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Aircraft Gas-Turbine Engine with Coolant Injection for Effective Thrust Augmentation as Controlled Object

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Additional information is available at the end of the chapter

Abstract

This chapter deals with some intensive methods regarding aircraft gas-turbine-engine performance enhancement, which are suitable alternatives for the most common temporarily thrust increasing method—the afterburning. Coolant injection method, into the compressor or into the combustor, realizes the desired thrust increase for a short period, when the flight conditions or other aircraft necessities require this. Both methods were studied from aircraft engine's point of view, considering it as controlled object. New engine's mathematical model was built up, following the thermo- and gas-dynamics changes and some quality studies were performed, based on engine's time behavior simulations; some control options and schemes were also studied. Quantitative studies were based on the model of an existing turbo-engine; mathematical model's coefficients are both experimentally determined (in the Aerospace Engineering Division labs) as well as estimated based on graphic-analytic methods. This approach and the presented methods could be applied to any other turbo-jet engine and used even in the stage of pre-design of a new engine, to estimate its stability and quality.

Keywords: gas turbine, control, cooling, injection, volatile, engine, compressor, combustor, fuel, thrust augmentation, step response

1. Introduction

Aircraft engine's performance enhancement is one of nowadays most studied issues. Since conventional methods (in terms of temperature and/or overall pressure ratio growth) have failed due to lack of suitable materials, some alternative methods have been sought. Among these, a few methods for temporarily engine thrust increase were designed and tested, such as the coolant injection (into the compressor or into the combustor), which is one of the most

effective, viable and reliable alternatives to the already classical afterburning. In fact, nowadays most efficient thrust augmentation method is the afterburning, but it is also the most expensive because of its significantly increased fuel consumption; moreover, in case of afterburning implementation, it is compulsory for the engine to have an afterburning chamber with variable area exhaust nozzle, heat insulation and noise dampers, as well as a fuel injection equipment with suitable control system(s), which implies important design and manufacturing issues and, consequently, additional costs.

In order to maximize the gas-turbine engine's thrust, especially of a turbojet, it is necessary that the energy of the hot gases evacuated through the exhaust nozzle is as high as possible [1, 2]. As far as the available energy of the hot gases leaving engine's combustion chamber is divided between the turbine and the exhaust nozzle, there are two methods to increase the nozzle fraction: (1) additional injection of fuel behind the turbine and burning it in a special combustion chamber, above-mentioned method called "afterburning" (which, obviously, is an extensive method); (2) the reduction of turbine power for the nozzle's benefit, by reducing the power required for the compressor, in fact reducing the air compression evolution mechanical work (which is an intensive method).

The second method may have two ways to be accomplished by (1) reducing compressor's specific mechanical work, but keeping the same pressure ratio value, or (2) reducing the compressor air mass flow rate, but keeping the same burned gases mass flow rate, as presented in [1].

The first way involves the conversion of the adiabatic air compression evolution into a polytropic one, by cooling the air flow through the engine's compressor. As long as for an aircraft engine, it is quite impossible to use an external air cooler (because of its prohibited dimension and volume), the only effective cooling method is the volatile coolant injection into the air flow, which realizes the heat extraction during its mixing with the air and vaporization.

The second way involves the injection of a coolant into the rear part of the engine's combustor, which, in order to keep constant the turbine's burned gases mass flow rate, leads to a decrease of the compressor's air mass flow rate and simultaneously, for a constant rotational speed, to the compressor's final pressure increase and to the compression mechanical work decrease.

Both the abovementioned ways are meant to increase the enthalpy drop of engine's exhaust nozzle, by reducing gases enthalpy drop into the turbine, as a consequence of a reduced necessity of mechanical work of the engine's compressor. Thus, the methods for temporarily engine thrust augmentation are based on both previously described operating modes.

2. Gas-turbine jet engine mathematical model

Equations which describe jet engine's operation are: the motion equation of the spool compressor and turbine rotor, characteristics chart(s) of the compressor and of the turbine, the heat balance equation of engine's combustor and, eventually, the mass flow rate equation, as presented in [3–5].

- Rotor motion equation [4]:

$$\frac{\pi J}{30}\frac{dn}{dt} = M_T - M_C,$$ (1)

- Compressor characteristic [2]:

$$\frac{i_2^*}{i_1^*} = \frac{T_2^*}{T_1^*} = 1 + \frac{(\pi_c^*)^{\frac{\chi-1}{\chi}} - 1}{\eta_C},$$ (2)

- Turbine characteristic [2]:

$$\frac{i_4^*}{i_3^*} = \frac{T_4^*}{T_3^*} = 1 - \frac{(\delta_T^*)^{\frac{\chi_g-1}{\chi_g}} - 1}{\eta_T^*(\delta_T^*)^{\frac{\chi_g-1}{\chi_g}}},$$ (3)

- Combustor equation [1, 6]:

$$\dot{m}_a c_{p_a} T_2^* + \dot{m}_c (i_c + \zeta_{CA} P_C) = \dot{m}_g c_{p_g} T_3^*,$$ (4)

- Mass flow rate equation [1, 2, 4]:

$$\dot{m}_g = \dot{m}_a - \dot{m}_{pr} + \dot{m}_c,$$ (5)

where J is turbo-compressor rotor inertia moment, $n-$ engine (rotational) speed, M_C, M_T- compressor and turbine torques, T_1^*, T_2^*- total temperature in front/behind the compressor, T_3^*- combustor total temperature (in front of the turbine), T_4^*- total temperature behind the turbine, π_c^*- compressor pressure ratio, δ_T^*- turbine pressure ratio, χ, χ_g- adiabatic exponents (for air and burned gases), η_C, η_T^*- compressor and turbine efficiency, $c_{p_g}, c_{p_a}-$ specific isobar heat of burned gases and air (assumed as equal), $\zeta_{CA}-$ burning process' perfection coefficient, P_c- fuel's chemical energy, $\dot{m}_g-$ burned gases mass flow rate, $\dot{m}_a-$ air mass flow rate, $\dot{m}_{pr}-$ bleed air mass flow rate (extracted from the compressor for aircraft's and/or engine's necessities), $\dot{m}_c-$ injected fuel mass flow rate, A_{5-} nozzle's effective exhaust area.

All of them are nonlinear equations, which makes them extremely difficult to be used for study. In fact, in abovementioned equations, all parameters X are multivariable ($X = X(u_i)$, $i = \overline{1, N}$), such as $\dot{m}_a = \dot{m}_a(n, p_2^*)$, $\dot{m}_g = \dot{m}_g(A_5, p_4^*, T_4^*)$, $M_C = M_C(\dot{m}_a, n, \pi_c^*)$, $M_T = M_T(\dot{m}_g, n, T_3^*, \delta_T^*)$, ... and so on. Consequently, one has to linearize them (around a steady state regime or operation point, using the finite difference method) and bring them to a dimensionless form (by suitable dividing), then apply the Laplace transformation (with null initial conditions) in order to obtain the useful form of the mathematical model and the possibility of the transfer function describing, as it was done in [3, 5]. So, any parameter noted as X, may be mathematically described as follows:

$$X = X_0 + \frac{\Delta X}{1!} + \frac{(\Delta X)^2}{2!} + \dots + \frac{(\Delta X)^n}{n!},$$ (6)

where X_0 is the value of X for a steady state regime (assumed as completely determined), ΔX is the static error (or the linear deviation). Assuming a small value of ΔX, terms containing $(\Delta X)^r, r \geq 2$, should be neglected. Consequently, the above-presented equation system has a new form, becoming a linear one. Moreover, dividing favorable $\frac{\Delta X}{X_0}$, the above mathematical model can be transformed into a dimensionless-one. In order to obtain the dimensionless linearized mathematical model, one has to apply the Laplace transformation.

For the first model's equation, Eq. (1), one observe that turbine and compressor torques may be expressed as.

$$M_T = M_{T_0} + \Delta M_T, M_C = M_{C_0} + \Delta M_C, \tag{7}$$

while

$$\Delta M_T = \left(\frac{\partial M_T}{\partial T_3^*}\right)_0 \Delta T_3^* + \left(\frac{\partial M_T}{\partial n}\right)_0 \Delta n + \left(\frac{\partial M_T}{\partial \delta_T^*}\right)_0 \Delta \delta_T^* + \left(\frac{\partial M_T}{\partial \dot{m}_g}\right)_0 \Delta \dot{m}_g, \tag{8}$$

and

$$\Delta M_C = \left(\frac{\partial M_C}{\partial n}\right)_0 \Delta n + \left(\frac{\partial M_C}{\partial \pi_c^*}\right)_0 \Delta \pi_c^* + \left(\frac{\partial M_T}{\partial \dot{m}_a}\right)_0 \Delta \dot{m}_a, \tag{9}$$

where $0-$ index refers to the steady-state regime. Meanwhile, compressor and turbine pressure ratios are expressed as $\pi_c^* = \frac{p_2^*}{p_1^*}$, $\delta_T^* = \frac{p_3^*}{p_4^*}$ and $p_3^* = \sigma_{CA}^* p_2^*$, so, for constant flight regimes (when $p_1^* = $ const.) their linear deviation become.

$$\Delta \pi_c^* = \left(\frac{\partial \pi_c^*}{\partial p_2^*}\right)_0 \Delta p_2^* = \frac{\Delta p_2^*}{p_{1_0}^*}, \Delta \delta_T^* = \left(\frac{\partial \delta_T^*}{\partial p_3^*}\right)_0 \Delta p_3^* + \left(\frac{\partial \delta_T^*}{\partial p_4^*}\right)_0 \Delta p_4^* = \frac{\Delta p_3^*}{p_{4_0}^*} - \frac{\delta_{T_0}^*}{p_{4_0}^*} \Delta p_4^*, p_{3_0}^* = \sigma_{CA}^* p_{2_0}^*, \tag{10}$$

