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The Nonlinear Analysis of Chiral Medium

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Abstract

The principle of calculation of a plate from a metamaterial with inductive type chiral inclusions is submitted. It is shown that distribution of an electromagnetic wave to such substance can be investigated with the help of introduction of a chiral parameter and on the basis of a detailed method of calculation. By comparison of two methods the dependence of chiral parameter from frequency of electromagnetic radiation falling on a plate is found. With the help of a detailed method the nonlinear equation for potential on the chiral plate is found. It is shown that this equation has solutions as solitary and standing waves but not running waves. The analysis of the received solutions of the nonlinear equation is carried out.

Keywords: metamaterial, chiral medium, chiral parameter, nonlinear equation, detailed method, solitary waves, standing waves

1. Introduction

Now the metamaterials (Greek “meta” outside), i.e. composite materials with the various inclusions allocated both chaotically, and periodically are widely applied in particular in a radio engineering, at designing space devices, in medicine, etc. [1–3]. Due to these inclusions the received materials have many useful physical, electric, optical and other properties which are not present at natural substances. Among metamaterials the substances with chiral properties [4] which capable to rotate a polarization plane of electromagnetic waves are distinguished. In optics as analogue of similar substances are optical active substances, for example, quartz, a solution of glucose etc.

However the methods of metamaterials calculation are enough limited [5]. Basically all calculations are based on the decision of the Maxwell’s equations and the material equations selected according to a problem.

The existing method has restrictions since are usually used only averaged characteristics of metamaterials for example chiral parameter.

In the present work attempt of more detailed approach to properties of the chiral inclusions into metamaterials is made also the analysis of these properties on interaction of chiral elements with the electromagnetic wave falling on a plate from a metamaterial is carried out.

2. Standard method of calculation of metamaterial with electromagnetic wave interaction

At research of metamaterials with chiral inclusions on the basis of Maxwell's equations usually use the material equations including so-called chiral parameter χ . In [6] the material equations in the following kind are offered:

$$\mathbf{D} = \varepsilon_a \mathbf{E} \mp i \frac{\chi}{V} \mathbf{H}, \quad (1)$$

$$\mathbf{B} = \mu_a \mathbf{H} \pm i \frac{\chi}{V} \mathbf{E}, \quad (2)$$

where \mathbf{D} and \mathbf{B} there are induction of electric and magnetic fields in the electromagnetic wave propagating in chiral medium, \mathbf{E} and \mathbf{H} —the strength of electric and magnetic components of wave, ε_a and μ_a —absolute electric penetrance and magnetic permeability of chiral medium, V —velocity of an electromagnetic wave in chiral medium, χ —chiral parameter, in this case dimensionless size.

In [6], it is shown that the material Eqs. (1) and (2) can be written down in more simple kind:

$$\mathbf{D} = (1 \mp \chi) \varepsilon_a \mathbf{E}, \quad (3)$$

$$\mathbf{B} = (1 \pm \chi) \mu_a \mathbf{H}. \quad (4)$$

In formulas (1)–(4) top signs concern to right-handed rotation chiral element bottom to left-handed rotation.

Using (3) and (4) it is possible to show [6] that if a chiral medium has only reactance the electromagnetic wave in it submits to the wave equations:

$$\Delta \mathbf{D} = \left(\frac{1 \pm \chi}{V} \right)^2 \frac{\partial^2 \mathbf{D}}{\partial t^2}, \quad (5)$$

$$\Delta \mathbf{B} = \left(\frac{1 \pm \chi}{V} \right)^2 \frac{\partial^2 \mathbf{B}}{\partial t^2}, \quad (6)$$

where t there is a time.

Further us the Eq. (5) will interest only. Substituting (3) in (5) and passing to scalar potential φ [7] we shall find:

$$\Delta \varphi = \left(\frac{1 \pm \chi}{V} \right)^2 \frac{\partial^2 \varphi}{\partial t^2}. \quad (7)$$

Let us search for the decision of the Eq. (7) as:

$$\varphi - \varphi_0 = \varphi(\mathbf{r}) \exp(i\omega t), \quad (8)$$

where φ_0 there is initial a potential level, \mathbf{r} —set of spatial coordinates, ω —a cyclic frequency of electromagnetic wave.

Substituting (8) in (7), we have:

$$\Delta \varphi(\mathbf{r}) + (1 \pm \chi)^2 k^2 \varphi(\mathbf{r}) = 0, \quad (9)$$

where $k = \frac{\omega}{V}$ there is a module of the wave vector of a falling electromagnetic wave.

