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Chapter 7

Stochastic Quantum Potential Noise and Quantum Measurement

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Abstract

Quantum measurement is the greatest problem in quantum theory. In fact, different views for the quantum measurement cause different schools of thought in quantum theory. The quandaries of quantum measurement are mainly concentrated in "stochastic measurement space", "instantaneous measurement process" and "basis-preferred measurement space." These quandaries are incompatible with classical physical laws and discussed many years but still unsolved. In this chapter, we introduce a new theory that provided a new scope to interpret the quantum measurement. This theory tells us the quandaries of quantum measurement are due to the nonlocal correlation and stochastic quantum potential noise. The quantum collapse had been completed by the noised world before we looked, and the moon is here independent of our observations.

Keywords: quantum measurement, quantum collapse, quantum potential noise, Feynman path integral

1. Introduction

Schrödinger cat was born from the thought experiment of Schrödinger in 1935. However, after more than 80 years, we still do not know whether it is dead or alive in its sealed box. According to the modern quantum mechanics, based on Copenhagen interpretation, the fate of this cat is entangled with the Geiger counter monitor in its box, and the cat is in a "mixed state"—both dead and alive—if we do not open the box to look at it. It is a miserable and mystical cat, which seems its fate depends on our look. Yes, it just seems, because we deeply doubt that the power of our glimpse can really make the cat alive or dead. This doubt is not

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the business about human self-confidence, but the fear of our fate determination. If the glimpse of us can determine the cat's fate, who determines our fate? Trouble never singly comes; many researches find that "the moon is not there" in experiments [1] responding to what Albert Einstein said, "I like to think that the moon is there even if I don't look at it." According to the physicist's research, Albert Einstein seems worried because the world is quantum world and all things obey quantum mechanics. This means all the definite statuses we have observed are due to "a glimpse" of us or the god. Really? Is really the moon not here if we do not look at it, does really the cat not exist if we do not look at it, and do we not exist if the god does not look at us?

It must be something to worry because the moon exists more than 4.5 billion years as the astronomer finding, which is much more than human history. We are not going to discuss the superpower of human and if the god exists or not in this book. We return to the fundamental of quantum mechanics and find that the hidden actor, quantum measurement, is the crime culprit that causes these puzzling questions.

There is a confliction in modern quantum physics after its birth. The confliction is concerning the full description between the superposition state for the behavior of matter on the microscopic level and the definite-status appearance as what we can observe on the macroscopic level in the real world. Schrödinger proposed Schrödinger cat in his essay to illustrate the "putative incompleteness" of quantum mechanics, but many researches show that quantum mechanics is still the best one of these "not satisfied theories." To alleviate the theory-to-world confliction, a new conception, quantum measurement, is brought out. It is the basic assumption in quantum mechanics, thought that the superposition state will be collapsed into one of the eigenstates with the square of amplification probability if we do a quantum measurement. Although the quantum measurement bridges the gap of the different behaviors of subatomic level and the macro-world, some problems still remain. For example, its physical mechanism is dim. We do not know what will lead to the quantum measurement and how the process that the quantum measurement undergoes. The words "stochastic", "instantaneous" and "irreversible" torment us more than 70 years, and we still have no way to integrate them into the "determinate", "time-costed" and "reversible" quantum evolution. In fact, the manual division for the world into two parts, quantum world and quantum measurement apparatus, is not satisfied, and we are finding a uniform description.

In this chapter, we will overview the mechanism of quantum measurement and the main kinds of interpretation of quantum measurement. Among these interpretations, a promised theory which can well interpret the quantum measurement quandaries—why the quantum state collapses into some eigenstates with "stochastic" and "instantaneity", and what causes the "basis-preferred"—is detailed. The advantage of this theory is it is just an extension of Feynman path integral (FPI) and is obviously compatible with the classical quantum theory. According the conclusions of this theory, we show that the "noise" world (or apparatus here when we do an experiment) causes the "random" and "nonlocal" mechanism of the quantum collapse. Actually, the world exists due to itself, and the god can go to have a rest.

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2. What is the quantum measurement?

Quantum measurement is different from the classical measurement, in which the measurement accuracy is dependent on the measurement instruments. It means, we could infinitely approach the "absolute exact value" by upgrading instruments or improving methods in the classical measurement realm. However, the things change when we access to the quantum world. In quantum world, the "accuracy" does not exist. We cannot speak that the velocity of an electron is 1376.5 m/s or the distance of two electrons is 20 nm, etc., because these physical quantities exist in the form of quantum states in quantum world. Objects are always in the superposition states of these kinds of the basis state, such as momentum, position, energy, spin and so on. We can just get one of the basis states under every measurement, and the "absolute exact value" is never revealed under one measurement unless the state of the object is in the basis state.

In quantum mechanics, the projection operator is defined as $\widehat{P}_{\varphi_i} = |\varphi_i\rangle\langle\varphi_i|$, where $|\varphi_i\rangle$ is an element of the basis-state set $\{|\varphi_k\rangle\}$. The measurement output for a mechanical quantity operator $\widehat{Q}$ under one quantum measurement is $Q_i = \left\langle \varphi_i \middle| \widehat{Q} \middle| \varphi_i \right\rangle = Tr\left(\widehat{P}_{\varphi_i}\widehat{Q}\right)$, and the initial state will instantaneously collapse into the basis state $|\varphi_i\langle$ with the probability $p_i = Tr\left(\widehat{P}_{\varphi_i}\widehat{\rho}_I\right)$, where $\widehat{\rho}_I$ is the initial density matrix of an object, after the quantum measurement. For multi-measurements, the output we get is the average value $\tilde{Q} = \sum_i p_i Q_i = Tr\left(\widehat{Q}\,\widehat{\rho}_I\right)$, and the final state of the many object systems becomes $\rho_O = \sum_i p_i \widehat{P}_{\varphi_i}$, which is very different from the initial state ρ_I.

