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# Electromechanical Co-Simulation for Ball Screw Feed Drive System

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Liang Luo and Weimin Zhang

Additional information is available at the end of the chapter

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## Abstract

Ball screw feed drive system is the most widely used linear drive system in the field of industrial automation. The continuous search for efficiency puts forward higher requests to the machine tool for high speed and high acceleration, which makes the feed drive system of lightweight-designed and large-size machine tools more likely to produce vibration during high-speed and high-acceleration feed operation. Electromechanical co-simulation for ball screw feed drive dynamics is an important technique for solving vibration problems occurring in the feed motion. This chapter elaborates on this technology from three aspects: modeling and simulation of dynamic characteristics of ball screw feed drive, modeling and simulation of servo control system, and the electromechanical co-simulation of ball screw feed drive system. In this chapter, the basic theoretical models, the establishment of simulation models and the comparison between simulation and experiment results of ball screw feed drive system are comprehensively introduced to provide technical references for readers.

**Keywords:** ball screw feed drive system, dynamic characteristics, electromechanical co-simulation, vibration, lumped mass model

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## 1. Introduction

Ball screw feed system is the most widely used linear drive system in the field of industrial automation [1]. In order to enhance the speed and accuracy of present systems further, current research focuses on the vibration reduction and avoidance of the feed drive. Additional damping modules or structures are integrated in the feed drive system to achieve this goal, such as semi-active damping system, set point filtering, etc. Active damping system only reacts once a vibration is present, and set point filtering can lead to path deformation [2–4]. Another

way to solve this problem is to generate a smoother trajectory. For this purpose, numbers of trajectory algorithms are found out, and the frequency contents of the trajectory orders are discussed and compared [5, 6]. The vibration caused by the trajectory is difficult to analyze on hardware because of the coupling factor of variety excitation sources. All these researches need a simulation method to help the researchers or engineers study or optimize the design and parameter setting of the feed drive system [10].

Finite element model of ball screw feed drive system can predict the accurate dynamic characteristics. However, it is difficult to integrate with the simulation model of servo control system. Lumped parameter model of ball screw feed drive system can simplify the simulation model by reducing the number of degrees of freedom (DOF) of the whole system. More importantly, it can easily integrate with the simulation model of the servo control system. A reasonable simplification of the lumped parameter model is the key to accurately predict the vibration of feed drive system [7–9].

In this chapter an electromechanical co-simulation method for ball screw feed drive system was established, which can be used to study the dynamic characteristics and vibration behavior of the feed drive system. An optimized dynamic modeling and simulation method of a ball screw feed drive based on the lumped mass model was firstly presented, and the optimized calculation method of the equivalent parameters was given. Then, a model of servo control system was built up, and based on it, the electromechanical co-simulation of ball screw feed drive system was established. Finally, a simulative and experimental test is conducted based on a ball screw feed drive system test bench. The result shows that electromechanical co-simulation of ball screw feed drive system could achieve a very good predictability.

## 2. Dynamic characteristic modeling and simulation of ball screw feed system

### 2.1. Lumped mass model of ball screw feed system

A typical ball screw feed system consists of a servomotor, coupling, ball screw, work table, and base (**Figure 1**). The ball screw is supported by two sets of bearing, which are fixed to the base. The servomotor torque is transmitted through a coupling onto the ball screw shaft to drive the work table. The linear guideway constrains the movement of the work table in an axial direction. The base is fixed on the machine bed or placed on the ground. The transformation from the rotational movement of the screw shaft into the linear motion of the work table is realized by the ball screw system with its transmission ratio  $i$ , which is defined as the distance of travel  $h$  during one revolution of the shaft as the following:

$$i = \frac{h}{2\pi} \quad (1)$$

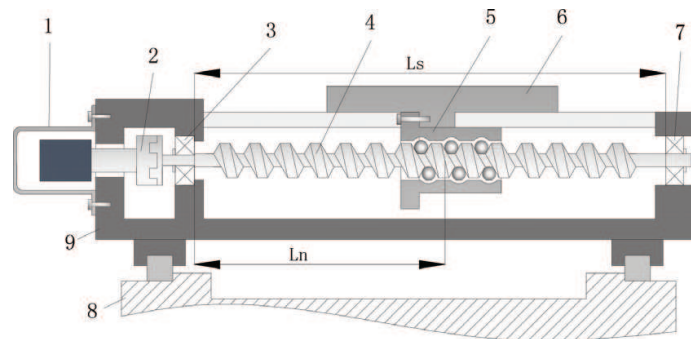
Low-order modes are the main factors affecting the dynamic characteristics of the ball screw feed drive system of machine tools. Typically, the first axial and rotational modes of the ball screw show a dominant influence on the overall dynamics, while the relevance of higher-order modes for most technical applications is rather small [8].

The lumped mass model can reasonably reduce the number of degrees of freedom (DOF) of the simulation model while preserving the low-order modes of the system to simplify calculations. **Figure 2** shows the lumped mass model of a ball screw feed drive system. The influence of the shaft on the rotational mode and axial mode of the drive system is explicitly included into the lumped mass model here. Therefore, the shaft is separated into two different branches, an axial branch and a rotational branch, while the coupling once more is realized using constrained equations. Since all components are expressed by discrete springs and dampers, the rigidity values of shaft, coupling, and bearing are combined to an overall axial  $K_{ax}$  and rotational value  $K_{rot}$ .

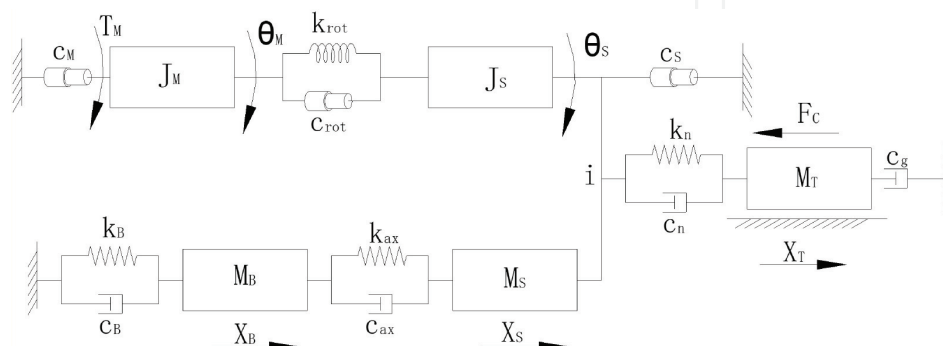
In this model the inertial component parameters are defined as the following: rotary inertia of servomotor  $J_M$ , screw shaft side equivalent rotary inertia  $J_S$ , mass of base  $M_B$ , screw shaft side equivalent mass  $M_S$ , and mass of the work table  $M_T$ .

The equivalent rigidity parameters in the model are defined as the following: equivalent torsional rigidity  $K_{rot}$ , equivalent axial rigidity  $K_{ax}$ , rigidity of ball screw nut  $K_n$ , and axial rigidity of the base  $K_B$ .

The equivalent damping parameters are defined as the following: servomotor torsional damping  $C_M$ , equivalent torsional damping  $C_{rot}$ , screw shaft side damping  $C_S$ , ball screw nut damping  $C_n$ , equivalent axial damping  $C_{ax}$ , axial damping of the base  $C_B$ , and axial damping of the guide  $C_g$ .



**Figure 1.** Typical structure of ball screw drive system. 1. Servomotor; 2. Coupling; 3. Fixed bearing; 4. Screw shaft; 5. Ball screw nut; 6. Work table; 7. Support bearing; 8. Machine bed; 9. Base.



