Chapter 6

Malmquist Index with Time Series to Data Envelopment Analysis

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Additional information is available at the end of the chapter

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Abstract

This chapter presents a new temporal data envelopment analysis (DEA) model that overcomes some weaknesses of the window analysis and Malmquist index. New model allows to work with time series. For each series the best of a set of ARIMA models is selected, and a forecast for two periods it is possible. Changes in efficiency of different decision making units (DMUs) are analyzed and the use of temporal series makes it easy to include Malmquist forecasts. The implementation of the new model in business administration or supply chain management can be useful because it considers more than two periods in contrast with classical Malmquist method, for that, control of efficiency over time is improved by changing deterministic univariate variables for time series. The last them have the structure of correlation and they get even more real modeling.

Keywords: data envelopment analysis, Malmquist index, DEA change over time, time series, forecast

1. Introduction

Data envelopment analysis (DEA) is a widely used methodology to evaluate the performance of different organizational systems. A good definition of DEA can be found in [1], it is a nonparametric technique used to evaluate the relative efficiencies of a set of DMUs (decision making units). Examples of DMUs can be a factories, countries, sections of factories, universities, supply chains and hospitals. For multiple applications, here are a few of the most notable, performance in healthcare sector [2], assessing the efficiency of wastewater treatment plants [3], environmental policy [4], oil refineries [5], finances institutions [6] and environmental performance [7, 8].

DEA was created by Charnes et al. [9] and the methodology consists in compares DMUs with a frontier of efficiency. The basic DEA model is the CCR (Charnes, Cooper and Rhodes). There are two CCR models, the primal form CCR and dual form CCR.

The dual form CCR is based on optimal weights (u^*, v^*). A linear programming obtain an optimal solution to maximize the ratio $\frac{virtual-output}{virtual-input} = \frac{u_1 y_{1o} + \ldots + u_s y_{so}}{v_1 x_{1o} + \ldots + v_m x_{mo}}$,

where s is the number of outputs, m is the number of inputs and (u, v) are the weights (multipliers) assigned to different input and output, y_{jo} is the jth output of DMU observed $(j = 1, \ldots, s)$, x_{io} is the ith input of DMU observed $(i = 1, \ldots, m)$, more details are given on [10].

The ratio above needs the constraints that the ratio of virtual output vs. virtual input should not exceed 1 for every DMU. So the complete CCR model is

$$\begin{gathered}
\theta = \frac{u_1 y_{1o} + \ldots + u_s y_{so}}{v_1 x_{1o} + \ldots + v_m x_{mo}} \\
\max_{u,\, v} \\
\textit{Subject to} \\
\frac{u_1 y_{1o} + \ldots + u_s y_{so}}{v_1 x_{1o} + \ldots + v_m x_{mo}} \le 1 \\
v_1, v_2, \ldots, v_m \ge 0 \\
u_1, u_2, \ldots, u_m \ge 0
\end{gathered} \tag{1}$$

The primal form of CCR (development form) is based on distances from frontier efficiency. The following model is a primal CCR input oriented in its matrix form:

$$\begin{gathered}
\min_{\lambda,\, \theta} \theta \\
\text{Subject to} \\
\theta x_o - X\lambda \ge 0 \\
Y\lambda \ge y_o \\
\lambda \ge 0 \\
\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T,
\end{gathered} \tag{2}$$

where n is the number of DMUs, X and Y are the matrixes of input and output respectively, λ is a semipositive vector in R^n. In DEA theory, the construction of primal CCR needs a production possibility set (PPS). The PPS, denoted by P, has 4 properties. The 4th propertie says "*Any semipositive linear combination of activities in P belongs to P*". In this property vector λ appears. But, if an intuitive explanation is wanted, vector λ helps building the frontier of efficiency. Details in [10].