Solving Eq. (9) with use of initial and boundary conditions it is possible to investigate processes of reflection, refraction, diffraction of an electromagnetic wave in the metamaterial.

3. Detailed method of calculation of metamaterial with electromagnetic wave interaction

Let us consider a plate of the metamaterial with chiral inclusions of the inductive type. The plate will consist of the dielectric in which are included the current-carrying chiral elements as spirals which axis is directed across a plate.

On **Figure 1** the irradiation of a plate by an electromagnetic wave is shown. We assumed as before that chiral inclusions have no active resistance.

Feature of a plate is the capacity distributed on its surfaces at dot inductive inclusions. Therefore to examine the interaction of separate chiral element having inductance and capacity with an electromagnetic wave is incorrectly.

At the irradiation on the plate there is a potential difference submitting to the Eq. (7). The density of a current through plate will look like:

$$j_m = C_m \frac{\partial \varphi}{\partial t} + (\varphi - \varphi_0)g_m, \tag{10}$$

where C_m there is capacity of the plate area unit, φ —potential on a plate concerning an initial level φ_0 , g_m —electrical conductivity of units of the plate area due to an inductive component.

The first term (10) reflects a capacitor bias current, the second term—an inductive current through the chiral elements.

For spiral chiral element it is possible to write down the equation of a voltage balance:

$$-L_i S_i \frac{\partial j_i}{\partial t} = (\varphi - \varphi_0), \tag{11}$$

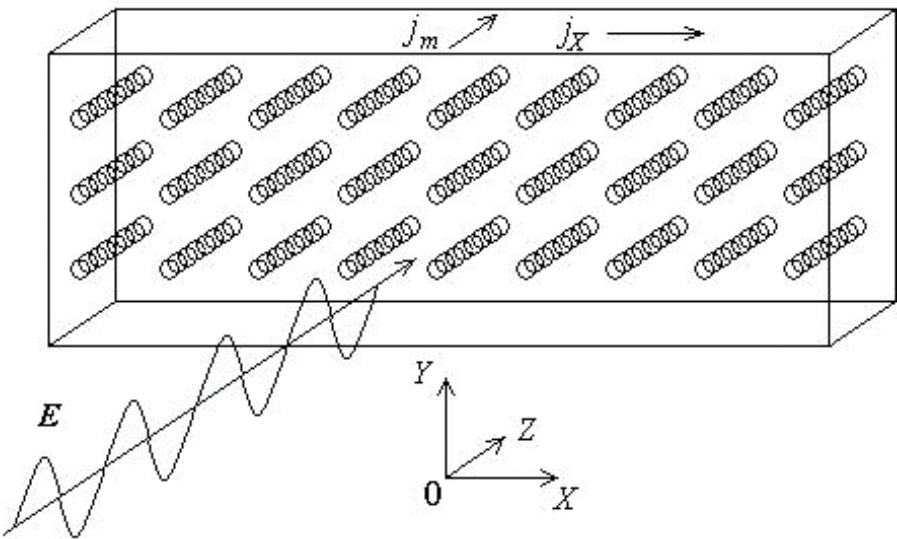


Figure 1.
The plate of metamaterials irradiated by an electromagnetic wave.

where j_i there is density of a current through the i -th chiral element, L_i —inductance i -th chiral element, S_i —the area of plate, falling one chiral element having inductive electrical conductivity g_i .

The density of a current j_i through the chiral element depends on a potential difference on a plate and electrical conductivity it chiral element g_i under the formula of the Ohm's law:

$$j_i S_i = g_i (\varphi - \varphi_0). \quad (12)$$

Substituting (12) in (11) we shall find:

$$g_i = -\frac{(\varphi - \varphi_0)}{L_i \frac{\partial \varphi}{\partial t}}. \quad (13)$$

Electrical conductivity falling unit area of a plate it is equal:

$$g_m = -\frac{(\varphi - \varphi_0)}{S_i L_i \frac{\partial \varphi}{\partial t}}, \quad (14)$$

where it is taken into account $g_i = g_m S_i$.