**Figure 2.** Lumped mass model of ball screw feed system.

The DOF parameters of the lumped mass model are defined as the following: angular rotation of the servomotor  $\theta_M$ , screw shaft angular rotation at the table position  $\theta_S$ , axial displacement of the base  $X_B$ , screw shaft axial displacement at the table position  $X_S$ , and work table position  $X_T$ . Therefore, the deformation of the equivalent springs is described as follows: equivalent torsional spring deformation  $\theta_M - \theta_S$ , equivalent axial spring deformation  $X_S - X_B$ , axial spring deformation of the base  $X_B$ , and screw nut contact deformation  $X_T - X_S - i\theta_S$ .

The speed parameters of equivalent damping are defined as the following: servomotor equivalent damping speed  $\dot{\theta}_M$ , equivalent torsional vibration damping speed  $\dot{\theta}_M - \dot{\theta}_S$ , equivalent damping speed of the screw  $\dot{\theta}_S$ , equivalent damping speed of the base  $\dot{X}_B$ , equivalent axial damping speed of screw  $\dot{X}_S - \dot{X}_B$ , equivalent damping speed of screw nut  $\dot{X}_T - \dot{X}_S - i\dot{\theta}_S$ , and speed of the work table  $\dot{X}_T$ .

According to the Lagrange's equations of the second kind, the dynamic model of the ball screw feed drive system is built up. The total kinetic energy  $T$ , the potential energy  $U$ , and the dissipation function of the system can be expressed using equations (2) through (4):

$$T = \frac{1}{2}J_M\dot{\theta}_M^2 + \frac{1}{2}J_S\dot{\theta}_S^2 + \frac{1}{2}M_B\dot{X}_B^2 + \frac{1}{2}M_S\dot{X}_S^2 + \frac{1}{2}M_T\dot{X}_T^2 \quad (2)$$

$$U = \frac{1}{2}k_{rot}(\theta_M - \theta_S)^2 + \frac{1}{2}k_B X_B^2 + \frac{1}{2}k_{ax}(X_S - X_B)^2 + \frac{1}{2}k_{nut}(X_T - X_S - i\theta_S)^2 \quad (3)$$

$$D = \frac{1}{2}C_M\dot{\theta}_M^2 + \frac{1}{2}C_{rot}(\dot{\theta}_M - \dot{\theta}_S)^2 + \frac{1}{2}C_S\dot{\theta}_S^2 + \frac{1}{2}C_B\dot{X}_B^2 + \frac{1}{2}C_{ax}(\dot{X}_S - \dot{X}_B)^2 + \frac{1}{2}C_n(\dot{X}_T - \dot{X}_S - i\dot{\theta}_S)^2 + \frac{1}{2}C_T\dot{X}_T^2 \quad (4)$$

According to the definition of the system lumped mass, we have the independent coordinates system  $\mathbf{q}$  as the following:

$$\mathbf{q} = (\theta_M \ \theta_S \ X_B \ X_S \ X_T)^T \quad (5)$$

The force inputs of the ball screw feed system are the servomotor torque  $T_M$  and cutting force  $F_C$ , and then the generalized forces  $\mathbf{Q}$  of the system can be expressed as the following:

$$\mathbf{Q} = (T_M \ 0 \ 0 \ 0 \ -F_C)^T \quad (6)$$

With  $L = T - U$ , the Lagrangian function of the system about the generalized coordinate  $\mathbf{q}$  and the generalized force  $\mathbf{Q}$  can be calculated according to Eq. (7). Then, the matrix form of the lumped mass model of the ball screw feed system can be established as in Eq. (8):

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\mathbf{q}}} \right) - \frac{\partial L}{\partial \mathbf{q}} - \frac{\partial D}{\partial \dot{\mathbf{q}}} = \mathbf{Q} \quad (7)$$

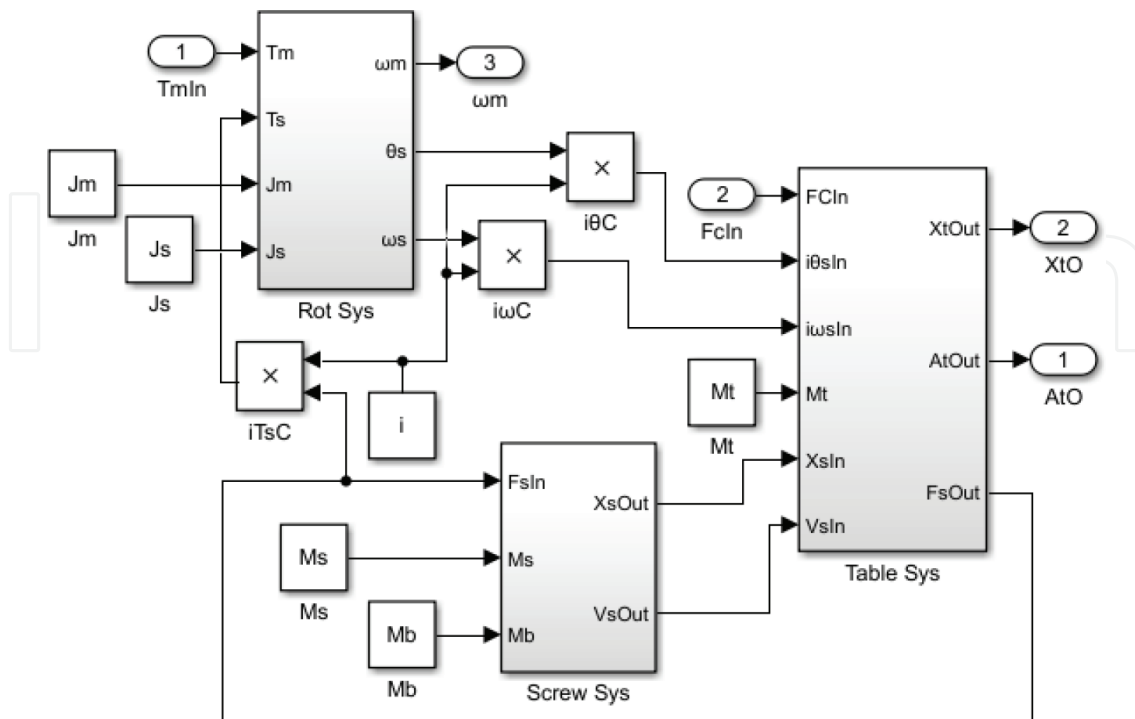
$$\mathbf{m}\ddot{\mathbf{q}} + \mathbf{c}\dot{\mathbf{q}} + \mathbf{k}\mathbf{q} = \mathbf{Q} \quad (8)$$

where,

$$\mathbf{m} = \begin{bmatrix} J_M & 0 & 0 & 0 & 0 \\ 0 & J_S & 0 & 0 & 0 \\ 0 & 0 & M_B & 0 & 0 \\ 0 & 0 & 0 & M_S & 0 \\ 0 & 0 & 0 & 0 & M_T \end{bmatrix} \quad \mathbf{k} = \begin{bmatrix} k_{rot} & -k_{rot} & 0 & 0 & 0 \\ -k_{rot} & k_{rot} - i^2 k_n & 0 & -ik_n & ik_n \\ 0 & 0 & k_{ax} + k_B & -k_{ax} & 0 \\ 0 & ik_n & -k_{ax} & k_{ax} + k_n & -k_n \\ 0 & -ik_n & 0 & -k_n & k_n \end{bmatrix}$$

$$\mathbf{c} = \begin{bmatrix} c_M + c_{rot} & -c_{rot} & 0 & 0 & 0 \\ -c_{rot} & c_{rot} - c_S - i^2 c_n & 0 & -ic_n & ic_n \\ 0 & 0 & c_{ax} + c_B & -c_{ax} & 0 \\ 0 & ic_n & -c_{ax} & c_{ax} + c_n & -c_n \\ 0 & -ic_n & 0 & -c_n & c_n + c_g \end{bmatrix}$$

The dynamic model of the ball screw feed system shown in Eq. (8) was decomposed into three subsystems: screw shaft torsional vibration system, screw shaft axial vibration system, and the table vibration system. The simulation model of the ball screw feed system can be established as **Figure 3**. The input of the simulation model is the motor torque  $T_M$  and the cutting force  $F_C$ , and the outputs are the table acceleration  $a_T$  and displacement  $X_T$ .