This kind of measurement, to be exact, is the projective measurement. A more general formulation of measurement is the positive-operator valued measure (POVM), which can be seemed as the partial measurement in the subsystem of a projective measurement system. No matter what kind of quantum measurements there is, it is the kind of destructive manipulations and irreversible. It destroys the old state and rebuilds a new mixed state. The definition of the quantum measurement is simple and definite, but the problem is that we do not know why the quantum measurement acts as these strange behaviors. The irreversibility and unpredictability are incompatible with the smooth Schrodinger differential equation and are hated by physicists. What kind of objects has priority to do the quantum measurement? Taking the experiment of two-slit interference of electrons, for example, the detector behind the slits usually is regarded as a measurement tool, but the detector itself, which may be a microcavity or atom ensemble, is also a physical system and obeys the quantum mechanism. Therefore, it seems that the process of a quantum measurement is the interaction between the detector and electrons and should be a "quantum evolution process". However, the quantum evolution process is non-destructive and reversible. In fact, in the real world, it is hard for us to distinguish strictly which is the quantum evolution operation and which is the quantum measurement.

The second problem is the space–time nonlocality in the quantum measurement process. This nonlocality exists not only in the correlation between particles but also in the wave function of single particle. We still take the experiment of two-slit interference of electrons, for example. If the detector behind the slits has detected the signal and we can distinguish which slit the electrons pass, then the interference phenomenon will disappear. In language of quantum mechanics, the diffused wave function $\psi(x,t)$ of the electron will collapse into $\delta(x_0,t)$ immediately after this measurement. This process is very fast and does not seem to need to cost time. How this process happens and whether this process violates the law of causation of relativity theory are still unclear for us.

The third problem is the basis-preferred problem. The basis-preferred problem refers to a quantum system that is measured which prefers to collapse to a set of eigenstates. For example, a spin system with an initial state $|\psi\rangle\rangle = a|\uparrow\rangle\rangle + b|\downarrow\rangle\rangle$ can collapse into the state of the set $\{|\uparrow\rangle\rangle, |\downarrow\rangle\rangle\}$, and it can also collapse into the state of the set $\left\{1/\sqrt{2}(|\uparrow\rangle\rangle + |\downarrow\rangle\rangle), 1/\sqrt{2}\,(|\uparrow\rangle\rangle - |\downarrow\rangle\rangle)\right\}$, but under a certain measurement, this state prefers one of these sets. Why the state prefers some basis set under quantum measurement? Does it have awareness?

Without any exaggeration, quantum measurement is one the most interesting and fascinating topics in quantum theory. There are too many unsolved mysteries in quantum measurement, and these spur us to further understand the quantum measurement and find the answers.

3. The main kinds of interpretation for quantum measurement

There are more than 10 kinds of interpretations for quantum measurement in quantum mechanics, such as Copenhagen interpretation, quantum logic, many worlds interpretation, stochastic interpretation, many-minds interpretation, etc. In this chapter, we just choose four of them to expound. According this section, we will know how difficult for physicists to solve these problems in one theory.

3.1. The Copenhagen interpretation

The Copenhagen interpretation was formed in 1925 to 1927 by Niels Bohr and Werner Heisenberg. In fact, it is still the most commonly taught interpretations of quantum mechanics today.

According to the Copenhagen interpretation, the physical law that microscopic objects obey are different from that the macroscopic objects obey. Microscopic objects can be in superposition states, but the macroscopic objects are forbidden. According to the Copenhagen interpretation, the statuses of macroscopic objects are definite. We can say a macroscopic object is in this status or not, but cannot say this macroscopic object is both in this status and not. Now that the laws in microscopic world and macroscopic world are different, then the Copenhagen interpretation assumes the existence of macroscopic measurement apparatuses that obey classical physics to make measurement for microscopic objects that obey quantum mechanics.

However, this assumption does not solve the problems of quantum measurement. It throws all the problems to the macroscopic apparatuses, but it even cannot answer how to distinguish the macroscopic object that obeys the classical laws and microscopic ensemble that obeys the quantum mechanics. Moreover it also cannot answer how the nonlocality produces in quantum measurement process because, there is no seed for nonlocality growing no matter in classical physics or quantum mechanics.

3.2. Many worlds interpretation

Many worlds interpretation was proposed by Hugh Everett in 1952. It supposes that there are a large, perhaps infinite, number of universes and every alternate state is in one of these universes [2, 3]. Many worlds interpretation denies the wave function collapse under quantum measurement. It asserts that the object that will be measured and the observer that will do the measurement are in a relative state. Each measurement will be a branch point and makes observer enter a universe. According to the thought of many worlds interpretation, the Schrödinger cat is alive in a universe and dead in the other universe. After the measurement, the observer will enter one of these two universes.

The advantage of this interpretation is that the discussion of collapse mechanism is avoided. However, the basis-preferred problem is still the big issue in many worlds interpretation although the quantum decoherence had been introduced into in the period of "post-Everett". Some researchers still think the many worlds interpretation of quantum theory exists only to the extent that the associated basis problem is solved [4–6]. Using the decoherence to define the Everett branches will lead to an approximate specification of a preferred basis and contradicts with the "exact" definition of the Everett branches.

