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Monitoring Changes in Operational Scenarios via Data Fusion in Sensor Networks

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1. Introduction

Sensor networks are designed to satisfy specific signal processing objectives, such as target recognition and identification, industrial quality control, community health sensing, multimedia systems & applications, etc. The satisfaction of each objective first requires careful stochastic modeling of the environment and deployment of the pertinent performance criterion; a optimal or best centralized (or coherent) signal processing procedure can be then determined, whose rate of convergence to the deployed performance criterion will be predictable. The centralized procedure utilizes unconstrained raw data, it is performed by a fusion center and attains the best possible convergence rate. In the distributed environment of sensor networks, however, the transmission of raw data to the fusion center induces a high communication cost (both in transmission power and bandwidth), where raw data are collected by local sensors. To reduce the communication cost, raw data are preprocessed by the local sensors. Reduced pertinent information is subsequently transmitted by the local sensors to possibly first cluster heads which in turn process the received information and transmit further reduced information to a fusion center. The fusion center executes the final steps of the now termed *decentralized* (or non-coherent) procedure for the satisfaction of the network objective. The arising issues here are: (a) The performance versus communication cost tradeoff arising when the centralized and the decentralized procedures are compared and (b) the effect of feedback on the convergence rate of the decentralized procedures.

We select as signal processing objective the monitoring of changes in operational scenarios. Such monitoring has numerous applications, including detection of anomalies in community health and industrial quality control. We propose, analyze and evaluate a sequential monitoring algorithm, including convergence, power and false alarm, as well as performance comparison with the corresponding centralized system.

The problem of detecting rapidly and accurately a change in the stochastic process that generates observation data has long history and numerous applications. The applications include industrial quality control, detection of edges in images, network quality control and traffic monitoring in data and sensor networks. The search for algorithms that detect changes in the underlying process which generates observation data has taken two distinct directions: Bayesian and non Bayesian. Since the assumed knowledge of prior probabilities in the Bayesian approaches is considered here unreasonable and unrealistic, we exclusively focus on non Bayesian solutions to the problem.

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Motivated by the application of industrial quality control, (Page 1954) first proposed a sequential algorithm to detect a possible change from a given stationary and memoryless process to another given such process, where it was only assumed that such a change may occur randomly in time. (Lorden 1971) analyzed Page's algorithm and proved its asymptotic optimality in the sense of stopping time. (Bansal et al 1986) extended Page's algorithm for stationary and ergodic processes with memory and proved optimality in the sense of asymptotic expected stopping time. (Bansal et al 1989) also "robustified" the algorithm for resistance to data outliers. Burrell et al (1998) extended the algorithm in (Bansal et al 1986) to sequentially detect reoccurring possible changes within a given set of stochastic processes, and analyzed asymptotic performance. (Lai 1995) considered a scenario similar to that in (Burrell et al 1998). (Veeravalli et al 1993) adopted the algorithm of (Page 1954) and that of (Bansal et al 1986) to analyze the effect of a fusion center processing outputs from a set of distributed-data collecting sensors. Some modification of the latter scenario where considered by (Mei 2005). (Burrell et al 2004) considered a distributed modification of the algorithm presented in (Burrell et al 1998), to monitor traffic in sensor networks, where partial decisions from neighboring sensors are incorporated into the sequential algorithmic processing at each sensor.

In this chapter, we consider the existence of a fusion center which processes partial decisions by distributed local sensors, to make the final decision as to the change of the underlying data generating process. The processes model adopted is that of (Burrell et al 1998). Feedback from the fusion center to the sensors is implicit and utilized in the algorithmic steps of the overall system.

The chapter is organized as follows: In Section 2, the system model is presented. In Section 3, preliminaries about the basic algorithms deployed by a centralized system are presented. In Section 4, the algorithmic system is presented and analyzed and the comparison of its performance with that of the centralized system is discussed. In Section 5, numerical evaluation scenarios are included. In Section 6, some conclusions are drawn.

2. System model

We consider discrete-time processes and we let time start at zero. Let x_1^n denote the sequence $x_1, \dots, x_n$ of n observations after time zero and let $\{\mu_i ; i = 0, 1, \dots, m-1\}$ denote the measures of m distinct and parametrically defined stochastic processes. The assumptions in the problem we consider are then as follows: the observation sequence is initially generated by the process μ_0, while it is possible that a shift to any one of the m-1 processes $\mu_i ; i = 1, \dots, m-1$ may occur at any point in time, where if a $\mu_0 \to \mu_i$ shift occurs, then the process μ_i remains active thereafter. The objective is to detect the occurrence of a $\mu_0 \to \mu_i$ shift as accurately and as timely as possible, including the detection of the process μ_i which μ_0 changed to. Let us denote by f_i ; $i = 0,1, .., m-1$ density or probability functions induced by the processes $\mu_i ; i = 0, 1, \dots, m-1$ and let us denote by $f_i(x_n \mid x_1^{n-1}) ; i = 0,1,\dots,m-1$ conditional density or probability functions at x_n, given the sequence x_1^{n-1}.

In a centralized system, the problem objective is satisfied by a single processor which collects all the data and processes them sequentially via the algorithm in (Burrell et al 1998). Here, a decentralized system is considered, however, where M physically distributed processors collect local data, in conjunction with possible feedbacks from a fusion center. (See Figure 1). The M sensors are identical, placed in identical stochastic environments; that is, possible changes of the local data generating processes occur simultaneously at all sensor sites. Each sensor deploys a sequential algorithm to detect a possible $\mu_0 \to \mu_i$, i = 0,1, .., m-1 change and transmits its local decisions to the fusion center. The fusion center makes the final decision as to a possible change in the data generating process, while it may be implicitly notifying the sensors as to its decision status at all times.