**Figure 3.** Simulation model of a ball screw feed drive.

## 2.2. Equivalent parameter calculation method of ball screw feed system lumped mass model

Accurate ball screw feed system dynamic model requires a reasonable equivalent parameter calculation method of the lumped mass model. As mentioned the shaft has influence on the rotational mode and the axial mode of the drive system; the shaft is separated into two different branches, an axial branch and a rotational branch, while the coupling is realized using constrained equations. The dynamic characteristics of the feed system should be analyzed to select the appropriate equivalent parameter calculation method. The inertia of the axial system and the inertia of rotational system are not only the mass or inertia of the component itself but also the mass or inertia converted to the independent coordinate system component of the dynamic system.

The equivalent rotary inertia of the screw  $J_S$  is composed of the rotary inertia of the screw  $J_{sc}$ , the rotary inertia of the coupling  $J_c$ , and the mass of the table  $M_T$  converted to the rotary inertia of the screw:

$$J_S = J_{sc} + J_c + M_T i^2 \quad (9)$$

With the material density  $\rho$ , namely, equivalent diameter  $d_s$  and length  $l_s$  of the screw shaft, the rotary inertia of the screw  $J_{sc}$  can be approximated using the following equation:

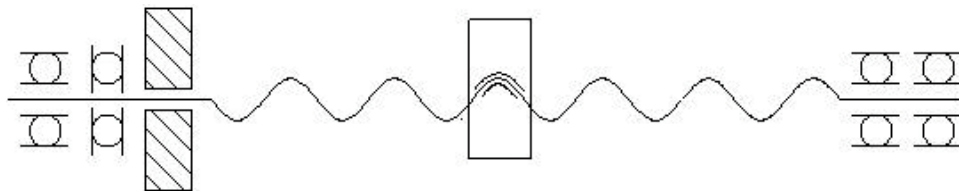
$$J_{sc} = \frac{\rho \cdot \pi}{32} d_s^4 l_s \quad (10)$$

The screw equivalent mass  $M_S$  is composed of screw mass  $M_{sc}$ , servomotor rotor mass  $M_m$ , and coupling mass  $M_c$ ; with the material density  $\rho$ , namely, equivalent diameter  $d_s$  and length  $l_s$  of the screw shaft, the rotary inertia of the screw values  $J_{sc}$  can be approximated using the following equations:

$$M_S = M_{sc} + M_m + M_c \quad (11)$$

$$M_{sc} = \frac{\rho \pi}{4} d_s^2 l_s \quad (12)$$

The axial rigidity of the ball screw feed system is related to the installation method of the screw. Here is an example of the screw-fixed-support method used on most machine tools (**Figure 4**). Servomotor side of the screw shaft uses fixed support to provide screw axial support, and the end of the shaft is free support. Therefore, the axial rigidity  $k_{ax}$  of the ball



**Figure 4.** Screw fixation-support installation diagram.



screw feed system consists of screw bearing rigidity  $k_b$  and the axial rigidity of the screw shaft  $k_{sax}$  is as follows:

$$k_{ax} = \left( \frac{1}{k_b} + \frac{1}{k_{sax}} \right)^{-1} \quad (13)$$

With the material elastic modulus  $E$ , screw cross-sectional area  $A$ , and ball screw length at table position  $l_n$ , the axial rigidity of the screw according to the position of the table  $k_{sax}$  can be established as shown in Eq. (14):

$$k_{sax} = \frac{EA}{l_n} = \frac{\pi d_s^2 E}{4l_n} \quad (14)$$

The torsional rigidity of the ball screw feed system  $k_{rot}$  consists of the torsional rigidity of the screw  $k_{srot}$  and the torsional rigidity of the coupling  $k_c$ . According to the position of the table, the torsional rigidity of the screw  $k_{srot}$  can be approximated with the shear modulus  $G$  and polar rotary inertia of the screw section  $I_p$  as shown in Eq. (16):

$$k_{rot} = \left( \frac{1}{k_c} + \frac{1}{k_{srot}} \right)^{-1} \quad (15)$$

$$k_{srot} = \frac{T}{\Delta\theta} = \frac{G \cdot I_p}{l_n} = \frac{G \cdot \pi \cdot d_s^4}{32 \cdot l_n} \quad (16)$$

With the nut reference rigidity  $K$ , nut axial load  $F_a$ , and basic dynamic load  $C_a$ , the contact rigidity of the screw nut  $k_n$  can be expressed as follows:

$$k_n = 0.8K \left( \frac{F_a}{0.1C_a} \right)^{\frac{1}{3}} \quad (17)$$

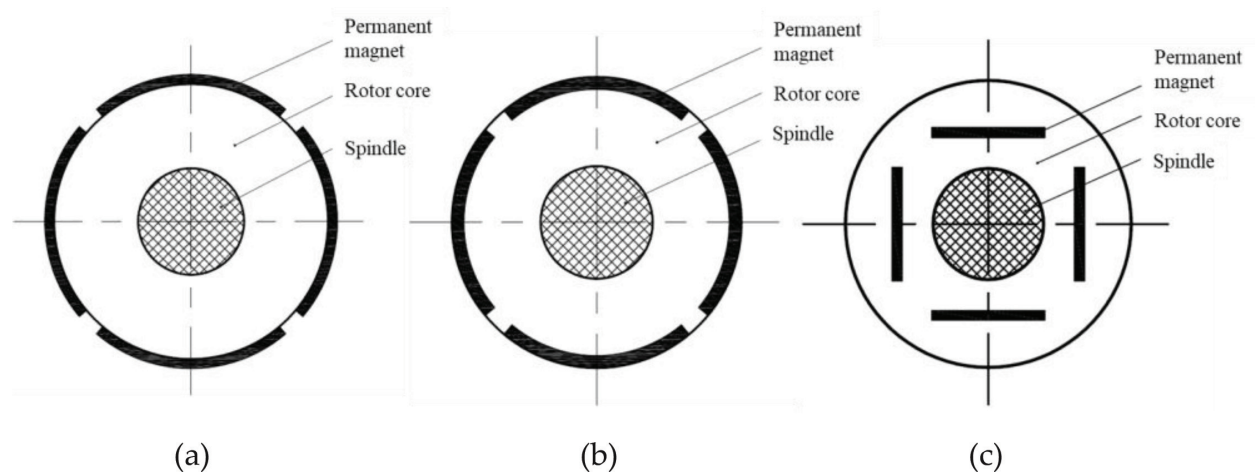
### 3. Servo control system modeling and simulation

#### 3.1. Modeling of permanent magnet synchronous motor

Permanent magnet synchronous motors can be divided into two types according to the rotor type, salient pole rotor and non-salient pole rotor. The structure is shown in **Figure 5**; in the surface-mounted permanent magnet synchronous motor (**Figure 5(a)**), the magnetic circuit of the rotor is symmetrical, and the magnetic permeability and air gap permeability of the permanent magnet material are approximately the same. In the rotor two-phase coordinate system, the direct-axis inductance and the quadrature-axis inductance are equal, that is  $L_d = L_q$ . It is named non-salient pole rotor permanent magnet synchronous motor.

