [1, 2], active galactic nuclei [3], gamma ray bursts (GRBs) [4], pulsar magnetospheres [5, 6] and the solar atmosphere [7]. These plasmas are also important in understanding extremely dense stars such as white dwarfs and pulsars, which are thought to be rotating neutron stars. The existence of these plasmas in neutron stars and in the pulsar magnetosphere is well documented [8]. The possibility for the co-existence of two types of cold and hot electron-positron populations in the pulsar magnetosphere has been suggested by [9] which was inspired by the pulsar model [10]. In their model, accelerated primary electrons moving on curved magnetic field lines emit curvature photons which produce electron-positron pairs. The secondary particles then produce curvature radiation, hence producing new electron-positron pairs, and so on. Therefore, both the electron and positron populations can be subdivided in two groups of distinct temperatures, one modeling the original plasma, and the second the higher-energy cascade-bred pairs. It is also

known that in astrophysical and cosmic plasmas, a minority of cold electrons and heavy ions exist along with hot electron-positron pairs [11]. Hence, the formation of two temperature multispecies plasmas is possible due to the outflow of the electron-positron plasma from pulsars entering into an interstellar cold, low-density electron-ion plasma [12].

Investigations into electron-positron plasma behavior have focused primarily on the relativistic regime. It is however plausible that nonrelativistic astrophysical electron-positron plasmas may exist, given the effect of cooling by cyclotron emission [13]. The study of nonrelativistic astrophysical electron-positron plasmas therefore plays an important role in understanding wave fluctuations. Due to the equal charge to mass ratio for these oppositely charged species, only one frequency scale exists and due to this symmetry, there exists different physical phenomena to the conventional electron-ion plasmas. Further, the frequent instabilities that arise in space plasma and astrophysical environments (e.g., solar flares and auroras), involve the growth of electrostatic and electromagnetic waves which gives rise to a growing wave mode. In particular, the linear behavior of the electrostatic modes using fluid and kinetic theory approaches allows one to understand the effect of plasma parameters such as the propagation angle, cool to hot temperature ratios, density ratios and the magnetic field strength on the waves.

Investigations conducted have focused on modulational instabilities and wave localization [14], envelope solitons [15], multidimensional effects [16]. Large amplitude solitons and electrostatic nonlinear potential structures in electron-positron plasmas having equal hot and cold components of both species have been studied by a number of authors [17–19]. In one such study [20], using the two-fluid model with a single temperature they investigated linear and nonlinear longitudinal and transverse electrostatic and electromagnetic waves in a nonrelativistic electron-positron plasma in the absence and presence of an external magnetic field. They found that several of the modes present in electron-ion plasmas also existed in electron-positron plasmas, but in a modified form. Collective modes in nonrelativistic electron-positron plasmas using the kinetic approach was studied by [21]. The author found that the dispersion relations for the longitudinal modes in the electron-positron plasma for both unmagnetized and magnetized electron-positron plasmas were similar to the modes in one-component electron or electron-ion plasmas. Moreover, the hybrid resonances present in the former are not found in an electron-positron plasma.

The understanding of nonlinear wave structures which gives rise to electrostatic solitary wave (ESWs) in space is important since it is known that satellite measurements using high-time resolution equipment aboard spacecraft S3-3 [22], Viking [23], Geotail [24], Polar [25], and Fast [26] have indicated the presence of Broad-band Electrostatic Noise (BEN) in the auroral magnetosphere at altitudes between 3000 km to 8000 km and beyond. These observations have shown the presence of electrostatic solitary waves (ESWs), which are characterized by their spiky bipolar pulses. Hence, the study of nonlinear wave behavior in electron-positron plasmas propagating at oblique angles to an ambient magnetic field is explored to understand electrostatic solitary waves in space. Specifically, the spiky nature of the electrostatic potential structures and the effects of the propagation angle, cold and hot drift velocities, cool to hot density and temperature ratios and Mach number on the ESWs are examined.

In this chapter a two-temperature magnetized four component electron-positron plasma model is used to study linear wave modes using both the fluid and kinetic approaches as well as the behavior of the nonlinear structures of these electrostatic solitary waves (ESWs) which plays an important role in space and astrophysical environments.

2. Linear waves in electron-positron plasmas: fluid theory approach

Let us consider a homogeneous magnetized, four component electron-positron plasma, consisting of cool electrons and cool positrons with equal temperatures and equilibrium densities denoted by T_c and n_{0c} , respectively, and hot electrons and hot positrons with equal temperatures and equilibrium densities denoted by T_h and n_{0h} , respectively. The temperatures are expressed in energy units and wave propagation is taken in the x -direction at an angle θ to the ambient magnetic field \mathbf{B}_0 , which is assumed to be in the x - z plane.

Assuming that the hot isothermal species are described by the Boltzmann distribution, their densities are, respectively

$$n_{eh} = n_{0h} \exp\left(\frac{e\phi}{T_h}\right) \quad (1)$$

and

$$n_{ph} = n_{0h} \exp\left(\frac{-e\phi}{T_h}\right), \quad (2)$$

where n_{eh} (n_{ph}) is the density of the hot electrons (positrons) and ϕ is the electrostatic potential.

Using Boltzmann distribution of hot electrons and positrons is justified provided they have sufficiently high temperatures, much greater than that of cooler species such that their thermal velocities parallel to the magnetic field exceed the phase velocity of the modes so that they are able to establish the Boltzmann distribution. The magnetic field effects on hot species are not felt since the perturbation wavelengths are shorter than their gyroradii such that both hot electrons and positrons follow essentially straight line orbits across the magnetic field direction.

The dynamics of cooler isothermal species are governed by fluid equations, namely the continuity equations,

$$\frac{\partial n_{jc}}{\partial t} + \nabla \cdot (n_{jc} \mathbf{v}_{jc}) = 0, \quad (3)$$

the equations of motion,

$$\frac{\partial \mathbf{v}_{jc}}{\partial t} + \mathbf{v}_{jc} \cdot \nabla \mathbf{v}_{jc} = -\varepsilon_j \frac{e}{m} \nabla \phi + \varepsilon_j \frac{e}{m} (\mathbf{v}_{jc} \times B_0) - \frac{\gamma T_c}{n_{jc} m} \nabla n_{jc}, \quad (4)$$

where $\varepsilon_j = +1(-1)$ for positrons (electrons), $j = e(p)$ for the electrons (positrons). The system is closed by the Poisson equation

$$\varepsilon_0 \frac{\partial^2 \phi}{\partial x^2} = -e(n_{pc} - n_{ec} + n_{ph} - n_{eh}). \quad (5)$$