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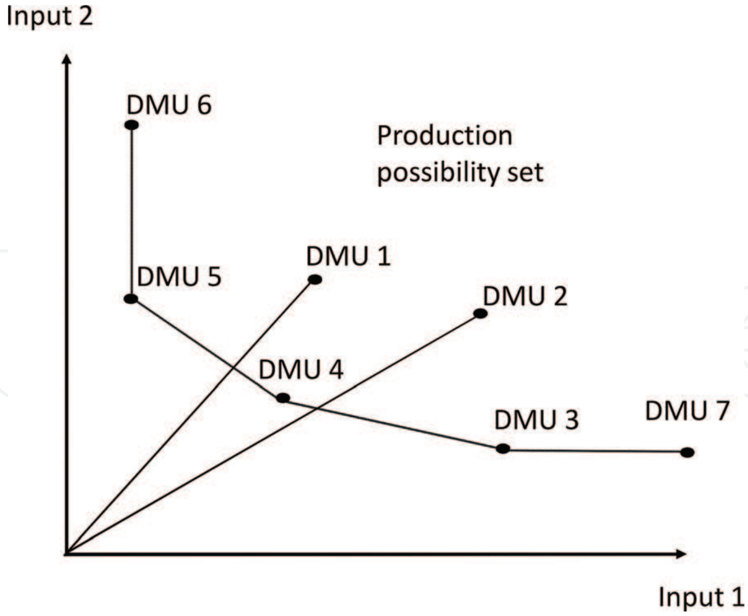

Figure 1. DMUs, frontier efficiency and production possibility set.

In order to have a better understanding, **Figure 1** shows a set of DMUs with 2 inputs and 1 output normalized to 1 (imagine it in the third dimension). Note that, in **Figure 1**, the frontier efficiency is formed by DMUs 3, 4, 5, 6 and 7 through λ_i, in this case, $i = 1, \dots, 7$. DMUs 1 and 2 are less efficient because they need a contraction to get the frontier. DMU 1 is more efficient than DMU 2, because the distance between DMU 1 and the frontier is shorter than the distance between DMU 2 and frontier. Note the production possibility set and note that the contraction is equivalent to θ in model Eq. (2). Details in [10].

The model Eq. (2) has a variation when it is output oriented. The following model is the CCR output oriented in its matrix form:

$$\begin{gathered}\max_{\eta,\,\mu} \theta \\ \text{Subject to} \\ x_o - X\mu \geq 0 \\ \eta y_o - Y\mu \leq 0 \\ \mu \geq 0 \\ \mu = (\mu_1, \mu_2, \dots, \mu_n)^T,\end{gathered} \tag{3}$$

There are many areas of research in DEA, but the most important have been classified in [11]:

- Several model to measure efficiency. This category includes CCR input and output oriented. Several models also includes radial and non-radial models, constant return scale, variable return scale [12] and additive model [13] which combines both, input oriented and output oriented.

- Methods with multiplier restrictions. This category includes absolute multiplier restrictions [14], cone ratio restrictions [15], assurance regions [16, 17].
- Special considerations regarding the status of variables. The variables non-controllable and non-discretionary variables [18] are include here. It also treats ordinal variable data [19–21] and categorical variables [19].
- Data variation. This category is divided in sensitivity analysis [23–26], data uncertainty and probability-based models [27], and time series data. In the last subcategory are the Malmquist index and window analysis.

According to classification above, one of the categories in which efficiency models are classified is the named variation in the data. In this category there are two models, the based in probabilities and data in time series. The last one, also known as efficiency change over time.

In classical DEA literature, the Malmquist index (MI) and window analysis are the unique methods of DEA change over time. There is some evidence that no new temporal DEA methods have appeared, only MI and window analysis [20, 21]. In recent years, temporal Malmquist with multiple periods has studied in [22], Authors calculated MI in years 2000–2009 for environmental assessment in Europe, Asia and America. Nevertheless, in this investigation, there is no MI that gathers the historical performance in a single measure, nor does it consider time series techniques and it does not make combinations of all the possible temporary changes according to the number of periods, that is, years 2000–2001, 2000–2002, 2000-2003, …, 2000–2009, 2001–2002, 2001–2003, …, 2001–2009, …, 2008–2009. That would be an interesting revision and a new vision of a temporal DEA.

This chapter propose a new temporal MI. This chapter is organized as follows. In Section 2 general considerations of MI are presented, the effects in efficiency and definition. Section 3 shows a brief description of window analysis. Section 4 presents the historical and forecast Malmquist that is a new methodology in DEA change over time. This section presents an application and it gives an example. In Section 5 the conclusions.