Substituting in Eq. (1) M_T and M_C with their expressions (Eqs. (8) and (9)), then eliminating the terms corresponding to the steady-state regime (identically satisfied), one obtains

$$\frac{\pi J n_0}{30 M_{C_0}} \frac{d}{dt}\left(\frac{\Delta n}{n_0}\right) + \frac{n_0}{M_{C_0}}\left[\left(\frac{\partial M_C}{\partial n}\right)_0 - \left(\frac{\partial M_T}{\partial n}\right)_0 + \left(\frac{\partial M_C}{\partial \dot{m}_a}\right)_0 \left(\frac{\partial \dot{m}_a}{\partial n}\right)_0\right]\frac{\Delta n}{n_0} = \frac{T_{3_0}^*}{M_{T_0}}\left[\left(\frac{\partial M_T}{\partial T_3^*}\right)_0 + \left(\frac{\partial M_T}{\partial \dot{m}_g}\right)_0 \left(\frac{\partial \dot{m}_g}{\partial T_3^*}\right)_0\right].$$

$$\cdot \frac{\Delta T_3^*}{T_{3_0}^*} + \frac{p_{2_0}^*}{M_{T_0}}\left[\sigma_{CA}^* \left(\frac{\partial M_T}{\partial \dot{m}_g}\right)_0 \left(\frac{\partial \dot{m}_g}{\partial p_3^*}\right)_0 + \frac{\sigma_{CA}^*}{p_{4_0}^*}\left(\frac{\partial M_T}{\partial \delta_T^*}\right)_0 - \frac{1}{p_{1_0}^*}\left(\frac{\partial M_C}{\partial \dot{m}_a}\right)_0 \left(\frac{\partial \dot{m}_a}{\partial \pi_c^*}\right)_0 - \frac{1}{p_{1_0}^*}\left(\frac{\partial M_C}{\partial \pi_c^*}\right)_0\right]\frac{\Delta p_2^*}{p_{2_0}^*} -$$

$$- \frac{\delta_{T_0}^*}{M_{T_0}}\left(\frac{\partial M_T}{\partial \delta_T^*}\right)_0 \frac{\Delta p_4^*}{p_{4_0}^*}, \tag{11}$$

then, after applying the Laplace transformation (for null initial conditions), the equation becomes

$$(T_1 s + \rho_1)\overline{n} - k_{1T3}\overline{T}_3^* - k_{1p2}\overline{p}_2^* + k_{1p4}\overline{p}_4^* = 0, \tag{12}$$

where $T_1 = \frac{\pi J n_0}{30 M_{C_0}}$, $\rho_1 = \frac{n_0}{M_{C_0}}\left[\left(\frac{\partial M_C}{\partial n}\right)_0 - \left(\frac{\partial M_T}{\partial n}\right)_0 + \left(\frac{\partial M_C}{\partial \dot{m}_a}\right)_0\left(\frac{\partial \dot{m}_a}{\partial n}\right)_0\right]$, $k_{1p4} = \frac{\delta_{T_0}^*}{M_{T_0}}\left(\frac{\partial M_T}{\partial \delta_T^*}\right)_0$, $k_{1T3} = \frac{T_{3_0}^*}{M_{T_0}}$

$\left[\left(\frac{\partial M_T}{\partial T_3^*}\right)_0 + \left(\frac{\partial M_T}{\partial \dot{m}_g}\right)_0\left(\frac{\partial \dot{m}_g}{\partial T_3^*}\right)_0\right]$, $k_{1p2} = \frac{p_{2_0}^*}{M_{T_0}}\left[\sigma_{CA}^*\left(\frac{\partial M_T}{\partial \dot{m}_g}\right)_0\left(\frac{\partial \dot{m}_g}{\partial p_3^*}\right)_0 + \frac{\sigma_{CA}^*}{p_{4_0}^*}\left(\frac{\partial M_T}{\partial \delta_T^*}\right)_0 - \frac{1}{p_{1_0}^*}\left(\frac{\partial M_C}{\partial \dot{m}_a}\right)_0\left(\frac{\partial \dot{m}_a}{\partial \pi_c^*}\right)_0 - \frac{1}{p_{1_0}^*}\right.$

$\left.\left(\frac{\partial M_C}{\partial \pi_c^*}\right)_0\right]$,

s – derivative operator, $\overline{X}(s)$–the Laplace transformation image of the dimensionless formal parameter $\frac{\Delta X}{X_0}$, or, as simplified notation

$$\overline{X}\left(\overline{X} = \overline{n}, \overline{T}_3^*, \ldots\right). \tag{13}$$

By doing the same for the other equations, one obtains their dimensionless linearized new forms, depending on the same parameters $\left(\overline{n}, \overline{T}_3^*, \overline{T}_4^*, \overline{p}_2^*, \overline{p}_4^*\right)$, as follows:

$$k_{2n}\overline{n} - k_{2T3}\overline{T}_3^* - k_{2p2}\overline{p}_2^* = 0, \tag{14}$$

$$-\overline{T}_3^* + \overline{T}_4^* - k_{3p2}\overline{p}_2^* - k_{3p4}\overline{p}_4^* = 0, \tag{15}$$

$$k_{4T3}\overline{T}_3^* + k_{4T4}\overline{T}_4^* - k_{4p2}\overline{p}_2^* - k_{4p4}\overline{p}_4^* = k_{4A}\overline{A}_5, \tag{16}$$

$$k_{5n}\overline{n} + k_{5T3}\overline{T}_3^* + k_{5p2}\overline{p}_2^* = k_{5c}\overline{\dot{m}}_c. \tag{17}$$

The above presented coefficients can be experimentally determined, or graphic-analytical estimated, based on engine's characteristics chart, as done in [3].

As presented in [5], engine's mathematical model built-up with Eqs. (12) and (14) to (17), may be expressed as an unique matrix eq. $[A] \times (u) = (b)$:

$$\begin{bmatrix} T_1 s + \rho_1 & -k_{1T3} & 0 & -k_{1p2} & k_{1p4} \\ k_{2n} & -k_{2T3} & 0 & k_{2p2} & 0 \\ 0 & -1 & 1 & -k_{3p2} & -k_{3p4} \\ 0 & k_{4T3} & k_{4T4} & k_{4p2} & k_{4p4} \\ k_{5n} & k_{5T3} & 0 & k_{5p2} & 0 \end{bmatrix} \times \begin{pmatrix} \overline{n} \\ \overline{T}_3^* \\ \overline{T}_4^* \\ \overline{p}_2^* \\ \overline{p}_4^* \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ k_{4A}\overline{A}_5 \\ k_{5c}\overline{\dot{m}}_c \end{pmatrix}. \tag{18}$$

Moreover, using the Cramer method, one can solve the equation and obtain the expressions of all the output parameters in vector (u). As an example, for engine's speed $\overline{n}$ parameter expression, it results in $\overline{n} = \frac{\det(A_n)}{\det(A)}$, or $\det(A)\overline{n} = \det(A_n)$, where $\det(A)$ is $[A]$– matrix determinant, while $\det(A_n)$ is the determinant of the matrix obtained by replacing the first $[A]$– matrix column with the input vector (b). Eventually, one obtains, formally, $(a_1 s + a_0)\overline{n} = f_0\overline{\dot{m}}_c + g_0\overline{A}_5$, or else

$$(\tau_m s + 1)\overline{n} = k_c \overline{\dot{m}}_c + k_A \overline{A}_5, \tag{19}$$

where the involved coefficients (a_1, a_0, f_0, g_0) and (τ_m, k_c, k_A) are used with their expressions determined in [5], depending on the values of $T_1, \rho_1, k_{1T3}, \ldots k_{5p2}, k_{4A}, k_{5c}$. If one considers the aircraft flight regime too (airspeed V and altitude H), Eq. (19) must be completed by a term containing both elements V and H, which is the pressure p_1^* (as determined in [3, 5]):

$$(\tau_m s + 1)\overline{n} = k_c \overline{\dot{m}}_c + k_A \overline{A}_5 - k_{HV} \overline{p}_1^*. \tag{20}$$

Together with the main output $\overline{n}$, secondary outputs may be obtained in the same way:

- Combustor temperature:

$$(\tau_m s + 1)\overline{T}_3^* = (l_{Tc} s + l_T)\overline{\dot{m}}_c + l_A \overline{A}_5 - k_{THV} \overline{p}_1^*, \tag{21}$$

- Engine thrust:

$$(\tau_m s + 1)\overline{F} = (l_{Fc} s + l_c)\overline{\dot{m}}_c + (l_{FA} s + l_A)\overline{A}_5 - k_{Fp} \overline{p}_1^*. \tag{22}$$

If the engine operates at low altitude and speed (such as during the take-off procedure), flight regime has no influence, as well as the exhaust nozzle area (which must be totally open), so $\overline{p}_1^* = 0$ and $\overline{A}_5 = 0$.

3. Coolant injection into gas-turbine engine's compressor

The first described thrust augmentation method consists of the injection of a special cooling fluid (distilled water, methanol, ammonia or mixtures), identified as *coolant*, into engine's axial compressor, in its front part. In order to achieve its task—the heat extraction by vaporization— it is compulsory that the coolant injectors are positioned in the front of the compressor to assure enough room for coolant-air mixing and coolant vaporization. If the coolant is a combustible fluid (such as methanol), not a neutral fluid (such as water), after its vaporization, it can participate into the burning evolution in the engine's combustor, which may reduce the fuel consumption increase due to the coolant injection [1].

Thrust augmentation is significant for an engine operating at uncommon atmospheric conditions (such as high temperatures, more than 30°C, low pressure and humidity), because injected coolant's vaporization is facilitated [1]. Method's disadvantages consist of icing hazard (air intake's and compressor's blades icing, or the suction of ice lumps into the compressor), as well as the necessity of on-board coolant carrying; injection running time is the one who limits the coolant necessary mass to be boarded, so aircraft's coolant injection system design and optimization must take into account aircraft's take-off payload, as well as aircraft mission and take-off atmospheric conditions.

In fact, the temporary thrust augmentation by this coolant injection method may be assimilated to an over-boost method for classic engines; this method is suitable for low or medium power jet-engines, as well as for turboprops (where the afterburning is impossible to be adapted).

3.1. Thermodynamic and gas-dynamic phenomena

The air compression evolution into the engine's compressor is an irreversible adiabatic (of adiabatic exponent $\chi = 1.4$), characterized by increasing air temperature (enthalpy), as depicted in **Figure 1**. The injected coolant vaporizing extracts a significant fraction of the produced compression heat; consequently, the compressed air becomes cooler, its mass flow rate grows, while its volume flow rate is kept constant. The extraction of a fraction of the compression heat (q_i in **Figure 1**) converts the air compression evolution into a polytropic-one (of a_i exponent, smaller than χ) and determines a significant decrease of the compression mechanical work; meanwhile, compressor's total pressure ratio changes, from π_c^* to $\pi_{c_i}^*$, as well as the compressor's working line on its universal characteristics (which moves much closer to the surge line).