In the above equations, n_j and v_j are the number densities and fluid velocities respectively of the j th species. In order to derive the linear dispersion relation, equations (3)–(5) are linearized. For perturbations varying as $\exp(i(kx - \omega t))$, $\partial/\partial t$ is replaced with $-i\omega$ and $\partial/\partial x$ with ik . Hence the perturbed densities for the electrons and positrons become

$$n_{ec} = -\left(\frac{n_{0c}ek^2\phi}{m}\right)\left(\frac{\omega^2 - \Omega^2 \cos^2\theta}{\omega^4 - \omega^2(3k^2v_{tc}^2 + \Omega^2) + 3k^2v_{tc}^2\Omega^2 \cos^2\theta}\right). \quad (6)$$

and

$$n_{pc} = \left(\frac{n_{0c}ek^2\phi}{m}\right)\left(\frac{\omega^2 - \Omega^2 \cos^2\theta}{\omega^4 - \omega^2(3k^2v_{tc}^2 + \Omega^2) + 3k^2v_{tc}^2\Omega^2 \cos^2\theta}\right). \quad (7)$$

From equations (1) and (2), the perturbed densities for the hot species are given by,

$$n_{eh} = n_{oh} \frac{e\phi}{T_h} \quad (8)$$

and

$$n_{ph} = -n_{oh} \frac{e\phi}{T_h}. \quad (9)$$

Substituting equations (6)–(9), into Poisson's equation (5), the general dispersion relation for the two temperature electron-positron plasma is found to be

$$\omega^2(\omega^2 - \Omega^2) - 3k^2v_{tc}^2(\omega^2 - \Omega^2 \cos^2\theta) - \frac{k^2v_{ea}^2}{1 + \frac{1}{2}k^2\lambda_{Dh}^2}(\omega^2 - \Omega^2 \cos^2\theta) = 0 \quad (10)$$

where $v_{ea} = (n_{0c}/n_{0h})^{1/2}v_{th}$ is the acoustic speed of the electron-positron plasma, analogous in form to the electron acoustic speed in an electron-ion plasma [27]. The thermal velocity of the cool species is $v_{tc} = (T_c/m)^{1/2}$, $\Omega_j = \Omega = q_j B_0/m$ is the gyro-frequency of the electrons and positrons and $\lambda_{dh} = (\epsilon_0 T_h/n_{0h}e^2)^{1/2}$ is the Debye length of the hot species.

It is noted that the study of linear electrostatic waves using a simple fluid model cannot handle the possible Landau damping of the modes. Hence, Landau damping is not significant since phase velocities are far away from the thermal velocities of either the hot or cooler species, i.e., $v_{th} \gg v_\phi \gg v_{tc}$ with $T_h \gg T_c$. The effects of the temperature variation on the acoustic mode in terms of Landau damping using kinetic theory are discussed in the next section.

For a single species electron-positron plasma, with temperature T_c , equation (10) reduces to,

$$\omega^4 - \omega^2(\Omega^2 + 3k^2v_{tc}^2) + 3k^2v_{tc}^2\Omega^2 \cos^2\theta = 0. \quad (11)$$

This is identical to the dispersion relation of [20] for their single temperature electron-positron model.

For wave frequencies much lower than the gyrofrequency and satisfying $\omega \ll \Omega \cos \theta$, the associated electron-acoustic (or positron-acoustic) mode is found to be,

$$\omega^2 = \frac{k^2v_{ea}^2 \cos^2\theta}{1 + \frac{1}{2}k^2\lambda_{Dh}^2} + 3k^2v_{tc}^2 \cos^2\theta. \quad (12)$$

Taking short wavelength limit ($k^2 \lambda_{Dh}^2 \gg 1$), the dispersion relation equation (10) reduces to,

$$\omega^4 - \omega^2(3k^2 v_{tc}^2 + \omega_{UH}^2) + (3k^2 v_{tc}^2 + 2\omega_{pc}^2)\Omega^2 \cos^2 \theta = 0, \quad (13)$$

where

$$\omega_{UH}^2 = \Omega^2 + 2\omega_{pc}^2 \quad (14)$$

is the upper hybrid frequency associated with the cooler species [20], with $\omega_{pc} = (n_{oc} e^2 / \epsilon_0 m)^{1/2}$ as the plasma frequency of the cooler species. If one solves equation (13) in the limit $(3k^2 v_{tc}^2 + \omega_{UH}^2)^2 \gg 4(3k^2 v_{tc}^2 \Omega^2 \cos^2 \theta + 2\omega_{pc}^2 \Omega^2 \cos^2 \theta)$, one obtains for the upper hybrid mode,

$$\omega_+^2 = (3k^2 v_{tc}^2 + \omega_{UH}^2) - \frac{(3k^2 v_{tc}^2 + 2\omega_{pc}^2)\Omega^2 \cos^2 \theta}{3k^2 v_{tc}^2 + \omega_{UH}^2}, \quad (15)$$

Taking the negative square root of equation (13) yields

$$\omega_-^2 = \frac{(3k^2 v_{tc}^2 + 2\omega_{pc}^2)\Omega^2 \cos^2 \theta}{3k^2 v_{tc}^2 + \omega_{UH}^2}, \quad (16)$$

In order to gain physical insight into the solution space of the dispersion relation, the two extreme limits of equation (10) will now be considered, viz. pure perpendicular and pure parallel propagations.

2.1 Case I: pure perpendicular propagation

Considering the pure perpendicular ($\theta = 90^\circ$) limit, the general dispersion relation (10), reduces to:

$$\omega^4 - \omega^2 \left(\Omega^2 + 3k^2 v_{tc}^2 + \frac{k^2 v_{ea}^2}{1 + \frac{1}{2} k^2 \lambda_{dh}^2} \right) = 0. \quad (17)$$

Hence the normal mode frequencies are, $\omega = 0$, which is a nonpropagating mode, and

$$\omega^2 = \Omega^2 + 3k^2 v_{tc}^2 + \frac{k^2 v_{ea}^2}{1 + \frac{1}{2} k^2 \lambda_{dh}^2}. \quad (18)$$

Taking the short wavelength limit ($k^2 \lambda_{dh}^2 \gg 1$) of the above relationship, one obtains,

$$\omega^2 = \omega_{UH}^2 + 3k^2 v_{tc}^2. \quad (19)$$

showing that the behavior of the upper hybrid mode for the two temperature model is due to the cooler species, where $\omega_{UH}^2 = \Omega_p^2 + 2\omega_{pc}^2$.