2. Malmquist index

The first contribution to DEA change over time is the MI, see [23]. The first construction of this index based on DEA methodology is the radial Malmquist [24], however there were efforts to consider non-radial Malmquist based in slacks measures [25, 26]. MI evaluates the productivity change of a DMU between two periods and is an example in comparative statistics analysis [10]. It is defined as the product of "Catch-up" and "Frontier-shift" terms. The catch up (or recovery) is the term that is calculated to study the effect of growth or deterioration in a DMU. The frontier shift (or innovation) term is used to verify the change in the efficient frontiers between the two time periods. In the following subsection these concepts will be explained in more detail.

2.1. Catch-up and frontier-shift effects

The best and clearest way to describe frontier-shift term is to consider a case with a single input and output, see **Figure 2**. In this one, the points $(x_0, y_0)^1$ and $(x_0, y_0)^2$ symbolize the vector of input and output of the same DMU observed in periods 1 and 2 respectively. The catch-up effect from period 1 to 2 is defined as following expression:

$$Catch - up = \frac{Efficiency\ of\ (x_0, y_0)^2\ with\ respect\ to\ period\ 2\ frontier}{Efficiency\ of\ (x_0, y_0)^1\ with\ respect\ to\ period\ 1\ frontier}$$

The catch-up effect in an input orientation can be expressed by:

$$\text{Catch} - \text{up} = \frac{DE/Dx_0^2}{AC/Ax_0^1}, \tag{4}$$

Where DE symbolizes, in **Figure 2**, the distance between D and E, Dx_0^2 symbolizes the distance between D and abscissa of the point $(x_0, y_0)^2$. Similarly AC is the distance between A and C, A x_0^1 is the distance between A and abscissa of the point $(x_0, y_0)^1$.

The change of $(x_0, y_0)^1$ from frontier of period 1 to frontier of period 2 is called the frontier shift effect at $(x_0, y_0)^1$ and it is evaluate by:

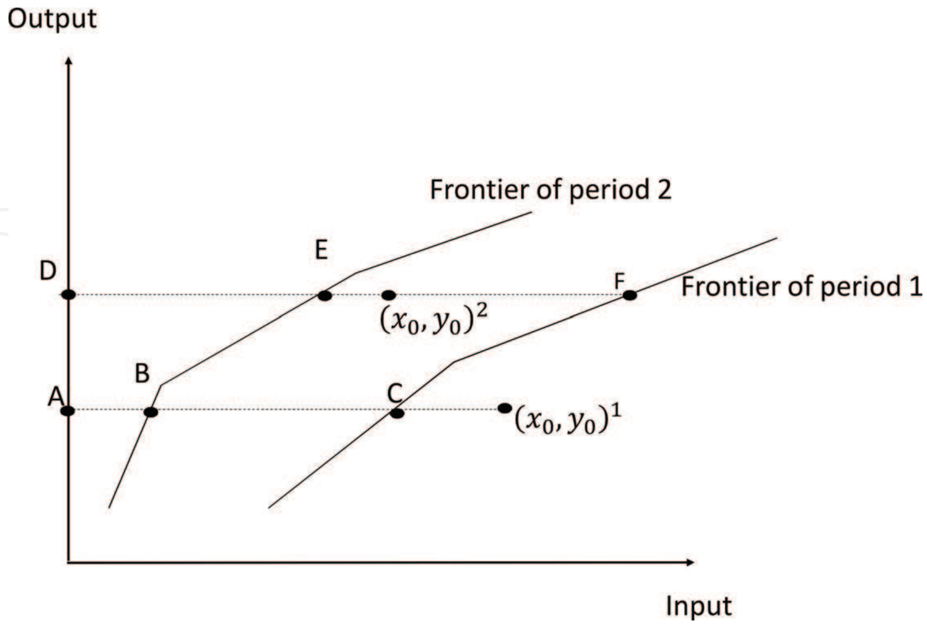

Figure 2. Vectors of inputs and outputs in different periods.

$$\phi_1 = \frac{AC}{AB}. \tag{5}$$

In Eq. (5), if numerator and denominator are divided by the distance between A and abscissa of point $(x_0, y_0)^1$, then:

$$\phi_1 = \frac{\frac{AC}{Ax_0^1}}{\frac{AB}{Ax_0^1}} = \frac{Efficiency\ of\ (x_0, y_0)^1\ with\ respect\ to\ period\ 1\ frontier}{Efficiency\ of\ (x_0, y_0)^1\ with\ respect\ to\ period\ 2\ frontier}. \tag{6}$$