The injected coolant may extract the heat $Q_i = \dot{m}_l r_f$, where r_f is coolant's latent heat of vaporization, $\dot{m}_l$— the injected coolant flow rate, which means that, for $\dot{m}_a$ the air flow rate through the compressor and for ξ_l— the injection fraction, one obtains for q_i

$$q_i = \frac{Q_i}{\dot{m}_a} = \frac{\dot{m}_l}{\dot{m}_a} r_f = \xi_l \cdot r_f. \tag{23}$$

Mechanical work's value should be the same for both evolutions (the adiabatic old one, as well as for the polytropic new one), so
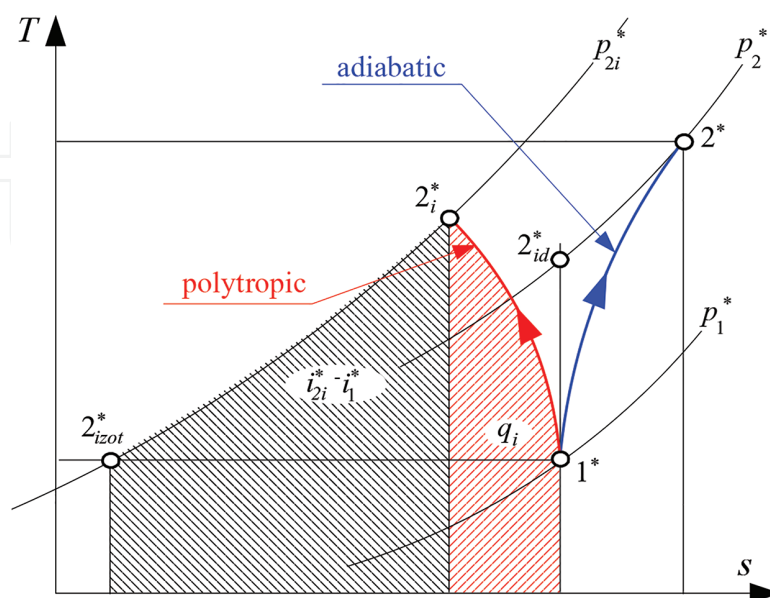

Figure 1. Air compression evolution with and without coolant injection.
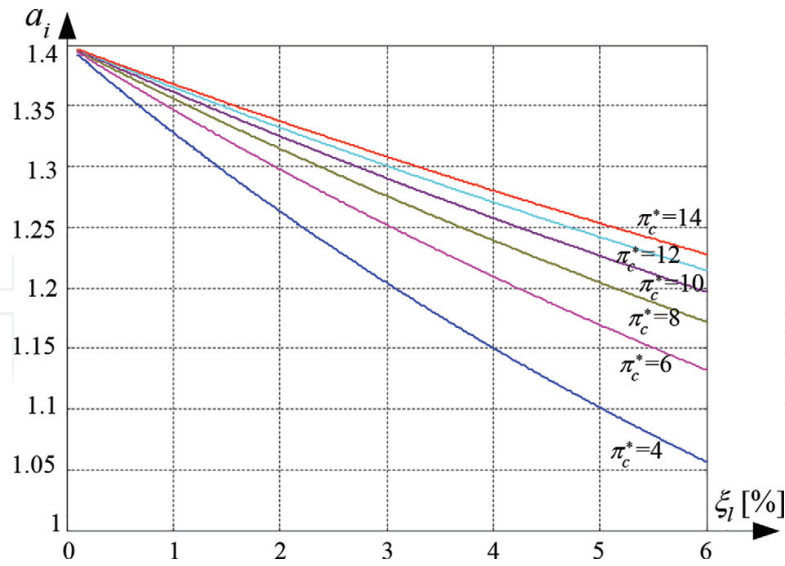
Figure 2. Polytropic exponent versus coolant injection fraction.

$$i_1^* \frac{a_i}{a_i - 1} \left[\left(\pi_{c_i}^* \right)^{\frac{a_i-1}{a_i}} - 1 \right] = i_1^* \left[\frac{\left(\pi_c^* \right)^{\frac{\chi-1}{\chi}} - 1}{\eta_c} \right] - q_i = i_1^* \left[\frac{\left(\pi_c^* \right)^{\frac{\chi-1}{\chi}} - 1}{\eta_c} \right] - \xi_l r_f, \tag{24}$$

where i_1^* is air's specific enthalpy in the front of the compressor, η_c— compressor's efficiency. This equation gives the connection between ξ_l and a_i, as shown in **Figure 2**. The bigger the coolant fluid flow rate $\dot{m}_l$ and its fraction ξ_l are, the lower the polytropic exponent is; its value tends to 1, which means that the limit evolution is the isothermic-one, extremely difficult to be achieved. Withal, the bigger compressor's total pressure ratio is, the bigger a_i value becomes.

The new value of the compressors total pressure ratio becomes

$$\pi_{c_i}^* = \left\{ 1 + \frac{a_i - 1}{a_i} \frac{\chi}{\chi - 1} \left[\frac{\left(\pi_c^* \right)^{\frac{\chi-1}{\chi}} - 1}{\eta_C} \right] \right\}^{\frac{a_i}{a_i-1}}. \tag{25}$$

In terms of the engine's working fluid mass flow rate, modifications also appear. For the basic engine, burned gases mass flow rate through the exhaust nozzle $\dot{m}_g$ is given by

$$\dot{m}_g = \dot{m}_a - \dot{m}_{pr} + \dot{m}_c, \tag{26}$$

As stated in [1, 2], the bleed air mass flow rate $\dot{m}_{pr}$ and the fuel mass flow rate $\dot{m}_c$ have near the same values, which lead to the conclusion that $\dot{m}_g \approx \dot{m}_a$. Therefore, if the coolant injection system is active, the burned gases flow rate $\dot{m}_{g_i}$ becomes

$$\dot{m}_{gi} = \dot{m}_{a_i} + \dot{m}_l, \tag{27}$$

where $\dot{m}_{a_i}$ is the compressor air mass flow rate amount when the coolant injection is active.

As long as nowadays gas-turbine-engines have critical flow in their turbines [1, 2, 8], their flow parameters should remain constant, even with coolant injection into the compressor. So, the condition $\frac{\dot{m}_g \sqrt{T_3^*}}{p_3^*} = idem$ for the turbine flow leads to

$$\frac{\dot{m}_g \sqrt{T_3^*}}{p_3^*} = \frac{\dot{m}_a \sqrt{T_3^*}}{\sigma_{CA}^* p_2^*} = \frac{\dot{m}_{a_i} \sqrt{T_3^*}}{\sigma_{CA}^* p_{2_i}^*}, \tag{28}$$

where p_3^*- burned gas pressure before the turbine (proportional to p_2^*- the air pressure after the compressor, $p_3^* = \sigma_{CA}^* p_2^*$), σ_{CA}^*- combustor's total pressure ratio, $p_2^* i-$ the air pressure after the compressor when the coolant injection is active.

Consequently, the new value for the compressor's air flow rate becomes

$$\dot{m}_{a_i} = \dot{m}_a \frac{\pi_{C_i}^*}{\pi_c^*}, \tag{29}$$

which, obviously, modifies the value of $\dot{m}_{g_i}$ given by Eq. (27).

3.2. Gas-turbine-engine with coolant injection into its compressor mathematical model

Coolant injection, as presented in previous section, has a significant influence above engine's behavior and, consequently, above its mathematical model form, as well as on its coefficients expressions and values.

First, the mass flow rate equation is modified because of the presence of $\dot{m}_l$; second, according to the coolant nature (combustible fluid or neutral fluid), the combustor's equation may, or may not, be also modified.

In terms of the mass flow equation, Eq. (27) may be written (according to [1, 9] and [10]) as

$$\dot{m}_{g_i}\left(p_3^*, T_3^*\right) = \dot{m}_{a_i}\left(p_2^*, n\right) + \dot{m}_l, \tag{30}$$

or, using the finite difference method for linearization, as

$$\left(\frac{\partial \dot{m}_{g_i}}{\partial p_3^*}\right)_0 \Delta p_3^* + \left(\frac{\partial \dot{m}_{g_i}}{\partial T_3^*}\right)_0 \Delta T_3^* = \left(\frac{\partial \dot{m}_{a_i}}{\partial p_2^*}\right)_0 \Delta p_2^* + \left(\frac{\partial \dot{m}_{a_i}}{\partial n}\right)_0 \Delta n + \Delta \dot{m}_l. \tag{31}$$

The above-determined equation, after some suitable amplifying and terms grouping, may be brought to a dimensionless form, then Laplace transformed, as described in [10], giving

$$\frac{p_{2_0}^*}{(\dot{m}_{a_i})_0}\left[\left(\frac{\partial \dot{m}_{g_i}}{\partial p_2^*}\right)_0 \sigma_{CA}^* - \left(\frac{\partial \dot{m}_{a_i}}{\partial p_2^*}\right)_0\right]\frac{\Delta p_2^*}{p_{2_0}^*} + \frac{T_{3_0}^*}{(\dot{m}_{a_i})_0}\left(\frac{\partial \dot{m}_{g_i}}{\partial T_3^*}\right)_0 \frac{\Delta T_3^*}{T_{3_0}^*} - \frac{n_0}{(\dot{m}_{a_i})_0}\left(\frac{\partial \dot{m}_{a_i}}{\partial n}\right)_0 \frac{\Delta n}{n_0} = \xi_l \frac{\Delta \dot{m}_l}{(\dot{m}_l)_0}, \tag{32}$$

equivalent to

$$k_{2n}^{/}\overline{n} - k_{2T3}^{/}\overline{T_3^*} + k_{2p2}^{/}\overline{p_2^*} = \xi_l\overline{\dot{m}_l}, \tag{33}$$

which is a new form for the second equation of the mathematical model, so the new coefficients in Eq. (33) left member should replace the old ones in the second line of the system matrix $[A]$. Withal, the input vector (b) should be completed on its second line by $\xi_l\overline{\dot{m}_l}$, the right member in Eq. (33).

In terms of the combustor equation, it shall be modified due to the supplementary presence of the coolant fluid's energy [10, 11], as follows

$$\dot{m}_{g_i}c_{p_g}T_3^* - \dot{m}_{a_i}c_{p_a}T_2^* = \dot{m}_c\zeta_{CA}P_c + \dot{m}_l\zeta_{CA}P_l, \tag{34}$$

where P_l—chemical energy of the injected fluid (if combustible), and T_2^*—air temperature behind the compressor, which may be expressed with respect to p_2^* as follows

$$\overline{T_2^*} = \frac{p_{2_0}^*}{T_{2_0}^*}\left(\frac{\partial T_2^*}{\partial \pi_c^*}\right)_0\left(\frac{\partial \pi_c^*}{\partial p_2^*}\right)_0\overline{p_2^*}, \tag{35}$$

while the term $\dot{m}_{a_i}$ shall be expressed from the compressor's characteristic with respect to its rotational speed n and to the pressure behind the compressor p_2^* [2, 8, 10].

Both cases, for neutral, respectively for combustible coolant, were studied in [10], the fifth equation of the mathematical model being modified according to each situation, as shown below.