Now taking the long wavelength limit ($k^2 \lambda_{dh}^2 \ll 1$) of the dispersion relation for perpendicular propagation, equation (18) reduces to

$$\omega^2 = \Omega^2 + k^2(3v_{tc}^2 + v_{ea}^2). \quad (20)$$

This is the cyclotron mode for the electron-positron plasma with contributions from both the thermal motion of the adiabatic cooler species and the acoustic motion due to the two species of different temperatures. To try and understand the physical implications, the above expression for the dispersion relation can be written as,

$$\omega^2 = \Omega^2 + k^2 v_{ea}^2 \left(1 + 3 \frac{T_c n_{0h}}{T_h n_{0c}} \right). \quad (21)$$

For $T_c/T_h \ll 1$, one requires $n_{0h} \gg n_{0c}$, i.e., a plasma dominated by the hot species, in order for the second term in brackets to affect the dispersive properties of the wave.

2.2 Case II: pure parallel propagation

Considering the limit of parallel propagation ($\theta = 0^\circ$), the general dispersion relation (10) reduces to,

$$\omega^4 - \omega^2 \left(\Omega^2 + 3k^2 v_{tc}^2 + \frac{k^2 v_{ea}^2}{1 + \frac{1}{2} k^2 \lambda_{dh}^2} \right) + \Omega^2 \left(3k^2 v_{tc}^2 + \frac{k^2 v_{ea}^2}{1 + \frac{1}{2} k^2 \lambda_{dh}^2} \right) = 0, \quad (22)$$

from which it can be shown

$$\omega^2 = \frac{1}{2} \left[\Omega^2 + 3k^2 v_{tc}^2 + \frac{k^2 v_{ea}^2}{1 + \frac{1}{2} k^2 \lambda_{dh}^2} \pm \left(\Omega^2 - 3k^2 v_{tc}^2 - \frac{k^2 v_{ea}^2}{1 + \frac{1}{2} k^2 \lambda_{dh}^2} \right) \right]. \quad (23)$$

There exist two possible solutions. Taking the positive sign of the relevant term in equation (23) as the first option yields,

$$\omega_+^2 = \Omega^2, \quad (24)$$

which is a constant frequency, nonpropagating cyclotron mode.

Now taking the negative sign of the term in equation (23) yields the normal mode frequency

$$\omega_-^2 = 3k^2 v_{tc}^2 + \frac{k^2 v_{ea}^2}{1 + \frac{1}{2} k^2 \lambda_{dh}^2}, \quad (25)$$

which may be written for $k^2 \lambda_{dh}^2 \ll 1$ as

$$\omega_-^2 = k^2 v_{ea}^2 \left(1 + 3 \frac{T_c n_{0h}}{T_h n_{0c}} \right), \quad (26)$$

which is identified fundamentally, as the electron-acoustic mode, with a correction term to its phase velocity due to the thermal motion of the cooler species.

In the limit $k^2 \lambda_{dh}^2 \gg 1$, one obtains

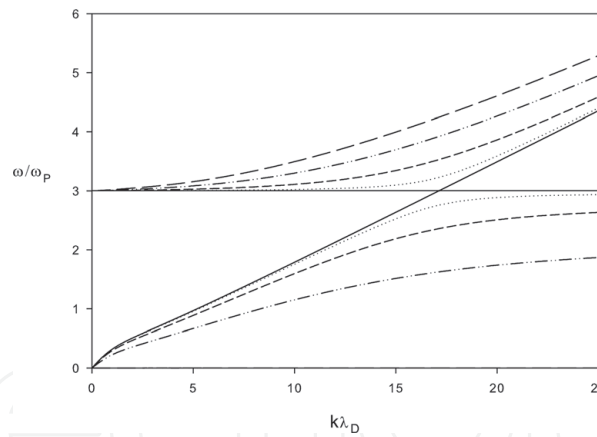


Figure 1. Normalized real frequency as a function of the normalized wavenumber showing the acoustic and cyclotron branches for various angles of propagation $\theta = 0^\circ$ (solid), 9° (dotted), 22.5° (broken), 45° (dashdot) and 90° (longbroken). The fixed plasma parameters are $R = 0.333$, $T_c/T_h = 0.01$ and $n_{0c}/n_{0h} = 0.11$.

$$\omega_-^2 = 3k^2 v_{tc}^2 + 2\omega_{pc}^2 \quad (27)$$

Equating equations (24) and (27) in the limit $k^2 \lambda_{dh}^2 \gg 1$, the critical k value for which the two modes may couple is determined to be,

$$(k\lambda_d)_{crit} = \left(\frac{T_h n_{0c}}{3T_c n_0} \right)^{1/2} \left(\frac{n_0}{n_{0c} R^2} - 2 \right)^{1/2}. \quad (28)$$

A numerical analysis of the general dispersion relation can be performed focusing on the effects of the density and temperature ratios of the hot and cool electrons and positrons. If one normalizes the fluid speeds by the thermal velocity $v_{th} = (T_h/m)^{1/2}$, the particle density by the total equilibrium plasma density $n_0 = n_{0c} + n_{0h}$, the temperatures by T_h , the spatial length by $\lambda_D = (\epsilon_0 T_h / n_0 e^2)^{1/2}$, and the time by $\omega_p^{-1} = (n_0 e^2 / \epsilon_0 m)^{-1/2}$ in equation (10), you get the normalized general dispersion relation,

$$\omega'^4 - \omega'^2 \left(\frac{1}{R^2} + 3k'^2 \frac{T_c}{T_h} + \frac{k'^2 n'_{0c}}{n'_{0h} + \frac{1}{2}k'^2} \right) + \frac{\cos^2 \theta}{R^2} \left(3k'^2 \frac{T_c}{T_h} + \frac{k'^2 n'_{0c}}{n'_{0h} + \frac{1}{2}k'^2} \right) = 0, \quad (29)$$

where $\omega' = \omega/\omega_p$, $k' = k\lambda_D$, $n'_{0h} = n_{0h}/n_0$, $n'_{0c} = n_{0c}/n_0$ and $R = \omega_p/\Omega$ is a measure of the plasma densities and the strength of the magnetic field. A typical result can be seen in **Figure 1** [28] for the normalized real frequency as a function of the normalized wavenumber showing the acoustic and cyclotron branches for a range of propagation angles.

3. Linear waves in electron-positron plasmas: kinetic theory approach

In this section the kinetic theory approach is used to study the acoustic mode that was investigated in the previous section using fluid theory. The focus is on this mode since it is a micro-instability arising from resonances in velocity space. This instability is kinetic in nature and the growth rate of the wave is a function of the slope of the velocity distribution function. When the wave phase velocity along \mathbf{B}_0 sees a negative slope of the velocity distribution ($\partial f_0 / \partial V_{\parallel} < 0$), the particles on

average will gain energy from the wave, consequently the wave losses energy and becomes damped, an effect known as Landau damping. The wave mode is hence subjected to Landau damping and wave enhancement. Therefore the focus in this section is primarily on the effect of the temperatures of the plasma species.