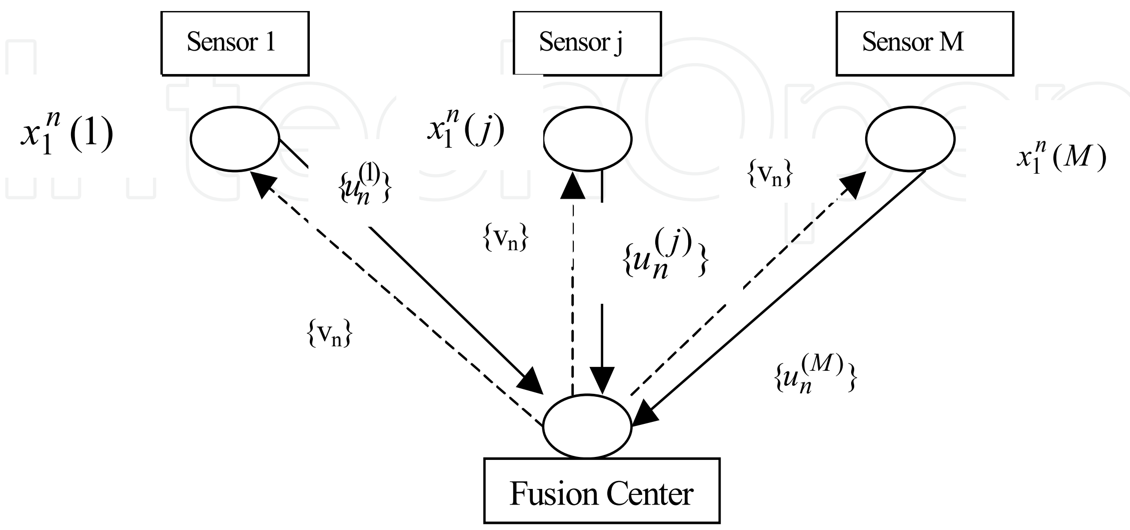

Fig. 1. Fusion Center

Let $x_1^n(i)$ denote a n-dimensional local to sensor i data sequence. Let $u_n^{(j)}$ denote the input of sensor j to the fusion center at time n. Note that sensor j does not transmit anything to the fusion center, until it makes a decision; if it makes a decision in favor of a $\mu_0 \to \mu_i$ shift at time n, it then transmits $u_n^{(j)}$ = i ; which also implies $u_k^{(j)}$ = i ; for all k > n ; before n the fusion center simply deduces that $u_l^{(j)} = 0$, $l \leq$ n-1, which means that senor j has not yet decided that a change from μ_0 has occurred. Let v_n denote the feedback of the fusion center to the sensors, at time n. Then, $v_n = 0$; for all n before a shift decision is made by the fusion center. Note, that the fusion center does not need to transmit any feedback to the sensors before it makes its shift decision: the sensors simply deduce then that $v_n = 0$ during such periods. At the time when the fusion center makes its decision, it simply "orders" the sensors to stop their local processing.

3. Preliminaries

Let us assume that all the processes μ_i ; i = 0, 1, … , m-1 are ergodic and stationary, where

$$L_{i,n} \stackrel{\Delta}{=} n^{-1} \log \frac{f_i(w_1^n)}{f_0(w_1^n)}$$

$$L_{0,n} \stackrel{\Delta}{=} n^{-1} \log \frac{f_0(w_1^n)}{f_i(w_1^n)}$$

$$I_{i,0} \stackrel{\Delta}{=} \lim_{n\to\infty} L_{i,n}$$

$$I_{0,i} \stackrel{\Delta}{=} \lim_{n\to\infty} L_{0,n}$$

and

$$p_{i,n}(v) \stackrel{\Delta}{=} P_{\mu_1}(L_{i,n} < v)$$

$$p_{0,n}(v') \stackrel{\Delta}{=} P_{\mu_0}(L_{0,n} < v')$$

and where the following conditions are satisfied, for all i = 1, …, m-1:
I_{i0} and I_{0i} exist (I_{i0}, I_{0i} $< \infty$) and

$$\mathrm{I}_{i0} = \mathrm{E}_{\mu_i}\{\mathrm{I}_{i0}\} \qquad \text{a.s.} \quad (\mathrm{P}_{\mu_i})$$
$$\mathrm{I}_{0i} = \mathrm{E}_{\mu_0}\{\mathrm{I}_{0i}\} \qquad \text{a.s.} \quad (\mathrm{P}_{\mu_0})$$

For $v \in (0, \mathrm{I}_{i0})$

$$\lim_{n\to\infty} n \; p_{in}(v) = 0 \quad \text{and} \quad \sum_{n\geq 1} p_{i,n} < \infty \qquad \text{(A)}$$

For $v' \in (0, \mathrm{I}_{0i})$

$$\lim_{n\to\infty} n \; p_{0n}(v') = 0 \quad \text{and} \quad \sum_{n\geq 1} p_{0,n} < \infty \qquad \text{(A')}$$

Then, denoting by x infinite generated sequences, we have from (Bansal et al 1986) the following results, regarding the centralized detection of a μ_0 to μ_i shift :
Defining the stopping variable

$$N_{0i}^{\delta}(x) \stackrel{\Delta}{=} \inf\left\{ n : T_n^{0i}(x_1^n) \geq \delta \right\}$$

for

$$\mathrm{T}_n^{0\,i}(x_1^n) \stackrel{\Delta}{=} \max_{1\leq k\leq n+1}\left(\sum_{l=k}^{n} \log \frac{f_i(x_l \mid x_k^{l-1})}{f_0(x_l \mid x_k^{l-1})} \right)$$

we have that,

$$\mathrm{E}_{\mu_0}\left\{ N_{0i}^{\delta}(x) \right\} > \frac{\delta}{2} \quad \text{as} \quad \delta \to \infty$$

$$\bar{\mathrm{E}}_{\mu_i}\left\{ N_{0i}^{\delta}(x) \right\} \sim \frac{\log \delta}{\mathrm{E}_{\mu_i}\left\{ I_{i0} \right\}} \quad \text{as} \quad \delta \to \infty \qquad (1)$$

If the μ_i ; i = 0,1, ….,m-1 stochastic processes possess in addition Lai's (1977) mixing conditions, then the stopping variable $\left\{ N_{0i}^{\delta}(x) \right\}$ can be closely approximated by the following stopping variable $N'_{\delta}(x)$ which possesses sequential properties while $N_{\delta}(x)$ does not.

$$N_{0i}^{'\delta}(x) = \max_{0\le k\le n}\left(\sum_{l=k+1}^{n}\log\frac{f_i(x_l \mid x_1^{l-1})}{f_0(x_l \mid x_1^{l-1})}\right)$$

$$T_{n+1}^{'0i}(x_1^{n+1}) = \max\left(0\,,T_{n+1}^{'0i}(x_1^{n}) + \log\frac{f_i(x_{n+1} \mid x_1^{n})}{f_0(x_{n+1} \mid x_1^{n})}\right) \tag{2}$$

where $\log\frac{f_i(x_{n+1} \mid x_1^{n})}{f_0(x_{n+1} \mid x_1^{n})}$ represents the algorithmic updating step at time n+1.

Denoting I_{ij} as I_{0i}, when μ_0 and μ_i are respectively substituted by μ_i and μ_j, and assuming that Lai's (1977) mixing conditions hold, we have from Burrell et al (1998):

$$\text{As } \delta \to \infty, \underset{\substack{j=0,1,..,m-1\\ i=1,...,m-1}}{E\{N_{0i}^{'\delta}(x) \mid \mu_j\}} : \begin{cases} \sim \left[I_{0j} - I_{i\ j}\right]^{-1}\log\delta & ;\text{if } I_{0j} \ge I_{i\ j} \\ \ge 2^{-1}\delta & ;\text{if } I_{0j} < I_{i\ j} \end{cases} \tag{3}$$

$$\text{As } \delta \to \infty, E\left\{N_{0j}^{'\delta}(x)|\mu_j\right\} \sim I_{0j}^{-1}\log\delta \;:\begin{cases} < E\left\{N_{0i}^{'\delta}(x)|\mu_j\right\} \sim \left[I_{0j} - I_{i\ j}\right]^{-1}\log\delta\ ;\forall i\ne j: I_{0j} > I_{ij} \\ < E\left\{N_{0i}^{'\delta}(x) \mid \mu_j\right\} \ge 2^{-1}\delta;\forall i\ne j: I_{0j} < I_{ij} \end{cases} \tag{4}$$

Expressions (1), (3) and (4) represent the asymptotic performance of the centralized system, where m-1 parallel algorithms as in (2) operate, with a common threshold δ, and where the first algorithm to cross this threshold determines the system decision: if the $\mu_0 \to \mu_k$ algorithm first crosses the threshold, then a $\mu_0 \to \mu_k$ shift is decided and the algorithmic system stops.