3.3. Many-minds interpretation

Many-minds interpretation is the extension of many worlds interpretation. It was proposed by Heinz-Dieter Zeh in 1970 to solve the "branch determining problem" and the puzzling concept of observers being in a superposition with themselves in many worlds interpretation [7–9]. The thought of this interpretation is when an observer measures a quantum system, then a state that is consistent with minds which produced by the observer brain, called mental states, will entangle with this quantum system. The mental state of the brain corresponding with this system is involving, and ultimately, only one mind is experienced, leading the others to branch off and become inaccessible. In this way, every sentient being is attributed with an infinity of minds, whose prevalence corresponds to the amplitude of the wave function. As an observer checks a measurement, the probability of realizing a specific measurement directly correlates to the number of minds they have where they see that measurement.

However, like the many worlds interpretation, the many-minds interpretation is still a local theory. Although the correlations of individual minds and objects could be the violation of Bell's inequality, the interactions between them that only take place are local, and only the separated events that are space-like separated could influence the minds of observers. Additionally, it tosses the basis-preferred problem to the mentality of observer and makes this physical problem fall into an endless discussion of mental state of human.

3.4. Dynamical reduction models

The theory of dynamical reduction models is a nonlinear and stochastic modification of the Schrödinger equation. It is proposed by Bassia and Ghirardia [10]. They integrated the master equation and linear Schrödinger equation and proposed a new nonlinear differential equation. This theory successfully solves the problems of "stochastic output" and "preferred basis" in quantum measurement and deduced the Born probability rule basing on the white noise model. However, it is still a nonrelativistic theory and remains the nonlocality problem.

4. The extended Feynman path integral and quantum measurement

4.1. Why is it concerning with the Feynman path integral?

As we know, in the history of the quantum theory, there are three equivalent expressions, namely, the differential equation of Schrödinger, the matrix algebra of Heisenberg and the path integral formulation of Feynman. However, these three expressions have their own focuses. The Schrodinger and Heisenberg expressions focus on the evolution of states and operations, respectively, whereas the path integral formulation of Feynman on the "correlation" of point to point as states is evolving [11]. On the other hand, in quantum mechanics, when do a measurement on a wave function diffusing in all of space, such as the measurement of the position of an electron in the experiment of double-slit interference, we will find that the whole wave function will instantaneously collapse to this position measured with some probabilities. Obviously there may be some inner "correlation" in wave function transferring the action of the measurement from local part to whole. These two "correlations" have common characters and may be unified to be one.

Moreover, we notice that the action integral in Feynman path integral formulation is the classical form. The classical physics is born to be a local theory and of course cannot exhibit the character of nonlocality. However, the relativity theory is different. In relativity theory, the time and space are coupling. Beyond the light cones in Minkowski space, the space-time causality is broken, and this may cause the nonlocality. The superluminal velocities are forbidden in real world, but for a connection description of virtual paths in the path integral theory, it might be practicable. What will happen when we extend the classical action to relativistic action? Could the superluminal trajectories included in possible paths to calculate quantum amplitude in the Feynman theory cause the nonlocality? How is the relationship between "unitary evolution operation" and "quantum measurement"? These questions will be revealed when we extend the Feynman path integral.

4.2. How to extend the Feynman path integral?

The formulation for Feynman path integral can be written as

$$K(r, r_0; t, t_0) = C\sum\nolimits_{allpaths} \exp\left(iS/\hbar\right) \tag{1}$$

where the coefficient C is a constant independent of paths and S is the action with classical form

$$S(t_0, t_1) = \int_{t_0}^{t_1} L(\acute{r}(t), r(t))dt \tag{2}$$

$K(r, r_0; t, t_0)$ in Eq. (1) is the propagator and defined into

$$K(r, r_0; t, t_0) = \langle\left\langle r\middle|\widehat{U}(t, t_0)\middle|r_0\rangle\right\rangle \tag{3}$$

Eq. (1) reveals an important assumption in Feynman path integral: the weights of different paths for propagator are the same. This assumption makes Feynman path integral very successful in nonrelativistic quantum theory, but it is also the top offender that impedes the integration between Feynman path integral and relativity in non-field theory. Why should this be?

For the extension, it is necessary to break up this assumption, and Eq. (1) should be written into a more general formulation in the following:

$$F(r, r_0; t, t_0) = R\sum\nolimits_{allpaths} W(\wp) \exp\left(iS/\hbar\right) \tag{4}$$

where R is the parameter that is independent of paths and $W(\wp)$ is the weight function with paths [13]. Additionally, some rules should be set to limit the range of choices for R and $W(\wp)$:

a. The formulation should be simple and concise.

b. It should obey the combination rule because the propagator is linear.

c. It is consisted by the four-dimension scalars, vectors and tensors.

d. It should be transformed into Feynman path integral in low-energy and low-velocity condition.

Under these four limitations, the forms of R and $W(p)$ are very few. The final forms of R and $W(p)$ chosen in extended Feynman path integral are

$$R = \frac{1}{\sqrt{2i\pi\hbar c^2}} \frac{H'}{\sqrt{mc^2 + H'}}; W(\wp) = \frac{\mathbb{P}(\wp)}{\mathcal{P}(\wp)} (\Delta\tau)^{-1/2} \tag{5}$$

The H' in Eq. (5) is the main Hamiltonian:

$$H' = \sqrt{m^2c^4 + (p - A_0)^2 c^2} \tag{6}$$

and

$$\mathbb{P}(\wp) = \int_{t_0}^{t} |P| d\tau; \mathcal{P}(\wp) = \int_{t_0}^{t} \left|\sqrt{2mT}\right| d\tau \tag{7}$$

P, T and $\Delta\tau$ are called the momentum, kinetic energy and proper time in terms of four-dimensional space–time, respectively:

$$|P| = \frac{mv}{\sqrt{1 - v^2/c^2}}, T = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2, \Delta\tau = \int_{t_0}^{t} \frac{1}{\sqrt{1 - v^2/c^2}} d\tau \tag{8}$$

The expressions of $W(\wp)$ and R are very interesting. As we can see, under the low-energy and low-velocity condition, $H' \ll mc^2$ and $v \ll c$, then $R = \frac{1}{\sqrt{2i\pi\hbar c^2}}$ and $W(\wp) = (t - t_0)^{1/2}$ because $|P| = \sqrt{2mT}$ in classical physical theory. This means Eq. (4) can be transformed into the Feynman path integral if we choose the formulations of $W(p)$ and R as shown in Eq. (5). What is concerning then for us is what we can get from Eq. (4) under very high energy and velocity.