The rotor magnetic paths of plug-in-type (**Figure 5(b)**) and built-in-type (**Figure 5(c)**) permanent magnet synchronous motors are asymmetrical, and the quadrature-axis inductance is



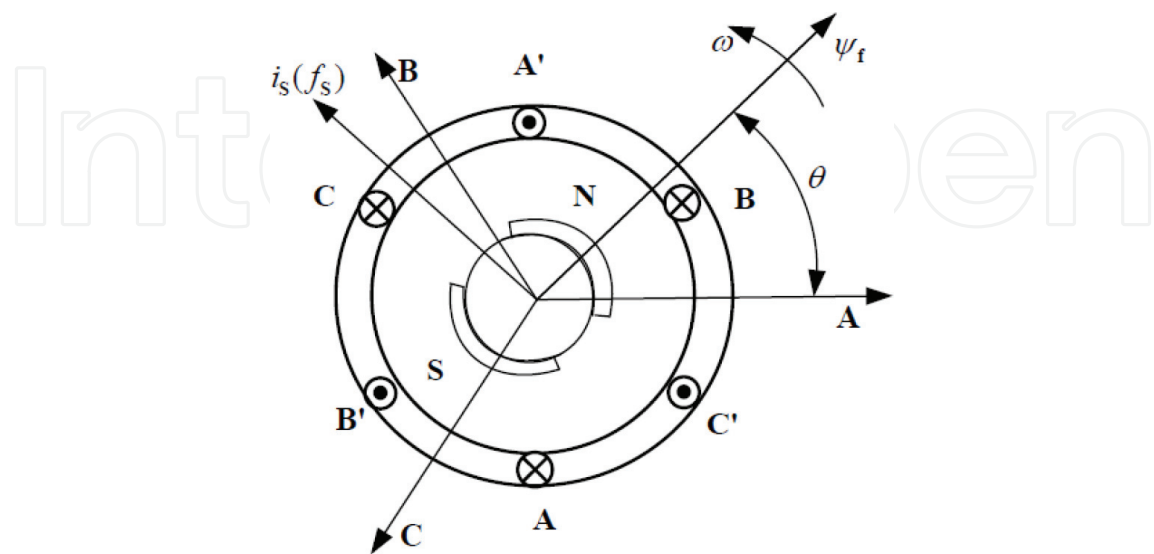


**Figure 5.** PMSM motor rotor structure. (a) Surface-mounted; (b) Plug-in; and (c) Interior-mounted.

greater than the direct-axis inductance, that is  $L_q > L_d$ . The rotor shows salient pole effect, which is called salient pole-type permanent magnet synchronous motor.

Taking non-salient pole rotor permanent magnet synchronous motor as an example, we simplify the motor model with the following conditions: neglecting the saturation of the motor core; no eddy current and hysteresis loss; permanent magnet material has zero conductivity; three-phase windings are symmetrical; and induced potential in the winding is sinusoidal. Then, a schematic diagram of the physical model of the motor shown in **Figure 6** can be obtained.

The axis of the sinusoidal magnetomotive wave generated by a flowing forward current through the phase winding is defined as the axis of the phase winding. Take axis A as the spatial reference coordinate of the ABC coordinate system. It is assumed that the positive direction of the induced electromotive force is opposite to the positive direction of the current (motor principle); take the counterclockwise direction as the positive direction of the speed and



**Figure 6.** PMSM physical model.

electromagnetic torque, and the positive direction of the load torque is the opposite. The physical model equation of permanent magnet synchronous motor is as follows:

$$\begin{cases} U_s = Ri_s + \frac{d\Psi_s}{dt} \\ \Psi_s = Li_s + \Psi_f \begin{bmatrix} \cos(\theta + \omega t) \\ \cos(\theta + \omega t + 2\pi/3) \\ \cos(\theta + \omega t - 2\pi/3) \end{bmatrix} \end{cases} \quad (18)$$

$$\begin{bmatrix} u_A \\ u_B \\ u_C \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \cdot \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} + \begin{bmatrix} \frac{d\varphi_A(\theta, i)}{dt} \\ \frac{d\varphi_B(\theta, i)}{dt} \\ \frac{d\varphi_C(\theta, i)}{dt} \end{bmatrix} \quad (19)$$

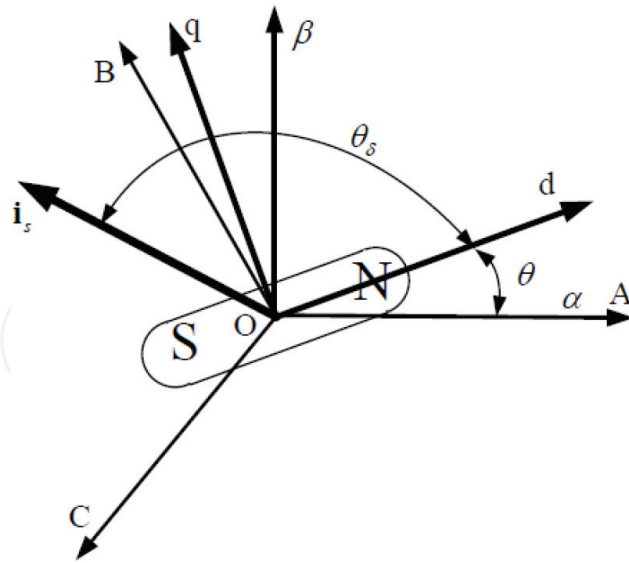
In Formula (18),  $R$  and  $L$  are matrices and their expressions are shown in Formula (20):

$$\begin{cases} U_s = [U_A \ U_B \ U_C]^T \\ i_s = [i_A \ i_B \ i_C]^T \\ R = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \\ L = \begin{bmatrix} L_A & M_{AB} & M_{AC} \\ M_{BA} & L_B & M_{BC} \\ M_{CA} & M_{CB} & L_C \end{bmatrix} \end{cases} \quad (20)$$

$U_A$ ,  $U_B$ , and  $U_C$  are phase voltages of PMSM stator three-phase windings.  $i_A$ ,  $i_B$ , and  $i_C$  are the phase currents of stator three-phase windings.  $R_s$  is the stator winding value.  $L_A$ ,  $L_B$ , and  $L_C$  are stator winding self-inductances.  $M_{AB} = M_{BA}$ ,  $M_{AC} = M_{CA}$ , and  $M_{BC} = M_{CB}$  are the stator winding mutual inductances.

### 3.2. Coordinate transformation

From the above physical model, we can see that in the ABC coordinate system, the PMSM rotor is asymmetric in the magnetic and electrical structures. The motor equation is a set of nonlinear time-varying equations related to the instantaneous position of the rotor, which makes the analysis of the dynamic characteristics of the PMSM very difficult. It is usually necessary to convert the motor equations by coordinate transformation to facilitate analysis and calculation. The coordinate system used in the vector control of the permanent magnet synchronous motor and their relationship is shown in **Figure 7**. In the figure, the  $\alpha - \beta$



**Figure 7.** The coordinate system used in the vector control and their relationship.

coordinate system is a two-phase stationary coordinate system, and the  $d - q$  coordinate system is a two-phase rotating coordinate system that is fixed to the rotor.