4. The algorithmic system

We assume identical sensors collecting mutually independent local data. We denote by $x_n(i)$ the n^{th} local datum at the i^{th} sensor. We denote by $x_1^{n-1}(i)$ the $(n-1)^{th}$ dimensional data sequence collected locally at senor i from time 1 to time n-1. The algorithms deployed by the sensors are identical, and utilize conditional densities or distributions. In addition to its local data, each sensor also utilizes the implicit fusion centers feedbacks $\{v_k = 0\}_k$ throughout its operation. Let $f_j\left(x_n(i), v_n = 0 \mid x_1^{n-1}(i), \{v_k = 0\}_{1\le k\le n-1}\right)$ denote the conditional density or distribution of the data at sensors i, given that the acting data process is μ_j. It is clearly seen that the $\{V_n\}$ sequence is a Markov Chain and that the data sequence $X_1^n(i)$ is independent of $\{V_n\}$. We can thus write,

$$f_j\left(x_n(i), v_n = 0 \mid x_1^{n-1}(i), \{v_k = 0\}_{1\le k\le n-1}\right) =$$

$$= f_j\left(\, v_n = 0 \mid v_{n-1} = 0, x_1^{n}(i)) \,\right) f_j\left(x_n(i) \mid x_1^{n-1}(i)\right) \tag{5}$$

We observe that the $\{v_k\}$ sequence is based only on the $\left\{\, u_k^{(i)} \right\}$ sequences of the sensor outputs rather than the data sequences collected by the sensors. We thus substitute

$f_{\mathrm{j}}\left(v_n = 0 | v_{n-1} = 0, x_1^n(i)) \right)$ by $f_{\mathrm{j}}\left(v_n = 0 | v_{n-1} = 0, u_n^{(i)} = 0 \right)$. Since the sensors are identical we drop the index i and we write,

$$f_{\mathrm{j}}\left(x_n, v_n = 0 | x_1^{n-1}, \{v_k\}_{1\le k\le n-1}\right) =$$

$$f_{\mathrm{j}}\left(v_n = 0 | v_{n-1}, u_n = 0 \right) f_{\mathrm{j}}\left(x_n | x_1^{n-1}\right) =$$

$$\frac{f_{\mathrm{j}}\left(v_n = 0 | u_n = 0 \right)}{f_{\mathrm{j}}\left(v_{n-1} = 0 | u_n = 0 \right)} f_{\mathrm{j}}\left(x_n | x_1^{n-1}\right) \tag{6}$$

$$\log \frac{f_{\mathrm{j}}\left(x_n, v_n = 0 | x_1^{n-1}, \{v_k\}_{1\le k\le n-1}\right)}{f_0\left(x_n, v_n = 0 | x_1^{n-1}, \{v_k\}_{1\le k\le n-1}\right)} =$$

$$\log \frac{f_{\mathrm{j}}\left(x_n | x_1^{n-1}\right)}{f_0\left(x_n | x_1^{n-1}\right)} + \log \frac{f_{\mathrm{j}}\left(v_n = 0 | u_n = 0 \right)}{f_0\left(v_n = 0 | u_n = 0 \right)} - \log \frac{f_{\mathrm{j}}\left(v_{n-1} = 0 | u_n = 0 \right)}{f_0\left(v_{n-1} = 0 | u_n = 0 \right)} \tag{7}$$

The expression in (7) represents the updating step of the $\mu_0 \rightarrow \mu_j$ shift detecting algorithm in (2) at time n, at any one of the M sensors, where x_1^n are the locally collected data. As compared to the centralized scheme, the terms

$$\log \frac{f_{\mathrm{j}}\left(v_n = 0 | u_n = 0 \right)}{f_0\left(v_n = 0 | u_n = 0 \right)} \quad \text{and} \quad -\log \frac{f_{\mathrm{j}}\left(v_{n-1} = 0 | u_n = 0 \right)}{f_0\left(v_{n-1} = 0 | u_n = 0 \right)}$$

are added to the updating step. Due to the latter terms, the algorithmic systems across the different sensors are mutually dependent, while the locally collected data are mutually independent, instead.

At the Fusion Center, m-1 parallel algorithms are operating, with a common threshold T, each monitoring a $\mu_0 \rightarrow \mu_j$ possible shift, for j = 1, … , m-1. These algorithms utilize the vectors $\bar{U}_n = \left[u_n^{(1)}, \ldots, u_n^{(m)} \right]^T$, where $u_n^{(i)}$ is the output of sensor i at time n. If at time n the sensor has not made a decision yet, then $u_n^{(i)} = 0$. If at time n the senor decides in favor of the process shift $\mu_0 \rightarrow \mu_j$, then $u_n^{(i)} = j$, and this value remains unchanged for all times after n. Due to the above discussed evolution of the $\left\{ u_n^{(i)} \right\}$ outputs, it is clear that the process $\left\{ \bar{U}_n \right\}$ is a Markov Chain. Thus, $f_j\left(\bar{U}_n | \bar{U}_k, 1 \le k \le n-1 \right) = f_j\left(\bar{U}_n | \bar{U}_{n-1} \right)$, where, in addition, the conditional probability $f_j\left(\bar{U}_n | \bar{U}_{n-1} \right)$ is determined solely by the transitions of the zero-valued components of $\bar{U}_{n-1}$. In fact, due to the identical nature of the sensors, $f_j\left(\bar{U}_n | \bar{U}_{n-1} \right)$ is determined by the number of sensors whose algorithms are still running at time n-1, and among them, from the numbers which transition to the states u_n = 1, … , m-1, at time n. For sensor i, let us denote the variable $d_n^{(i)}$ as, $d_n^{(i)} = 0$ if $u_n^{(i)} = 0$ and $d_n^{(i)} = 1$ if $u_n^{(i)} = 1, 2, \ldots, m-1$.

Let $\bar{D}_n$ be the vector whose components are $d_n^{(i)}; i=1,2,...,M$. Then, we can first write $f_j\left(\bar{U}_n|\bar{U}_{n-1}\right) = f_j\left(\bar{U}_n|\bar{D}_{n-1}\right) = f_j\left(\bar{U}_n|\bar{D}_n,\bar{D}_{n-1}\right)\; f_j\left(\bar{D}_n|\bar{D}_{n-1}\right)$ and,

$$\log\frac{f_j\left(\bar{U}_n|\bar{U}_{n-1}\right)}{f_0\left(\bar{U}_n|\bar{U}_{n-1}\right)} = \log\frac{f_j\left(\bar{U}_n|\bar{D}_n,\bar{D}_{n-1}\right)}{f_0\left(\bar{U}_n|\bar{D}_n,\bar{D}_{n-1}\right)} + \log\frac{f_j\left(\bar{D}_n|\bar{D}_{n-1}\right)}{f_0\left(\bar{D}_n|\bar{D}_{n-1}\right)} \tag{8}$$