4.3. The new differential equation and Klein-Gordon equation

It is hard to directly calculate the value of Eq. (4) because the path integral is not normal integral term and the normal integral method is invalid for Eq. (4). A way to get some results from Eq. (4) is to follow the method that Feynman used [11, 12]. We consider a minimal evolution time process, $t = t_0 + \varepsilon$, where $\varepsilon \to 0$. In this process:

$$\psi(r, t_0 + \varepsilon) = \int_{-\infty}^{\infty} \psi(r_0, t_0) F(r, r_0; t_0 + \varepsilon, t_0) dr_0 = R \int_{-\infty}^{\infty} \psi(r_0, t_0) W(\wp) dr_0 \tag{9}$$

When $\varepsilon \to 0$, the weight function $W(\wp)$ can be simply expressed the term of

$$W(\wp) = \frac{\left(1 + \sqrt{1 - v^2/c^2}\right)^{1/2}}{\varepsilon^{1/2}\sqrt{1 - v^2/c^2}} \tag{10}$$

where $v = (r - r_0)/\varepsilon$. This value can be greater than the superluminal velocity, and $F(r, r_0; t_0 + \varepsilon, t_0)$ therefore will become the complex function when $v > c$. The integral form should be departed into two parts: the part that contains the low-velocity paths and the part that contains superluminal-velocity paths:

$$I = \int_{-\infty}^{\infty} \psi(r_0, t_0) F(r, r_0; t_0 + \varepsilon, t_0) dr_0 = \int_{-ct}^{ct} \cdots dr_0 + \left(\int_{ct}^{\infty} \cdots dr_0 + \int_{-\infty}^{-ct} \cdots dr_0 \right) = I_0 + I_1 \tag{11}$$

This can be exactly calculated. The amazing thing is **the final result calculated for I that contains the term** $\sqrt{m^2c^4 + (-i\hbar\nabla + A_0)^2 c^2}$. In the following context, we will detail this calculation in 1D space for simplification. The methods of the calculation in 2D and 3D are the same. Before this calculation, we define two parameters as $\tau_0 = \hbar/(mc^2)$ and $\varepsilon_0 = \varepsilon/\tau_0$:

$$
\begin{aligned}
I_0 &= \int_{-ct}^{ct} \cdots dr_0 = \varepsilon \int_{-ct}^{c} \cdots dv = 2R\tau_0^{1/2} \int_{-ct}^{ct} \frac{\left(1+\sqrt{1-v^2/c^2}\right)^{1/2}}{\varepsilon_0^{1/2}\sqrt{1-v^2/c^2}} \varepsilon_0 \exp\left(-i\sqrt{1-v^2/c^2}\varepsilon_0\right)\psi(r_0,t_0)dv \\
&= \int_{-\infty}^{\infty} \varphi_p dp \left(2R\tau_0^{1/2} \int_0^c \frac{\left(1+\sqrt{1-\frac{v^2}{c^2}}\right)^{1/2}}{\varepsilon_0^{1/2}\sqrt{1-\frac{v^2}{c^2}}} \varepsilon_0 \exp\left(-i\sqrt{1-\frac{v^2}{c^2}\varepsilon_0}\right) \exp\left(-ipv\varepsilon/\hbar\right) dv \right) \exp\left(ipx/\hbar\right) \\
&= \int_{-\infty}^{\infty} \varphi_p dp \left(2R\tau_0^{1/2} \int_0^1 (1-u)^{-1/2} \varepsilon_0^{1/2} \exp\left(-iu\varepsilon_0\right) \sum\nolimits_m \left(i\frac{p\varepsilon}{\hbar}\right)^{2m} \frac{\left(1-u^2\right)^m}{2m!} du \right) \exp\left(ipx\right) \\
&= \sum\nolimits_m 2R\left(i\frac{pc}{\hbar}\right)^{2m} \frac{c\varepsilon^{2m+1/2}}{2m!} \int_{-\infty}^{\infty} \varphi_p \exp\left(ipx\right) dp \left(\int_0^1 u^{\frac{-1}{2}} \left(1-(1-u)^2\right)^m \exp\left(iu\varepsilon_0 - i\varepsilon_0\right) du \right)
\end{aligned}
\tag{12}
$$

Similarly, we can also get the expression of I_0:

The contour integral is used in the last step as shown in **Figure 1**.