### 3.2.1. Clarke transformation

Clarke transformation simplifies the voltage loop equations on the original three-phase windings into the voltage loop equations on the two-phase windings, from the three-phase stator ABC coordinate system to the two-phase stator  $\alpha - \beta$  coordinate system, as shown in Eq. (21); its inverse transform is shown in Eq. (22). However, after the Clarke transformation, the torque still depends on the rotor flux. In order to facilitate control and calculation, Park transformation is also required:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} \quad (21)$$

$$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \sqrt{\frac{3}{2}} \begin{bmatrix} 1 & 0 \\ -1/2 & \sqrt{3}/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad (22)$$

### 3.2.2. Park transformation

In the physical sense, the Park transformation is equivalent to projecting the currents  $i_a, i_b, i_c$  onto the  $d - q$  axis. The transformed coordinate system rotates at the same speed as the rotor, and the d-axis and the rotor flux have the same position. The Park transformation is shown in Eq. (23), and the inverse transformation is shown in Eq. (24). This transformation also holds for the three-phase voltage and flux linkage:

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad (23)$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad (24)$$

### 3.2.3. Mathematical model of permanent magnet synchronous motor in coordinate system

The mathematical model of the permanent magnet synchronous motor in the ABC coordinate system can be transformed into any two-phase coordinate system through coordinate transformation, so that it is possible to simplify the decoupling of the motor flux linkage equation and the electromagnetic torque equation. If the mathematical model of the motor is transformed into a  $d - q$  coordinate system fixed on a permanent magnet rotor, the motor flux equation and the electromagnetic torque equation will be greatly simplified.

With the equivalent flux  $\psi_d$  and  $\psi_q$ , the equivalent inductance  $L_d, L_q$  of the motor in the  $d - q$  coordinate system, and the rotor permanent magnet flux linkage  $\psi_f$ , the stator flux equation can be obtained in Eq. (25):

$$\begin{cases} \psi_d = L_d i_d + \psi_f \\ \psi_q = L_q i_q \end{cases} \quad (25)$$

Take the rotor permanent magnet flux linkage  $\psi_f$  as constant; the voltage of stator in the  $d - q$  coordinate system is as follows:

$$\begin{cases} u_d = R i_d + L_d \frac{di_d}{dt} - \omega_r L_q i_q \\ u_q = R i_q + L_q \frac{di_q}{dt} + \omega_r L_d i_d + \omega_r \psi_f \end{cases} \quad (26)$$

where  $u_d$  and  $u_q$  are the stator-side equivalent voltages of the motor in the  $d - q$  coordinate system,  $R$  is the stator winding resistance per phase, and  $\omega_r$  is the electrical angular velocity of the rotor rotation.

The electromagnetic torque equation in the  $d - q$  coordinate system can be expressed as follows, where  $p$  is the number of pole pairs,  $\psi$  is the flux synthesis vector, and  $\mathbf{i}$  is the current composition vector:

$$T_e = 1.5p\psi \times \mathbf{i} \quad (27)$$

Using the components of the  $d - q$  coordinate system to represent the flux linkage and current vector shown in Eq. (28), the electromagnetic torque can be expressed as shown in Eq. (29):

$$\begin{cases} \mathbf{i} = i_d + j i_q \\ \psi = \psi_d + j \psi_q \end{cases} \quad (28)$$

$$T_e = 1.5p \left[ \psi_f i_q + (L_d - L_q) i_d i_q \right] \tag{29}$$

### 3.3. Servo control modeling of ball screw feed system

In order to model the servo control of the ball screw feed system, the modeling of the three-loop cascade control architecture of the vector control and servo control system of the permanent magnet synchronous servomotor is studied, which is commonly used in the ball screw feed system.

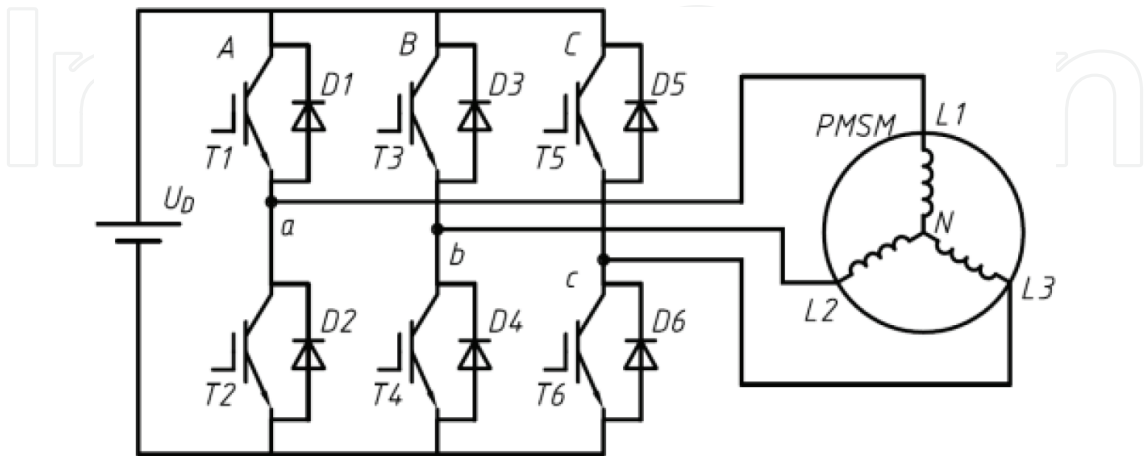
#### 3.3.1. Modeling of permanent magnet synchronous motor vector control.

The principle of space vector pulse width modulation (SVPWM) is based on vector equivalents. The magnitude and direction of the current vector can be indirectly controlled by the timing of the six switching elements of the inverter through the three-phase winding of the permanent magnet synchronous motor, so that the winding produces a constant amplitude circular magnetic field that rotates according to a given demand, thus dragging the permanent magnet to rotate. The voltage inverter circuit is shown in **Figure 8**. The simulation model of SVPWM control system for permanent magnet synchronous motor is shown in **Figure 9**.

#### 3.3.2. Modeling of cascade control system

The SVPWM control system for permanent magnet synchronous motor is based on the three-phase current information and rotor position information fed back by the motor. AC motor is equivalent to a direct current motor by formula transformation to control the position and amplitude of the stator current.

The control system schematic is shown in **Figure 10(a)**. The system includes a cascaded control structure with a P-position controller, a PI-velocity controller, and a PI-current controller. In the cascade control system, the servomotor feedback speed  $\omega_M$  and the work table feedback



**Figure 8.** Circuit of voltage bridge inverter.

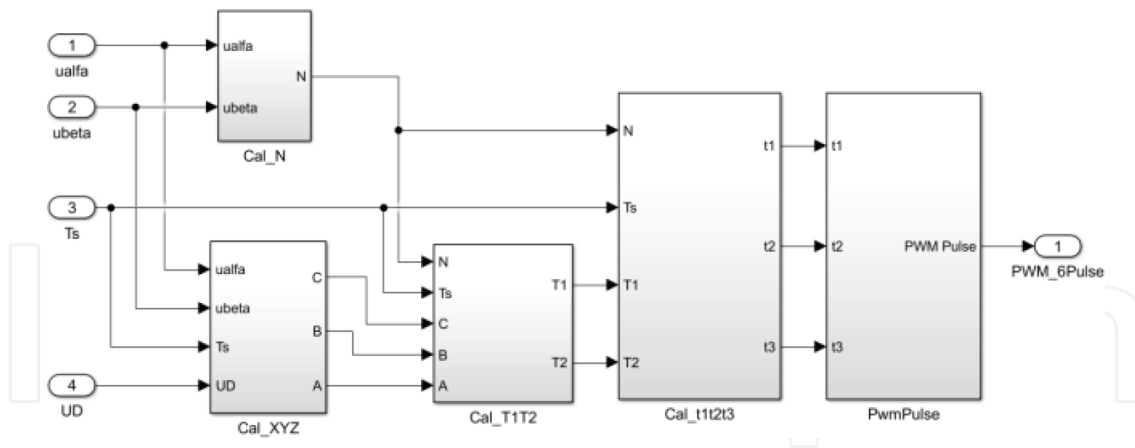


Figure 9. Simulation model of SVPWM control system.