Similarly, the change of $(x_0, y_0)^2$ from frontier of period 1 to frontier of period 2 is called the frontier shift effect at $(x_0, y_0)^2$ and it is evaluate by Eq. (7) or Eq. (8).

$$\phi_2 = \frac{DF}{DE}. \tag{7}$$

$$\phi_2 = \frac{\frac{DF}{Dx_0^2}}{\frac{DE}{Dx_0^2}} = \frac{Efficiency\ of\ (x_0, y_0)^2\ with\ respect\ to\ period\ 1\ frontier}{Efficiency\ of\ (x_0, y_0)^2\ with\ respect\ to\ period\ 2\ frontier}. \tag{8}$$

Using ϕ_1 and ϕ_2, in [10] define the frontier-shift effect by the geometric mean of them, that is:

$$\text{Frontier} - \text{shift} = \phi = \sqrt{\phi_1 \phi_2}. \tag{9}$$

2.2. Definition of MI

The MI is computed as the product of Catch-up and Frontier shift terms, that is:

$$MI = \text{Catch} - \text{up} \times \text{Frontier shift}. \tag{10}$$

According to Eqs. (4)–(10), MI can be calculated by:

$$MI = \frac{Ax_0^1}{Dx_0^2}\sqrt{\frac{DFDE}{ACAB}},$$

where the first term represents the relative change in performance and the second represents the relative change in frontier used to evaluate these performance, see [10].

The Eq. (4) can be expressed by other, using another notation for efficiency score of DMU as follows:

$$Catch - up = \frac{\delta^2\left((x_0, y_0)^2\right)}{\delta^1\left((x_0, y_0)^1\right)}, \tag{11}$$

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where $\delta^2\left((x_0, y_0)^2\right)$ denotes the efficiency of DMU observed in period 2 measures by the frontier technology 2 and $\delta^1\left((x_0, y_0)^2\right)$ denotes the efficiency of DMU observed in period 2 measures by the frontier technology 1. In Eq. (11), δ^1 refers frontier efficiency of period 1 and δ^2 refers frontier efficiency of period 2. According to Eqs. (6), (8) and (9), the frontier-shift effect can be expressed by

$$\text{Frontier} - \text{shift} = \left[\frac{\delta^1\left((x_0, y_0)^1\right)}{\delta^2\left((x_0, y_0)^1\right)} \times \frac{\delta^1\left((x_0, y_0)^2\right)}{\delta^2\left((x_0, y_0)^2\right)}\right]^{\frac{1}{2}}. \tag{12}$$

With Eqs. (11) and (12) the following formula is obtained to calculate the MI:

$$MI = \left[\frac{\delta^1\left((x_0, y_0)^2\right)}{\delta^1\left((x_0, y_0)^1\right)} \times \frac{\delta^2\left((x_0, y_0)^2\right)}{\delta^2\left((x_0, y_0)^1\right)}\right]^{1/2}. \tag{13}$$

Using the Eq. (13); the facts that $s = 1, 2$ and $t = 1, 2$; and the notation of a DEA model; the input-oriented radial MI is obtained by the scores of θ given the following 4 linear programs (making s = 1, 2 and t = 1, 2):

$$\begin{gathered}
\delta^s\left((x_0, y_0)^t\right) = \min_{\theta,\lambda} \theta \\
\text{subject} \\
\theta x_0^t \geq X^s \lambda \\
y_0^t \leq Y^s \lambda \\
L \leq e\lambda \leq U \\
\lambda \geq 0,
\end{gathered} \tag{14}$$

where the vector $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_N)'$; variables λ_i help build the envelopment frontier of efficiency; the vector $e = (1, 1, \dots, 1)$ size $1 \times N$; X is the input matrix and Y is the output matrix. The matrixes X and Y are arranged such the number of rows are the number of inputs and outputs respectively, and the number of columns are the number of DMUs. For each pair of values (s, t), the model Eq. (14) is calculated N times, where N is the number of DMUs.

In Eq. (14), if $(L, U) = (1, 1)$ then it is a BCC model (Banker Charnes and Cooper), it means that efficiencies are calculated with variable return scale (VRS), see [27], which makes model Eq. (14) suitable to compare DMUs with different sizes. If $(L, U) = (0, \infty)$ then Eq. (14) is a CCR (Charnes, Cooper and Rhodes), it means that efficiencies are calculated with constant return scale (CRS), which makes it suitable to compare DMUs with similar sizes.