From the discussion above, it is evident that the sufficient statistics for the term $\log\frac{f_j\left(\bar{D}_n|\bar{D}_{n-1}\right)}{f_0\left(\bar{D}_n|\bar{D}_{n-1}\right)}$ are $M^{-1}\sum_{i=1}^{M} d_n^{(i)}(1-d_{n-1}^{(i)})$ and $M^{-1}\sum_{i=1}^{n}(1-d_n^{(i)})(1-d_{n-1}^{(i)})$. As to the conditional probability $f_j\left(\bar{U}_n|\bar{D}_n,\bar{D}_{n-1}\right)$, j=0,1, … , m-1, it represents the probability of the number of sensors deciding in favor of the $\mu_0 \to \mu_k; k=1,..,m-1$ shifts at time n, given that their algorithmic systems stop at time n; this probability equals 1 if m=2, since then, $\bar{D}_n = \bar{U}_n$. The sufficient statistics for the probability $f_j\left(\bar{U}_n|\bar{D}_n,\bar{D}_{n-1}\right)$ and the term log $f_j\left(\bar{U}_n|\bar{D}_n,\bar{D}_{n-1}\right) / f_0\left(\bar{U}_n|\bar{D}_n,\bar{D}_{n-1}\right)$ in (8) are, $M^{-1}\sum_{i=1}^{M} d_n^{(i)}(1-d_{n-1}^{(i)})$ and $M^{-1}\sum_{i=1}^{M} d_n^{(i)}(1-d_{n-1}^{(i)}) \prod_{\substack{1\le k\le m-1\\ k\ne l}} (u_n^{(i)}-k)\,; 1\le l\le m$. The expression in (8) represents the updating step of the $\mu_0 \to \mu_j$ shift detecting algorithm in (1) and (2) at time n, as implemented by the fusion center. Let now us denote,

$\underset{j=1,\dots,m-1}{p_n^j}$: The probability that the algorithmic system at a sensor stops at time n (the common threshold is first crossed at time n), given that the data generating process is μ_j.

$\underset{k,j;1,\dots,m-1}{p_{nk}^j}$: The probability that, given the data generating process μ_j, the algorithmic system at a senor stops at time n, where the $\mu_0 \to \mu_k$ shift detecting algorithm is the one that first crosses the threshold at n.

$\underset{j=1,\dots,m-1}{\beta_n^j}$: The probability that the algorithmic system at a sensor stops before or at time n, given that the data generating process is μ_j.

α_n: The probability that the algorithmic system at a senor stops before or at time n, given that the data generating process is μ_0.

We note that $p_n^j = p_{nk}^j$; if m=2. Also, $p_n^j = \beta_n^j - \beta_{n-1}^j$; j =1, …, m-1 and $p_n^0 = \alpha_n - \alpha_{n-1}$.

We now express a theorem, whose proof is in the Appendix.

<u>Theorem 1:</u>

Let the probabilities $f_j\left(v_n = 0 | u_n = 0\right); j=1,...,m-1$ and $f_0\left(v_n = 0 | u_n = 0\right)$ be such that there exist constants n_0, c_j ; j=1, …, m-1 and c_0 such that

$$f_j\left(v_n=0\,|\,u_n=0\right)\;/\;f_j\left(v_{n-1}=0\,|\,u_{n-1}=0\right)=\mathrm{c_j}\,;\ \mathrm{j=1,\ \ldots,\ m\text{-}1},\ \forall\,\mathrm{n>n_0}$$

$$f_0\left(v_n=0\,|\,u_n=0\right)\;/\;f_0\left(v_{n-1}=0\,|\,u_{n-1}=0\right)=\mathrm{co}\,;\ \forall\,\mathrm{n>n_0} \tag{9}$$

Then, the algorithmic systems across different scenarios are mutually independent for all $n > n_0$. In addition, if the μ_j ; j=0,1, …,m-1 processes are stationary, ergodic and satisfying conditions (A) and (A′) in Section III, as well as (Lai's 1977) mixing conditions, then the performances of the sensors algorithmic systems are asymptotically ($n > n_0$) identical and as in (3) and (4). Finally, the updating step of the $\mu_0 \to \mu_j$ shift detecting algorithm at the fusion center in (8) takes then the following form :

$$S_{FS}^{nj} \stackrel{\Delta}{=} U(m-2)\sum_{l=1}^{m-1} M^{-1}\sum_{i=1}^{M} d_n^{(i)}(1-d_{n-1}^{(i)}) \prod_{\substack{1\le k\le m-1\\ k\ne l}} (u_n^{(i)}-k)\log\frac{\mathrm{p}_{nl}^{j}/\mathrm{p}_n^{j}}{\mathrm{p}_{nl}^{0}/\mathrm{p}_n^{0}}$$

$$+M^{-1}\sum_{i=1}^{M} d_n^{(i)}(1-d_{n-1}^{(i)})\log\frac{\mathrm{p}_n^{j}\,|\,(1-\beta_{n-1}^{j})}{\mathrm{p}_n^{0}\,|\,(1-\alpha_{n-1})}+M^{-1}\sum_{i=1}^{M}(1-d_n^{(i)})(1-d_{n-1}^{(i)})\log\frac{(1-\beta_n^{j})\,|\,(1-\beta_{n-1}^{j})}{(1-\alpha_n)\,|\,(1-\alpha_{n-1})}\,;\ \mathrm{n>n_0} \tag{10}$$

; where $\mathrm{U(n)} = \begin{cases} 1\,;\ \mathrm{n} > 0 \\ 0\,;\ \mathrm{n} \le 0 \end{cases}$ □

The expected value of the updating step in (10), subject to the data generating process being μ_i is found by straight substitution as follows:

$$E\left\{S_{FS}^{nj}\,|\,\mu_i\right\}_{i=1,\ldots,m-1} = U(m-2)\sum_{l=1}^{m-1}\frac{p_{nl}^{i}}{p_n^{i}}\log\frac{p_{nl}^{j}\,|\,p_n^{j}}{p_{nl}^{0}\,|\,p_n^{0}}+\frac{p_n^{i}}{1-\beta_{n-1}^{i}}\log\frac{p_n^{j}\,|\,(1-\beta_{n-1}^{j})}{p_n^{o}\,|\,(1-\alpha_{n-1})}+$$

$$+\frac{(1-\beta_n^{i})}{(1-\beta_{n-1}^{i})}\log\frac{(1-\beta_n^{j})\,|\,(1-\beta_{n-1}^{j})}{(1-\alpha_n)\,|\,(1-\alpha_{n-1})}\quad;\quad n>n_0 \tag{11}$$

$$E\left\{S_{FS}^{nj}\,|\,\mu_0\right\} = U(m-2)\sum_{l=1}^{m-1}\frac{p_{nl}^{0}}{p_n^{0}}\log\frac{p_{nl}^{j}\,|\,p_n^{j}}{p_{nl}^{0}\,|\,p_n^{0}}+\frac{p_n^{0}}{(1-\alpha_{n-1})}\log\frac{p_n^{j}\,|\,(1-\beta_{n-1}^{j})}{p_n^{o}\,|\,(1-\alpha_{n-1})}+$$

$$+\frac{(1-\alpha_n)}{(1-\alpha_{n-1})}\log\frac{(1-\beta_n^{j})\,|\,(1-\beta_{n-1}^{j})}{(1-\alpha_n)\,|\,(1-\alpha_{n-1})}\quad;\quad n>n_0 \tag{12}$$