Having substituted (14) in (10) we shall find:

$$j_m = C_m \frac{\partial \varphi}{\partial t} - \frac{(\varphi - \varphi_0)^2}{S_i L_i \frac{\partial \varphi}{\partial t}}. \quad (15)$$

Using $C_i = C_m S_i$ —capacity of the plate falling one chiral element, and designating $\omega_0^2 = \frac{1}{C_i L_i}$ we shall find:

$$\frac{j_m}{C_m} \frac{\partial \varphi}{\partial t} = \left(\frac{\partial \varphi}{\partial t} \right)^2 - (\varphi - \varphi_0)^2 \omega_0^2. \quad (16)$$

Let us consider a plate consisting of chiral elements one lines, **Figure 2**.

Along this plate the inductive current flows.

The law of an electromagnetic induction for this current looks like:

$$-L \frac{\partial I_X}{\partial t} = \varphi - \varphi_0, \quad (17)$$

where $I_X = \gamma_X \frac{S}{l} (\varphi - \varphi_0)$ there is a longitudinal inductive current, γ_X —specific inductive electrical conductivity a single-row plate, L —its inductance, S —the area of cross-section of a single-row plate, l —its length.

Hence:

$$-\gamma_X S L_1 \frac{\partial \varphi}{\partial t} = \varphi - \varphi_0, \quad (18)$$

where $L_1 = \frac{l}{l}$ there is inductance of a single-row plate unit of length.

Under the Ohm's law for density of a longitudinal current we have:

$$j_X = -\gamma_X \frac{\partial \varphi}{\partial X}. \quad (19)$$

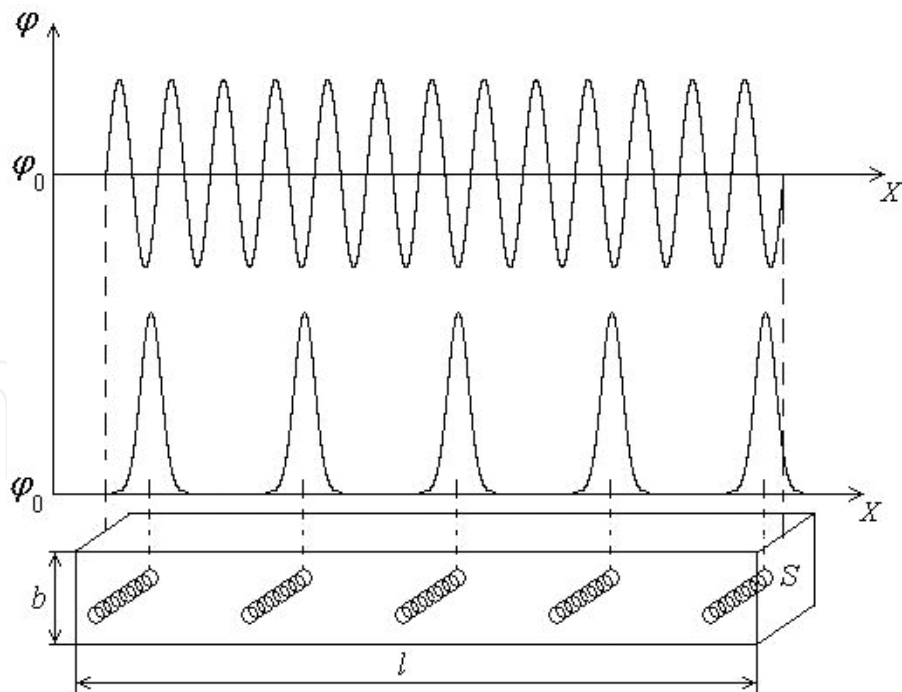


Figure 2.
 The single-row chiral plate.

Hence:

$$dj_X = -\gamma_X \frac{\partial^2 \varphi}{\partial X^2} dX. \quad (20)$$

Having divided (20) on (18), and having reduced on γ_X we shall find:

$$dj_X = \frac{\varphi - \varphi_0}{SL_1 \frac{\partial \varphi}{\partial t}} \frac{\partial^2 \varphi}{\partial X^2} dX. \quad (21)$$

On the other hand taking into account that a longitudinal current is defined only presence of a cross-section current (or on the contrary) we have:

$$Sdj_X = j_m b dX, \quad (22)$$

where b there is width of a single-row plate.