3.2.1. Neutral coolant injection

When the injected coolant is a neutral fluid$(P_l = 0)$, the term $\dot{m}_l\zeta_{CA}P_l$ becomes null. Consequently, considering Eq. (35) and applying the same above described method (as in [4, 10]), Eq. (34) becomes

$$\begin{aligned}
&\frac{c_p\left(T_{3_0}^* - T_{2_0}^*\right)n_0}{\dot{m}_{c_0}\zeta_{CA}P_c}\left(\frac{\partial \dot{m}_a}{\partial n}\right)_0\overline{n} + \frac{c_p(\dot{m}_{a_0} - \dot{m}_{l_0})T_{3_0}^*}{\dot{m}_{c_0}\zeta_{CA}P_c}\overline{T_3^*} \\
&+\left[\frac{c_p\left(T_{3_0}^* - T_{2_0}^*\right)p_{2_0}^*}{\dot{m}_{c_0}\zeta_{CA}P_c}\left(\frac{\partial \dot{m}_a}{\partial p_2^*}\right)_0 - \frac{c_p(\dot{m}_{a_0} - \dot{m}_{l_0})}{\dot{m}_{c_0}\zeta_{CA}P_c}p_{2_0}^*\left(\frac{\partial T_2^*}{\partial \pi_c^*}\right)_0\left(\frac{\partial \pi_c^*}{\partial p_2^*}\right)_0\right]\overline{p_2^*} = \overline{\dot{m}_c} - \frac{\dot{m}_{l_0}}{\dot{m}_{c_0}\zeta_{CA}P_c}\overline{\dot{m}_l}
\end{aligned} \tag{36}$$

The coefficient $\frac{\dot{m}_{l_0}}{\dot{m}_{c_0}\zeta_{CA}P_c}$ of $\overline{\dot{m}_l}$ in the above-determined equation has a very small value (10^4 times less than any other coefficient), so it may be neglected, which simplifies equation's final form

$$k_{5n}\overline{n} + k_{5T3}^{/}\overline{T_3^*} + k_{5p2}^{/}\overline{p_2^*} = \overline{\dot{m}_c}, \tag{37}$$

so the fifth line in matrix $[A]$ of Eq. (18) must be rewritten. Consequently, one obtains a new form of engine's mathematical model, as follows:

$$\begin{bmatrix} T_1s + \rho_1 & -k_{1T3} & 0 & -k_{1p2} & k_{1p4} \\ k_{2n}^{/} & -k_{2T3}^{/} & 0 & k_{2p2}^{/} & 0 \\ 0 & -1 & 1 & -k_{3p2} & -k_{3p4} \\ 0 & k_{4T3} & k_{4T4} & k_{4p2} & k_{4p4} \\ k_{5n}^{/} & k_{5T3}^{/} & 0 & k_{5p2}^{/} & 0 \end{bmatrix} \times \begin{pmatrix} \overline{n} \\ \overline{T}_3^* \\ \overline{T}_4^* \\ \overline{p}_2^* \\ \overline{p}_4^* \end{pmatrix} = \begin{pmatrix} 0 \\ \xi_l \overline{\dot{m}}_l \\ 0 \\ 0 \\ k_{5c}^{/} \overline{\dot{m}}_c \end{pmatrix}, \tag{38}$$

which gives for the main output parameter, by Cramer-method solving, an equation similar to Eq. (19)

$$(\tau_{m1}s + 1) \cdot \overline{n} = k_{c1} \overline{\dot{m}}_c - k_l \overline{\dot{m}}_l. \tag{39}$$

3.2.2. Combustible fluid injection

When the coolant is a combustible fluid, Eq. (34) shall be reconsidered and rewritten; consequently, its right member becomes

$$\left(1 - \frac{P_l}{P_c} \frac{\dot{m}_{l_0}}{\dot{m}_{c_0}}\right) \overline{\dot{m}}_c - \frac{\dot{m}_{l_0} c_p \left(T_{3_0}^* - T_{2_0}^*\right)}{\dot{m}_{c_0} \zeta_{CA} P_c} \overline{\dot{m}}_l = k_{5c}^{/} \overline{\dot{m}}_c - k_{5l}^{/} \overline{\dot{m}}_l, \tag{40}$$

so, the fifth equation of the model shall have a new form, similar to Eq. (37), but considering the coolant heat effect too:

$$k_{5n} \overline{n} + k_{5T3}^{/} \overline{T}_3^* + k_{5p2}^{/} \overline{p}_2^* = k_{5c}^{/} \overline{\dot{m}}_c - k_{5l}^{/} \overline{\dot{m}}_l. \tag{41}$$

In order to eliminate the term containing the coolant injection flow rate parameter and simplify the input vector's form, one has to multiply the second model's equation by $\frac{k_{5l}^{/}}{\xi_l}$ and to add it to Eq. (40). It results, for the last equation of the model, a simplified form

$$\left(k_{2n}^{/} \frac{k_{5l}^{/}}{\xi_l} + k_{5n}\right) \overline{n} + \left(k_{2T3}^{/} \frac{k_{5l}^{/}}{\xi_l} + k_{5T3}\right) \overline{T}_3^* + \left(k_{2p2}^{/} \frac{k_{5l}^{/}}{\xi_l} + k_{5p2}\right) \overline{p}_2^* = k_{5c}^{/} \overline{\dot{m}}_c \tag{42}$$

and, consequently, the new form of the main matrix $[A_2]$ of engine's mathematical model becomes

$$A_2 = \begin{bmatrix} T_1s + \rho_1 & -k_{1T3} & 0 & -k_{1p2} & k_{1p4} \\ k_{2n}^{/} & -k_{2T3}^{/} & 0 & k_{2p2}^{/} & 0 \\ 0 & -1 & 1 & -k_{3p2} & -k_{3p4} \\ 0 & k_{4T3} & k_{4T4} & k_{4p2} & k_{4p4} \\ k_{2n}^{/} \frac{k_{5l}^{/}}{\xi_l} + k_{5n} & k_{2T3}^{/} \frac{k_{5l}^{/}}{\xi_l} + k_{5T3} & 0 & k_{2p2}^{/} \frac{k_{5l}^{/}}{\xi_l} + k_{5p2} & 0 \end{bmatrix}, \tag{43}$$

while the vectors (u) and (b) remain the same as in Eq. (38).

3.3. About system's quality

The gas-turbine-engine with coolant injection into its compressor as controlled object, according to its above-determined mathematical model and as **Figure 3** shows, it is a first-order system; it has two inputs (the fuel mass flow rate $\overline{\dot{m}_c}$ and the coolant mass flow rate $\overline{\dot{m}_l}$), but multiple outputs (such as engine's rotation speed $\overline{n}$, combustor's temperature $\overline{T}_3^*$, engine's thrust $\overline{F}$, etc.); obviously, only one output is the main one (in this case $\overline{n}$), the others being secondary outputs.

System's quality evaluation consists of the study of system's time behavior, in fact the study of system's step response(s); system's input(s) is (are) replaced by step functions (Heaviside), and the outputs are evaluated from their behavior point of view, knowing how the system responds to sudden input(s) and gathering information on system's stability, as well as on its ability to reach one stationary state when starting from another [4, 7, 11].

A quantitative study was performed and described in [10], based on an existing VK-1A-type engine (jet-engine with constant geometry exhaust nozzle), excluding its built-in control systems (such as rotation speed controller or combustor's temperature limiter), in order to evaluate only the basic engine as possible controlled object.

One has chosen for study as outputs: the main output – engine's speed parameter $\overline{n}$, as well as two secondary, but very important, outputs – combustor temperature's parameter $\overline{T}_3^*$ and engine's thrust parameter $\overline{F}$. Outputs' quantitative expressions (determined in [10]) are:

a. engine with neutral coolant injection (distilled water):

$$\overline{n}(s) = \frac{1.283\,\overline{\dot{m}_c}(s) - 0.092\,\overline{\dot{m}_l}(s)}{1.6348\,s + 4.5158}, \tag{44}$$

$$\overline{T}_3^*(s) = \frac{(1.474\,s + 2.748)\overline{\dot{m}_c}(s) - (0.047\,s + 0.034)\overline{\dot{m}_l}(s)}{1.6348\,s + 4.5158}, \tag{45}$$

$$\overline{F}(s) = \frac{(1.385\,s + 4.516)\overline{\dot{m}_c}(s) - (0.089\,s + 0.315)\overline{\dot{m}_l}(s)}{1.6348\,s + 4.5158}; \tag{46}$$

b. engine with combustible coolant injection (methanol):

$$\overline{n}(s) = \frac{1.43\,\overline{\dot{m}_c}(s) - 0.184\,\overline{\dot{m}_l}(s)}{2.161\,s + 4.7973}, \tag{47}$$

$$\overline{T}_3^*(s) = \frac{(1.816\,s + 2.467)\overline{\dot{m}_c}(s) - (0.053\,s + 0.047)\overline{\dot{m}_l}(s)}{2.161\,s + 4.7973}, \tag{48}$$

$$\overline{F}(s) = \frac{(1.584\,s + 6.317)\overline{\dot{m}_c}(s) - (0.113\,s + 0.803)\overline{\dot{m}_l}(s)}{2.161\,s + 4.7973}. \tag{49}$$

Engine outputs' time behavior is graphically represented in **Figures 4–6**, for the basic engine (blue lines), as well as for the engine with coolant injection (green lines for neutral coolant, red lines for combustible coolant). All the obtained curves are proving that the studied system is a stable-one, with asymptotic stabilization and static errors.
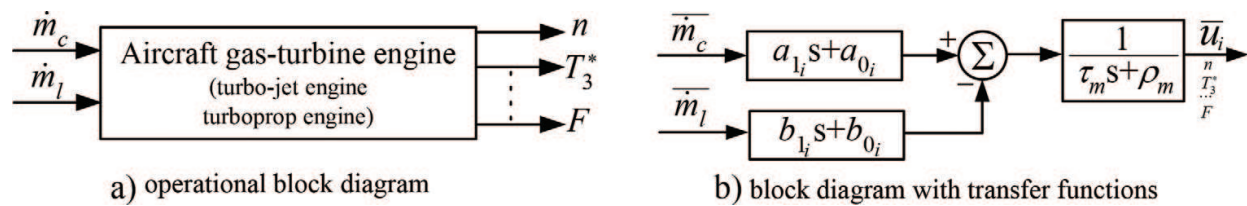
a) operational block diagram   b) block diagram with transfer functions

Figure 3. Formal description of a gas-turbine-engine with coolant injection.

In terms of engine's time constant value, whatever the method was, it remains nearly the same, settling time values being kept around (2.5–3.0) s, similar to the basic engine.

As **Figure 4** shows, whatever the nature of the coolant is, the speed parameter behavior is little influenced, the deviation from basic engine's curve being very small; the biggest value is obtained for combustible coolant (obviously, because its supplementary contribution to the burning process and supplementary heat transfer).

Figure 5 shows temperature parameters' initial overshoots, followed by asymptotic settlings. When the coolant is neutral, one can observe a significant growth of the $\overline{T}_3^*-$ parameter's value, as a consequence of air mass flow rate increase, which requires a supplementary fuel injection, in order to avoid temperature and thrust decrease. When the coolant is a combustible fluid, because of its own heat input, the supplementary fuel injection is smaller and, consequently, one obtains a smaller temperature increase, even if the initial overshoot is comparable to the first case.