For asymptotically many sensors ($M \to \infty$), the updating step in (10) converges to the expected values in (11) and (12), depending on the acting data generating process. Let $K(p\,|\,q)$ denote the Kullback-Leibler information number of a Bernoulli trial with parameter p, with respect to a Bernoulli trial with parameter q. Let $K(\{p_l\}_{1\le l\le m-1}\,|\{q_l\}_{1\le l\le m-1})$ denote the Kullback-Leibler number of a distribution with probabilities $\{p_l\}_{1\le l\le m-1}$, with respect to a distribution with probabilities $\{q_l\}_{1\le l\le m-1}$. Then, from expressions (11) and (12), we easily deduce the expressions (13) and (14) below.

$$E\left\{S_{FS}^{nj} \mid \mu_i\right\}_{i=1,\dots,m-1} = U(m-2)[\mathrm{K}\left(\left\{\frac{\mathrm{p}_{nl}^{i}}{p_n^{i}}\right\}_{1\le l\le m-1} \middle| \left\{\frac{\mathrm{p}_{nl}^{0}}{p_n^{0}}\right\}_{1\le l\le m-1}\right) -$$

$$-\mathrm{K}\left(\left\{\frac{\mathrm{p}_{nl}^{i}}{p_n^{i}}\right\}_{1\le l\le m-1} \middle| \left\{\frac{\mathrm{p}_{nl}^{j}}{p_n^{j}}\right\}_{1\le l\le m-1}\right)] +$$

$$+\mathrm{K}\left(\frac{1-\beta_n^{i}}{1-\beta_{n-1}^{i}} \middle| \frac{1-\alpha_n}{1-\alpha_{n-1}}\right) - \mathrm{K}\left(\frac{1-\beta_n^{i}}{1-\beta_{n-1}^{i}} \middle| \frac{1-\beta_n^{j}}{1-\beta_{n-1}^{j}}\right) \tag{13}$$

$$E\left\{S_{FS}^{nj} \mid \mu_0\right\} = -U(m-2)\,\mathrm{K}\left(\left\{\frac{\mathrm{p}_{nl}^{0}}{p_n^{0}}\right\}_{1\le l\le m-1} \middle| \left\{\frac{\mathrm{p}_{nl}^{j}}{p_n^{j}}\right\}_{1\le l\le m-1}\right) -$$

$$-\mathrm{K}\left(\frac{1-\alpha_n}{1-\alpha_{n-1}} \middle| \frac{1-\beta_n^{j}}{1-\beta_{n-1}^{j}}\right) \tag{14}$$

We note that the quantities $\left\{\frac{\mathrm{p}_{nl}^{i}}{p_n^{i}}\right\}_{1\le l\le m-1}$; i = 0,1, …, m-1, $\frac{1-\beta_n^{i}}{1-\beta_{n-1}^{i}}$; i = 1, …, m-1 and $(1-\alpha_n) / (1-\alpha_{n-1})$ in expression (13) and (14) all represent performance metrics per single sensor. We now state a theorem whose proof is included in the Appendix.

Theorem 2

Let the sequences $\left\{\frac{\mathrm{p}_{nl}^{i}}{p_n^{i}}\right\}_{1\le l\le m-1}$; i = 0,1, …, m-1, $\frac{1-\beta_n^{i}}{1-\beta_{n-1}^{i}}$; i = 1, …, m-1 and $(1-\alpha_n) / (1-\alpha_{n-1})$ converge asymptotically. Then, the algorithmic system at the fusion center has the following asymptotic performance characteristics, where N_{0j}^{T} denotes the stopping variable of the $\mu_0 \to \mu_j$ shift monitoring algorithm in the system when the common threshold is T, and where $E\left\{S_{FS}^{j} \mid \mu_k\right\} \stackrel{\Delta}{=} \lim_{n\to\infty} E\left\{S_{FS}^{nj} \mid \mu_k\right\}$.

$$\text{As } \mathrm{T}\to\infty,\quad \mathrm{E}\left\{\mathrm{N}_{0j}^{\mathrm{T}} \mid \mu_i\right\}: \begin{cases} \sim \mathrm{E}^{-1}\left\{S_{FS}^{j} \middle| \mu_i\right\}\log \mathrm{T}\,; & \text{if } \mathrm{E}\left\{S_{FS}^{j} \middle| \mu_i\right\} > 0 \\ \ge 2^{-1}\,\mathrm{T}\,; & \text{if } \mathrm{E}\left\{S_{FS}^{j} \middle| \mu_i\right\} < 0 \end{cases} \tag{15}$$

$$\text{As } \mathrm{T}\to\infty,\quad \mathrm{E}\left\{\mathrm{N}_{0j}^{\mathrm{T}} \mid \mu_j\right\} < \mathrm{E}\left\{\mathrm{N}_{0i}^{\mathrm{T}} \mid \mu_j\right\}\ \forall\, i \ne j \tag{16}$$

In addition, the conditions in Theorem 1 for mutual independence across the various sensors hold, for asymptotically many sensors. □

From the results in Theorem 2, we clearly observe that the asymptotic performance of the algorithm deployed at the fusion center is determined by the performance of the algorithms deployed by the individual sensors, which are determined, in turn, by the Kullback-Leibler numbers among the various acting processes. Furthermore, each individual sensor may be viewed as a representation of a centralized system; thus, comparison between a decentralized and a centralized systems translates to comparison of the fusion center performance to that of a single sensor. The asymptotic performance of the fusion center is controlled by the limits of the expectations in expressions (13) and (14), which are, in turn, determined by the limits of Kullback-Leibler numbers among power and false alarm quantities induced at a single sensor; the latter numbers are functions of the Kullback-Leibler numbers among the various acting processes. The asymptotic performance of a single sensor, on the other hand, is directly controlled by the Kullback-Leibler numbers among the acting processes. As the latter numbers increase, both sensor and fusion center performances enhance.

5. Numerical evaluations

In this section, we examine metrics for the non-asymptotic performance of the algorithms in the system. We first state the general experimental setup. Then, we present numerical results, for a specific scenario.

5.1 Experimental setup

In the construction of our experimental setup, we follow the steps listed below :

1. We select specific processes, $\mu_1, \ldots, \mu_{m-1}$.
2. We construct the specific updating step for each of the parallel algorithms $\mu_0 \rightarrow \mu_k$; $k = 1,\ldots, m - 1$that are ran at each sensor, as per expression (7) in Section IV.
3. Via the construction in step 2, we compute numerically the quantities $\{P_{0k}^j(n)\}$, $\{\beta_{0k}^j(n)\}$ and $\{\alpha_{0k}(n)\}$ in a recursive fashion, where :

   $\{P_{0k}^j(n)\}$: Given that the data generating process is μ_j , the probability that the $\mu_0 \rightarrow \mu_k$ monitoring algorithm crosses the threshold at n.