The same plasma model as in the previous section is considered, i.e., a four component magnetized electron-positron plasma, consisting of cool electrons and cool positrons with equal temperatures and equilibrium densities denoted by T_c and n_{0c} respectively, and hot electrons and hot positrons with equal temperatures and equilibrium densities denoted by T_h and n_{0h} , respectively.

We begin by deriving the general dispersion relation where each species j has an isotropic, drifting Maxwellian velocity distribution with temperatures T_j drifting parallel to the magnetic field $\mathbf{B}_0 = B_0 \hat{z}$, with drift velocities V_{oj} .

Hence, the equilibrium velocity distribution for the electron and positron species is chosen to be,

$$f_{\alpha 0} = \frac{n_{\alpha 0}}{(2\pi v_{tj}^2)^{\frac{3}{2}}} \exp \left\{ -\frac{[V_x^2 + V_y^2 + (V_z - V_{oj})^2]}{2v_{tj}^2} \right\}, \quad (30)$$

The Vlasov equations are,

$$\frac{\partial f_{\alpha}}{\partial t} + \mathbf{V} \cdot \nabla f_{\alpha} + \frac{q_{\alpha}}{m} (\mathbf{E} + \mathbf{V} \times \mathbf{B}) \cdot \frac{\partial f_{\alpha}}{\partial \mathbf{V}} = 0, \quad (31)$$

and the equations of motion for the electrons and positrons is given by,

$$m \frac{d\mathbf{V}}{dt} = q_{\alpha} \{\mathbf{E} + \mathbf{V} \times \mathbf{B}\}, \quad (32)$$

where $j = c(h)$ for the cool (hot) species and $\alpha = ec, pc, eh$ and ph for the cool electrons, cool positrons, hot electrons and hot positrons respectively, and

$v_{tj} = (T_j/m)^{1/2}$ is the thermal velocity of the j^{th} species.

Following standard techniques for electron-ion plasmas [29], the general kinetic dispersion relation for the four component, two temperature electron-positron plasma is given by

$$\begin{aligned} k^2 + \frac{2}{\lambda_{Dc}^2} \left[1 + \frac{\omega - \mathbf{k} \cdot \mathbf{V}_{oc}}{\sqrt{2} k_{\parallel} v_{tc}} \sum_{p=-\infty}^{\infty} Z(z_{pc}) \Gamma_{pc} \right] \\ + \frac{2}{\lambda_{Dh}^2} \left[1 + \frac{\omega - \mathbf{k} \cdot \mathbf{V}_{oh}}{\sqrt{2} k_{\parallel} v_{th}} \sum_{p=-\infty}^{\infty} Z(z_{ph}) \Gamma_{ph} \right] = 0, \end{aligned} \quad (33)$$

where $\lambda_{Dc,h} = (\epsilon_0 T_h / n_{0c,h} e^2)^{1/2}$ is the Debye length for the cool (hot) species and z_{pj} is the argument of the plasma dispersion function or Z-function [30] and is given by,

$$z_{pj} = \frac{\omega - \mathbf{k} \cdot \mathbf{V}_{oj} - p\Omega_j}{\sqrt{2} k_{\parallel} v_{tj}}, \quad (34)$$

where,

$$\Gamma_{pj} = e^{-\alpha_j} I_p(\alpha_j), \quad (35)$$

and

$$\alpha_j = \frac{k_{\perp}^2 v_{tj}^2}{\Omega_j^2}, \quad (36)$$

where I_p is the modified Bessel function of order p . The components of \mathbf{k} parallel (perpendicular) to \mathbf{B}_0 are given by k_{\parallel} (k_{\perp}) respectively, while \mathbf{V}_{oc} and \mathbf{V}_{oh} are the drift velocities of the cool (hot) species, respectively.

3.1 Approximate solutions of the kinetic dispersion relation

The general dispersion relation (33) can be numerically solved without any approximations. However, to get some insight into the solutions, here, approximate expansions of the plasma dispersion function are used to obtain analytical expressions for the frequency and growth rate of the acoustic mode.

In proceeding, for the temperatures it is assumed that $T_h \gg T_c (\sim 0)$. In addition low frequency modes satisfying $|\omega| \ll \Omega$ are considered. The series expansion of the Z-function [30] is given by

$$Z(z) = i\sqrt{\pi}e^{-z^2} - 2z \left[1 - \frac{2z^2}{3} + \frac{4z^4}{15} - \dots \right] \text{ for } |z| \ll 1 \text{ and} \quad (37)$$

$$Z(z) = i\sqrt{\pi}\delta e^{-z^2} - \frac{1}{z} \left[1 + \frac{1}{2z^2} + \frac{3}{4z^4} + \dots \right] \text{ for } |z| \gg 1. \quad (38)$$

$$\text{where for } |z| \gg 1, \delta = \begin{cases} 0, & \text{Im}(z) > 0 \\ 1, & \text{Im}(z) = 0 \\ 2, & \text{Im}(z) < 0 \end{cases}$$

Assuming the drift of the electrons and positrons to be weak (i.e., small V_{oc} and V_{oh}) [31] and $|\omega| \ll \Omega$,

$$z_{pc} = \frac{\omega - \mathbf{k} \cdot \mathbf{V}_{oc} - p\Omega}{\sqrt{2}k_{\parallel}v_{tc}} \approx \frac{-p\Omega}{\sqrt{2}k_{\parallel}v_{tc}} \text{ for } p \neq 0 \quad (39)$$

and

$$z_{ph} = \frac{\omega - \mathbf{k} \cdot \mathbf{V}_{oh} - p\Omega}{\sqrt{2}k_{\parallel}v_{th}} \approx \frac{-p\Omega}{\sqrt{2}k_{\parallel}v_{th}} \text{ for } p \neq 0. \quad (40)$$

Then for the cool species,

$$\sum_{p=-\infty}^{\infty} Z(z_{pc})\Gamma_{pc} \approx Z\left(\frac{\omega - \mathbf{k} \cdot \mathbf{V}_{oc}}{\sqrt{2}k_{\parallel}v_{tc}}\right)\Gamma_{oc} + \sum_{p=1}^{\infty} \left\{ Z\left(\frac{p\Omega}{\sqrt{2}k_{\parallel}v_{tc}}\right) + Z\left(\frac{-p\Omega}{\sqrt{2}k_{\parallel}v_{tc}}\right) \right\} \Gamma_{pc}. \quad (41)$$

