Chapter 4

Measuring 'Big G', the Newtonian Constant, with a Frequency Metrology Approach

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Additional information is available at the end of the chapter

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Abstract

A new approach is described and discussed to the determination of the Newtonian gravitational constant G, which is based on the very powerful measurement of the frequency difference between two similar oscillators. The rate of change of time delay between the two is equal to their relative frequency difference, and small variations of either one can then be detected via delay monitoring with resolution limited only by time resolution and frequency stability of the two oscillators. The latter should be highly sensitive to gravitational field, to measure *G*, which triggers the choice of simple pendulums as field detectors. Since the relative effect on frequency readily obtainable in the lab by well-controlled variations of the gravitational field is on the order of 10^{-7}, stabilities on the order of 10^{-12} are needed of the relative frequency difference if measurement of the fifth decimal digit of *G* is the target of the experiment. It is argued that such high stability is possible with a pendulum properly designed for being locally isochronous and showing an adequately high *Q* factor. The latter is projected to reach possibly 10^7 or more with the discussed design.

Keywords: Newtonian constant, simple pendulum, pendulum frequency stability, time stability, isochronous pendulum

1. Introduction

The presently official value of the Newtonian constant *G* is listed in the most recent CODATA report (2014) as $6.674\ 08 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$, with a quoted relative uncertainty of 4.7×10^{-5}, which still makes it the least well known of all constants of nature, despite improvements derived from a flurry of efforts undertaken in the last decades.

Several different approaches have been followed in the realization of experiments aimed at its determination. A short summary can be found in the introduction of [1], where the experiment illustrated in this chapter was proposed, and for a deeper insight, a well-done recent comprehensive review [2] can be used for reference and comparison. It makes metrological sense to devise different experiments for the purpose, so that the set of possible systematic errors be not the same for all and the risk of undetected coherent biases among various G determinations be minimized. While refurbished and modernized versions of the original Cavendish torsion balance are still the most commonly adopted sensing device and at least one of them has demonstrated extremely high accuracy [3], experiments based on other configurations have also been developed, and a few of them have yielded some of the best results to date. The latter include a measurement based on a beam balance [4] and one based on a pair of simple pendulums used in the static mode [5]. Both achieved accuracy in the low 10^{-5} region. These three determinations of G agree within their stated uncertainty and are the most influential in the 2014 CODATA value, which, however, was attributed higher uncertainty due to the excessive disagreement of other results. A coordinate effort is being led by the recently established working group WG13 of UIAP, stimulated by a NIST initiative, aimed at improving the status of G metrology. The experiments coordinated in this effort are mainly based on the torsion balance approach because of its favorable S/N ratio, hoping to put to fruition the enormous amount of information on systematics affecting it, with the target of improving accuracy by possibly an order of magnitude. However, other approaches are also encouraged, and experiments based differently are monitored or even supported. The free-fall gravimeter [6–8] still appears very promising due to its unique absence of difficult-to-evaluate systematics, but results are still hard to come by, mainly due to the inherently low S/N ratio of these experiments. The experiment presented in this chapter is supported by NIST through its Precision Measurements Grant Program and is based on the adoption of a pair of simple pendulums as a detection device. The target is the determination of G with an accuracy of 10^{-5}. The concept of the experiment has evolved from a pilot experiment carried on at Politecnico di Torino from 1998 to 2005, which used a single pendulum in vacuum and yielded preliminary results at 3% accuracy level [9–11].

2. The dynamic dual simple-pendulum approach

The experiment illustrated here is based on a high-resolution technique, well known in frequency metrology [12], to measure very accurately small frequency differences between two almost synchronous sources. In fact, such small differences Δv produce a variable time delay between the two waveforms, which add up to a full cycle in a time interval $1/\Delta v$. The rate of change of the time delay yields directly the relative frequency difference.

Simple pendulums appear attractive for a G measurement based on this approach, because their small oscillation resonance frequency is directly proportional to the square root of the Earth's gravitational acceleration g, as is well known, which makes them particularly sensitive to a gravitational field variation induced in a controlled way by a displacement of field masses. We will call y the relative frequency change produced in this way. Resolution in this measurement is limited only by time delay resolution and differential frequency stability of the two sources. For example, if time resolution is 1 ns, a 1000 s run allows to determine the relative frequency difference to 10^{-12}, provided its stability is adequate. This means that the two frequencies can wander around in parallel by more than that but their difference should not. This is important in considering the use of pendulum oscillators, because the gravity acceleration g is not constant in time due to a variety of causes, and so will be their frequency, which will then show instabilities not much below the 10^{-7} level [13]. Nevertheless, since such instabilities affect in a similar way all pendulum oscillators, particularly if they are in the same location, it can be expected to be quite possible that the differential instability of two equal pendulums oscillating not far from each other may be adequate for the projected resolution of the experiment under discussion.

In **Figure 1**, a sketch is shown of the expected evolution of time delay as the active field mass distribution is shifted back and forth between a geometrical configuration in which it increases the frequency of one pendulum and another antisymmetric one in which it increases the frequency of the other one. Suppose one pendulum is slightly slower than the other one (it always will be the case as two exactly equal lengths are very unlikely and even undesirable to avoid coupling). As time goes by, this slower oscillator will show increasing time delay with respect to the other, as indicated in **Figure 1** by the broken trend line. Now, when its frequency is increased by the field masses, it will get closer to that of the faster one, and its time delay rate of change (*DROC*) will decrease. The opposite will happen when the field masses increase the frequency of the other pendulum. The relevant information in this measurement is the difference between *DROC*s in the two configurations.

If v_{s0} and v_{f0} are the undisturbed frequencies of slow and fast pendulums ($v_{f0} - v_{s0} = \Delta v_0 > 0$), their difference will be modified by field masses as in Eq. (1) below, when the latter are next to the slow pendulum, and as in Eq. (2) when they are next to the fast one.

$$\Delta\nu|_s = \nu_{f0}(1 + y_{far}) - \nu_{s0}(1 + y_{near}) \tag{1}$$

$$\Delta\nu|_f = \nu_{f0}(1 + y_{near}) - \nu_{s0}(1 + y_{far}) \tag{2}$$

The DROC measurement is performed by measuring the time delay accumulated in n periods of the slow pendulum and dividing it by nT_s, as illustrated in **Figure 2**.

Therefore, it turns out that the relationship between measured DROC and actual frequencies of the two oscillators is.

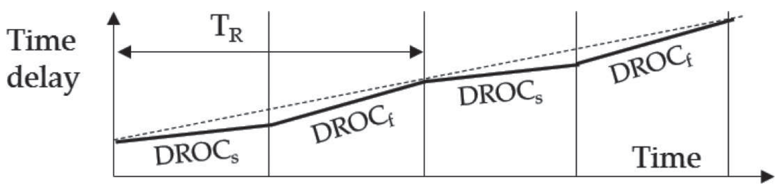

Figure 1. Time delay slope changes as field masses are moved back and forth between the two pendulums with repetition period T_R.