The output oriented MI is shown in the following model:

$$\left((x_0, y_0)^t\right) = \min_{\theta,\lambda} \theta$$
$$\text{subject}$$
$$x_0^t \geq X^s \lambda$$
$$\left(\frac{1}{\theta}\right) y_0^t \leq Y^s \lambda \tag{15}$$
$$L \leq e\lambda \leq U$$
$$\lambda \geq 0,$$

3. Window analysis

The second contribution of temporal DEA is the window analysis (WA) created by G. Klopp in his doctoral thesis in 1985, see [10]. WA treats each DMU as if it were different in each period of time. Having N time periods, n DMUs and w windows for each DMU, the total of efficiencies to calculate would be $n \times N \times w$, but in WA, a "length of window" p is selected and $n \times p \times w$ efficiencies are calculated instead of total of them. Then, the statistical measures are calculated like mean and variance. In this way, it is possible to verify if a DMU shows stability, deterioration or improvement. In **Table 1**, just to illustrate, there are 2 DMUs of n of them. Each DMU is analyzed in 8 periods, length of window is 5 and there are 4 windows. Note that it is not necessary to evaluate the whole of efficiencies because the window keeps sliding and it is possible to calculate descriptive measures like mean, variance and Range.

According with **Table 1**, it is easy to observe that window analysis does not consider the correlation structure of efficiencies and it does not use statistical technique to estimate efficiencies. Window analysis is a DEA change over time, but there are not time series in input and output variables.

DMU	P1	P2	P3	P4	P5	P6	P7	P8	Mean	Var	Range
1	1.00	0.90	0.93	0.8	0.98						
		0.99	0.90	1.00	1.00	1.00			0.956	0.0032	0.2
			0.85	1.00	1.00	1.00	0.93				
				0.98	0.92	1.00	1.00	0.95			
2	0.78	0.97	0.88	0.92	1.00						
		0.89	0.92	0.98	0.99	0.93			0.941	0.0034	0.22
			1.00	1.00	0.85	0.92	0.93				
				1.00	1.00	1.00	0.92	0.95			
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Table 1. Window analysis for two of n DMUs.

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4. Historical and forecast Malmquist

Some weaknesses can be highlighted in temporal DEA with its MI and WA techniques. Those techniques are designed for short periods of time, they do not consider random error in the variables and they do not use the dependence structure to estimate the efficiencies. For example, MI works with two periods and forecast is not possible.

There is not a methodology in DEA literature that calculates efficiency taking into account the history of input and output variables, although it could be argued that WA makes it, the history of efficiency is considered in this technique, but not the history of variables neither its correlation structure. There is no MI that gathers, in a single measure, all the possible changes of efficiency period to period.

Given the weaknesses above, a historical and forecast Malmquist is presented. The advantage of this approach is the consideration of stationary and non-stationary time series. These can be large time series and using the possibility to make forecast.

The historical Malmquist is defined in [28] as:

$$Mh = \sqrt[c]{M_{1,2}M_{1,3}\dots M_{1,n}M_{2,3}M_{2,4}\dots M_{2,n}\dots M_{n-2,n-1}M_{n-1,n}}, \qquad (16)$$

where $M_{i,j}$ is the classical Malmquist index calculated in periods i and j, $i = 1, 2, \dots, n-1$; $j = 1, 2, \dots, n$; c is the number of combinations of periods given by $\binom{n}{2}$. The principal assumption is that all time series (input and output) have the same number of periods. In the new Mh, the different $M_{i,j}$ are obtained, specifically $\binom{n}{2}$ of them, and each one is calculated by model Eq. (14) or Eq. (15) depending on orientation, but now, $t = 1, 2, \dots, n$ and $s = 1, 2, \dots, n$. To calculate Mh in Eq. (16) the geometric mean is chosen because it is appropriate to average indexes.

4.1. Application in a context

To illustrate the application of historical Malmquist, the five largest industrial centers of Colombia were chosen. They are Antioquia, Atlántico, Bogotá, Cundinamarca (without Bogotá) and Valle. The information was obtained from "Departamento Administrativo Nacional de Estadística de Colombia (DANE)" which translates "National Administrative Department of Statistics of Colombia" from yearly manufacturing survey conducted from 1992 to 2010.