From the definition of the Z-function, $Z(\xi) + Z(-\xi) = 0$, hence

$$\sum_{p=-\infty}^{\infty} Z(z_{pc})\Gamma_{pc} \approx Z(z_{oc})\Gamma_{oc}. \quad (42)$$

Taking the cooler species to be stationary, V_{oc} is therefore set to zero, allowing only the hot species to drift. Then,

$$z_{oc} = \frac{\omega}{\sqrt{2}k_{\parallel}v_{tc}}. \quad (43)$$

For modes satisfying $\omega/k_{\parallel} \gg v_{tc}$, one may assume $|z_{oc}| \gg 1$, i.e., the wave phase speed along \mathbf{B}_0 is much larger than the cool electron thermal speed. For instability (i.e., a growing wave with $\text{Im}(z) > 0$), δ is set equal to zero in equation (38). Hence using the series expansion equation (38), equation (41) becomes

$$\sum_{p=-\infty}^{\infty} Z(z_{pc})\Gamma_{pc} \approx \left[-\frac{1}{z_{oc}} - \frac{1}{2z_{oc}^3} - \frac{3}{4z_{oc}^5} \right] \Gamma_{oc}. \quad (44)$$

Similarly, using the series expansion equation (37) (where $e^{-z_{oh}^2} \approx 1$ for $|z_{oh}| \ll 1$), we have for the hot species,

$$\sum_{p=-\infty}^{\infty} Z(z_{ph})\Gamma_{ph} \approx \left(i\sqrt{\pi} - 2z_{oh} + \frac{4z_{oh}^3}{3} \right) \Gamma_{oh}. \quad (45)$$

It is noted that for relatively high temperature T_h , the thermal velocity of the hot species is much larger than the wave phase velocity. Hence, for large T_h , we have assumed that $|z_{oh}| \ll 1$.

Substituting (44) and (45), λ_D , λ_{Dc} and λ_{Dh} , into the dispersion relation (33), whereas before $\lambda_D = (\epsilon_0 T_h / n_0 e^2)^{1/2}$, gives

$$k^2 \lambda_D^2 + 2 \frac{n_{0c}}{\frac{n_0}{T_c} \frac{T_h}{T_h}} \left[i\sqrt{\pi} z_{oc} e^{-z_{oc}^2} - \frac{1}{2z_{oc}^3} - \frac{3}{4z_{oc}^5} \right] + 2 \frac{n_{0h}}{n_0} [1 + i\sqrt{\pi} z_{oh} \Gamma_{oh}] = 0. \quad (46)$$

For the cool species we have assumed $|\alpha_c| = |k_{\perp}^2 v_{tc}^2 / \Omega^2| = k^2 \rho_c^2 \ll 1$ (where ρ_c is the gyroradius of the cool species), i.e., long wavelength fluctuations in comparison to ρ_c . Since in general for $|x| \ll 1$ we can write $\Gamma_p(x) = e^{-x} I_p(x) \approx (x/2)^p (1/p!)(1-x)$, hence we have $\Gamma_{oc} \approx 1$.

Second and higher order terms in z_{oh} are also neglected since we have assumed $|z_{oh}| \ll 1$. Setting $\omega = \omega_r + i\gamma$ and assuming $\gamma/\omega_r \ll 1$ one may write

$$\frac{1}{\omega^2} \approx \frac{1}{\omega_r^2} \left(1 - \frac{2i\gamma}{\omega_r} \right). \quad (47)$$

Using the above manipulation the dispersion relation equation (46) becomes

$$\begin{aligned} k^2 \lambda_D^2 + 2 \frac{n_{0c}}{\frac{n_0}{T_c} \frac{T_h}{T_h}} \left[i\sqrt{\pi} \left(\frac{\omega_r + i\gamma}{\sqrt{2}k_{\parallel}v_{tc}} \right) e^{-z_{oc}^2} - \frac{k_{\parallel}^2 v_{tc}^2}{\omega_r^2} \left(1 - \frac{2i\gamma}{\omega_r} \right) - \frac{3k_{\parallel}^4 v_{tc}^4}{\omega_r^4} \left(1 - \frac{2i\gamma}{\omega_r} \right)^2 \right] \\ + 2 \frac{n_{0h}}{n_0} \left[1 + i\sqrt{\pi} \left(\frac{\omega_r + i\gamma - \mathbf{k} \cdot \mathbf{V}_{oh}}{\sqrt{2}k_{\parallel}v_{th}} \right) \Gamma_{oh} \right] = 0. \end{aligned} \quad (48)$$

Taking the real part of equation (48) with the charge neutrality condition $n_{oc} + n_{oh} = 1$, gives

$$\omega_r^2 = \frac{k^2 v_{ea}^2 \cos^2 \theta}{1 + \frac{1}{2} k^2 \lambda_{Dh}^2} + 3k^2 v_{tc}^2 \cos^2 \theta, \quad (49)$$

where $\cos \theta = k_{\parallel}/k$ and $v_{ea} = (n_{0c}/n_{0h})^{1/2} v_{th}$ is the acoustic speed of the electron-positron plasma. It is noted that equation (49) is consistent with the expression (12) obtained from fluid theory.

The approximate solution of the growth rate is determined by taking the imaginary part of equation (48), and hence solving for γ , one finds

$$\gamma = \frac{\frac{\omega_r^4}{k_{\parallel}^3} \left(\frac{\pi}{8}\right)^{1/2} \left(\frac{m}{T_h}\right)^{3/2} \left[-\left(\frac{T_h}{T_c}\right)^{3/2} e^{-z_{0c}^2} + \left(\frac{n_{0h}}{n_{0c}}\right) \left(\frac{\mathbf{k} \cdot \mathbf{V}_{oh}}{\omega_r} - 1\right) \Gamma_{oh} \right]}{\left[1 + \frac{6k_{\parallel}^2 T_c}{\omega_r^2 m} \right]}. \quad (50)$$

We note that in equation (50), it is the cooler species that provides the Landau damping, i.e., the velocity distribution function sees a negative slope ($\partial f_0/\partial V_{\parallel} < 0$). It is also seen from equation (50) that for an unstable mode ($\gamma > 0$), it is necessary that $V_{oh} > \omega_r/k_{\parallel}$, i.e., the drift velocity parallel to \mathbf{B}_0 of the hot species has to be larger than the phase velocity to overcome the damping terms.