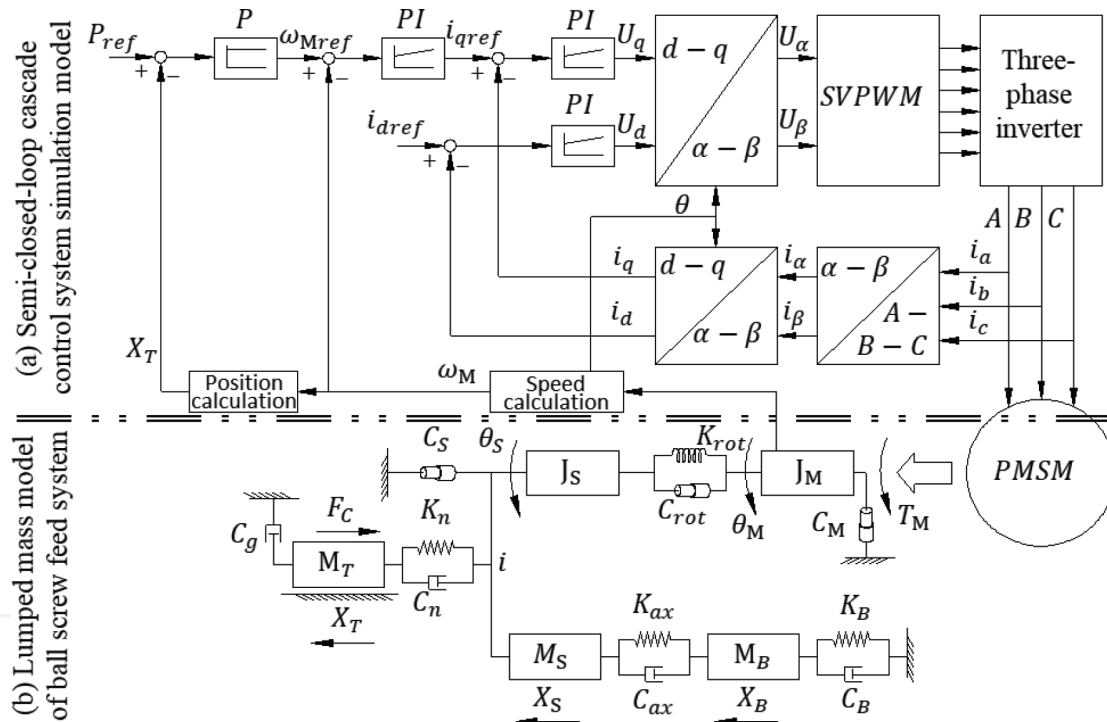
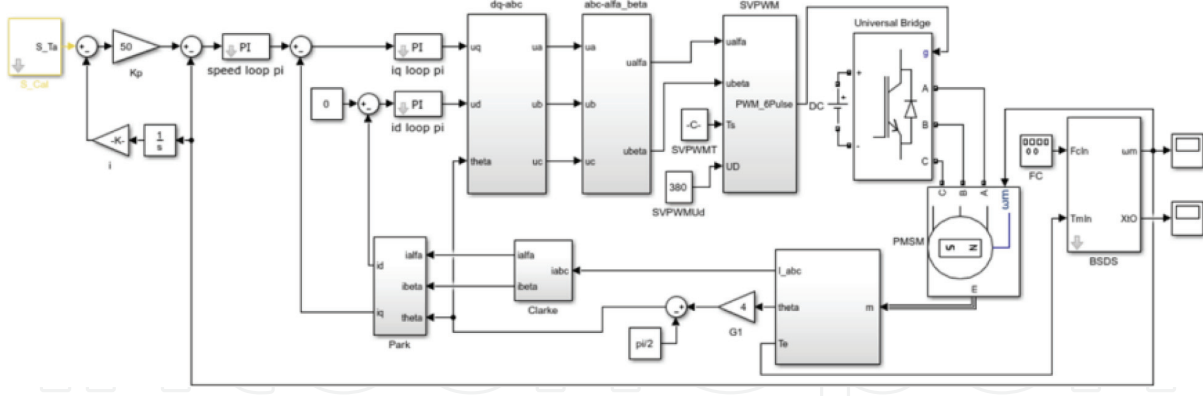


Figure 10. Schematic of ball screw feed drive system electromechanical co-simulation.

position  $X_T$  are calculated by the rotor position detected by the encoder on the servomotor. The input  $P_{ref}$  is given by the CNC system according to the feed motion command, the input  $P_{ref}$  and work table feedback position  $X_T$  are compared, and the reference speed  $\omega_{Mref}$  is given by the position controller. Then, the reference speed  $\omega_{Mref}$  is compared with the feedback speed  $\omega_M$ , and the velocity controller gives the reference current  $i_{qref}$  for the  $q$ -axis and the reference current  $i_{dref} = 0$  for the  $d$ -axis of the stator. The three-phase current of the servomotor is detected and converted into  $i_d$  and  $i_q$  in  $d-q$  coordinate system through the Clark and Park





**Figure 11.** Electromechanical co-simulation model of half-closed ball screw feed system.

transformation.  $i_{dref}$  and  $i_{qref}$  are compared with the feedback  $i_d$  and  $i_q$ , respectively, and the current controller calculates the given voltages  $U_d$  and  $U_q$  of the  $d$  and  $q$  axes; then, they are converted into  $U_\alpha$  and  $U_\beta$  in the  $\alpha - \beta$  coordinate system by Park inverse transformation. Finally, the SVPWM module generates six-phase PWM to drive the three-phase inverter. The inverter outputs ABC three-phase voltage to servomotor stator, which generates rotating magnetic field and produces magnetic torque on the servomotor rotor. This magnetic torque is the output torque  $T_M$  of the servomotor and drives the rotor to rotate under the dynamic relations of ball screw feed system.

### 3.4. Electromechanical co-simulation modeling of ball screw feed drive system

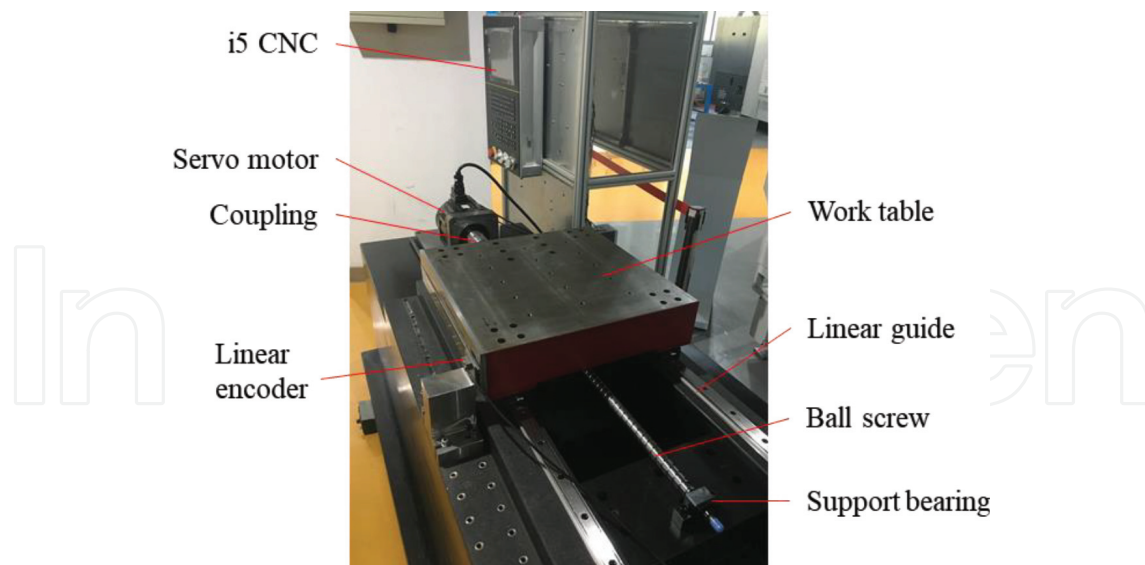
Based on the lumped mass model of ball screw feed system and the servo control system simulation model, an electromechanical co-simulation model of the ball screw feed drive system was constructed. The co-simulation schematic is shown in **Figure 10**; as described above (a) is the semi-closed-loop cascade control system simulation model, while (b) is the lumped mass model of ball screw feed system. The inverter outputs ABC three-phase voltage to servomotor stator, which generates rotating magnetic field and produces magnetic torque on the servomotor rotor. This magnetic torque is the output torque  $T_M$  of the servomotor and drives the rotor to rotate under the dynamic relations as shown in Eq. (8).