In terms of thrust parameter's behavior, as **Figure 6** shows, after a sudden initial increase, the curves have similar asymptotic aspects. Most significant thrust increase is obtained when the coolant is a combustible-one because of engine's specific thrust increase (due to the additional heat input), as well as because of air mass flow rate increase, while thrust increase for neutral coolant injection is moderate.
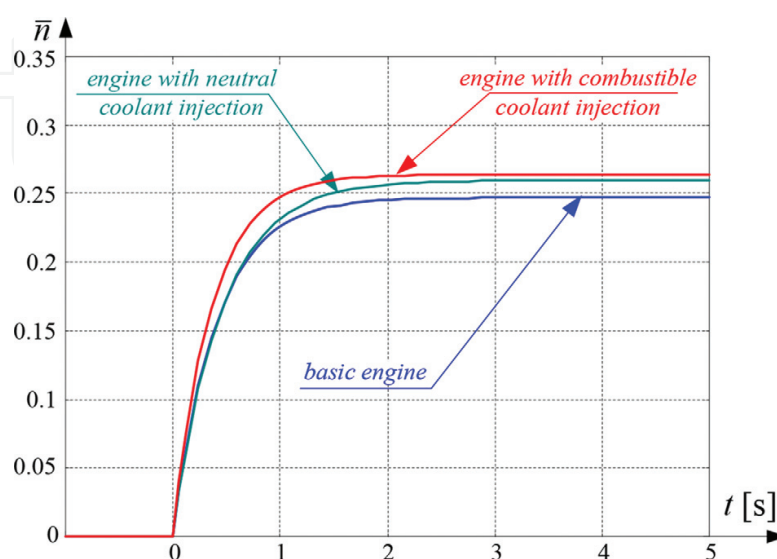

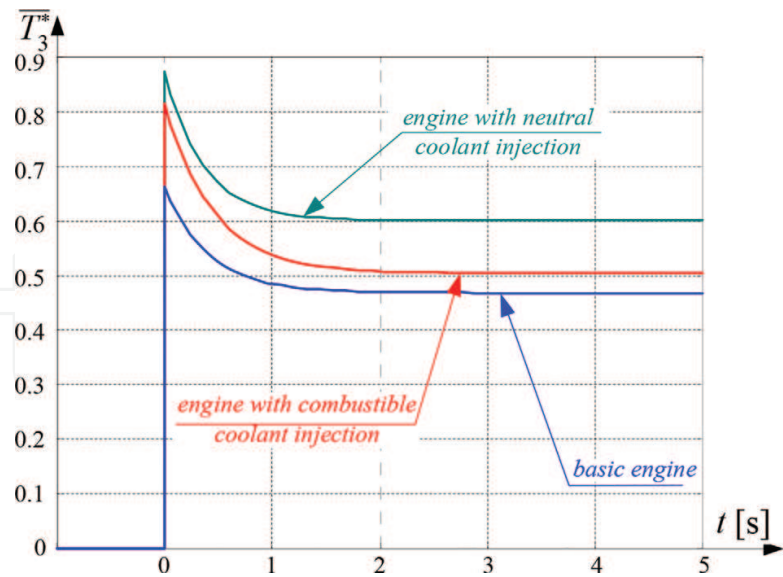**Figure 4.** Comparative step response of engine rotational speed parameters.

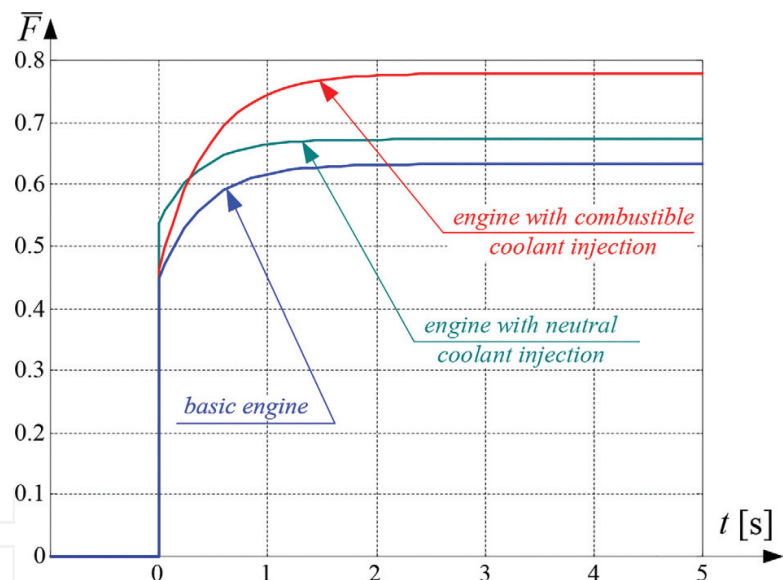**Figure 5.** Comparative step response of engine combustor temperature parameters.

Figure 6. Comparative step response of engine thrust parameters.

3.4. Jet engine with coolant injection controller

The coolant injection into the compressor is done by means of a pump, which may be electrically driven (by a DC motor, supplied by the aircraft electric system), or it may be connected to the engine's gearbox and, consequently, driven by engine shaft [10, 12, 14]. As long as an electric driven pump has a constant rotation speed, both its mass flow rate and its injection pressure are constant, while engine shaft-driven pump's speed is equal (or proportional) to engine's speed n, thus its flow rate behavior is also proportional to n, which introduces another feedback into the engine's control system, as shown in **Figure 7**. In fact, in order to assure an
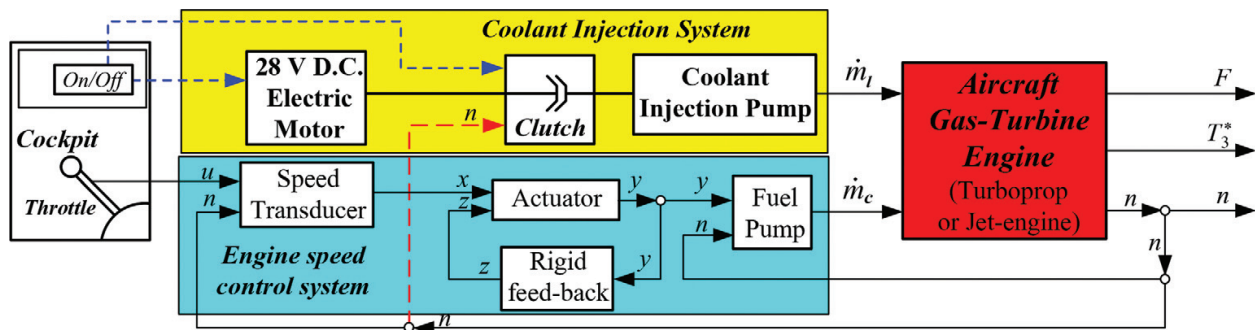
Figure 7. Gas-turbine-engine with coolant injection formal block-diagram.

appropriate engine operation, not only the fuel mass flow rate, but also the coolant mass flow rate must be controlled; this one must be correlated to the compressor provided air mass flow rate. Engine shaft-driven pump assures such a correlation, much better than the electric driven pump (as presented in [12]). Thus, gearbox-driven pump speed follows engine's speed, so the coolant mass flow rate grows smoothly (as engine's speed does); in terms of temperature and thrust parameters, they also show similar behavior to basic engine's parameters. Independent driven injection pump (by an electric DC motor) is a more convenient solution (sometimes easier to manufacture and less expensive); although it does not affect engine's stability, it worsens its time behavior, as stated in [12].

However, some engine manufacturers (based on operational, economic and efficiency reasons) prefer to implement a coolant injection controller (CIC) consisting of an electric driven pump, assisted by a coolant flow rate controller (which correlates the coolant flow rate to the air flow rate, following the compressor's total pressure ratio). Such an embedded system, presented and studied in [14], consists of a gas-turbine-engine, its speed controller (ESCS - based on its fuel mass flow rate control) and the coolant injection controller (CIC – built up with a dosage valve, an actuator and a pressure ratio transducer), as formally depicted in **Figure 8**.
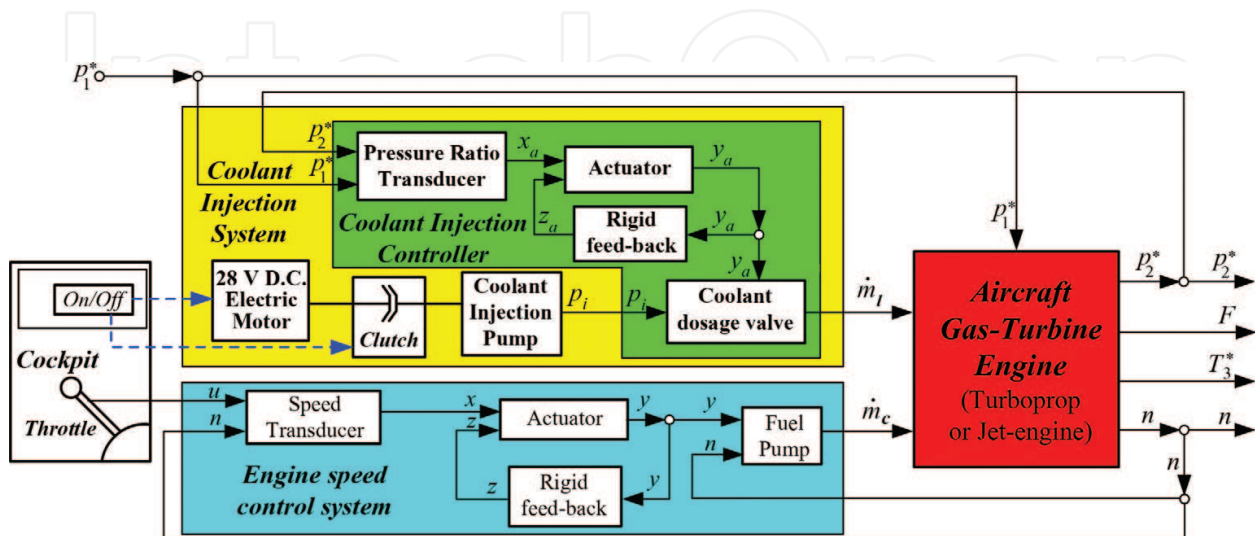

Figure 8. Gas-turbine-engine with speed control system and coolant injection controller.

Embedded system's mathematical model has the following equations:

a. ESCS model (as determined in [4, 13]):

- Fuel pump:

$$\overline{\dot{m}}_c = k_{pn}\overline{n} + k_{py}\overline{y}, \tag{50}$$

- Throttle:

$$\overline{u} = k_{u\alpha}\overline{\alpha}, \tag{51}$$

- Speed transducer:

$$\overline{x} = k_u\overline{u} - k_{es}\overline{n}, \tag{52}$$

- Actuator:

$$\tau_s s\overline{y} = \overline{x} - \overline{z}, \tag{53}$$

- Rigid feedback

$$\overline{z} = \rho_s\overline{y}, \tag{54}$$

b. CIC model (similar to the one studied in [15]):

- Pressure ratio transducer:

$$\overline{x}_a = k_{px}\left(\overline{p}_1^* - \overline{p}_R\right), \tag{55}$$

$$k_{2R}\overline{p}_2^* - k_{xR}(\tau_{ax}s + 1)\overline{x}_a = (\tau_R s + 1)\overline{p}_R, \tag{56}$$

- Actuator:

$$\tau_{sl}s\overline{y}_a = \overline{x}_a - \overline{z}_a, \tag{57}$$

- Rigid feedback

$$\overline{z}_a = \rho_l\overline{y}_a, \tag{58}$$

- Dosage valve:

$$\overline{\dot{m}}_l = k_{ly}\overline{y}_a. \tag{59}$$

c. jet engine (with operational coolant injection) main output parameter equation, determined in previous subsections, together with engine's secondary output parameter $\overline{p}_2^*$ equation (which is used as CIC main input, as Eq. (56) shows):

$$(\tau_{m1}s + 1) \cdot \overline{n} = k_{c1}\overline{\dot{m}_c} - k_l\overline{\dot{m}_l} - k_{HV}\overline{p_1^*}, \tag{60}$$

$$(\tau_{m1}s + 1) \cdot \overline{p_2^*} = (l_{p2c}s + l_{p2})\overline{\dot{m}_c} - (l_{p2l}s + l_{l2})\overline{\dot{m}_l}, \tag{61}$$

where the new term $k_{HV}\overline{p_1^*}$ should introduce the effect of flight regime (airspeed and altitude).