Normalizing the fluid speeds by the thermal velocity $v_{th} = (T_h/m)^{1/2}$, the particle density by the total equilibrium plasma density $n_0 = n_{0c} + n_{0h}$, the temperatures by T_h , the spatial length by $\lambda_{dj} = \left(\frac{\epsilon_0 T_j}{n_{0j} e^2}\right)^{1/2}$, and the time by $\omega_p^{-1} = \left(\frac{n_0 e^2}{\epsilon_0 m}\right)^{-1/2}$, one may write the normalized real frequency as,

$$\omega_r^2 = \frac{2n_{0c} k_{\parallel}^2 \lambda_d^2}{2(1 - n_{0c}) + k_{\parallel}^2 \lambda_d^2} + 3k_{\parallel}^2 \lambda_d^2 \frac{T_c}{T_h}, \quad (51)$$

and the approximate normalized growth rate as,

$$\gamma_r = \frac{\frac{\omega_r^4}{k_{\parallel}^3 \lambda_d^3} \left(\frac{\pi}{8}\right)^{1/2} \left[\left(\frac{1 - n_{0c}}{n_{0c}}\right) \left(\frac{\vec{k} \cdot \vec{V}_{oh}}{\omega_r} - 1\right) \Gamma_{oh} \right]}{\left[1 + \frac{6k_{\parallel}^2 T_c}{\omega_r^2 T_h} \right]}, \quad (52)$$

For a fixed value of $k\lambda_d$, the real frequency increases with an increase in the cool to hot temperature ratio. This can be seen from the approximate analytical expression (51). **Figure 2** displays the normalized growth rate as a function of the normalized wavenumber for varying cool to hot species temperature ratios T_c/T_h . It is noted that as the T_c/T_h decreases, the growth rate increases, implying that the

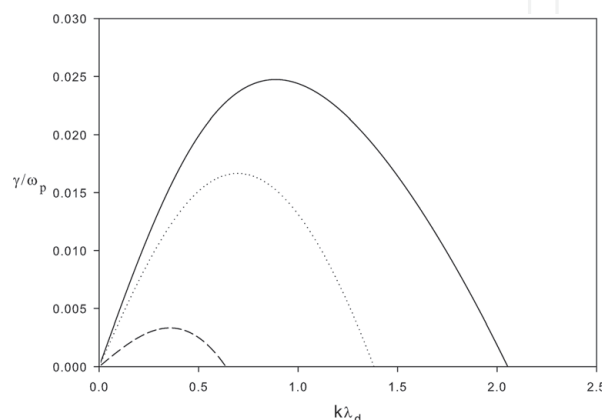


Figure 2. Normalized growth rate as a function of the normalized wavenumber. The fixed parameters are $R = 0.333$, $V_{oh} = 0.5$, $n_{0c} = 0.1$ and $\theta = 45^\circ$. The parameter labeling the curve is the cool to hot temperature ratio $T_c/T_h = 0.005$ (solid), 0.01 (dotted), and 0.02 (broken).

instability is more easily excited with lower temperature ratios. This may be explained as follows. As the temperature of the cooler species is increased, the associated Landau damping increases, resulting in a reduction of the overall growth rate. It is noted that a cutoff $k\lambda_d$ value is reached beyond which the mode is damped.

4. Nonlinear electrostatic solitary waves in electron-positron plasmas

The study of nonlinear effects in electron-positron plasmas is important since these plasmas exhibit different wave phenomena as compared to electron-ion plasmas. It is therefore important to understand the nonlinear structures, especially the solitary waves that exist in electron-positron plasmas. Satellite observations in the Earth's magnetosphere have shown the existence of electrostatic solitary waves which forms part of broadband electrostatic noise (BEN) and electrostatic solitary waves (ESWs) in various regions of the Earth's magnetosphere. The characteristic features of these ESWs are solitary bipolar pulses and consist of small scale, large amplitude parallel electric fields. These large amplitude spiky structures have been interpreted in terms of either solitons [32] or isolated electron holes in the phase space corresponding to positive electrostatic potential [33]. Given that electron-positron plasmas are increasingly observed in astrophysical environments, as well as in laboratory experiments [34], the above mentioned satellite observations also lead one to explore if such nonlinear structures are also possible in electron-positron plasmas. There is a distinct possibility that a pulsar magnetosphere can support coexistence of two types of cold and hot electron-positron populations [10, 35, 28]. In this section we investigate nonlinear electrostatic spiky structures in a magnetized four component two-temperature electron-positron plasma.

4.1 Basic equations

The model considered, as in the previous section is a homogeneous magnetized, four component, collisionless, electron-positron plasma, consisting of cool electrons (ec) and cool positrons (pc) with equal temperatures T_c and initial densities ($n_{ec0} = n_{pc0}$), and hot electrons (eh) and hot positrons (ph) with equal temperatures T_h and densities ($n_{eh0} = n_{ph0}$). Wave propagation is taken in the x -direction at an angle θ to the magnetic field \mathbf{B}_0 , which is assumed to be in the x - z plane.

The continuity and momentum equations for the four species are given by

$$\frac{\partial n_j}{\partial t} + \frac{\partial(n_j v_{jx})}{\partial x} = 0 \quad (53)$$

$$\frac{\partial v_{jx}}{\partial t} + v_{jx} \frac{\partial v_{jx}}{\partial x} + \frac{1}{n_j m} \frac{\partial p_j}{\partial x} = -\frac{\epsilon_j e}{m} \frac{\partial \phi}{\partial x} + \epsilon_j \Omega v_{jy} \sin \theta \quad (54)$$

$$\frac{\partial v_{jy}}{\partial t} + v_{jx} \frac{\partial v_{jy}}{\partial x} = \epsilon_j \Omega v_{jz} \cos \theta - \epsilon_j \Omega v_{jx} \sin \theta \quad (55)$$

$$\frac{\partial v_{jz}}{\partial t} + v_{jx} \frac{\partial v_{jz}}{\partial x} = -\epsilon_j \Omega v_{jy} \cos \theta, \quad (56)$$

where $\epsilon_j = +1(-1)$ for positrons (electrons) and $j = ec, pc, eh, ph$ for the cool electrons, cool positrons, hot electrons, and the hot positrons, respectively.

The density of the cool electrons (positrons) is n_{ec} (n_{pc}), and that of the hot electrons (positrons) is n_{eh} (n_{ph}).

The general equation of state for the four species is given by

$$\frac{\partial p_j}{\partial t} + v_{jx} \frac{\partial p_j}{\partial x} + 3p_j \frac{\partial v_{jx}}{\partial x} = 0, \quad (57)$$

The system is closed by the Poisson equation

$$\epsilon_0 \frac{\partial^2 \phi}{\partial x^2} = -e(n_{pc} - n_{ec} + n_{ph} - n_{eh}). \quad (58)$$

In the above equations, n_j , \mathbf{v}_j and p_j are the densities, fluid velocities and pressures, respectively, of the j^{th} species. $\Omega = \Omega_e = \Omega_p = eB_0/m$ is the cyclotron frequency. Here $m = m_e = m_p$ is the common mass of the electrons and the positrons. Adiabatic compression, $\gamma = (2 + N)/N = 3$, is assumed, where $N = 1$ implies one degree of freedom.