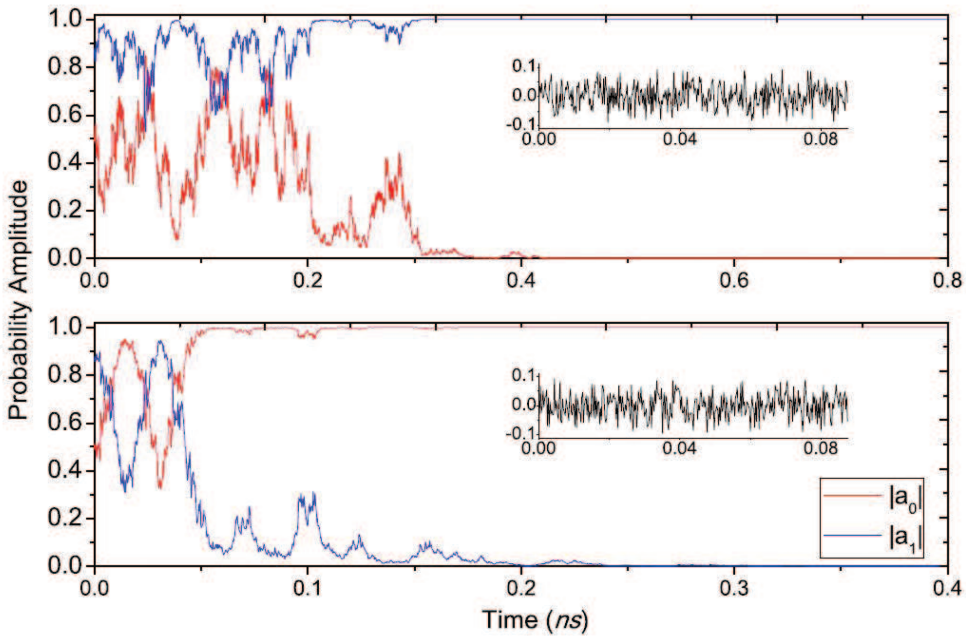

Figure 1. Contour integral. This figure shows the contour integral in a complex plane. The black line in figure denotes the integral $-\int_1^{1+i\infty} \cdots du - \int_0^1 \cdots du$; the blue line denotes $\int_0^\infty \cdots du$. The integral on the red line is always zero when $|z| \to \infty$. For this contour integral, there is no singular point, and of course the total integral value is zero. Therefore, $\int_1^{1+i\infty} \cdots du = \int_1^\infty \cdots du$:

$$\begin{aligned} I_0 &= \int_{ct}^{\infty}\cdots dr_0 + \varepsilon\int_{-\infty}^{-ct}\cdots dr_0 = 2R\tau_0^{1/2}\int_{ct}^{\infty}\frac{\left(1+\sqrt{1-v^2/c^2}\right)^{1/2}}{\varepsilon_0^{1/2}\sqrt{1-v^2/c^2}}\varepsilon_0\exp\left(-i\sqrt{1-v^2/c^2}\varepsilon_0\right) \\ &\times(\psi(r_0,t_0)+\psi(-r_0,t_0))du = \sum_m 2R\left(i\tfrac{pc}{\hbar}\right)^{2m}\frac{c\varepsilon^{2m+1/2}}{2m!}\int_{-\infty}^{\infty}\varphi_p\exp(ipx) \\ &\times dp\left(\int_1^{1+i\infty}u^{\frac{-1}{2}}\left(1-(1-u)^2\right)^m\exp(iu\varepsilon_0-i\varepsilon_0)du\right) \\ &= \sum_m 2R\left(i\tfrac{pc}{\hbar}\right)^{2m}\frac{c\varepsilon^{2m+1/2}}{2m!}\int_{-\infty}^{\infty}\varphi_p\exp(ipx)dp\left(\int_1^{\infty}u^{\frac{-1}{2}}\left(1-(1-u)^2\right)^m\exp(iu\varepsilon_0-i\varepsilon_0)du\right) \end{aligned} \tag{13}$$

Integrating Eq. (12) and Eq. (13), we get the conclusion finally:

$$\begin{aligned} I &= \sum_m 2R\left(i\tfrac{pc}{\hbar}\right)^{2m}\frac{c\varepsilon^{2m+1/2}}{2m!}\int_0^{\infty}\varphi_p\exp(ipx)dp\left(\int_0^{\infty}u^{\frac{-1}{2}}\left(1-(1-u)^2\right)^m\exp(iu\varepsilon_0-i\varepsilon_0)du\right) \\ &= \int_0^{\infty}\sum_m 2R\left(i\tfrac{pc}{\hbar}\right)^{2m}\frac{c\varepsilon^{2m+\frac{1}{2}}}{2m!}\Gamma\left(2m+\frac{1}{2}\right)M\left(-m,\frac{1}{2}-2m,-2i\varepsilon_0\right)\varphi_p\exp(ipx)dp \end{aligned} \tag{14}$$

The function $M(a,b,z)$ is the Kummer's function (confluent hypergeometric function) and equals

$$M\left(-m,\frac{1}{2}-2m,-2i\varepsilon_0\right) = \sum_n\frac{m!}{n!}\frac{(4m-1)!}{n!}(-i\varepsilon_0)^n \tag{15}$$

Summation in Eq. (14) is then

$$\sum_m 2R\left(i\frac{pc}{\hbar}\right)^{2m}\frac{c\varepsilon^{2m+\frac{1}{2}}}{2m!}\Gamma\left(2m+\frac{1}{2}\right)M\left(-m,\frac{1}{2}-2m,-2i\varepsilon_0\right) = \exp\left(\frac{-i\sqrt{m^2c^4+p^2c^2}\varepsilon}{\hbar}\right) \tag{16}$$

And Eq. (14) can be further simplified:

$$\begin{gathered} I = \int_0^{\infty}\exp\left(\frac{-i\sqrt{m^2c^4+p^2c^2}\varepsilon}{\hbar}\right)\varphi_p\exp(ipx)dp \\ \exp\left(\frac{-i\sqrt{m^2c^4+(-ic\hbar\nabla_x)}\varepsilon}{\hbar}\right)\int_0^{\infty}\varphi_p\exp(ipx)dp = \exp\left(\frac{-i\sqrt{m^2c^4+(-ic\hbar\partial_x)}\varepsilon}{\hbar}\right)\psi(x,t_0) \end{gathered} \tag{17}$$