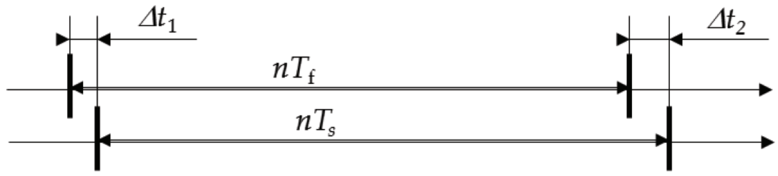

Figure 2. Measurement scheme of the *DROC* (time delay rate of change).

$$DROC = \frac{\Delta t_2 - \Delta t_1}{nT_s} = \frac{n(T_s - T_f)}{nT_s} = \frac{\Delta \nu}{\nu_f}, \tag{3}$$

and the difference between DROCs in the two configurations is then given by.

$$DROC_f - DROC_s = \frac{\nu_{f0} + \nu_{s0}}{\nu_f}\left(y_{near} - y_{far}\right) \approx \left(2 - \frac{\Delta \nu_0}{\nu_f}\right) KG \tag{4}$$

In Eq. (4), the concept was introduced that relative frequency variations induced on pendulums by the field masses are proportional to the Newtonian constant *G* through a proportionality factor *K*. The analysis needed to identify the value of *K* is sketched in the next section. The value of G can be obtained by inverting Eq. (4) and is.

$$G = \frac{DROC_f - DROC_s}{K\left(2 - \frac{\Delta \nu_0}{\nu_f}\right)}. \tag{5}$$

Clearly, *K* must be known with relative uncertainty smaller than the 10^{-5} target *G* accuracy of this experiment. In fact, this may well be the ultimate accuracy limit of this approach.

As for measurement resolution, it is also shown in the next section that the relative effect on pendulum frequency obtainable by a geometrical change in mass distribution around the bob can be on the order of 10^{-7}. It appears therefore clear that a target differential frequency stability of at least 10^{-12} should be looked for in designing the two oscillators. Other than that, the requirements for time interval measurement resolution are instead benign, both because the S/N ratio of pendulum signals is expected to be quite good (more on this in the following) and because the DROC type A uncertainty can be expected to improve as the averaging time to the power 3/2, much faster than the typical power ½ of averaging on white noise [1]. The intuitive explanation for this is in the fact that a linear regression on the scatterplot of delay data versus time will in fact yield a statistical uncertainty improving as the square root of the number of measurements (which is proportional to time), but then the result is divided by elapsed time to get the DROC, which produces the power 3/2 improvement law.

3. Field mass configuration

For the calculation of the gravitational effect on frequency, the relative extra acceleration a_M given to the bob by the system of field masses is the relevant parameter. In fact, for small

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oscillations of the bob along x, its angular frequency is given by the square root of $(a_g + a_M)/x$, with $a_g = gx/L$. The relative frequency change y induced by field masses will then be $(a_M/a_g)/2$. A peculiarity of the experiment discussed here, with field masses centered on both sides of the bob, is that both a_g and a_M vanish at rest position, but their ratio does not, as both are linear in x for small displacements. This fact gives this scheme a great advantage over other approaches, because it maximizes the effect exactly where field masses are closest to sensors. For example, this is not the case for free-fall experiments, which see the effect vanish along with a_M where the sensing object spends most of its time, at the apogee of its parabolic flight.

An analysis of the arrangement under discussion, with two equal masses symmetrically centered on either side of the bob rest position, yields for the effect on pendulum frequency.

$$y = \frac{\rho_M}{\rho_E}\frac{L}{R_E}\left(\frac{R}{a}\right)^2 \Gamma(0) \ , \tag{6}$$

where L is the pendulum length, R and a are radius and half distance of field masses between centers, R_E is the Earth's radius, and shown densities are those of the Earth and field masses. $\Gamma(0)$ is the value at $x = 0$ of a geometrical shape factor which is discussed below. It is interesting to point out in Eq. (6) that only the density of field masses is relevant for the size of the effect and not their total mass, other than in the fact that, for a given gap between them, the ratio R/a depends slightly on mass size. Also interesting is to notice that, other than hidden in the size of the gap that must host it, the test mass (which is the bob) does not appear in Eq. (6). This is because neither gravitational acceleration in play, from the Earth or from field masses, depends in any way on the mass of the bob.

The shift of Eq. (6) is expected for small oscillations. However, neither acceleration is strictly linear, which yields the well-known non-isochronism of the simple pendulum plus, relevant for this experiment, a nontrivial tie with the extra gravitational pull. So, while it is easy to find frequency and shift for small oscillations, as the relative extra acceleration can then be considered constant over the swing, nontrivial calculations are necessary for wider swings.

The field masses adopted for the experiment are cylinders of heavy metal positioned, for the "near" configuration, at either side of the bob as shown in **Figure 3a**. The metal could be platinum or, more cheaply, tungsten, but copper is chosen for budget reasons in the preliminary phases. The reason for adopting a cylindrical shape lies in the much higher uniformity of the additional recalling acceleration provided by this shape to a bob displaced from the rest point, with respect to the case of a spherical field mass shape. In **Figure 3b** a plot is given of such additional acceleration (relative to a_g) versus bob displacement in a 1 m pendulum, calculated for two tungsten field masses 85 mm in diameter and 117 mm long, spaced by an 8 mm gap to host a 5 mm spherical bob in between.

The resulting fractional frequency increase y_{near} can be calculated with a suitable integration, which is quite straightforward for oscillation amplitudes not exceeding the uniformity region. The expression of $\Gamma(x)$ used in **Figure 3b**, written with $\eta = w/a$ and $\xi = x/a$, is.

$$\Gamma(x) = \frac{3}{4}\frac{a}{R\xi}\left(\frac{1}{\sqrt{1+(\eta-\xi)^2}} - \frac{1}{\sqrt{1+(\eta+\xi)^2}}\right) \ , \tag{7}$$

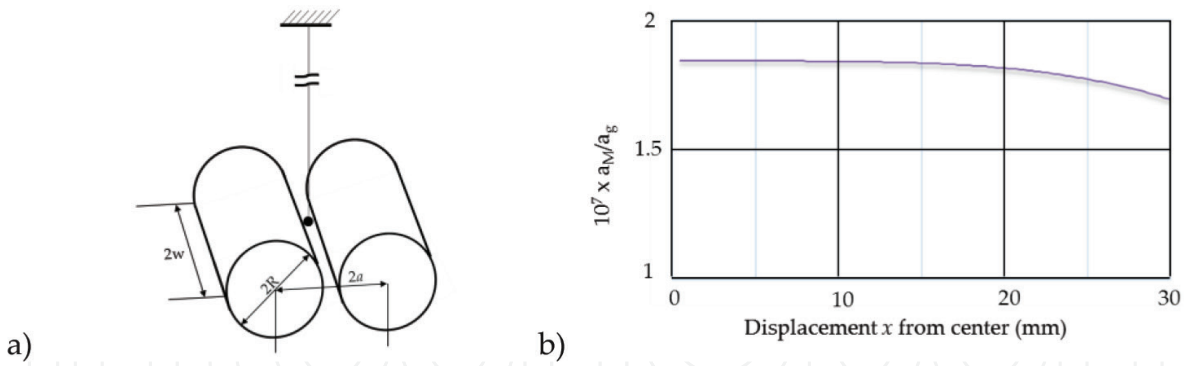

Figure 3. (a) Sketch of the near arrangement of two cylindrical field masses and (b) variations along x, elongation of the bob from rest position, of the recall acceleration a_M produced by the two field masses divided by the relevant *g* component a_g.