Upon linearizing and combining equations (53)–(58) and taking the limit $v_{tc} \ll \omega/k \ll v_{th}$, where $v_{th} = (T_h/m)^{1/2}$ and $v_{tc} = (T_c/m)^{1/2}$ are the thermal velocities of the hot (cool) species, the dispersion relation equation for a magnetized two-temperature four component electron-positron plasma, where all species are governed by the fluid equations is,

$$\omega^4 - \omega^2(\Omega^2 + 2\omega_s^2 + 3k^2v_{tc}^2) + 2\omega_s^2\Omega^2 \cos^2\theta = 0. \quad (59)$$

where $\omega_{pc,ph} = (n_{0c,h}e^2/\epsilon_0m)^{1/2}$ are the plasma frequencies of the cool and hot species respectively and $\omega_s = \omega_{pc}/(1 + 2/3k^2\lambda_{Dh}^2)^{1/2}$ and $\lambda_{Dh} = (\epsilon_0T_h/n_{oh}e^2)^{1/2}$.

Solving the above dispersion relation gives the cyclotron mode,

$$\omega_+^2 = (\Omega^2 + 2\omega_s^2 + 3k^2v_{tc}^2) - \frac{2\omega_s^2\Omega^2 \cos^2\theta}{\Omega^2 + 2\omega_s^2 + 3k^2v_{tc}^2} \quad (60)$$

and the acoustic mode,

$$\omega_-^2 = \frac{2\omega_s^2\Omega^2 \cos^2\theta}{\Omega^2 + 2\omega_s^2 + 3k^2v_{tc}^2}, \quad (61)$$

4.2 Nonlinear analysis

In the nonlinear regime, a transformation to a stationary frame $s = (x - Vt)(\Omega/V)$ is performed, and v , t , x and ϕ are normalized with respect to v_{th} , Ω^{-1} , $\rho = v_{th}/\Omega$, and T_h/e , respectively. V is the phase velocity of the wave. In equations (53)–(57), $\partial/\partial t$ is replaced by $-\Omega(\partial/\partial s)$ and $\partial/\partial x$ by $(\Omega/V)(\partial/\partial s)$, and the diving electric field amplitude is defined as $E = -(\partial\psi/\partial s)$, where $\psi = e\phi/T_h$.

Integrating equation (53) and using the initial conditions $n_{ec0} = n_0$ and $v_{ecx} = v_0$ at $s = 0$, yields the normalized velocity for the cool electrons in the x-direction.

$$v_{ecx} = -\left(\frac{n_{eco}}{n_{ec}}\right)(V - v_0) + V \quad (62)$$

Similarly the cool positrons, hot electrons and hot positrons velocities are determined. Substituting these into the normalized form of equations (53)–(57), gives

the following set of nonlinear first-order differential equations for the cool electron species in the stationary frame.

$$\frac{\partial \psi}{\partial s} = -E \quad (63)$$

$$\frac{\partial E}{\partial s} = R^2 M^2 (n_{pcn} - n_{ecn} + n_{phn} - n_{ehn}) \quad (64)$$

$$\frac{\partial n_{ecn}}{\partial s} = \frac{n_{ecn}^3 [E + M \sin \theta v_{ecyn}]}{\left(\frac{n_{ec0}}{n_0}\right)^2 (M - \delta_c)^2 - 3 \frac{T_c}{T_h} p_{ecn} n_{ecn}} \quad (65)$$

$$\frac{\partial v_{ecyn}}{\partial s} = \frac{M n_{ecn}}{(M - \delta_c)} \left(\frac{n_0}{n_{ec0}}\right) \left[- \left(M - \frac{(M - \delta_c)}{n_{ecn}} \left(\frac{n_{ec0}}{n_0}\right) \right) \sin \theta + v_{eczn} \cos \theta \right] \quad (66)$$

$$\frac{\partial v_{eczn}}{\partial s} = - \left(\frac{n_0}{n_{ec0}}\right) \frac{n_{ecn} v_{ecyn} M \cos \theta}{(M - \delta_c)} \quad (67)$$

$$\frac{\partial p_{ecn}}{\partial s} = \frac{3 p_{ecn} n_{ecn}^2 [E + M \sin \theta v_{ecyn}]}{\left(\frac{n_{ec0}}{n_0}\right)^2 (M - \delta_c)^2 - 3 \frac{T_c}{T_h} p_{ecn} n_{ecn}} \quad (68)$$

The set of differential equations for the cool positrons are given by,

$$\frac{\partial n_{pcn}}{\partial s} = \frac{n_{pcn}^3}{(M - \delta_c)^2} \left(\frac{n_0}{n_{pc0}}\right)^2 [-E - M \sin \theta v_{pcyn}] \quad (69)$$

$$\frac{\partial v_{pcyn}}{\partial s} = \frac{M n_{pcn}}{(M - \delta_c)} \left(\frac{n_0}{n_{pc0}}\right) \left[\left(M - \frac{(M - \delta_c)}{n_{pcn}} \left(\frac{n_{pc0}}{n_0}\right) \right) \sin \theta - v_{pczn} \cos \theta \right] \quad (70)$$

$$\frac{\partial v_{pczn}}{\partial s} = \left(\frac{n_0}{n_{pc0}}\right) \frac{n_{pcn} v_{pcyn} M \cos \theta}{(M - \delta_c)} \quad (71)$$

$$\frac{\partial p_{pcn}}{\partial s} = \frac{3 p_{pcn} n_{pcn}^2 [-E - M \sin \theta v_{pcyn}]}{\left(\frac{n_{pc0}}{n_0}\right)^2 (M - \delta_c)^2 - 3 \frac{T_c}{T_h} p_{pcn} n_{pcn}} \quad (72)$$

Similar sets of differential equations can be derived for the hot electrons and hot positron species. The velocities are normalized with respect to the thermal velocity of the hot species $v_{th} = (T_h/m)^{1/2}$ and the densities with respect to the total density n_0 . The equilibrium density of the cool (hot) electrons is n_{ec0} (n_{eh0}), and that of the cool (hot) positrons n_{pc0} (n_{ph0}), with $n_{ec0} + n_{eh0} = n_{pc0} + n_{ph0} = n_0$. $R = \omega_p/\Omega$, where $\omega_p = (n_0 e^2 / \epsilon_0 m)^{1/2}$ is the total plasma frequency, $M = V/v_{th}$ is the Mach number and $\delta_{c,h} = v_{0c,0h}/v_{th}$ is the normalized drift velocity of cool (hot) species at $s=0$. The system of nonlinear first-order differential equations can now be solved numerically using the Runge-Kutta (RK4) technique [36]. The initial values can be determined self consistently where the actual normalized electric fields are given by $E_{norm} = -(1/M)(\partial \psi / \partial s)$ and wave propagation is taken almost parallel to the ambient magnetic field \mathbf{B}_0 .