It is, namely:

$$\psi(x,t_0+\varepsilon) = \exp\left(\frac{-i\sqrt{m^2c^4+(-ic\hbar\partial_x)^2}\varepsilon}{\hbar}\right)\psi(x,t_0) \tag{18}$$

Hence, the new differential equation we get in this extended Feynman path integral is

$$i\hbar\frac{d}{dt}\psi(x,t)=\sqrt{m^2c^4+(-ic\hbar\partial_x)^2}\psi(x,t) \tag{19}$$

The more general formulation in 3D is

$$i\hbar\frac{d}{dt}\psi(r,t)=\left(\sqrt{m^2c^4+(-i\hbar\nabla-A_0)^2c^2}+V(r)\right)\psi(r,t) \tag{20}$$

It is more complicated to get Eq. (20), and we will not detail it in this chapter. The detailed deduction can be seen in supplementary online material of the reference [13].

It should be mentioned that Eq. (20) is not a covariant equation under the Lorentz transformation. To construct a Lorentz covariant, the antiparticle wave function should be introduced. The antiparticle wave function is denoted as ϕ_- to be distinguished from the particle wave function ϕ_+. ϕ_+ satisfied the relation that Eq. (20) has shown and ϕ_- is satisfied

$$\left(i\hbar\frac{d}{dt}-V(r)\right)\phi_-=-\sqrt{m^2c^4+(-i\hbar\nabla-A_0)^2c^2}\phi_- \tag{21}$$

Combining Eqs. (20) and (21), we get these two equations:

$$\left(i\hbar\frac{d}{dt}-V(r)\right)\psi_+=-\sqrt{m^2c^4+(-i\hbar\nabla-A_0)^2c^2}\psi_+ \tag{22}$$

$$\left(i\hbar\frac{d}{dt}-V(r)\right)\psi_-=-\sqrt{m^2c^4+(-i\hbar\nabla-A_0)^2c^2}\psi_- \tag{23}$$

where $\psi_+=1/\sqrt{2}(\phi_++\phi_-)$ and $\psi_-=1/\sqrt{2}(\phi_+-\phi_-)$. Eqs. (22) and (23) are the Klein-Gordon equation.

In 1926, Oskar Klein and Walter Gordon proposed this relativistic wave equation. However, it was found later that this equation is not suitable for one particle because the probability density is not a positive quantity, which means the particle can be created and annihilated arbitrarily in Klein-Gordon equation [14]. The extended Feynman path integral shows the explanation for this non-positive probability density here. The wave function that is determined by Klein-Gordon equation is the mixed state of the particle and its antiparticle. Because particles and antiparticles can be annihilated each other to a vacuum state, and the vacuum state can produce particles and antiparticles, so the mixed state with superposition state of a particle and an antiparticle is a matter of course of a non-positive quantity. This is the physical interpretation for Klein-Cordon equation by EFPI theory.

4.4. The extended Feynman path integral and density-flux equation

In quantum mechanics, the continuity equation describes the conservation of probability density in the transport process. It is a local form of conservation laws. It says the probability cannot be created or annihilated and, at the same time, also cannot be teleported from one place to another. However, in the extended Feynman path integral, the density-flux equation will be revised, and the local conservation is broken.

In extended Feynman path integral, the density-flux equation can be written as the following formula:

$$\frac{\partial\rho(r,t)}{\partial t} + \nabla\cdot j + \sum\nolimits_{n=2}^{\infty} B_n \nabla^n \cdot Q_n(r,t) = 0 \tag{24}$$

where $Q_n(r,t) = \psi^*\nabla^n\psi - \psi\nabla^n\psi^*$ and $B_n = -(-i\hbar)^{2n-1}c^2 n/\left(mc^2\right)^{2n-1}$. The last term in the right of Eq. (24) is caused by relativistic effect and breaks the local conservation.

4.5. The wave function collapse in extended Feynman path integral

From the theory of Neumann, the difficulties of understanding collapse are the probability, which seems incompatible with the deterministic time-evolution equation, and the instantaneity, which seems that it breaks the special relativity theory. In this section, we will show that these puzzling characters are due to the potential noise and nonlocal correlation (or relativistic effect).

Let us return to Eq. (9). The superluminal paths are included when we calculate the propagator. The superluminal paths will support complex phases in Eq. (9), and these phases cannot be canceled by each other like the real phases in Feynman path integral theory. These complex phases are the main culprits that cause the nonlocal correlation.

To describe this mechanism concisely, the nonlocal correlation produced in 1D space is just detailed here. Assume a system in the potential field with the scalar potential $U(x)$ and vector $A_0(x)$. A potential noise $A_I(t)$ is under this system and satisfies the white noise equations, namely:

$$\langle A_I(t_1)A_I(t_0)\rangle = \frac{2mk_bT}{\eta}\delta(t_1 - t_0); \langle A_I(t)\rangle = 0 \tag{25}$$

The Hamiltonian of this system is then

$$H = \sqrt{m^2c^4 + \left(-i\hbar\partial_x - (A_0 + A_I)\right)^2 c^2} + V(x) \tag{26}$$

And we define a new Hamiltonian without potential noise as

$$H_0 = \sqrt{m^2c^4 + \left(-i\hbar\partial_x - A_0\right)^2 c^2} + V(x) \tag{27}$$

We will see later that H_0 is very important in quantum measurement, because it determines the basis-state-space that the wave function collapses into. The basis-preferred problem puzzles us for many years; we do not know why the system measured prefers to collapse into some set of basis state. According to the extended Feynman path integral theory, the preferred basis is depended by the Hamiltonian H_0. This will be detailed in the following.