It should be pointed out here that shape factor of the cylindrical field masses was chosen in this calculation to optimize the uniformity of the effect, as shown in **Figure 3b**. The absolute dimension of the masses, instead, was designed to best fit the chosen geometry of the vacuum chamber, whose relevant part of the realization is shown in **Figure 4**. The chamber is realized with commercial 10 inch ConFlat flanges for the vertical body assembly, which will host both pendulums. Two thin steel tubes were welded across it to provide tunnels for the passage of the movable field mass system. These tubes are 100 mm in diameter and cannot therefore host cylinders greater than say 90 mm in diameter, together with their cradle which will be necessary for their management.

While the expression of Eq. (7) is valid for one pendulum in the "near" configuration, the effect on the other pendulum of field masses in that position must also be studied because it

Figure 4. Detail of the lower chamber of the UHV vacuum system, showing the two thin steel tubes that allow management of field masses without feedthroughs by keeping them outside the vacuum vessel.

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gives raise to the relative frequency change y_{far} of the "far" configuration which appears in Eq. (4). As a matter of fact, this effect is not so small, given the fact that the two pendulums are contained in the same vacuum vessel of **Figure 4**.

In order to facilitate this calculation, while increasing the signal by a factor of two, the idea was conceived to design the field mass system as a periodic structure. In fact, it can easily be shown that increasing w, the length of the field mass cylinders, would cause a signal reduction which would take the signal to vanish if the length is taken to infinity. This happens because such a structure would produce no field gradient in the longitudinal direction. Only a modulation along x of the mass density can produce a field gradient. The periodic structure which is planned, with a density switch between ρ_M and zero (or the lower density of another material) for every length of 2 w, will produce a periodic field gradient along x which vanishes at the center of all regions of uniform density. A pendulum centered at such vanishing gradient points will experience an increased frequency when positioned in correspondence with the higher density material and a symmetrically decreased frequency when positioned in correspondence with the lower density one.

By placing the two pendulums inside the vacuum vessel at a distance 2 w from each other, within a periodic field mass system so conceived, as shown in **Figure 5**, the measured DROC will be doubled because while one pendulum is pulled up, the other one is pulled down. The opposite will then happen after the whole field mass system is displaced by 2 w to invert the centering of the two pendulums.

In practice, an infinite length of the field mass system cannot obviously be deployed, and the structure must be truncated at some point. In **Figure 6**, a calculation is shown of the expected relative gravitational extra acceleration in the case of a nine-mass-long truncated periodic structure. The material of field masses was assumed to be tungsten, dimensions were the same of **Figure 3**, and the density of air in between masses was neglected.

Details of acceleration uniformity around the rest point are given in **Figure 7** for both positions of the two pendulums, at the center of the middle field masses (upper curve) and at the center of the first air gap at their right (lower curve). It can be noticed that the latter is asymmetric. This is because the truncated periodic structure is asymmetric with respect to that point, with five masses on one side and only four on the other one. However, the effect on frequency of such asymmetry is expected to vanish to first order, as long as the rest point of the pendulum is correctly centered. In any case, centering of the pendulums will be important for accuracy as much as uniformity of the extra acceleration. Nevertheless, it can be noticed that a subtraction of the slanting baseline in the lower curve will make it appear very similar

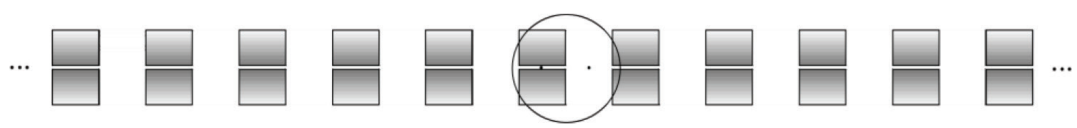

Figure 5. Scheme of the periodic field mass principle. Rest positions of the two bobs are shown (black dots). The circle in the middle represents the outline of the lower vacuum chamber through whose tunnels, shown in **Figure 4**, the field mass systems go.

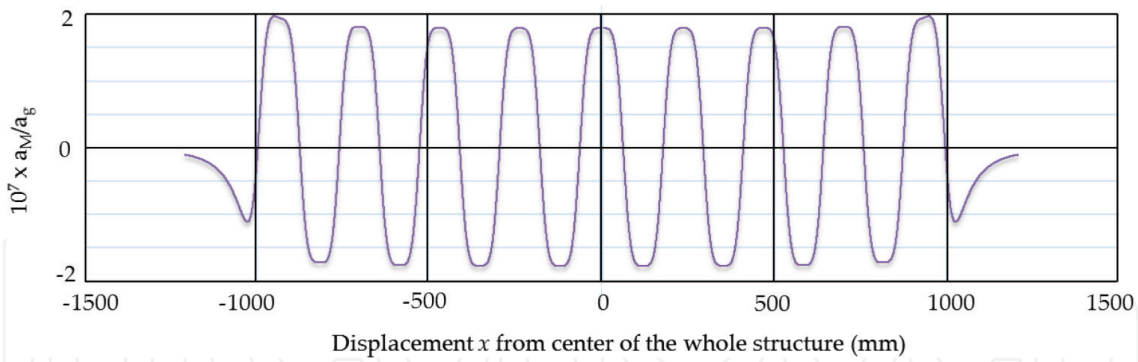

Figure 6. Calculated relative extra acceleration for a pendulum positioned at x from the center of a periodic field mass structure truncated to nine masses.