   $\alpha_{0k}(n)$: $\sum_{l=1}^{n} P_{0k}^0(l)$

   $\beta_{0k}^j(n)$: $\sum_{l=1}^{n} P_{0k}^j(l)$, $j = 1, \ldots, m-1$
4. Via the computed quantities in step 3, we compute the quantities P_{nk}^j, P_n^j, α_n and β_n^j defined in Section IV, as follows :

   $$\underset{k,j\,;1,\ldots,m-1}{P_{nk}^{j}} = P_{0k}^j(n) \prod_{\substack{1 \le l \le m-1 \\ l \ne k}} \left(1 - \beta_{0l}^j(n)\right); \; j = 0, 1, \ldots, m-1$$

   $$P_n^j = \sum_{k=1}^{m-1} P_{nk}^j \; ; \; j = 0, 1, \ldots, m-1$$

   $$\alpha_n = \sum_{l=1}^{n} P_l^0$$

   $$\beta_n^j = \sum_{l=1}^{n} P_l^j \; ; \; j = 1, \ldots, m-1$$
5. The quantities computed in step 4 are used to compute the updating steps of the parallel algorithms ran by the Fusion Center, as the former are determined by expression (10) in Section IV.
6. The number of sensors is selected. Data are independently generated at each senor by the same process μ_l, where μ_l is one of the processed selected in step 1. Given μ_l, the overall system-sensors/fusion center – is simulated, where the system algorithmic thresholds have been a priori selected. The performance metrics computed are metrics at the Fusion Center. In particular, the computed metrics for each given μ_l are:

$\underset{1 \le k \le m-1}{h^{l}_{kn}}$: Given that the data at the sensors are generated by the process μ_l, the percentage of simulated runs that led the Fusion Center to decided at time n that μ_k started acting.

$\underset{1 \le k \le m-1}{T^{l}_{k}}$: The average time to decided in favor of process μ_k, given that the data generating process is μ_l, where the decision is by the Fusion Center.

r^{l}_{kn} : Given that the data generating process at the sensors is μ_l, the probability that the Fusion Center decides in favor of process μ_k before or at time n.

where,

$$r^{l}_{kn} = \sum_{p=1}^{n} h^{l}_{kn} \quad \text{and} \quad T^{l}_{k} = \sum_{p=1}^{n} n \cdot h^{l}_{kn}$$

m-1 sets of simulations are ran, each corresponding to one of the processes μ_l; $l = 1, \dots, m-1$ that generate the actual data at each sensor. □

In step 6, we stated that the algorithm thresholds of the system are a priori selected. The methodology for this selection is as follows.

A. The thresholds across different sensors are identical, since the sensors are considered identical. Per sensor, we test a number of different thresholds, $\delta_1, \dots, \delta_p$. For each given threshold, δ_i, we evaluate numerically the metrics $\{\alpha_{0k}(n)\}$ and $\{\beta^{k}_{0k}(n)\}_{1 \le k \le m-1}$, where the latter metrics are defined in step 3. Given δ_i, we plot the m-1 pairs of curves $\{\alpha_{0k}(n), \ \beta^{k}_{0k}(n)\}_{1 \le n \le N}$ for some pre-selected N. We then decide on a value no and select a lower bound βo for powers and a upper-bound αo for false alarms. We select as the operational algorithmic threshold, the minimum among the tested thresholds such that all powers at are above βo and all false alarms at no are below αo. That is, operational selected thresholds attains :

$$\min_{1 \le k \le m-1} \beta^{k}_{0k}(n_0) \ge \beta_0 \quad \text{and} \quad \max_{1 \le k \le m-1} \alpha_{0k}(n_0) \le \alpha_0 .$$

B. The thresholds for the Fusion Center are evaluated and selected, as in (A).

5.2 Specific simulation scenario

We selected homogeneous Poisson processes $\mu_0, \dots, \mu_{m-1}$ with specific different rates. The simplification of the updating step in (7), Section 4, in this case, as well as the computation of the quantities $\{P^{j}_{0k}(n)\}$, $\{\beta^{j}_{0k}(n)\}$ and $\{\alpha_{0k}(n)\}$ in Step 3, Section 5.1, was included in (Burrell et. al. 1998a) and is also included in the Appendix.

We specifically selected six homogenous Poisson processes, $\mu_0, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5$, with corresponding rates per unit time : $r_0 = 0.1$, $r_1 = 0.25$, $r_2 = 0.35$, $r_3 = 0.5$, $r_4 = 0.65$, $r_5 = 0.8$. We tested several thresholds for the algorithm systems ran by the sensors, and finally selected a common threshold equal to 300. For the latter threshold, all induced powers attained values above 0.97 at time 200 and all false alarms remained below the value 0.005 at the same time. We simulated the overall system, for 30 and 50 sensors and for fusion center threshold values 10, 20, 100 and 300. To exemplify our results, we plot some power and false alarm curves in Figures 2 and 3 below. Specifically, in Figure 2, we plot the power and false alarm curves induced by the algorithm that monitors a change from Poisson rate 0.1 to Poisson rate 0.25 at the fusion center, when the number of sensors is 30 and the algorithmic threshold values are 10. 20, 100 and 300. In Figure 3, we plot the same curves when the number of sensors is 50. From studying the two figures, we observe that, as the number of sensors increases from 30 to 50, low false alarm and high power are simultaneously attained for less than 100 data, when the threshold value at the fusion center is 10.

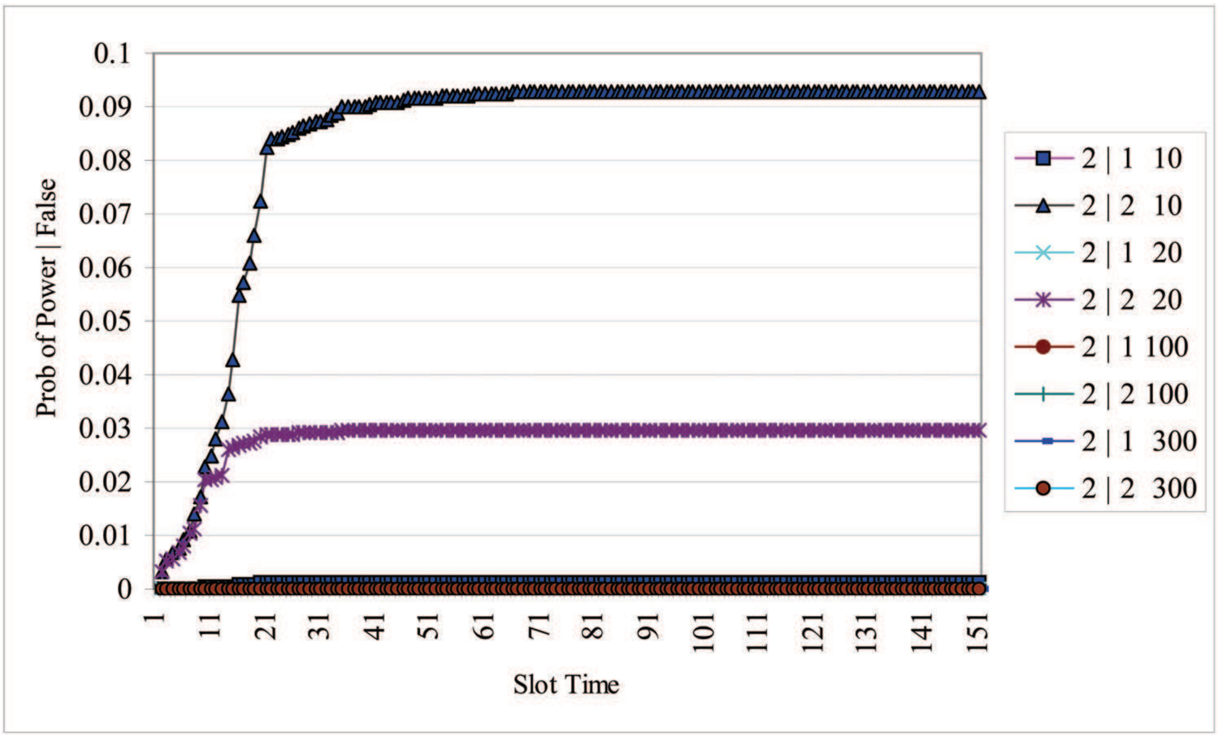

Fig. 2. Power & False Alarm Curves for 0.1 → 0.25 Monitoring Algorithm at the Fusion Center, for 30 Sensors. Legend: 2 | 1 : False Alarm 2 | 2 : Power