Based on above-presented equations, one has built-up embedded controller block diagram with transfer functions, depicted in **Figure 9**; the diagram in **Figure 9a** corresponds to the case when the coolant injection pump is driven by engine's shaft, while the diagram in **Figure 9b** — to the electrically driven pump case (formally depicted in **Figure 8**). One may observe that both embedded control systems have as unique input the engine's power lever angle parameter $\overline{\alpha}$. In fact, engine's power lever (throttle) is the only way for the pilot to command the engine, so the throttle should "control the controllers."

As long as the coolant injection operates only for take-off and at very low flight altitudes, one may consider that p_1^* has a negligible variation; consequently, $\overline{p_1^*} = 0$, while the terms giving the flight regime's influence ($k_{HV}\overline{p_1^*}$ in Eq. (50) and $k_{px}\overline{p_1^*}$ in Eq. (56)) may be neglected.

A quantitative study was performed and described in [14], based on a VK-1A-type jet engine and on some studies concerning the involved controllers, developed in [4, 9, 11, 13]. The quantitative expression of the embedded system mathematical model is given by the equations:
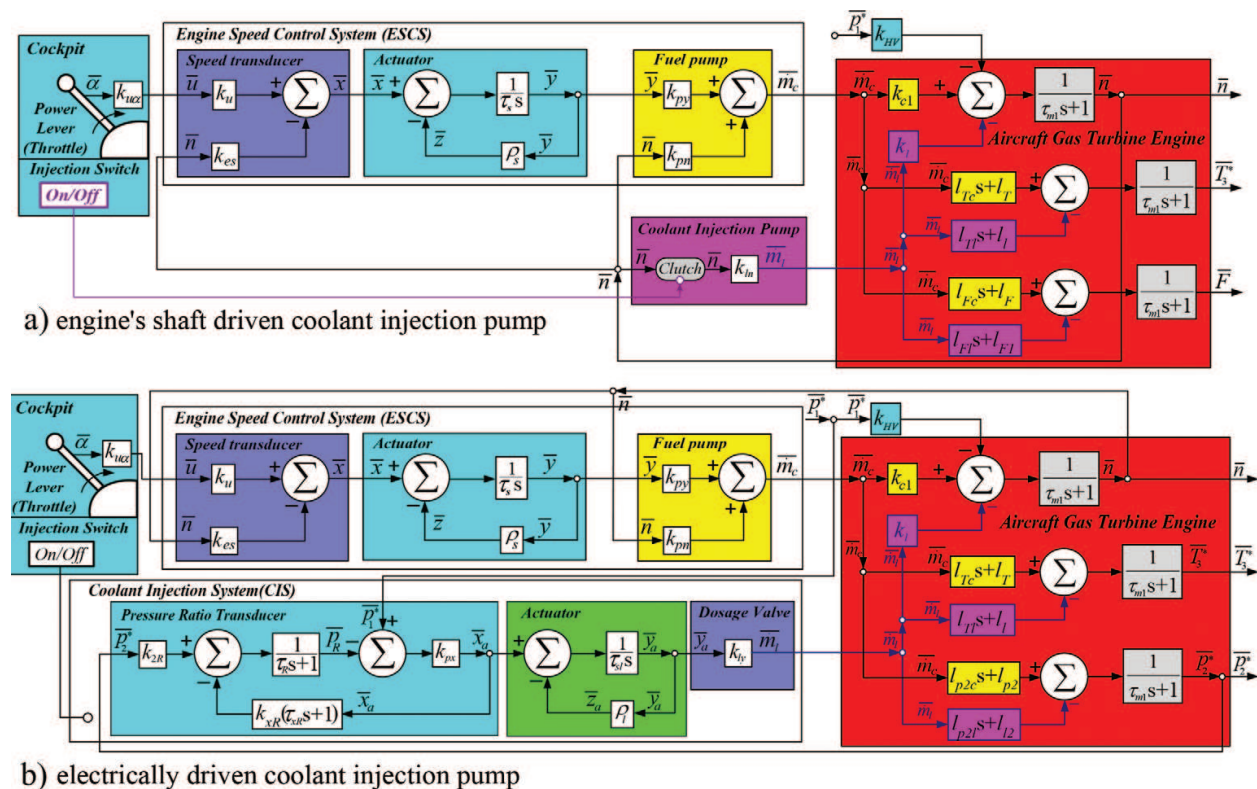

Figure 9. Embedded control systems block diagrams with transfer functions.

$$\overline{n}(s) = \frac{1.283\,\overline{\dot{m}}_c(s) - 0.092\overline{\dot{m}}_l(s)}{1.6348\,s + 4.5158}, \tag{62}$$

$$\overline{p_2^*}(s) = \frac{(0.3425\,s + 2.6718)\overline{\dot{m}}_c(s) - (0.0672\,s + 0.9211)\overline{\dot{m}}_l(s)}{1.6348\,s + 4.5158}, \tag{63}$$

$$\overline{T_3^*}(s) = \frac{(1.474\,s + 2.748)\overline{\dot{m}}_c(s) - (0.047\,s + 0.034)\overline{\dot{m}}_l(s)}{1.6348\,s + 4.5158}, \tag{64}$$

$$\overline{F}(s) = \frac{(1.385\,s + 4.516)\overline{\dot{m}}_c(s) - (0.089\,s + 0.315)\overline{\dot{m}}_l(s)}{1.6348\,s + 4.5158}, \tag{65}$$

$$\overline{x}(s) = 0.317\overline{a}(s) - 0.439\overline{n}, \tag{66}$$

$$\overline{y}(s) = \frac{1}{1.81 \cdot s + 5.306}\overline{x}(s), \tag{67}$$

$$\overline{\dot{m}}_c(s) = 0.5\overline{n}(s) + 0.5\overline{y}(s), \tag{68}$$

$$\overline{x}_a(s) = \frac{0.346}{0.8101 \cdot s + 3.1221}\overline{p_2^*}(s), \tag{69}$$

$$\overline{y}_a(s) = \frac{1}{1.364 \cdot s + 4.617}\overline{x}_a(s), \tag{70}$$

$$\overline{\dot{m}}_l(s) = 0.615\overline{y}_a(s). \tag{71}$$

Simulation was performed based on these equations and on the block diagrams with transfer functions in **Figure 9**, considering as single input the power lever angle (PLA) or the throttle angle
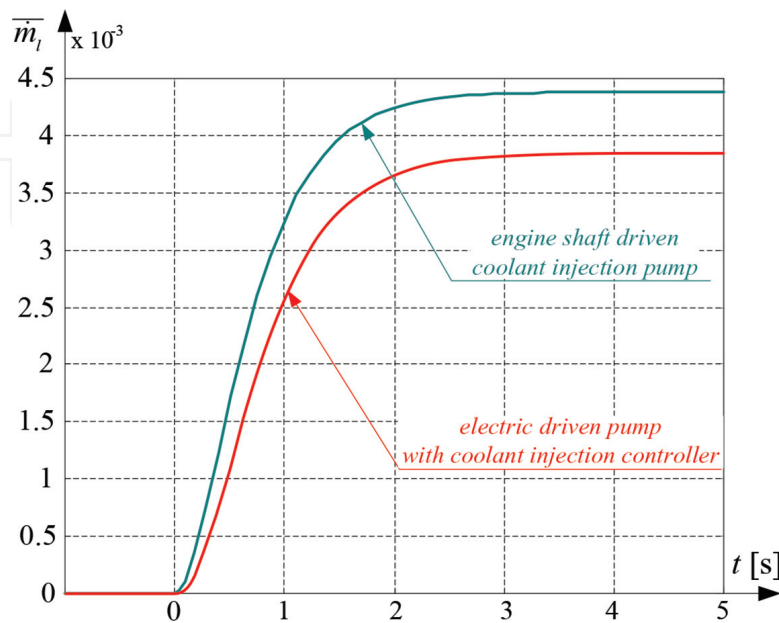

Figure 10. Coolant flow rate parameter step response.

parameter $\overline{\alpha}$, while as main output - engine's speed parameter $\overline{n}$; secondary outputs were also considered, such as coolant flow rate parameter $\overline{\dot{m}_l}$, combustor temperature parameter $\overline{T}_3^*$ and total thrust parameter $\overline{F}$. Embedded system step response is graphically presented in **Figures 10–13** for both of abovementioned pump driving options (red lines – electric-driven pump with CIS, green lines – engine shaft-driven pump). In terms of coolant flow rate, as shown in Figure, one has obtained for the electric-driven pump with CIS a suitable behavior, very similar to the engine shaft-driven pump, with smaller static error (around 0.38%), but with a little bigger settling time (around 3.2 s). Comparing to the basic engine, speed parameter has a different behavior (see **Figure 11**); for engine's shaft-driven injection pump one can observe a similar speed parameter
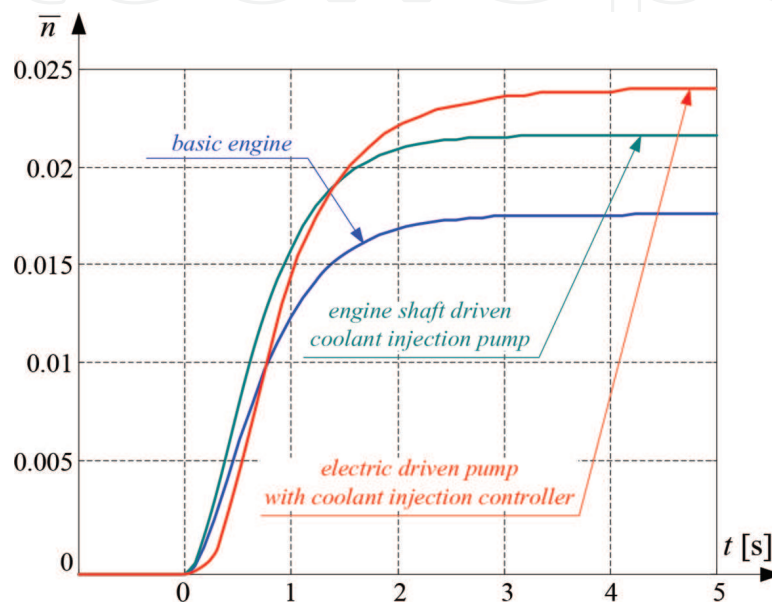

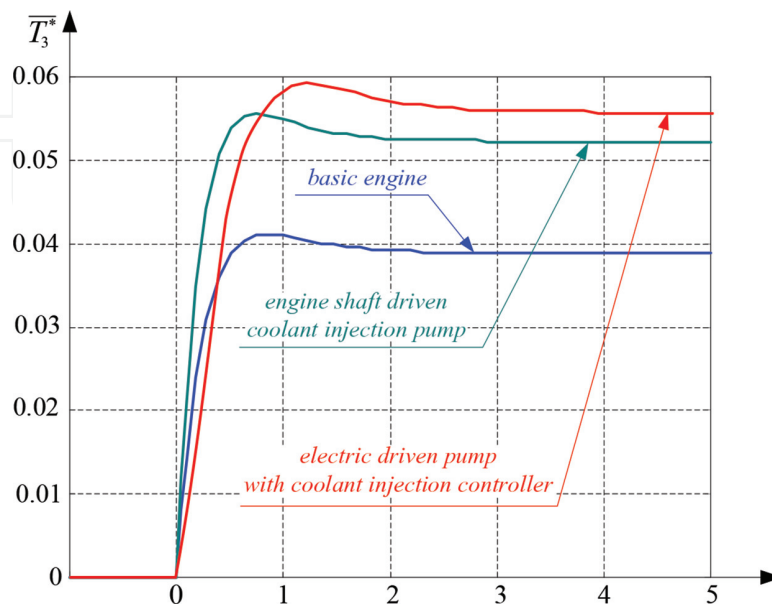**Figure 11.** Comparative step response of engine speed parameters.