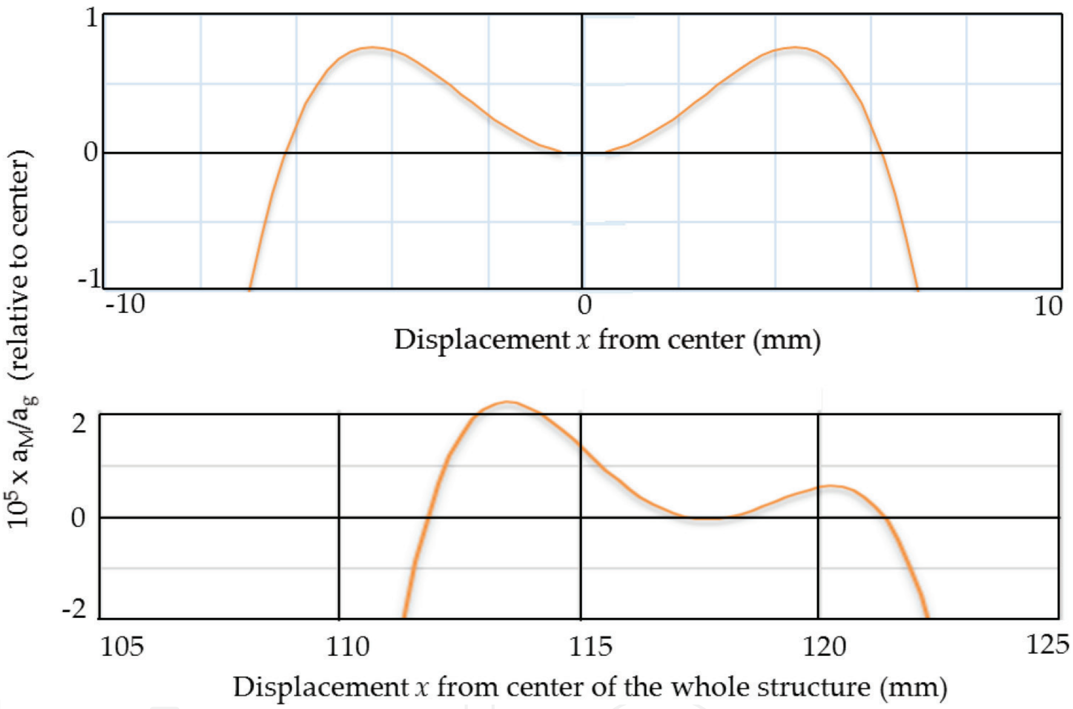

Figure 7. Relative uniformity of the extra acceleration for a pendulum positioned at the center of the middle masses (upper display) and one positioned at the center of the first gap (lower display), as a function of x, distance from the center of a nine-mass-long periodic structure. Uniformity was optimized here by trimming $\eta = w/a$.

to the upper curve for what concerns uniformity. Since both remain within +/− 10^{-5} up to 7 mm either side of the center, integration of the extra gravitational effect will be straightforward for peak pendulum oscillation amplitudes up to 7 mm if the target accuracy is at the 10^{-5} level.

In any case, all geometrical characteristics of the field mass system affect the proportionality constant K of Eq. (4), including the uniformity of their mass density and their stability in operational environmental conditions (like temperature expansion or deformation under mechanical stress). Adequate care must then be taken in design, realization, and handling of the field mass system.

To be truthful, in this respect, the experiment presented here is no different from any other experiment that was or will be tried to measure G. Revisiting the geometry of the field mass system for accuracy optimization will then be necessary after the concept is proven, which

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is the real target of the present work. Such an operation will most likely belong to a national metrology institute and not to a university. What this effort wants to prove is that no real obstacle exists in this approach on the way to an accuracy of 10^{-5}, other than problems that may come from accuracy and stability of the field mass system.

More benign is the requirement on positioning of the bob's trajectory with respect to active masses. In fact, it turns out that both in the horizontal and vertical direction, the extra acceleration features an extreme versus trajectory positioning, as shown in **Figures 8** and **9**, respectively: a minimum in the center for the lateral direction and a maximum a little above masses' gravity centers for the vertical.

The vertical displacement of the maximum is due to the extra vertical pull down that field masses exert if they are moved lower than the bob, which adds to Earth's gravity and hence to recall force, until they get too far down to be relevant. Such maximum is $(a^2 + w^2)/3\,L$ above the masses' gravity centers, which turns out to be almost 3 mm for the assumed masses. The

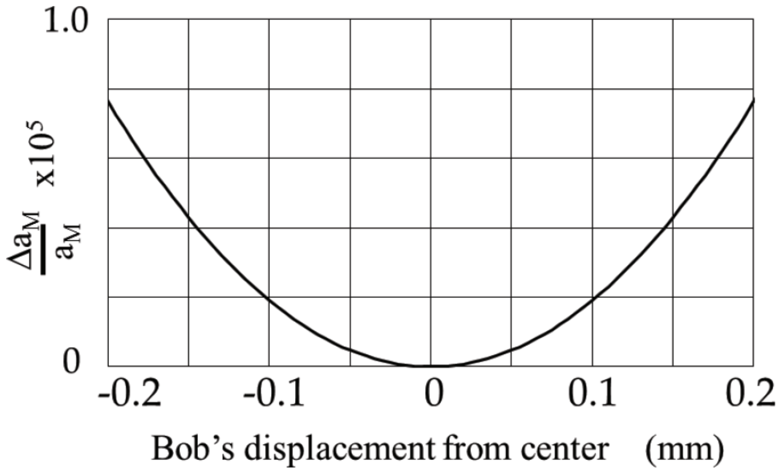

Figure 8. Relative variation of extra pull for lateral displacement of the bob's trajectory from the symmetry plane between field masses.

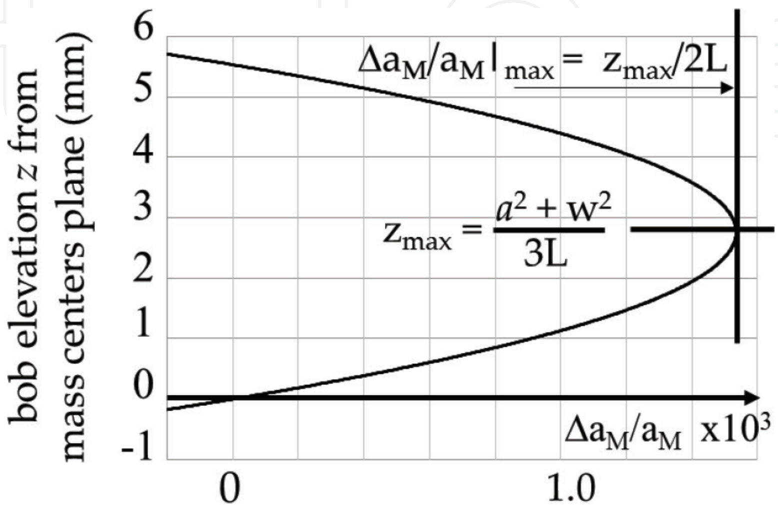

Figure 9. Relative variation of the relevant effect for vertical displacements of the bob's trajectory from the plane of mass centers.

relative shift is below $7 \cdot 10^{-4}$, which must be evaluated only to 1% for an accuracy contribution well below 10^{-5}, and the vertical positioning tolerance is 0.2 mm either side of the maximum, just like that of transverse horizontal positioning.

4. Pendulum design and optimization

The pendulums to be used in the experiment should be designed with the double target in mind of maximizing both accuracy and differential stability.