For each industrial center, input and output variables are selected, each of them, with values in time. Each of DMU (Antioquia, Atlántico, Bogotá, Cundinamarca and Valle) has three output variables and five input variables. The following input variables are defined (i) electric energy consumption in Kilowatts-hour, (ii) total assets, (iii) intermediate consumption, (iv) social benefits and (v) salaries. The following output variables are defined (i) permanent remunerated staff

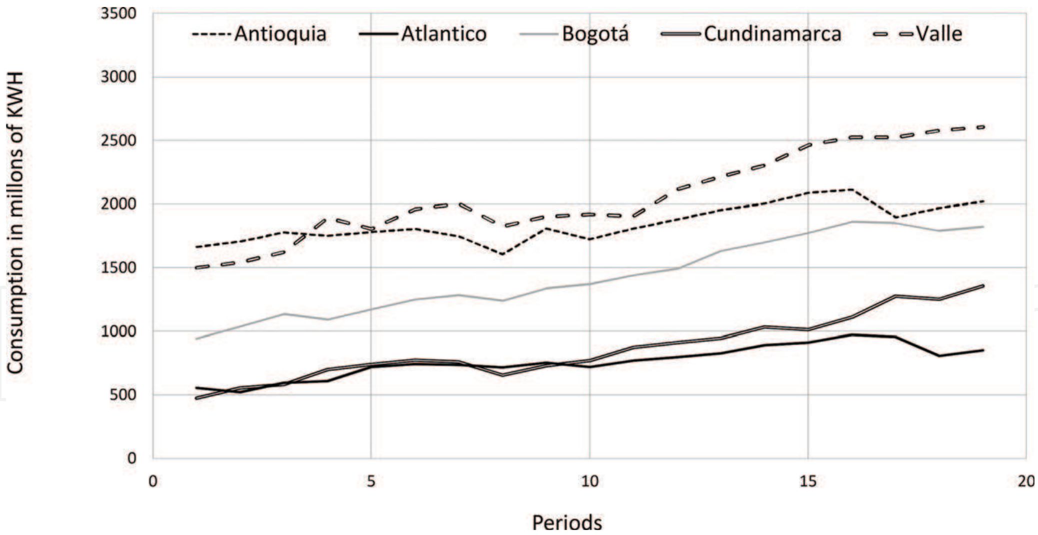

Figure 3. Time series of consumption in KWH of 5 industrial centers.

(PRS), (ii) gross production (GP) and (iii) value added (VA). For details of meaning of variables see [29]. It is important to know that each input or output variable is a time series, then, there are 40 time series (5 DMUs $\times$ 8 variables).

The **Figure 3** shows 5 of 40 time series analyzed, these 5 series correspond to consumption of electric energy in millions of Kilowatts-Hour in 5 industrial center (5 DMUs).

4.2. Using time series for application of historical Malmquist

To simplify calculations, all values of variables are divided by 1,000,000, except PRS that was divided by 1000. All the $M_{i,j}$ were calculated suing model Eq. (15) obtaining Mh with $n = 19$ periods. Also Mh is calculated with $n = 21$ periods, in other words including no observed periods 20 and 21, for that, forecasts were established from the following set ARIMA models: ARIMA(1,1,0), ARIMA(2,1,0), ARIMA(1,2,0), ARIMA(2,2,0), ARIMA(0,0,1), ARIMA(0,1,1), ARIMA(0,1,2), ARIMA(0,2,1) and ARIMA(0,2,2).

Each of the 40 time series was modeled through all the above ARIMA and for forecast, in each series, the model with less mean square error (MSE) was chosen.

Table 2 shows the estimations of ARIMA models for time series of Antioquia in PRS variable. This table also shows that ARIMA(0,1,2) model is the best, with less (MSE) and less Bayesian information criterion (BIC). The same procedure for the others two output variables and five input variables was applied, as well as the others industrial centers, for a total of 40 tables like **Table 2**.