Figure 12. Comparative step response of engine combustor temperature parameters.
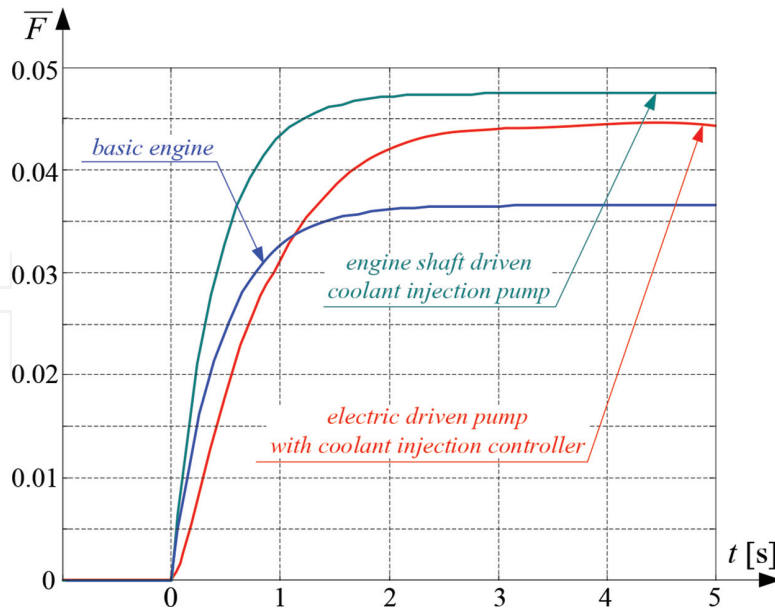
Figure 13. Comparative step response of engine thrust parameters.

behavior, but with bigger static error (2.1% than 1.7%), while for an electric driven-pump assisted by a flow rate controller (with respect to engine's compressor pressure ratio) the static error grows up to 2.35%. Settling times are also growing, from 2.5 to 3.5 s. Engine thrust parameter has a similar behavior (see **Figure 13**), following the engine speed; these are the reasons why the presence of a CIS is mandatory when an electric-driven pump is used.

Combustor temperature parameter's behavior curves show asymptotic stabilization for all studied cases (**Figure 12**), but they all have small initial overshoots; settling time has the same values as for engine speed parameter, but static errors are bigger.

3.5. Engine performance enhancement

Engine thrust expression shows that it depends on working fluid mass flow rate and velocity [1]:

$$F = \dot{m}_g C_5 - \dot{m}_a V = \dot{m}_a (\xi_g C_5 - V), \tag{72}$$

where C_5- exhaust gases velocity, $V-$ airspeed, ξ_g- gases mass fraction. Consequently, when the coolant injection is active, the terms $\dot{m}_g$ and C_5 grow, as shown in [1]: working fluid flow rate grows because of the coolant injection, while exhaust gases velocity grows because of nozzles enthalpy drop increase, as a consequence of turbine's enthalpy drop decrease. As long as the coolant injection is used only during the aircraft take-off maneuver, for short time periods, at low flight speeds and altitudes, the $V-$ term becomes very small, thus negligible, so, when the coolant injection is active, it results

$$F_i = \dot{m}_{a_i} \xi_g C_{5i} = \dot{m}_{a_i} F_{sp_i} = \dot{m}_a \frac{\pi^*_{c_i}}{\pi^*_c} F_{sp_i}, \tag{73}$$

where the $i-$ index refers to the parameters of the engine with coolant injection.
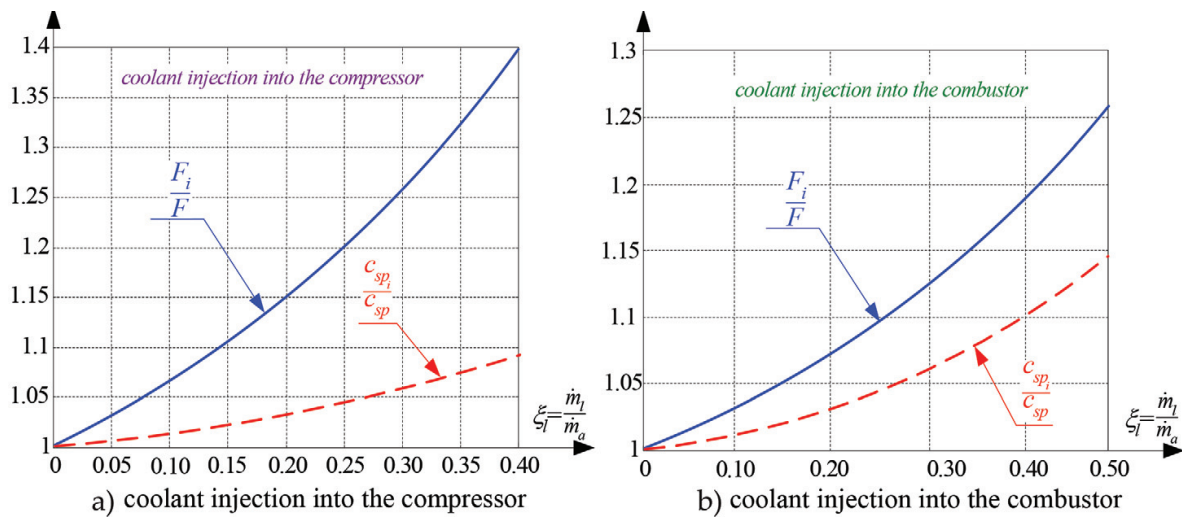
Figure 14. Gas-turbine-engine performance enhancement with respect to coolant fraction.

Figure 14a [1] presents engine's performance (thrust and specific fuel consumption) versus the coolant flow rate fraction, for a neutral injection coolant (such as distilled water). It is worth noting that, even for small injection fractions $\left(\xi_l = \frac{\dot{m}_l}{\dot{m}_a} \leq 0.04 \right)$, engine's thrust increases up to 40%, while the specific fuel consumption has a moderate increase (less than 8%), which makes this method a successful one. Fuel consumption grows because of T_2^*- temperature decrease, which forces engine's fuel control system to take action to restore combustor's T_3^*- temperature and compensate the loss by additional fuel injection.

Combustible coolant injection (e.g., methanol, water–methanol mixture or other combustible fluids mixture) also generates T_2^*- temperature's decreases, but engine's fuel control system has to compensate less fuel than for neutral coolant injection, because of the coolant burning, which brings its own heat into the engine's combustor.

4. Coolant injection into gas-turbine jet-engine's combustor

Coolant injection into the combustor is the other thrust augmentation method; it is accomplished by an injection system positioned in the rear part of the burner can, near its wall (as shown in **Figure 15**). It assures a burner's wall supplementary cooling, and it facilitates the mixing of the vaporized coolant into the burned gases. If the compressor's air mass flow rate exceeds the combustor's necessary (as a consequence of the coolant injection), in order to prevent compressor's stall or surge, as well as an unstable engine operation, the combustor may have an air flow rate by-pass duct (see **Figure 15**), to evacuate the air excess [16].

In most of the practical situations, the injected coolant is distilled (pure) water, which means a neutral fluid, the injection of a combustible fluid being unnecessary, even prohibited [1, 15].

Both the mass flow and the exhaust nozzle burned gases' velocity increase cause thrust augmentation, but only up to 25%, for a coolant flow rate fraction of 5% (see **Figure 14b** [1]),
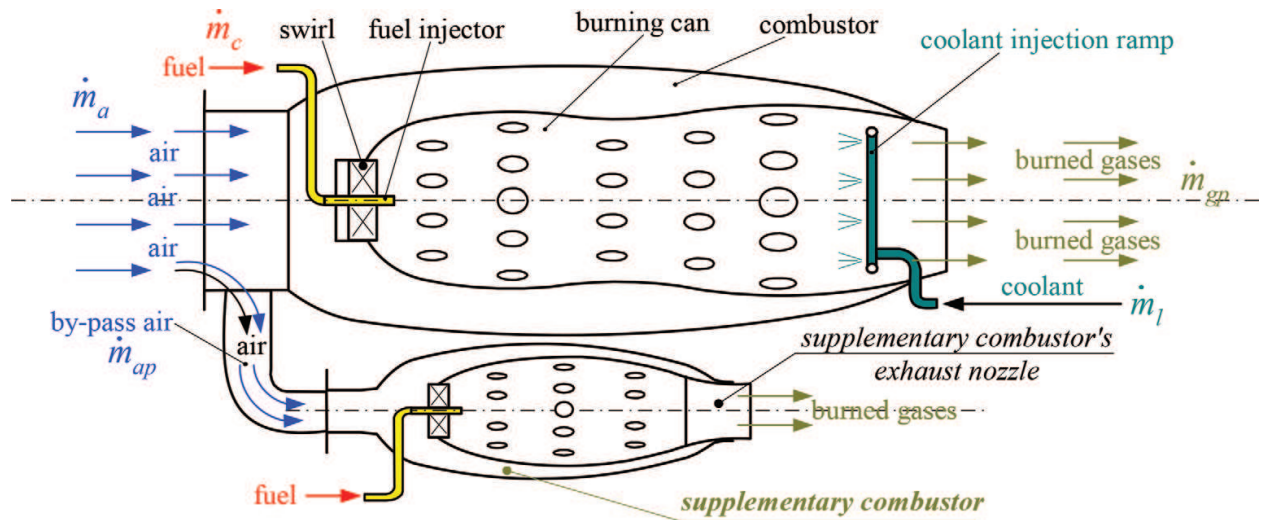
Figure 15. Gas-turbine-engine combustor with coolant injection system, air bypass and supplementary combustor.

while the specific fuel consumption grows moderately, up to 15%, which is an acceptable value considering the thrust increase advantage [1, 15]. One can observe that, comparing to the previous described injection method, this one offers less performance enhancements; however, it offers some other advantages such as constructive simplicity, icing occurring hazard elimination, as well as less blades' corrosion hazard. Method's major disadvantage [1] consists of the fact that an uncontrolled coolant injection could obstruct the burning duct, and consequently, it could impede burning deployment or even extinguish the combustor's flame.

4.1. Thermodynamic and gas-dynamic phenomena

As long as the flow in the turbine of the engine is a critical one, the flow parameter should remain constant, as Eq. (28) states; that means that, for a constant engine operation regime, the hot gases flow rate through engine's turbine must remain constant, no matter if the coolant injection operates or not. Consequently, the greater the coolant mass flow rate $\dot{m}_l$ is, the smaller the compressor air flow $\dot{m}_a$ rate must be and the gases mass flow rate must keep its value; meanwhile, it leads to an important pressure increase (both p_2^* and p_3^*) and engine's regime becomes closer to the stall limit, which is an undesirable phenomenon. Therefore, the burned gases mass flow rate equation should be

$$\dot{m}_g = \dot{m}_{ai} - m_{pr} + \dot{m}_l + \dot{m}_c, \tag{74}$$

where $\dot{m}_{ai}$ is the new compressor air mass flow rate value, smaller than the initial value $\dot{m}_a$.