For accuracy, they should be "ideal," meaning that in their behavior they should not differ from the description that can be made with a mathematical model, supported by an adequate experimental characterization, in a way that can make G measurements uncertain by more than the desired accuracy. To this aim, all non-idealities affecting differential measurements between the two configurations of the field mass system should be considered. The main problem in this respect seems to be the uncertainty in the position of the center of mass of the pendulum given by the nonvanishing mass of the suspension relative to the bob. This shifts high the effective center of mass in a different way for the attraction of the Earth and that of the field mass system.

For differential frequency stability, which should exceed 10^{-12} for a full repetition period T_R, three main characteristics should be optimized in design and realization. They are:

1. Environmental sensitivity, especially versus temperature
2. Amplitude-to-frequency conversion, which induces frequency variations if the oscillation amplitude is not constant
3. Quality factor Q of the resonator, which is relevant in two ways: to obtain a long time constant in case of free decay operation [1] and to maximize stability with a given S/N ratio in case of sustained oscillations

Stability against environmental changes may be particularly critical for temperature, if not addressed, because at least a few ppm per Kelvin must be assumed for the linear expansion coefficient of the suspension, unless some kind of compensation is made. This is a quite common practice in pendulum clocks, but the demand in this application is severe. Even in the case of a successful compensation by a factor of ten of a low-expansion suspension material (e.g., tungsten, with its 4.3 ppm/K), the requirement would still be for a temperature differential between the two pendulums of the order of a few K for the necessary 10^{-12} differential frequency stability. It is true that the two pendulums are contained in the same vacuum vessel, which can be temperature stabilized, but an excellent thermalization scheme must certainly be devised for the purpose. It is assumed here that gold plating of the inner surface of the vacuum vessel or if necessary cylindrical gold-plated mirrors focusing the two suspensions [14] onto each other are the best bet to this aim, but a detailed discussion of the problem is out of the scope of this paper. What instead can be done at the pendulum design phase is implementation of a temperature compensation scheme. To this aim, tungsten is used for the suspension of the bob, and aluminum is deployed in an expansion compensation structure as shown below.

Amplitude-to-frequency (or period) conversion is a well-known problem of pendulum clocks, because period-to-period instability of the kick turns into frequency instability through such connection, and famous in this perspective is the solution proposed by Christiaan Huygens in his 1657 patent of making an initial ribbon section of the suspension wrap on cycloidal profiles each side as it swings back and forth. However, neither Salomon Coster (who built the first such device, still shown in Boerhaave Museum in Leiden) nor anyone later appeared to be able to take full advantage of Huygens' idea, presumably because realizing a faithfully cycloidal profile is very difficult, as its curvature diverges in the cusp, where the shape is most important for small oscillations, which is where pendulum clocks are operated for wear minimization and consequent long-term stability.

In the model chosen for this experiment, pictured in **Figure 10a** with the bob in between field masses, the pendulum suspensions are made of tungsten wires hanging between two cylinders on which they wrap and unwrap. The wires are two for each pendulum, converging on the bob, for removal of the degeneracy of the two orthogonal modes, and the wire section above the cylinders is dimensioned for temperature compensation in a scheme that includes an aluminum structure to fix the length of the upper part of the wires.

Cylinders are technologically very easy to fabricate, contrary to the cycloidal case, and very good ones are common in modern machines, which makes them easy to obtain and cheap. In this work, dowel pins and specifically wrist pins are employed. The latter are very well rectified and have a hard surface because they must bear high forces with little friction in connecting pistons to rods in ICE power trains. As for amplitude-to-frequency conversion, deploying circular profiles does not realize a completely isochronous pendulum like Huygens showed true for a cycloidal profile; nevertheless, they produce a period vs. amplitude curve which shows a minimum at a certain amplitude value which is related to the diameter D of the cylinders. For that magic amplitude, the pendulum is then locally isochronous, and operation exactly at that amplitude shows no amplitude-to-frequency conversion. This means that the effect on frequency of amplitude variations vanishes if the amplitude is set correctly and that it depends quadratically on the amplitude error from that magic value in a way that makes it possible to achieve the necessary stability.

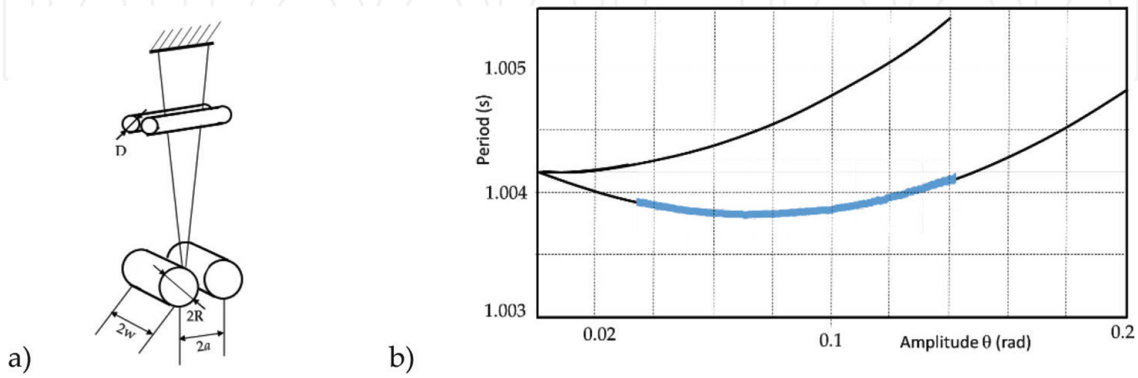

Figure 10. (a) Picture of the pendulum configuration chosen for this work, with the bob hanging between field masses, and (b) period versus amplitude curve of such pendulum compared to the one of a mathematical pendulum. Length is about 250 mm and D is 22 mm. The experimental points are superimposed on the measured section of the curve.

The period versus amplitude curve is compared to that of a mathematical pendulum in **Figure 10b**. The shape is still parabolic, but the vertex is moved from the vanishing amplitude point to an amplitude which can be chosen and adapted to the desired pendulum energy by suitably designing *D*, as shown in [15]. Period measurements, compared to the theory in **Figure 10b**, were taken on a 0.25-m-long pendulum with cylinders of 22 mm in diameter.