Numerical results to investigate the effect of parameters such as the electric driving force E_0 , densities n_{ec0} and n_{ph0} , temperature ratio T_c/T_h , Mach number M , drift velocities $\delta_{c,h}$ and propagation angle θ on the wave can be explored. A typical numerical result is seen in **Figure 3a–d** [37] showing the evolution of the system for various driving electric field amplitudes E_0 . It is seen that as E_0 increases, the electric field structure evolves from a sinusoidal wave to a sawtooth structure. For a

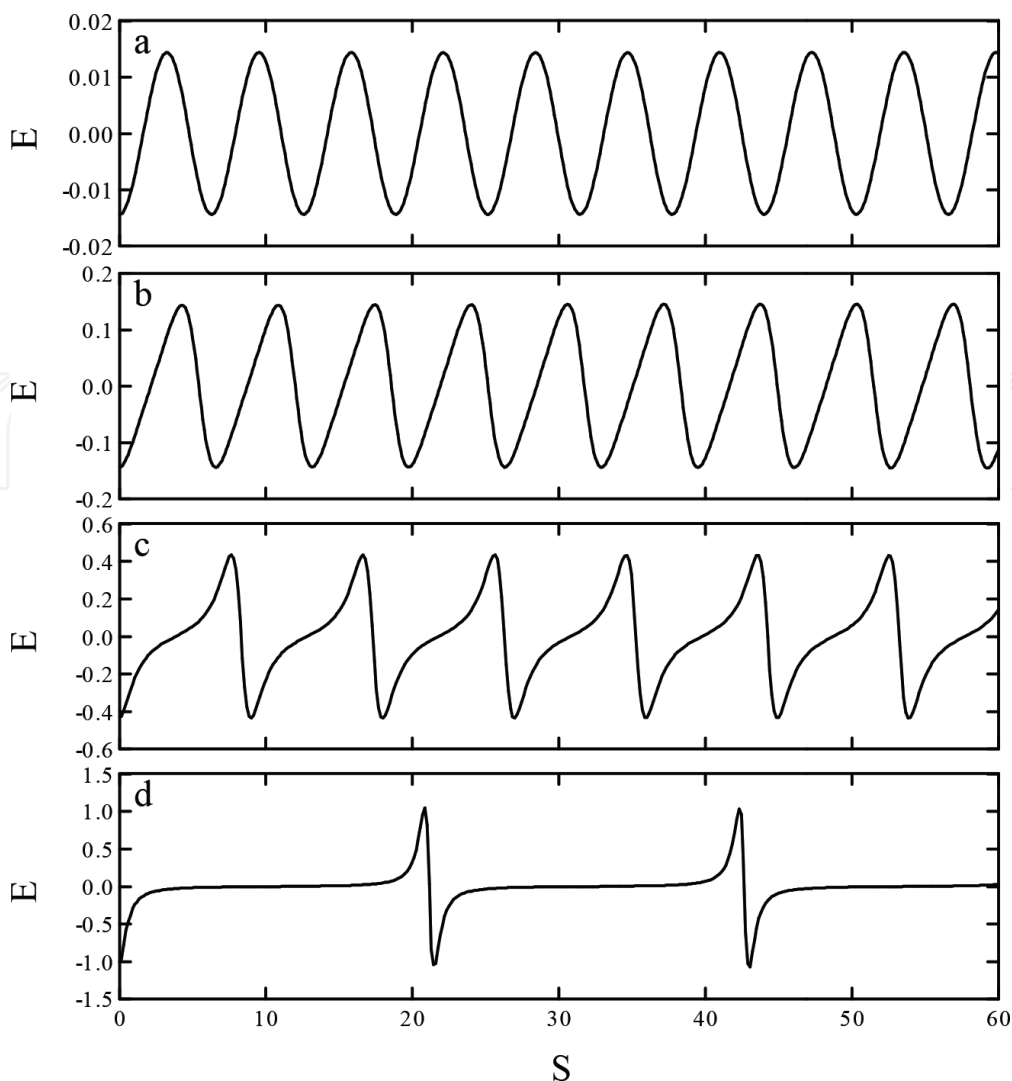


Figure 3. Numerical solution of the normalized electric field for the parameters $M = 3.5$, $\theta = 2^\circ$, $R = 10.0$, $\delta_c = \delta_h = 0.0$, $n_{ec0}/n_0 = n_{pc0}/n_0 = 0.5$, $T_c/T_h = 0.0$, and $E_0 =$ (a) 0.05 [linear waveform], (b) 0.5 [sinusoidal waveform], (c) 1.5 [sawtooth waveform] and (d) 3.5 [bipolar waveform].

higher E_0 value of 3.5, the potential structure has a spiky bipolar form showing that as the period of the wave increases and the frequency of the wave decreases.

5. Conclusion

Linear and nonlinear electrostatic waves in a magnetized four component two-temperature electron-positron plasma have been investigated. In the linear analysis fluid and kinetic theory approaches are employed to describe the wave motion. The fluid theory approach focused on the wave dynamics of both the acoustic and cyclotron branches. Solutions of the dispersion relation from fluid theory yielded electron-acoustic, upper hybrid, electron plasma and electron cyclotron branches. Perpendicular and parallel wave propagation was examined showing its influence on the dispersive properties of the wave. The kinetic theory approach further examined Landau damping effects on the acoustic mode, analyzing the frequency and growth rate of the wave. The analysis shows that a large enough drift velocity (V_{oh}) is required to produce wave growth. Both fluid and kinetic theory show excellent agreement for the real frequencies of the acoustic mode and solutions of the corresponding dispersion relation can be explored as a function of several

plasma parameters. In the nonlinear analysis, the two-fluid model is used to derive a set of differential equations for the electrostatic solitary waves in a magnetized two-temperature electron-positron plasma. In particular, electrostatic solitary waves and their electric fields, similar to those found in the Broadband Electrostatic Noise are explored. For the onset of spiky ESWs, it is noted that as the wave speed increases, a larger driving electric field is required.

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