The electromechanical co-simulation model of the ball screw feed drive system is shown in **Figure 11**. The S\_Cal module on the left side generates the trajectory command for the feed drive system according to the acceleration/deceleration strategy. Under the cascade control system, which consists of position controller, velocity controller, and current controller, the servomotor drive and the ball screw accomplish the motion command accordingly.

## 4. Experimental verification of the electromechanical co-simulation model of ball screw feed drive system

The electromechanical co-simulation model in this chapter has been tested on a single-axis ball screw drive system test bench shown in **Figure 12**. The test bench uses an i5 CNC system and





**Figure 12.** Single-axis ball screw feed drive test bench.

servo system of Shenyang Machine Group, which use a semi-closed-loop cascade control structure. The specifications of the test bench are listed in **Table 1**, which are either obtained from the manufacturers' catalogs, approximated from prior knowledge, or calculated from computer-aided design (CAD). According to the modeling method described in Chapter 2, the lumped mass model of this ball screw feed system test bench was built up. The equivalent parameters of the lumped mass model were calculated by using the specifications in **Table 1**, and the other calculated lumped parameters are listed in **Table 2**.

Taking the servomotor torque as input and the axial acceleration of work table as output, the frequency response characteristics of the lumped parameter model of the test bench are analyzed. The bode diagram is shown in **Figure 13**, and simulation result shows that the work table has four-order natural frequencies, which are 26.2, 76.7, 247, and 633 Hz. Further study shows that 76.7 Hz is the main axial vibration frequency of the work table, 26.2 Hz is the main axial vibration frequency of the base, and 247 and 633 Hz are the rotational vibration frequencies.

Parameter of the component	Value	Parameter of the component	Value
Work table mass $M_T$ (kg)	206	Rotary inertia of coupling $J_C$ ( $\text{kg} \cdot \text{m}^2$ )	$1.09 \times 10^{-4}$
Base mass $M_B$ (kg)	3820	Rotary inertia of motor $J_M$ ( $\text{kg} \cdot \text{m}^2$ )	$6.75 \times 10^{-3}$
Coupling mass $M_c$ (kg)	1.18	Torsional rigidity of coupling $k_c$ (N/m)	$1.4 \times 10^3$
Motor rotor mass $M_m$ (kg)	10.9	Screw bearing rigidity $k_b$ (N/m)	$1 \times 10^8$
Screw pitch length $h$ (m)	$1.6 \times 10^{-2}$	Nut reference rigidity $K$ (N/m)	$6.12 \times 10^8$
Screw diameter $d_s$ (m)	$2.5 \times 10^{-2}$	Nut basic dynamic load $C_a$ (N)	37.4
Screw length $l_s$ (m)	1	Ball screw length at table position $l_n$ (m)	0.35

**Table 1.** Specifications of the test bench.

Parameter of the component	Value	Parameter of the component	Value
Screw equivalent mass $M_S$ (kg)	11.28	Equivalent rotary inertia of screw $J_S$ (kg · m <sup>2</sup> )	$1.7 \times 10^{-3}$
Axial rigidity of screw $K_{ax}$ (N/m)	$0.743 \times 10^8$	Rotary rigidity of screw $K_{rot}$ (N · m · rad <sup>-1</sup> )	$3.14 \times 10^3$
Axial rigidity of base $K_B$ (N/m)	$1 \times 10^8$	Contact rigidity of the screw nut $K_n$ (N/m)	$9.8 \times 10^7$

Table 2. Calculated parameters used in the lumped mass model of test bench.

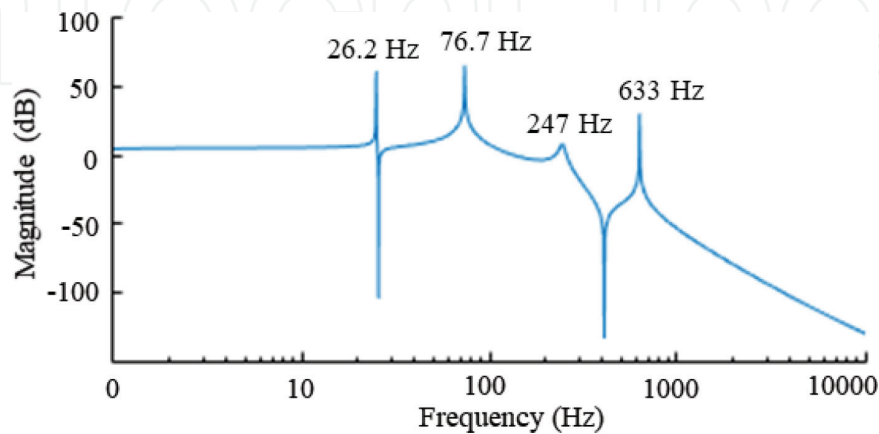


Figure 13. Bode diagram of the lumped parameter model of ball screw feed system.

To establish the simulation model of servo control system, the servomotor parameters are needed as shown in Table 3.

In order to compare and verify the simulation results with the experimental results, the motion command parameters and the control parameters of the experimental test and simulation are set in Table 4.

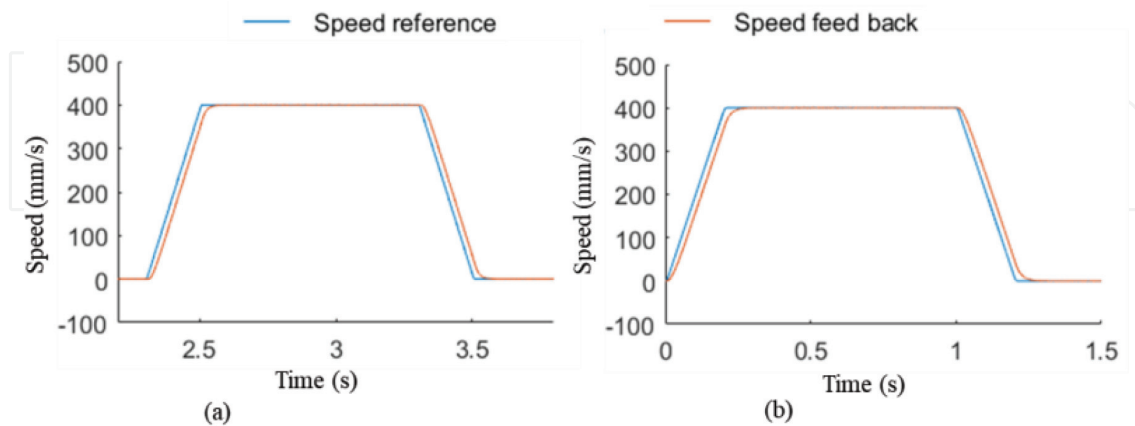
Parameter name	Value	Parameter name	Value
Rated power (kW)	4.4	Number of pole pairs	4
Rated torque (N · m)	18.6	Stator resistance per phase (Ω)	1.44
Rotor inertia (kg · m <sup>2</sup> )	$6.75 \times 10^{-3}$	Inductance $L_d, L_q$ (H)	$8.15 \times 10^{-3}$
Rated speed (r/min)	1500	Permanent magnetic flux $\psi_f$ (wb)	0.21

Table 3. Parameters of the servomotor.