In order to avoid unstable regimes and to keep the flow rate balance, even when the coolant injection system operates, one has to extract the surplus air mass flow rate $\dot{m}_{ap}$ and to by-pass it before the engine's combustor; thus, the mass flow rate equation becomes

$$\dot{m}_{gi} = \dot{m}_a - m_{pr} - \dot{m}_{ap} + \dot{m}_l + \dot{m}_c \equiv \dot{m}_g. \tag{75}$$

This surplus air mass flow rate should not be a loss for the engine; it may be used into another external, independent, supplementary combustor; it has its own exhaust nozzle and it is

supplied with fuel by an additional fuel system. One has obtained a supplementary propulsion system (a complementary thrust augmentation method), which offers its own thrust, added to the basic engine's thrust.

4.2. Engine's mathematical model and quality study

One may observe that, comparing to the basic engine's model, the equations which change are the same as for the previous described method, meaning that both mass flow rate equation and combustor's equation are to be modified. Therefore, formally, mathematical model's equations are the same in subSection 3.2.1, but the coefficients' values should be calculated considering the new values of the air flow rate, injection fraction, temperature(s) and pressure (s), as presented in [16].

A quantitative study was performed and described in [16], based on a VK-1A-type jet engine, but a completion of the study, concerning the possibility of the supplementary combustor adding (with its own fuel control system), was performed and presented in [15].

One has studied the common case of a neutral coolant (distilled water) injection into the rear part of the engine's combustor [16], which gave as results for outputs as follows:

$$\overline{n}(s) = \frac{1.411\,\overline{\dot{m}_c}(s) - 0.167\overline{\dot{m}_l}(s)}{2.3761\,s + 4.817}, \tag{76}$$

$$\overline{T_3^*}(s) = \frac{(1.8732\,s + 2.847)\overline{\dot{m}_c}(s) - (0.0823\,s + 0.0764)\overline{\dot{m}_l}(s)}{2.3761\,s + 4.817}, \tag{77}$$

$$\overline{F}(s) = \frac{(1.583\,s + 5.167)\overline{\dot{m}_c}(s) - (0.0834\,s + 0.4725)\overline{\dot{m}_l}(s)}{2.3761\,s + 4.817}. \tag{78}$$

Engine step responses are depicted in **Figures 16–18**; one has realized a comparison between the basic engine behavior (as described in [5]) and the engine with thrust augmentation systems, as described by the Eqs. (44)–(46), respectively by the Eqs. (76)–(78).

No matter the coolant injection method, engine's speed parameter is less influenced, as the small $n-$ parameter increases as shown in **Figure 16**. Coolant injection into the combustor brings less speed changes than coolant injection into the compressor, but with a little longer settling time (about 0.5 s); that means that, comparing to the basic engine, it has become a little bit slower. From the response time point of view, coolant injection into the compressor makes the engine a little, but insignificant, faster.

In terms of temperature's parameter behavior (see **Figure 17**), it is noteworthy that, no matter the coolant injection method, it has the same trend as for the basic engine, but bigger initial overshoots, following the initial supplementary fuel injection, meant to restore combustor's T_3^*- temperature.

Engine's thrust parameter behavior (see **Figure 18**) has moderate increases, no matter the injection method. A cause should be the coolant nature, a neutral coolant could not assure additional heat into engine's combustor, thus the thrust augmentation is realized only by the
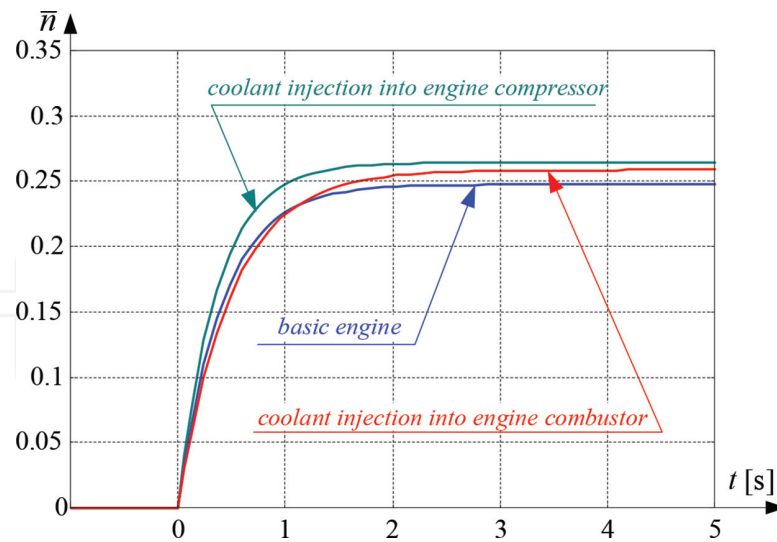
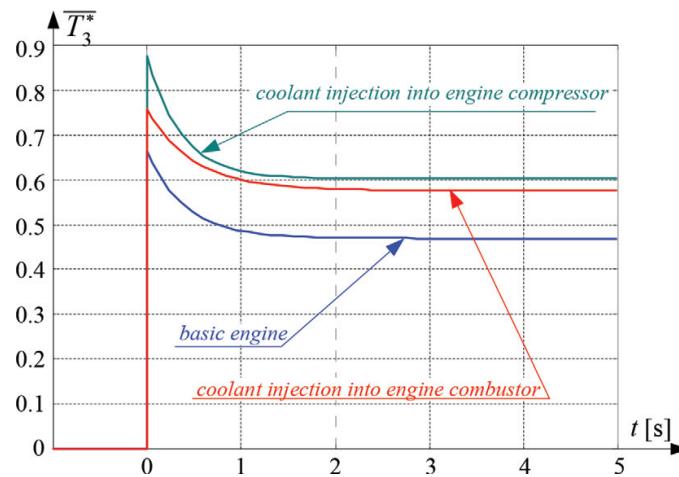**Figure 16.** Comparative step response of engine speed parameters.

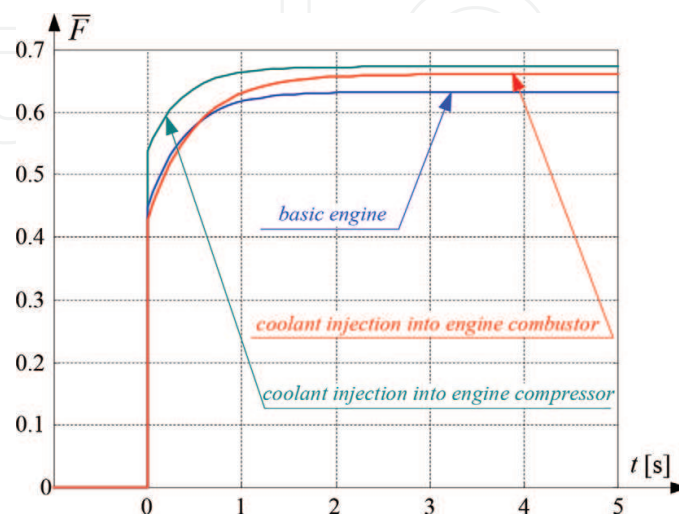**Figure 17.** Comparative step response of engine combustor temperature parameters.

Figure 18. Comparative step response of engine thrust parameters.

airflow and gases velocity increases. Coolant injection into the compressor assures a more rapid thrust increase, while growing percentage is near the same for both described injection methods.

5. Conclusions

Between the nowadays issues of aircraft engine design and manufacturing, thrust augmentation, along with operational safety, are some of the most important ones; if the classic over-boost method by afterburning is impossible or too expensive to be adapted (e.g., for turbo-props or twin-jet turbofans), coolant injection method(s) shall be used, for temporarily thrust increase. As a result, when the coolant injection system is operational, engine performance temporarily improves, which means a spectacular thrust increase (from 25–40%) and moderate specific fuel consumption increase (8% up to 15%), depending on the used injection method [1].

Comparing to the afterburning, which assures spectacular thrust augmentation (up to 65% for single jets, up to 100% for twin jets) but also significant specific fuel consumption increases (from 50% up to 110%) [1, 2, 4], coolant injection is a less expensive augmentation method, even only from the fuel consumption point of view, offering acceptable performance enhancement and less design and constructive issues. In fact, the afterburning is the choice for supersonic aircraft engines; today it is widespread among combat aircraft engines (for supersonic fighters) and practically misses from transport aircraft, no supersonic jetliner being actually in service. Moreover, supersonic aircraft engines uses the afterburning at low regimes even for cruise flights, which makes of it another propulsion system, with its own controller, but using the same fuel and the working fluid provided by the basic jet engine. Obviously, afterburning's implementation involves significant design effort and manufacturing expenses, so its usage is entitled by the specific necessity of the engine, according to aircraft destination(s) and mission(s).

Coolant injection remains a suitable alternative, meant to equip gas-turbine engines without afterburning adapting possibilities, but who need temporarily thrust increasing, especially for take-off, when total thrust is affected by excessive atmospheric conditions, or/and aircraft's payload has high values, close to its maximum limit. Injection system design, manufacturing and implementation is significantly less expensive than for the afterburning; however, the coolant is, obviously, a different fluid than the fuel, which should be stored in a special tank; consequently, aircraft or/and engine architecture must change and, because of coolant stored additional mass, aircraft fuel storage capacity must diminish, which reduces aircraft maximum flight range. These are reasons for optimization studies, concerning the coolant injection mass flow rate, injection strategy and duration and storage tank's capacity.

One has studied aircraft jet engine with coolant injection as a controlled object, from system's theory point of view, identifying its input(s) and output(s), as well as some control architectures implementation possibilities. Both coolant injection methods' implementation involves thermo- and gas-dynamics engine changes, concerning the air and burned gases mass flow rate, as well as the heat exchange balance. Consequently, engine's model as controlled object changes too, as well as its stability and quality. One has highlighted the mathematical model modifications of the engine with coolant injection, starting from basic engine's model, then

describing engine's control schemes by block diagram with transfer functions; based on it, some simulations were performed and comparative studies of stability and quality were realized.

Studies have proven that small-to-moderate used injection fraction values and suitable control methods and architectures keep the engine as a stable system; static errors have grown, as well as settling times, but still remaining in the acceptable rank of values. Based on the emphasized engine's changes, embedded engine control systems may be designed and optimized, even during the early design stages (pre-design).

Coolant injection into the compressor, in spite of its icing hazard and other disadvantages, if assisted by a properly designed pump and controller, is an excellent thrust increase method for take-off (especially for turboprops), better rated than coolant injection into the combustor (considered suitable for turbojets and higher altitudes flights) [1, 2].

Studies were performed for a single-spool single-jet engine at sea-level atmospheric conditions and take-off air speed, but can be extended for other engine-types (two-spool, twin jets, turbofans) or for other altitudes and cruise speeds.

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