Considering a minimum time-evolution process, the propagator is

$$F(x_1, x_0; t_0 + \varepsilon, t_0) = \hat{R}\sqrt{\frac{c}{i\eta}}\exp(-mc|\eta|\hbar^{-1} + i\hbar^{-1}\int_{x-\eta}^{x} A_0(x_0, t)dx_0) \tag{28}$$

Because the term $\int_{x-\eta}^{x} A_0(x_0, t)d_0$ exists in the integral formula of Eq. (28), then $\lim_{\varepsilon\to 0} F(x_1, x_0; t_0 + \varepsilon, t_0) \neq \delta(x_1 - x_0)$. This is different from the normal propagator $K(x, x_0; t_0 + \varepsilon, t_0)$ shown in Eq. (2), because $\lim_{\varepsilon\to 0} K(x, x_0; t_0 + \varepsilon, t_0) = \delta(x - x_0)$. This difference, caused by relativistic effect of paths, is the root that produces the nonlocality in quantum measurement process.

In fact:

$$\int_{t_0}^{t_0+\varepsilon} -mc^2\sqrt{1 - v^2/c^2}\, dt = \int_{t_0}^{t_0+\varepsilon} -mc^2\sqrt{(dt)^2 - dx^2/c^2} = imc(\Delta x);$$

$$\int_{t_0}^{t_0+\varepsilon} (-U(x) + Av)dt = A\Delta x$$

Therefore

$$F(x_1, x_0; t_0 + \varepsilon, t_0) = \frac{1}{\sqrt{2i\pi\hbar c^2}}\frac{H'}{\sqrt{mc^2 + H'}}\sqrt{\frac{c}{i\eta}}\exp(-mc|\eta|\hbar^{-1} + i\hbar^{-1}\int_{x-\eta}^{x} A_0(x_0, t)dx_0) \tag{29}$$

$$\psi_- = 1/\sqrt{2}(\phi_+ - \phi_-)$$

$\lim_{\varepsilon\to 0} F(x_1, x_0; t_0 + \varepsilon, t_0) \neq \delta(x_0 - x_0)$ means the change of arbitrary point should spend time to propagate the other point and exhibit stronge nonlocal space-time character. If the value of wave function at $x = x_0$ changes, the whole wave function will change for the nonlocal propagator. In the followings, we will detail this character.

We define $\widehat{R}_0 = \frac{1}{\sqrt{2i\pi\hbar c^2}}\frac{H_0}{\sqrt{mc^2 + H_0}}$; then

$$\widehat{R} \approx \widehat{R}_0\left(1 - \frac{A_I c^2(\widehat{p} - A_0)}{H_0}\right) \tag{30}$$

After this definition, we will show how the measurement happens under the potential noise. Considering an initial state with the form $\psi(x, t_0) = \sum_m a_m \varphi_m(x)$, where φ_m is the eigenstate of H_0, if we put the potential noise in this system, the initial state will change. We denote the evolution state in arbitrary time t as $\psi(x, t)$. The $\psi(x, t)$ can be expanded with basis states φ_m *as* $\psi(x, t) = \sum_m a_m \varphi_m$. The task for us is to find out the varying value of a_m under each perturbational noise:

$$
\begin{aligned}
a_n(t+\delta) &= \int_{-\infty}^{\infty} \varphi_n(u,t) * \phi(u,t)du \\
&= \int_{-\infty}^{\infty} \varphi_n(u,t) * c^{\frac{1}{2}}\Re(u,t) * \int_{-\infty}^{\infty} (i\eta)^{-\frac{1}{2}} \exp\left(-\frac{mc|\eta|}{\hbar}\right) \exp(\xi_I)\phi(u-\eta,t)\, d\eta \\
&= \sum_m a_m(t)\, \lambda_{n,m}(t-\delta)\left(1+\frac{A_I c^2 p_n}{E_n}\delta_{n,m}\right)
\end{aligned}
$$

After rearranging the equation above, we get

$$a_n(t+\delta) = \sum\nolimits_m a_m(t) D_{m,n}(t-\delta) \tag{31}$$

where

$$D_{m,n}(t-\delta) = \lambda_{n,m}(t-\delta)\left(1+\frac{A_I c^2 p_n}{E_n}\delta_{n,m}\right)$$

$$\lambda_{n,m}(t-\delta) = \int_{-\infty}^{+\infty} \varphi_n(x) R(x, t-\delta) * \widehat{R}_0^{-1} \varphi_m(x) dx$$

$$R(x,t) = \frac{\psi(x,t)}{\widehat{R}_0^{-1}\psi(x,t)}$$

δ is the time interval of the neighbor potential noise pulses. In fact, to simulate the process of quantum measurement under potential noise, we let

$$A_I = \sum\nolimits_{n=0}^{\infty} \left(\frac{2mk_b T}{\eta\Delta}\right)^{1/2} Random(n)(\theta(t-n\delta) - \theta(t-(n-1)\delta)) \tag{32}$$

We simulate the collapse process of a wave function with the form $|\psi\rangle = 1/2|0\rangle + \sqrt{3}/2|1\rangle$, where $|0\rangle$ and $|1\rangle$ are the harmonic-oscillator basis. According the simulation, we show the $|\psi\rangle$ will randomly collapse into $|0\rangle$ or $|1\rangle$ quickly (**Figure 2**).