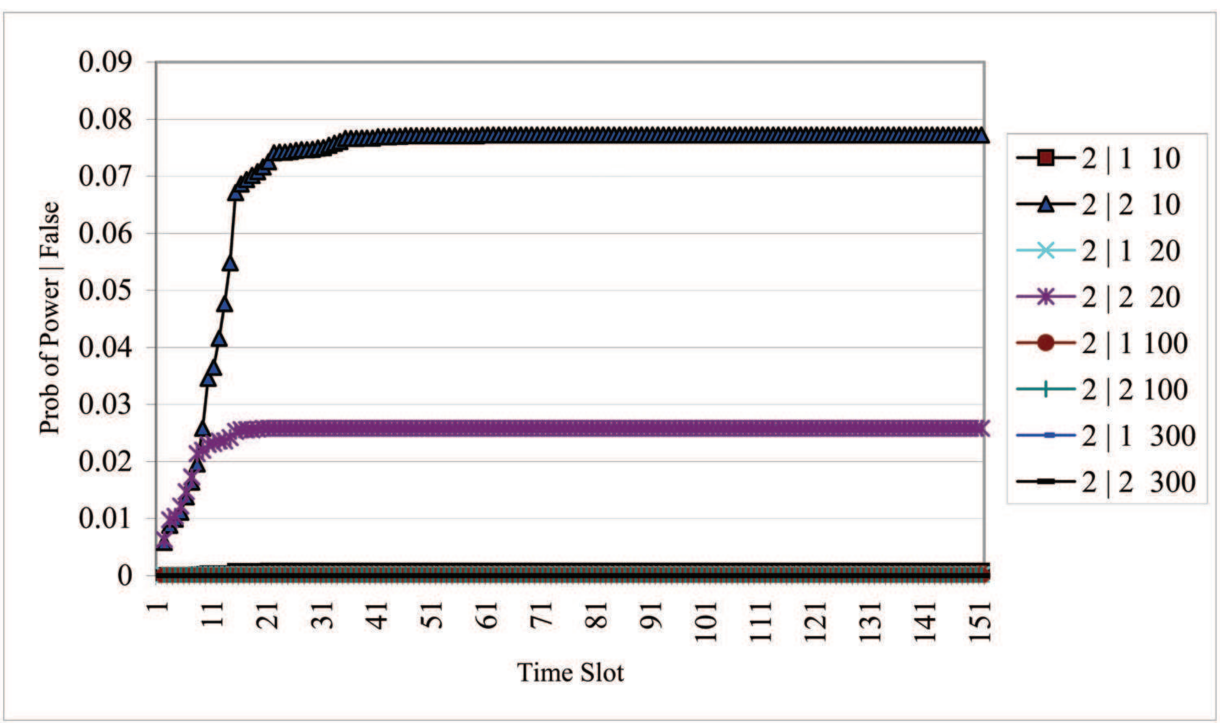

Fig. 3. Power & False Alarm Curves for 0.1 → 0.25 Monitoring Algorithm at the Fusion Center, for 50 Sensors. Legend : 2 | 1 : False Alarm 2 | 2 : Power

In Tables 1 and 2 below, we include our computed T_k^l values for 30 and 50 sensors respectively, for the Poisson model explained above, and for fusion center threshold values 10, 20, 100 and 300. From the values in the tables, we note that the system approaches its expected asymptotic performance, as the number of sensors increases from 30 to 50, and for fusion threshold value 10.

T- Files, Threshold 300, Sensors = 30

k \ l	1	2	3	4	5	6	Diagonal
2	0	0	0	0	0.0008	0.0077	0
3	0	0.0037	0.0073	0.0527	0.1207	0.2117	0.0073
4	514.26	3.8112	0.2629	0.4904	0.4103	0.2794	0.4904
5	1290.03	36.1046	1.1842	0.8225	0.4612	0.2874	0.4612
6	27.0723	13.2926	2.4171	1.311	0.5245	0.2861	0.2861

T- Files, Threshold 100, Sensors = 30

k \ l	1	2	3	4	5	6	Diagonal
2	0	0.0003	0.0044	0.026	0.0541	0.1286	0.0003
3	0	0.0779	0.2877	0.4659	0.3382	0.2296	0.2877
4	398.395	2.5276	0.7833	0.7369	0.3909	0.2342	0.7369
5	1049.44	23.7907	2.892	1.1396	0.4044	0.2441	0.4044
6	247.986	33.4387	4.1832	1.207	0.4336	0.2378	0.2378

T- Files, Threshold 20, Sensors = 30

k \ l	1	2	3	4	5	6	Diagonal
2	0.741587	0.2708	0.662	0.5768	0.305	0.2103	0.2708
3	5.93519	7.8534	2.0764	0.927	0.3568	0.2206	2.0764
4	344.015	7.3547	3.1896	0.9682	0.3356	0.2115	0.9682
5	865.765	7.1572	3.2217	0.9998	0.3564	0.2233	0.3564
6	380.923	11.6246	3.2076	0.9974	0.3569	0.2035	0.2035

T- Files, Threshold 10, Sensors = 30

k \ l	1	2	3	4	5	6	Diagonal
2	9.00174	1.422	1.7969	0.8834	0.3555	0.2188	1.422
3	23.6324	7.3706	2.453	0.9198	0.3502	0.2176	2.453
4	364.822	7.7948	3.3048	1.002	0.357	0.2125	1.002
5	809.686	4.6126	2.8298	1.0032	0.3709	0.2204	0.3709
6	379.928	4.4201	2.8206	0.9947	0.3622	0.2112	0.2112

Table 1. T_k^l Values at the Fusion Center for 30 Sensors

Legend:
1: Rate 0.1 2: Rate 0.25
3: Rate 0.35 4: Rate 0.5
5: Rate 0.65 6: Rate 0.8

T- Files
Threshold 300
Sensors 50

$k \backslash l$	1	2	3	4	5	6	Diagonal
2	0	0	0	0	0	0.0013	0
3	0	0	0.0037	0.017	0.0642	0.1898	0.0037
4	493.968	3.2876	0.1555	0.2853	0.2617	0.2593	0.2853
5	1783.71	43.4664	1.1773	0.6799	0.4089	0.282	0.4089
6	11.1052	10.7968	2.069	1.1099	0.4433	0.2776	0.2776

T- Files
Threshold 100
Sensors 50

$k \backslash l$	1	2	3	4	5	6	Diagonal
2	0	0	0.0001	0.0072	0.0539	0.1522	0
3	0.972617	0.0393	0.1549	0.234	0.1958	0.207	0.1549
4	545.46	1.6381	0.8906	0.6919	0.3144	0.2224	0.6919
5	1247.48	20.5153	2.3331	0.7516	0.321	0.2155	0.321
6	220.568	39.1443	4.8733	1.0118	0.3317	0.2109	0.2109