In fact, since at the apex of the parabola the two curves of **Figure 10b** show the same curvature, their local description is well known to be

$$\frac{\Delta T}{T_{min}} = \frac{1}{16}(\Delta\theta)^2 \tag{8}$$

which means that amplitude deviations $\Delta\theta$ from the minimum period spot must not exceed 4 μ rad to keep the first term at the 10^{-12} level. On the other hand, a period-to-period amplitude reduction is unavoidable due to energy loss and is related to it by

$$\frac{\Delta E}{E_{min}} = 2\frac{\Delta\theta}{\theta_{min}} = \frac{2\pi}{Q} \tag{9}$$

which means that the minimum pendulum quality factor Q_m necessary to keep the amplitude from decreasing more than the acceptable limit $\Delta\theta$ in one period is

$$Q_m > \pi\frac{\theta_{min}}{\Delta\theta} \tag{10}$$

For example, if θ_{min} = 0.075 rad, as in the case of **Figure 10b**, the quality factor must be greater than about 10^4 to keep the period (and the frequency) within 10^{-12} for one period, and a *Q* of 10^7 will keep the desired frequency stability for less than an hour at most. Luckily, because it's only the differential frequency stability that must be very stable, this requirement applies only to the difference of the two pendulum quality factors, provided they are both oscillating at the sweet amplitude spot. If it can be assumed that both quality factors are the same within say 10%, a *Q* of 10^7 would be enough to guarantee that the desired differential stability is kept for a full working day. This would be a long enough time for two full cycles of the repetition rate of the experiment if the system of field masses is kept in one position for a couple of hours and then moved to the other position for another couple of hours. Such is the situation for an experiment based on a pair of pendulums operated in free decay mode, and it could possibly be improved more if the two quality factors are within 1% of each other, in which case the experiment could go on for almost a week. Modeling out the effect may also be possible to some extent, as silently assumed in [1], and might further increase the useful duration of the experiment between periodic operations of amplitude reset, but this gets more complicated.

Alternatively, at the light of the experimented difficulty in obtaining consistently the extremely high *Q* values which are needed for the discussed reasons if the free decay mode must be adopted, a sustained oscillation approach can be tried for the two pendulums. In this perspective, a synchronous forcing term must be applied to the pendulum, designed to exactly recover the energy lost by friction. The best for stability and most efficient way of doing this would be a sine-wave force *F* applied in phase with the velocity *u* of the bob. This approach avoids pulse timing and duration problems often encountered in the past by pendulum clock makers. The amplitude of such forcing term can be regulated in closed loop, by an automatic gain control (AGC) arrangement, to exactly compensate the dissipated power P_d at the desired swing amplitude. This requires the average delivered mechanical power $< Fu >$ to be

$$< Fu >= P_d = \frac{\omega \mathbf{e}}{Q}. \tag{11}$$

Since the energy **e** stored in the pendulum swing is proportional to the amplitude squared and u is linear with amplitude, it appears that the desired force is proportional to amplitude, as it might be intuitively expected. However, this is true only if Q is constant with amplitude, which turns out not to be the case for the adopted pendulum design. An analysis of what were felt to be the two main dissipation mechanisms for this structure was given in [1] and showed that in both cases the Q limitation is proportional to some power κ of the amplitude. In detail, periodic stretching of the wires under varying tension and their bending as they wrap and unwrap on the cylinders produce Q limitations which are inversely proportional to the square of the amplitude for the former ($Q_s \propto \theta^{-2}$, where s stands for stretching) and proportional to the amplitude's three-halves power for the latter ($Q_b \propto \theta^{3/2}$, where b stands for bending). Within that simplified theory, cyclic length variations of wires were overlooked, and only stretching under varying tension and bending on the cylinders were analyzed for small oscillations. Expressions obtained for the corresponding Q limitations (Q_s and Q_b, respectively) were

$$\frac{Q_s}{Q_f} = \frac{16}{9\sqrt{\varepsilon_0}\theta^2} \tag{12}$$

$$\frac{Q_b}{Q_f} = \left(\frac{LD}{\phi^2}\varepsilon_0\theta\right)^{3/2}, \tag{13}$$

which shows that $\kappa_s = -2$ and $\kappa_b = 3/2$. Here ε_0 is the static strain imposed on wires by the weight of the bob, ϕ is the wires' diameter, and Q_f is the intrinsic Q of the wire material. The total Q of the pendulum can then be obtained as

$$\frac{1}{Q} = \frac{P_{ds}+P_{db}}{\omega E} = \frac{1}{Q_s} + \frac{1}{Q_b}, \tag{14}$$

and features a maximum Q value at an angular swing amplitude $\theta_{\max}$ which can both be calculated from Eqs. (12) through (14).

An example of such a Q dependence on amplitude is given in **Figure 11a** as calculated from Eq. (14) for a pendulum which could be suitable for the G experiment (L = 1 m and D = 4 mm), built with 4 μm Tungsten wires and a spherical 4.5 mm tungsten bob. The resulting peak force that is necessary to keep the bob swinging at the given amplitude according to Eq. (11) is shown in **Figure 11b**, where the strange effect appears that, in the branch before the minimum, weaker forcing terms are needed to maintain greater amplitudes.

A comprehensive campaign to confirm the theory in all conditions has not been completed yet as this book is going in press. In particular, Q values in excess of several millions were

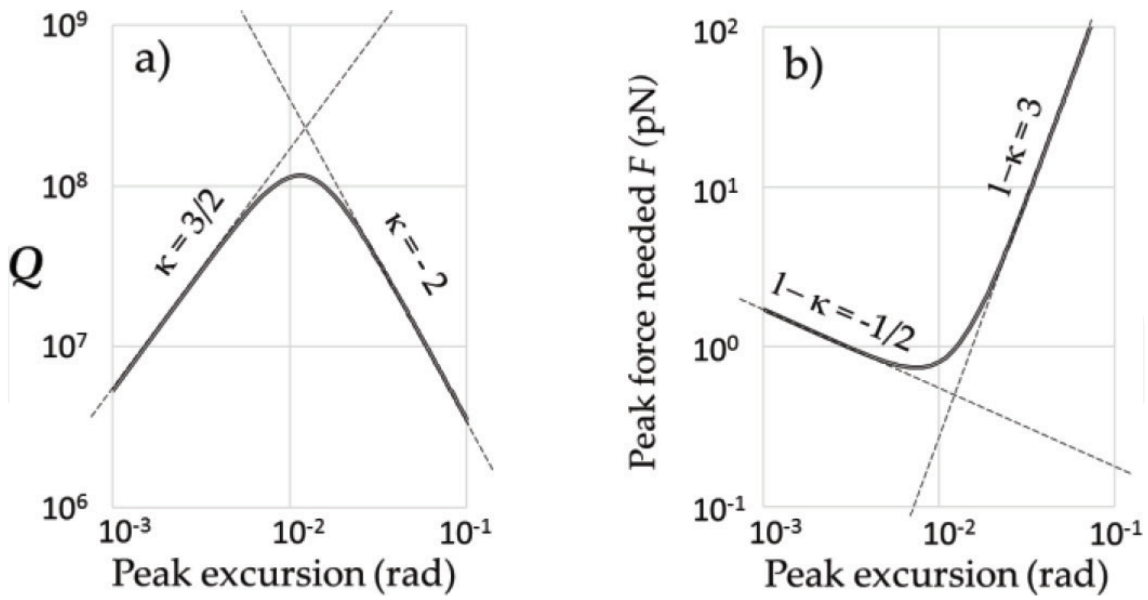

Figure 11. (a) Q as a function of angular oscillation amplitude and (b) peak force value of the sinusoidal forcing term necessary to maintain the corresponding amplitude, calculated for a pendulum with 4-mm-diameter suspending cylinders and 1 m length, made with 4 μm Tungsten wires and a spherical 4.5 mm tungsten bob.

not observed yet in the limited range of configurations that were staged, which suggests that there may be other dissipation mechanisms worth studying, but it seems unlikely that a more complete analysis may not confirm the general shape of these curves. In fact, experimental results obtained by analyzing free decay ringdown amplitude data clearly show that such Q maximum exists. Further experiments are in progress, as well as the analysis of such additional dissipative phenomena as belt friction, squeeze film energy loss [16], and more trivially dissipation in the structure holding the experimental arrangement.