Substituting (21) in (22) we have:

$$j_m = \frac{\varphi - \varphi_0}{L_1 b \frac{\partial \varphi}{\partial t}} \frac{\partial^2 \varphi}{\partial X^2}. \quad (23)$$

Further substituting (23) in (16) we shall find:

$$\frac{\varphi - \varphi_0}{C_m L_1 b} \frac{\partial^2 \varphi}{\partial X^2} = \left(\frac{\partial \varphi}{\partial t} \right)^2 - (\varphi - \varphi_0)^2 \omega_0^2. \quad (24)$$

Taking into account $C_1 = C_m b$ —capacity of a single-row plate unit of length and $V^2 = \frac{1}{C_1 L_1}$ —a square of an electromagnetic field along a plate velocity we have:

$$V^2 (\varphi - \varphi_0) \frac{\partial^2 \varphi}{\partial X^2} = \left(\frac{\partial \varphi}{\partial t} \right)^2 - (\varphi - \varphi_0)^2 \omega_0^2. \quad (25)$$

The nonlinear Eq. (25) can be transformed to a kind correct for spatial geometry:

$$V^2 \Delta \varphi + \omega_0^2 (\varphi - \varphi_0) = \frac{1}{\varphi - \varphi_0} \left(\frac{\partial \varphi}{\partial t} \right)^2. \quad (26)$$

Linearization of Eqs. (26) can be carried out by a ratio (8):

$$\Delta \varphi(\mathbf{r}) + k_S^2 \varphi(\mathbf{r}) = 0, \quad (27)$$

where is $k_S^2 = \frac{\omega_0^2 + \omega^2}{V^2} = k_0^2 + k^2$.

4. Various kinds of the equation solution of a metamaterial and an electromagnetic wave interaction

Eqs. (27) and (9) reflect the same physical process – propagation of electromagnetic fluctuations on the chiral plate. Distinction consists that at a deduction (27) as against (9) was not necessity to use the material Eqs. (1)–(4) i.e. the chiral parameter was not used.

On the basis of Eqs. (27) and (9) identity it is possible to put down:

$$k_S^2 = k_0^2 + k^2 = (1 \pm \chi)^2 k^2. \quad (28)$$

Hence the chiral parameter can be written down as:

$$\pm \chi = \sqrt{1 + \frac{k_0^2}{k^2}} - 1. \quad (29)$$

If $k_0 \ll k$ or $\omega_0 \ll \omega$ formula (29) becomes simpler:

$$\chi = \pm \frac{k_0^2}{2k^2} = \pm \frac{\omega_0^2}{2\omega^2}. \quad (30)$$

Let us notice that quantum calculation of an optical active substance [8, 9] results in the formula for chiral parameter:

$$\chi = \frac{2V\delta}{3\hbar} \frac{\omega_{0j}}{\omega_{0j}^2 - \omega^2}, \quad (31)$$

where \hbar there is Planck's reduced constant, δ - size proportional to product of the real parts electric and magnetic dipole moments of an optical active molecule power transition excited by a light of the wave given length, ω_{0j} —in this case the frequency corresponding to power transition $0 \rightarrow j$ [10].

The increase in a degree of frequency dependence ω_0 up to square-law in formula (30) in comparison with (31) is characteristic at transition from quantum area in classical.

On **Figure 2** the illustrative graph of the potential fluctuations on the chiral plate is shown according to the oscillatory solutions satisfying Eqs. (9) and (27). Character of fluctuations will be investigated below.

The nonlinear Eq. (25) has at least one more solution as a solitary wave:

$$\varphi - \varphi_0 = \varphi_{\max} \exp \left(-\frac{(k_0(X - X_0) \pm \omega_0(t - t_0))^2}{2} \right). \quad (32)$$

whereas before $k_0 = \frac{\omega_0}{V}$, φ_{\max} there is a peak value of potential, t_0 —an initial of a time reading, X_0 —coordinate of the chiral element center. The sign a minus concerns to a wave spreading from left to right, a sign plus from right to left.

The illustrative graph of the solution (32) as function of coordinate X also is shown on **Figure 2**. Growth of potential above chiral inclusions is caused by proportionality of the chiral inclusions reactance their inductivities

$$\varphi - \varphi_0 \sim X_{Li} = \omega L_i.$$

From the analysis of both curves it is possible to conclude that the top curve, **Figure 2**, concern to often enough inclusions of the chiral elements in a plate, and bottom to more rare. Therefore into the solution (32) to enter a chiral parameter it is irrational.

Obviously for the nonlinear Eqs. (25) or (26) there should be a multiwave solution. However to find such solution it is extremely difficult. Multiwave solutions are found for very much limited circle of the nonlinear wave equations [11, 12]. The multiwave solution should depend on concentration of the chiral elements in a plate. Only with its help it is possible to understand under what conditions it is possible is proved to enter the chiral parameter, i.e. to understand borders of the material Eqs. (1)–(4) applicability.