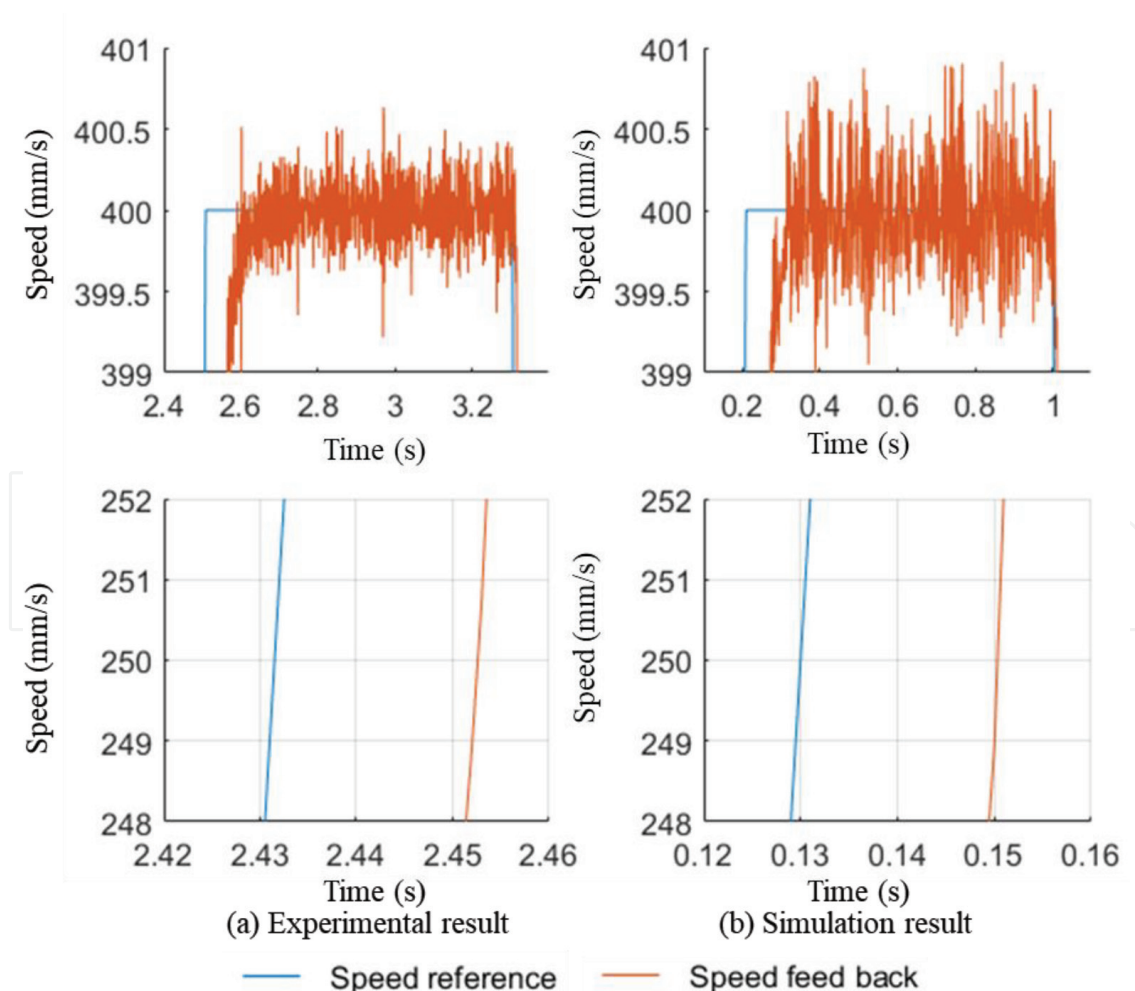
Parameter name	Value	Parameter name	Value
Position instruction (mm)	400	Position loop gain $k_p$	50
Maximum velocity (mm/s)	400	Velocity loop gain $k_v$	10
Maximum acceleration (mm/s <sup>2</sup> )	2000	Current loop gain $k_i$	30
Maximum jerk (mm/s <sup>3</sup> )	20,000		

Table 4. Motion command parameters and the control parameters settings.

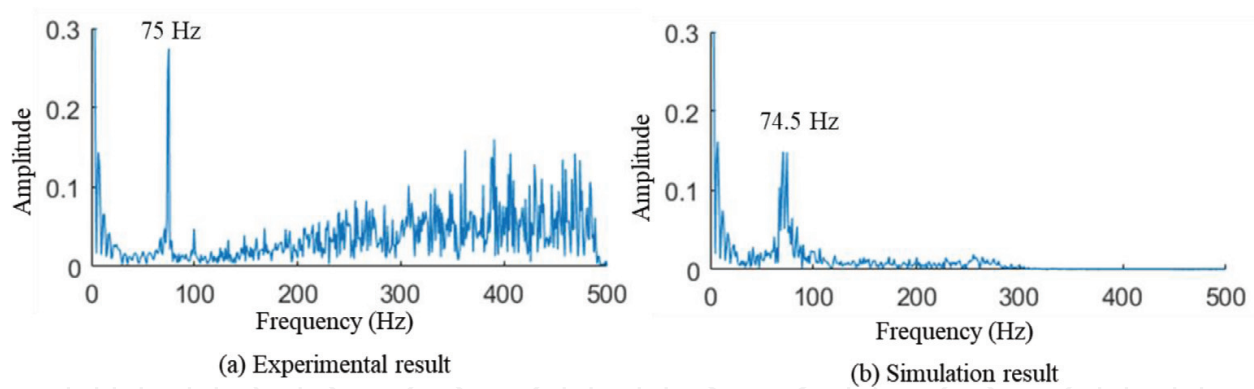
Using the same operation parameters as set in the simulation model, a feed motion experiment was conducted and the work table position was measured. The simulation results are compared to the experimental results. **Figures 14** and **15** exemplarily show simulated and measured



**Figure 14.** Reference velocity and feedback velocity.



**Figure 15.** Detailed reference velocity and feedback velocity.



**Figure 16.** Frequency contents of work table acceleration.

reference velocity and feedback velocity of the servomotor at the given operating conditions. The simulation result has a similar curve to the experimental result.

**Figure 16** shows frequency contents of the work table acceleration signals from simulation result and the experimental result. Comparing the simulation result with the experimental result, the co-simulation model of ball screw feed drive system can predict the vibration that occurs in the feed operation. Both results show that in this case the second-order natural frequency (about 75 Hz) but not the first-order natural frequency is the main factor influencing the performance of feed drive system.

## 5. Conclusions

In this chapter an electromechanical co-simulation model of the ball screw feed drive system was constructed based on lumped mass model of ball screw feed system and the servo control system simulation model, which can be used to study the dynamic characteristics and vibration behavior of the feed drive system. Simulative and experimental tests were conducted based on a ball screw feed drive system test bench. The result shows that the co-simulation model of ball screw feed drive system can predict the vibration that occurs in the feed operation. Because of the integration of lumped parameter model into the detailed modeled cascade control simulation model, the electromechanical co-simulation of ball screw feed drive system could achieve a very good predictability for control performance and vibration behavior study of ball screw feed drive system, which may be affected by the servo controller, ball screw feed system, or the coupling between them.

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## Author details

Liang Luo and Weimin Zhang\*

\*Address all correspondence to: [iamt.tongji.edu.cn](mailto:iamt.tongji.edu.cn)

School of Mechanical Engineering, Tongji University, Shanghai, China

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