T- Files
Threshold 20
Sensors 50

$k \backslash l$	1	2	3	4	5	6	Diagonal
2	1.19472	0.1307	0.3722	0.3605	0.2338	0.2041	0.1307
3	5.97928	5.2579	1.3822	0.5765	0.2408	0.1975	1.3822
4	382.187	7.9517	2.6174	0.6448	0.2468	0.2036	0.6448
5	1012.35	5.0408	2.5545	0.7314	0.2501	0.21	0.2501
6	396.057	8.7207	2.6216	0.7072	0.2418	0.1933	0.1933

T- Files
Threshold 10
Sensors 50

$k \backslash l$	1	2	3	4	5	6	Diagonal
2	4.34465	0.8419	1.1619	0.545	0.2322	0.2034	0.8419
3	10.8702	4.293	1.7951	0.6618	0.2538	0.2085	1.7951
4	362.561	6.9178	2.5261	0.6414	0.2377	0.1919	0.6414
5	889.308	4.4505	2.1631	0.6447	0.2438	0.1974	0.2438
6	469.714	3.7693	2.215	0.6622	0.2446	0.2073	0.2073

Table 2. T_k^l Values at the Fusion Center for 50 Sensors

Legend:
1: Rate 0.1 2: Rate 0.25
3: Rate 0.35 4: Rate 0.5
5: Rate 0.65 6: Rate 0.8

6. Conclusions

We studied a fusion center structure, for the detection of change in the underlying data generating process. We established the pertinent algorithms and stated conditions for the asymptotic optimality of the overall system. In general terms, we showed that the relevant algorithms converge in logarithmic time. We also established metrics for the study of non-asymptotic performance and presented numerical results for a specific Poisson data traffic scenario.

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APPENDIX:

Proof of Theorem 1

When the conditions in (9) hold, the two last terms in the updating step in (7) reduce to a constant, for all $n > n_0$. The sensor algorithmic system becomes then identical to that of a centralized system, when no implicit feedback from the fusion center exists. The latter sensor systems are mutually independent, since the local data are. The performance of these independent systems are then as in (Bansal et al 1989) and (Burrell et al 1998). The expression in (10) is derived from (8) in a straight forward fashion, via the mutual independence of the sensors and the sufficient statistics at the fusion center.

Proof of Theorem 2

When the sequences in the Theorem converge, the Markov Chain $\left\{\bar{U}_n\right\}$ becomes asymptotically stationary, and the algorithmic system is optimal in the sense of (Bansal et al 1989); expression (15) is a direct consequence of this optimality. A direct inspection of expression (13) leads to the conclusion expressed by (16) in a straight forward fashion. When the number of sensors is asymptotically large, the updating steps of the algorithms at the fusion converge to the expected values in (13) and (14). Via the assumptions in the Theorem, the latter values converge asymptotically to constants. The latter fact leads directly to the satisfaction of the assumptions in Theorem 1.

The Computation of Useful Probabilities

In this part of the Appendix, we express the useful quantities needed in the simulation scenario of Section 5.2.

Consider the algorithm which monitors a change from μ_0 to μ_k , where the process μ_0 and μ_k homogeneous Poisson, with respective rates r_0 to r_k. Then, be dividing both threshold and the updating step of the monitoring algorithm by the scaling factor | ln (r_k / r_0) |, we obtain the following simplified form of the updating step in (7), Section 4, for

$$\operatorname{sgn} x \stackrel{\Delta}{=} \begin{cases} 1 & ; \quad x > 0 \\ -1 & ; \quad x < 0 \end{cases} \quad : \quad (\mathrm{m} \cdot \mathrm{s\text{-}t}) \operatorname{sgn}\left(\ln \frac{\mathrm{r_k}}{\mathrm{r_0}}\right)$$

where

m : number of arrivals within a time unit.

t, s : $\dfrac{r_k - r_0}{\ln(r_k / r_0)} \approx \dfrac{\mathrm{t}}{\mathrm{s}} < 1$, t, s, natural numbers.

Let us denote by v the (without loss in generality) integer threshold value of the $\mu_0 \to \mu_k$ monitoring algorithm. If V_0 is an integer common threshold for the algorithmic system, then $\mathrm{v} = \left\lfloor \dfrac{\mathrm{V_0}}{(\ln(\mathrm{r_k}/\mathrm{r_0}))} \right\rfloor$. Let us define :

$P^j_{0k}(y,n)$: Given that all data are generated by the Poisson process with rate r_j, the probability that at time n the $\mu_0 \to \mu_k$ monitoring algorithm has the value y and it has not crossed its threshold before or at n-1.

We can then express the time/value evolution of the probabilities { $P^j_{0k}(y,n)$ } as follows:

$$P^j_{0k}(0,0) = 1$$

$$n \geq 1 \; ; \; P^j_{0k}(0,n) = \sum_{x=0}^{\min(t,v-1)} P^j_{0k}(x, n-1)\, e^{-r_j}$$

$$; \begin{matrix} n \geq 1 \text{ and } v > 1+t \\ 1 \leq y \leq v\text{-}1\text{-}t \end{matrix} \; ; \; P^j_{0k}(y,n) = \sum_{m=0}^{\left\lfloor \frac{y+t}{s} \right\rfloor} P^j_{0k}(y\text{-}ms+t, n\text{-}1)\, e^{-r_j} \frac{(r_j)^m}{m!} \qquad \text{(A.1)}$$

$$; \begin{matrix} n \geq 1 \\ v\text{-}t \leq y \leq v\text{-}1 \end{matrix} \; ; \; P^j_{0k}(y,n) = \sum_{m=\left\lceil \frac{y+t-v+1}{s} \right\rceil}^{\left\lfloor \frac{y+t}{s} \right\rfloor} P^j_{0k}(y\text{-}ms+t, n\text{-}1)\, e^{-r_j} \frac{(r_j)^m}{m!}$$

Then, the quantities $P^j_{0k}(n)$, $\beta^j_{0k}(n)$ and $\alpha_{0k}(n)$ are computed as follows :

$$P^j_{0k}(n) = \sum_{y=0}^{v-1} P^j_{0k}(y,n\text{-}1) \left\{ 1 - \sum_{0 \leq m \leq \left\lceil \frac{v-y+t}{s} \right\rceil - 1} e^{-r_j} \frac{(r_j)^m}{m!} \right\} \qquad \text{(A.2)}$$

$$\alpha_{0k}(n) = \sum_{l \leq n} P^0_{0k}(l) \quad \text{and} \quad \beta^j_{0k}(n) = \sum_{l \leq n} P^j_{0k}(l) \qquad \text{(A.3)}$$

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Data fusion is a research area that is growing rapidly due to the fact that it provides means for combining pieces of information coming from different sources/sensors, resulting in ameliorated overall system performance (improved decision making, increased detection capabilities, diminished number of false alarms, improved reliability in various situations at hand) with respect to separate sensors/sources. Different data fusion methods have been developed in order to optimize the overall system output in a variety of applications for which data fusion might be useful: security (humanitarian, military), medical diagnosis, environmental monitoring, remote sensing, robotics, etc.