5. Conclusions

Measurement, in quantum theory, is not just a theory concerning the Schrödinger cat that is alive or dead, or the moon being here or not, but also the key and basis to the problem of the interpretation of quantum mechanics. In fact, the different views for the quantum measurement yield different interpretation for quantum mechanics, such as the Copenhagen interpretation, relative-state interpretation, Bohmian mechanics and so on. It has attracted many

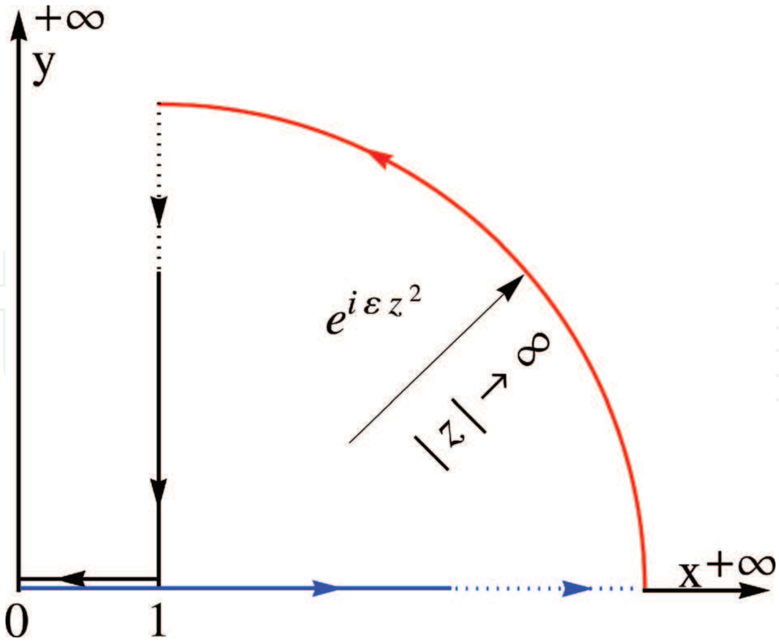

Figure 2. The process of collapse under a "potential noise". (a) The red line denotes the absolute value of probability amplitude $a_0(t)$ with the initial value 1/2, and the blue one denotes $a_1(t)$ with the initial value $\sqrt{3}/2$. The black oscillatory line is the function of potential. The different sets of noise cause the different collapse results. According the simulation, the process time of collapse is 0.3 ns in the top picture and 0.1 ns in the bottom picture. (b) The function of A_I shown in Eq. (32).

attentions of physicists since the beginning of the quantum theory establishment, but there is still no consensus. The measurement problem blocks up the way for us to understand the nonlocality and manipulate quantum state. Can the quantum measurement be controlled? Can we get the definite output we want under every measurement? If the quantum measurement can be controlled, the teleportation without classical communication channel can be realized, and the aim of superfast manipulation for quantum state will arrive. We can even transfer the energy thought nonlocality under controlled quantum measurement and make more novel encryption scheme for quantum communication. However, the key problem is "can we control the quantum measurement?" If yes, how? If no, why?

The extended Feynman path integral mechanism answered this question. According to this mechanism, the character, "stochastic output" and "instantaneous collapse process" of quantum measurement are rooted in the "random" potential noise and "nonlocal" wave function inner correlation. The "nonlocality" is caused by the "relativistic effect" of superluminal paths in path integral theory. The superluminal paths will support a complex action function S in Eq. (4) for the expression $\sqrt{1 - v^2/c^2}$ of S. This complex action that acted as a phase in integral theory cannot be canceled and makes $F(x_1, x_0; t_0, t_0) \neq \delta(x_1 - x_0)$. This relation reveals that the propagator is no longer a local correlation. All points in space are correlated simultaneously, and any local perturbation will simultaneously transfer into the whole space. The extended Feynman path integral gives a simulation for two-energy-level system and exhibits that the potential noise can indeed lead to the collapse state randomly and rapidly. Therefore, the key to control the quantum measurement is to control the potential noise exactly. “Potential noise” is caused by thermal fluctuation of potential filed or irregularity potential boundary. How to control this potential noise is still an unsolved topic.

The extended Feynman path integral mechanism also solves the “basis-preferred” problem in quantum measurement. It exhibits the reason that the state prefers to collapse some set of basis states, which is due to the main Hamiltonian H_0 defined in Eq. (27). H_0 is the Hamiltonian that contains no noise. The eigenstates are the basis state that wave function prefers to collapse into.

The extended Feynman path integral mechanism shows the relation between “quantum measurement” and “unitary evolution operation”. They are one and the same thing but are departed by jumpy potential noise. In mathematics, the function of potential noise is nowhere differentiable functions, and therefore, the path integral shown in Eq. (4) is not the regular path integral function under a noised potential. This is the main difference between “quantum measurement” and “unitary evolution operation” in mathematics. In physics, each potential noise point can be quickly absorbed by wave function through the nonlocality correlation, and the amounts of noise points will quickly accumulate to be a big quantity to change the whole wave function $|\widehat{R}|$.

Additionally, besides the potential noise, the condition that the quantum measurement happens is that the interaction of system and environment should be big enough to distinguish the preferred basis state “$\{\varphi_n\}$”. If the interaction is not big enough, $\left\langle \varphi_n|\widehat{R}|\varphi_n \right\rangle \approx \left\langle \varphi_m|\widehat{R}|\varphi_m \right\rangle$ and then $D_{mn} \to \delta_{m,n}$ in Eq. (31), then the collapse will not happen. In other words, the instrument that can realize the quantum measurement should be “macro” enough to produce enough noise and have big enough energy gaps of a system measured.

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