What must be underlined here is that, due to the Q behavior shown in **Figure 11**, Eq. (11) points to a criticality for AGC stability, because amplitude stabilization cannot be reached if increasing the amplitude requires a reduction of the forcing term. The derivative of the required force with respect to amplitude must be positive for AGC stability, which imposes the selection of a swing amplitude in a region where the dominant dissipation mechanism is such that $\kappa < 1$. If $\kappa > 1$ the AGC will be unstable, and if $\kappa = 1$ it will not be effective because the necessary force does not depend on amplitude. Conversely, given a desired swing amplitude, as dictated, for example, by the range of acceptable field uniformity of **Figure 7**, the design of pendulum suspensions should observe the specification of placing the desired amplitude in a range where AGC stability is guaranteed.

The best choice in this respect appears to be a design which positions the θ_{max} at the desired oscillation amplitude, which is what was tried in the simulation of **Figure 11**, where the amplitude of maximum Q was made to correspond at 10 mm with a bob peak excursion which can be judged desirable from the calculated effect uniformity shown in **Figure 7**. However, it must be pointed out here that this design problem is still open because also the minimum of the period, as illustrated in **Figure 8**, must be placed by design at the same oscillation amplitude, which implies a tight restriction on the acceptable values of D, the diameter of suspension

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cylinders. These values are much smaller than the one adopted in the simulation of **Figure 11** to force the position of the Q maximum. Clearly, full confirmation of the Q theory must be carried out before the pendulum design can be finalized.

Another detail which is obviously relevant to this effect is the material of the suspension wires, because both Q_f and ε_0 in Eqs. (12) and (13) are material dependent, as well as the diameter ϕ of the wires, in its own way. Unfortunately, mechanical characterization of fibers is less than complete in the open literature, particularly for what concerns mechanical losses summarized by Q_f. Therefore, tests were carried out in the laboratory with a purposely built apparatus [17] on promising candidates, mainly aiming at characterization of mechanical losses, creep, and linearity [18]. Para-aramid, SiC, basaltic, and carbon fibers were analyzed [19], as well as steel and tungsten metal wires. Glass and fused quartz are still waiting in line. No doubt, a final decision on this important item must be integrated with the whole design of the pendulum, as both analyzed loss mechanisms depend on ε_0, and hence on ϕ, while the bending loss, in particular, depends also explicitly on ϕ.

Three more very important items must be considered in the design of the pendulum because they have an impact on the operation of the device, if not on its effectiveness in detecting the gravitational field modulation. They are the mode map of the pendulum, the oscillation detection system, and the excitation mechanism in case of forced oscillations' operation mode.

The first one may affect obtainable Q values and introduce fastidious coherent noise in the detection signal. In fact, if undesired oscillation modes get excited, albeit weakly, they can easily increase the effective total damping by sucking energy into dissipative mechanisms which do not belong to the main pendulum mode, lowering its Q as a consequence, and on the other hand, they force detection data processing to face spurious coherent signals which may reduce S/N ratio and ultimately affect resolution. Getting rid of spurious signals is impossible by the Nyquist theorem because of aliasing if the sampling frequency is not at least double the highest undesired mode frequency, which forces the handling of a massive amount of data in a full sine-wave detection system. The most difficult undesired modes to deal with, however, are the ones that are closest in frequency to the pendulum mode [20], because they are the ones that are most easily excited. In particular, the transverse mode, whose degeneracy with the pendulum mode is removed by the double-wire suspension structure, remains close to it in frequency if the angle between wires is not too big.

Other modes that should be focused on are the double pendulum mode and the similar balance wheel mode, which are more separated in frequency but are easily excited as soon as some imperfection appears in the suspension structure or in the excitation system, if present.

Given the boundary conditions emerging from the panorama spelled out here, care must be taken in designing and realizing excitation and detection systems for the two pendulums, to minimize the risk of getting undesired modes excited and affecting in this way damping and measurement resolution. Both optical and electromagnetic methods have been analyzed for both. All tested methods have their own advantages and problems, but all can serve the purpose if well implemented.

5. Attitude control

One final problem must be addressed here to give a complete picture of the complexity of this apparently very simple experiment: the attitude of the whole apparatus with respect to the vertical direction, as defined by the Earth's local gravity vector, can affect the operation of pendulums and must therefore be guaranteed to be adequately accurate and stable in time, if necessary by active attitude control. Two different problems must be addressed in this respect as both the absolute tilt and its stability are relevant, in different ways.

The absolute verticality is important because the two cylinders must be guaranteed to be horizontal for the symmetry of the swing, which in turn guarantees the positioning of the minimum period in amplitude space (the curve of **Figure 10** was calculated for the case of two cylinders at the same level). Because the pendulums are two, this issue is complicated by the need to have both pairs of suspension cylinders aligned on the same horizontal plane.

The attitude stability is particularly critical in the case of low sampling-rate detection, like simple flyby time stamping at half periods, because of the heavy aliasing of seismic angular noise [1] that it produces. In **Figure 12**, a series of background seismic power spectrums is reported, as collected in different locations of the global seismographic network [21]. A peak at about 0.2 Hz appears in all of them, which is produced by low damping surface Rayleigh waves excited by ocean waves hammering the shores, extended with reduced intensity at higher frequencies. Because of their low frequency, it is very difficult to filter out such seismic angular noise contributions.