Let us consider in more detail a kind of the wave arising on single-row chiral plate at falling on it of an electromagnetic wave.

The nonlinear Eqs. (25) and (26) can be solved a method of variables division [13]. We search for the solution of Eq. (25) as:

$$\varphi - \varphi_0 = \varphi(X)T(t). \quad (33)$$

where $\varphi(X)$ there is function only coordinates X , $T(t)$ —a function only time t . Having substituted (33) in (25) we shall find:

$$V^2 \varphi(X)T^2(t) \frac{\partial^2 \varphi(X)}{\partial X^2} = \left(\varphi(X) \frac{\partial T(t)}{\partial t} \right)^2 - \varphi^2(X)T^2(t)\omega_0^2. \quad (34)$$

Let us divide both parts of the equation on $\varphi^2(X)T^2(t)$. In result we shall receive:

$$V^2 \frac{1}{\varphi(X)} \frac{\partial^2 \varphi(X)}{\partial X^2} + \omega_0^2 = \left(\frac{1}{T(t)} \frac{\partial T(t)}{\partial t} \right)^2 = -\alpha^2. \quad (35)$$

where α there is a constant.

Eq. (35) breaks up to two independent equations. The equation dependent on X looks like:

$$\frac{\partial^2 \varphi(X)}{\partial X^2} + \left(k_0^2 + \frac{\alpha^2}{V^2} \right) \varphi(X) = 0. \quad (36)$$

Comparing (36) and (27) we notice that $k_s^2 = k_0^2 + \frac{\alpha^2}{v^2}$. Hence $k^2 = \frac{\alpha^2}{v^2}$, and hence $\alpha = \omega$.

The solution of Eq. (36) we shall write down as:

$$\varphi(X) = \varphi(0)\exp(ik_s X). \quad (37)$$

where $\varphi(0)$ there is value of function $\varphi(X)$ in the beginning of coordinates.

The second equation of equality (35) looks like:

$$\frac{\partial T(t)}{\partial t} = i\omega T(t). \quad (38)$$

Solving this equation we shall find:

$$T(t) = T(0)\exp(i\omega t), \quad (39)$$

where $T(0)$ there is initial value of function $T(t)$.

Using (33) and the real parts of solutions (37), (39) we shall find the solution of Eq. (25):

$$\varphi - \varphi_0 = \varphi_A \exp(i\omega t) \exp(ik_s X) = \varphi_A \cos \omega t \cos k_s X = \varphi_A \cos \omega t \cos \frac{2\pi X}{\lambda}. \quad (40)$$

where it is designated $\varphi_A = T(0)\varphi(0)$ —a peak value of potential on a plate, λ —a wave length.

Formula (40) represents the equation of a standing wave.

Condition of the nodes occurrence in a standing wave $X_{ns} = \pm(2n + 1)\frac{\lambda}{4}$, where $n = 0, 1, 2, \dots$

On the ends of the single-row chira plate, **Figure 2**, should be nodes of a standing wave. If excitation of a wave occurs in the center of a plate the number of the maximal node can be found under the formula $\pm \frac{l}{2} = \pm(2n_{\max} + 1)\frac{\lambda}{4}$ or $n_{\max} = (\frac{l}{\lambda} - \frac{1}{2})$.

It is necessary to note that running waves $\varphi - \varphi_0 = \frac{\varphi_A}{2} \cos(k_s X \pm \omega t)$ are not the solution of Eq. (25) therefore formula (40) from the physical point of view cannot be presented as the sum of the direct, and reflected from borders plate waves though mathematical this procedure is simple for making. It is consequence of Eq. (25) nonlinearity.

5. Conclusion

Distribution of potential to a plate from a metamaterial with inductive chiral inclusions is investigated as with use of the material equations together with the Maxwell's equations, and on the basis of a detailed method of calculation of the chiral elements and an electromagnetic wave interaction. Comparison of two approaches has allowed to find out that introduction of the chiral parameter is correct only at enough high concentration of the chiral inclusions. On the basis of comparison of two methods results the frequency dependence of chiral parameter is found. At use of a detailed method of calculation the nonlinear equation for the potential having solutions as standing waves and solitary waves is received. Running waves are not the solution of this equation. Necessity of the multiwave solution existence of the nonlinear equation which should depend on concentration of the chiral elements in a metamaterial is marked.

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