Work done to attack this problem includes passive and active attitude control [22, 23]. However, passive filtering was quickly understood to be inadequate for the purpose, not only because of its awkwardness at such low frequencies but also because of the need for stiffness of the structure holding pendulums to prevent detrimental effects of recoil from pendulums on attitude stability and damping itself. Active stabilization was then decided to be necessary, and work

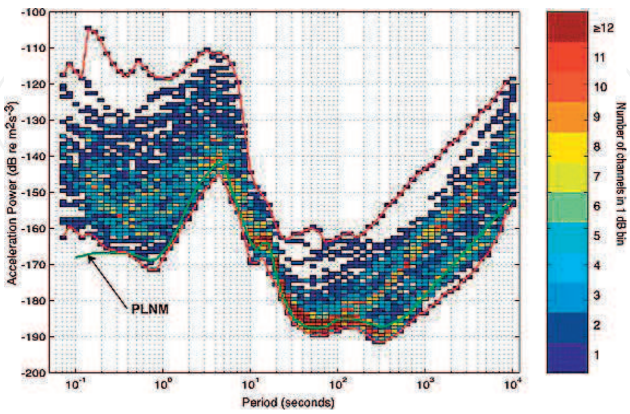

Figure 12. Background seismic power spectrums, as collected in different locations of the global seismographic network. High noise at periods above 1s (i.e. Fourier frequencies below 1 Hz) is caused by far travelling Rayleigh surface-waves excited by ocean waves.

was done to realize for the purpose a high-sensitivity tiltmeter [24] with sub-nanoradian resolution and special half-pole actuators based on thermoelectric devices [25]. The problem of substantial angular stability improvement by active control is not trivial because a tiltmeter with the necessary sensitivity was never seen with resonance-limited bandwidth above, say, 10 Hz, which provides not much more than a decade range in the Fourier frequency domain for open-loop gain increase from the zero-dB level of the loop bandwidth allowed by a typical 45° phase margin to the desired open-loop gain at the seismic peak sub-Hertz frequency. The half-pole actuator was developed for this reason, so that a loop-gain roll-off of 30 dB/dec could be achieved without resorting to a digital control, allowing in this way a guaranteed-stability servo operation with low-frequency open-loop gain in excess of 40 dB.

Nevertheless, the seismic angular noise problem is expected to be much more benign in case of acquisition of the whole sine-wave swing signal at a conveniently high sampling rate. For this reason, and because acquisition of the full sine wave is necessary anyway for oscillation support, unless the free ringdown approach is adopted, the choice was made to develop a suitable detection method for this purpose. The latter can be electromagnetic, based on the generation of an e.m.f. in a wire swinging through a magnetic field together with the suspenders of the bob, or optical, based on a linear position-sensitive detector deployed to translate the bob's position in an electrical signal.

The first approach has the advantage of measuring the bob's velocity, with which the forcing term should be in phase, and is therefore to be preferred for phase accuracy of the oscillation loop but implies the risk of introducing in the experiment undesired electromagnetic forces which could be greater than the weak gravitational force that must be measured.

The second approach, on the contrary, is less risky of introducing undesired forces but requires the generation of a sine-wave signal in quadrature with the detected position for the implementation of the forcing term. This must be performed very accurately to obtain oscillations at exactly the resonance frequency because any error from quadrature would introduce a frequency shift, which in turn may build an error on G if it's not adequately common moded between the two pendulums.

Similar considerations hold for the forcing sine wave, which can be realized with just a current-carrying wire attached to the suspenders in a voice coil type of device in the first approach and if Q is high enough could be implemented by radiation pressure in the second.

6. Accuracy budget

A tentative accuracy budget for the experiment described here is given in [1]. Because of the highly efficient time and frequency metrology approach, only geometrical uncertainties are expected to be relevant at the level of 10^{-5}, provided the necessary differential stability of 10^{-12} can be achieved. This is clearly a big "if," as discussed above, because it assumes that seismic and mode leakage problems are adequately solved. However, it can be in principle obtained if the limitation is electronic noise. It must be noted here for completeness that the

Effect	Relative bias	Uncertainty	Conditions
Shift at bob's vertical position	$6.7 \cdot 10^{-4}$	$<10^{-6}$	< 50 μm uncertainty in *a*, w
Bob's vertical position	0	$2 \cdot 10^{-6}$	0.2 mm full tolerance
Bob's lateral position	0	$1.7 \cdot 10^{-6}$	0.2 mm full tolerance
Non-isochronism	$-1.8 \cdot 10^{-5}$	$< 10^{-7}$	Operation at minimum period
Spacing between twin masses	0	$6 \cdot 10^{-6}$	0.4 μm gap uncertainty
Field masses' dimensions	0	$6 \cdot 10^{-6}$	1 μm uncertainty
Field masses' density	0	$5 \cdot 10^{-6}$	

Table 1. Accuracy budget projection based on 1-m-long 4 μm tungsten fibres, 6-mm-diameter suspension cylindrical profiles, a swing amplitude of 0.01 rad, and a 5 mm tungsten bob. The position of field masses' gravity center is assumed known with <300 nm uncertainty.

best pendulum clocks ever realized [13] were probably not differentially stable better than 10^{-9} at the target 10^4 s averaging time, which means that 60 dB improvement is necessary. Although this is granted on paper by projected S/N and *Q*, actually achieving it is still a big challenge. On the positive side, it is worth pointing out here that energy-induced amplitude changes [11] do not affect frequency if operation is kept at the minimum period isochronous point and that an approach to oscillation support aimed at overcoming pulse stability problems by moving to a sine-wave excitation system similar to that employed in high-stability quartz oscillators will remove one of the worst contributions to instability.

This said, it can be seen in **Table 1** that most geometrical contributions to uncertainty impose quite loose requirements at the target accuracy level of 10^{-5}, with the sole exception of size and positioning of field masses, which must be guaranteed at high accuracy. While other contributions enjoy relaxed specifications granted by parabolic minima which are specific of this configuration, the latter do not and must comply with specs which are similar to any other big G experiment. However, the expectation that accuracy on G may be limited by control on this single geometry contribution ushers the possibility of doing even better than 10^{-5} if resources were to become available to improve the accuracy of field masses. A summary of such uncertainties is reported in **Table 1**, as listed in Ref. [1], where the reader can find more details and a deeper discussion on accuracy.

7. Conclusions

A new experiment was presented for the determination of the Newtonian constant. It is based on a time and frequency metrology approach consisting in the measurement of the small frequency difference between two freely oscillating pendulums via their time delay rate of change. A system of dense field masses is moved back and forth between the two, alternately increasing one frequency and reducing the other and vice versa. The increase in resolution by averaging is fast in this case because the limiting noise is white delay noise, which yields $\sigma_y(\tau)$

proportional to $\tau^{-3/2}$. This fact is unique among experiments for the determination of *G* and offsets the poor signal size problem allowing to focus the design on accuracy rather than S/N ratio. It remains to be shown that differential stability in the 10^{-12} region can be obtained with consistency for two similar pendulums of the design which has been sketched here. This seems to be a long shot when considering the absolute stability achieved by the best Shortt clock [13], because it requires an improvement of more than three orders of magnitude with respect to it, at the target few hours (T_R) averaging time. However, it is not unreasonable to think that two adequately similar pendulums can be realized, and if they are within 100 mm of each other, it can be expected that *g* uniformity may be adequately stable in time to support the assumption. A description of the apparatus and a discussion of pendulum design optimization for this experiment were offered in detail, pointing out problems and possible solutions. Work is in progress on the preparation of the experiment, considering both a free decay solution and pendulum operation with active support of oscillations and amplitude control. It is expected that an accuracy of 10^{-5} may be obtained for *G* with the proposed approach, limited only by the accuracy of field masses' size and positioning, and that it may be possible in a metrology laboratory to reduce limiting geometrical uncertainties enough to push it into the 10^{-6} range.

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