Table 3 shows the forecast for Antioquia time series and PRS variable, in addition, it shows the 95% confidence intervals. The same procedure for the others two output variables and five input variables was applied, as well as the others industrial centers, for a total of 40 tables like **Table 3**. Each of these forecasts were used in Eq. (13), model Eqs. (15) and (16), and their results

	MSE	Coefficients		p value	BIC
ARIMA(1,1,0)	17.928	AR 1	0.651	0.005	3.024
		Constant	−0.293	0.777	
ARIMA(2,1,0)	17.159	AR 1	0.410	0.114	3.071
		AR 2	0.403	0.113	
		Constant	0.045	0.968	
ARIMA(1,2,0)	17.770	AR 1	−0.496	0.041	2.951
		Constant	−0.160	0.878	
ARIMA(2,2,0)	15.778	AR 1	−0.339	0.19	2.918
		AR 2	0.433	0.095	
		Constant	−0.040	0.967	
ARIMA(0,0,1)	134.09	MA 1	−0.947	0.000	5.097
		Constant	98.070	0.000	
ARIMA(0,1,1)	22.830	MA 1	−0.333	0.175	3.266
		Constant	−1.761	0.257	
ARIMA(0,1,2)	11.475	MA 1	−0.800	0	2.669
		MA 2	−0.972	0	
		Constant	−2.177	0.252	
ARIMA(0,2,1)	20.322	MA 1	0.287	0.265	3.085
		Constant	−0.150	0.85	
ARIMA(0,2,2)	18.723	MA 1	0.308	0.247	3.089
		MA 2	−0.337	0.208	
		Constant	−0.170	0.878	

Table 2. ARIMA models for Antioquia time series, variable PRS.

allow to calculate the historical Malmquist with forecast, so, the observed periods and no observed periods (20 and 21) were taken into account.

Table 4 shows the estimations of parameters of ARIMA models selected with less mean square error, they correspond to Antioquia department with three output and five input variables. With models and parameters given in **Table 4**, a forecast is estimated like **Table 3**. The same procedure for the others industrial centers was applied. With these estimations, a good quality forecasts are available for each of temporal input and output variable in each industrial center. Using these forecasts in Eq. (13), model Eqs. (15) and (16), can also be applied to answer the following questions: What can be the historical performances of the DU taking into account the observed and predicted periods?, What the change of efficiency of all DMUs will be in the following two periods, although they do not yet observed? What the instant efficiencies will be in the following two periods, although they do not yet observed? These answers facilitate the

Period	Forecast	Lower limit	Upper limit
20	88.566	81.925	95.206
21	88.123	74.447	101.800

Table 3. Forecast for Antioquia time series, variable PRS.

	Variable	Model	coefficient		p-value	Constant	p-value
Output variable	PRS	ARIMA(0,1,2)	MA 1	−0.800	0.000	−2.177	0.252
			MA 2	−0.972	0.000		
	GP	ARIMA(0,2,1)	MA 1	0.893	0.006	61.650	0.276
	VA	ARIMA(1,2,0)	AR 1	−0.514	0.035	−2.500	0.980
Input variable	Consumption KWH	ARIMA(0,2,1)	MA 1	0. 949	0.004	0.710	0.874
	Total assets	ARIMA(0,1,1)	MA 1	0.892	0.000	857.660	0.000
	Intermediate consumption	ARIMA(0,2,1)	MA 1	0.891	0.005	52.050	0.222
	Social benefits	ARIMA(0,2,1)	MA 1	1.120	0.000	−0.894	0.193
	Salaries	ARIMA(0,2,1)	MA 1	0.9569	0.000	0.812	0.538

Table 4. Estimation of parameters in ARIMA model for Antioquia with 5 variables.

business and operational decision making processes because they would allow flagging of possible problems or failures in output or input levels.

4.3. Using historical Malmquist

Figure 4 shows the levels of Malmquist $M_{i,j}$, with $i = 1, \ldots, 18$ and $j = 1, \ldots, 19$; where $i = 1$ corresponds to year 1992 and $j = 19$ corresponds to year 2010. In total, in this figure there are $c = \binom{19}{2} = 171$ indexes, in other words, the Malmquist calculated with all the possible combinations of periods (1–2, 1–3, …, 1–19, 2–3, …, 2–19, …, 10–11, …, 10–19, …, 18–19). In **Figure 4**, the horizontal axis indicates the position of combinations taken in pairs of periods. The following code helps building this figure.

For *DMU =1 to 5 # for DMUs Antioquia, Atlántico, Bogotá, Cundinamarca and Valle*

  For *i=1 to 18*

    For *j=2 to 19*

      If *(j > i)* ***then***

        Run model Eq. (15) and obtain $\theta = M_{i,j}$

      Else

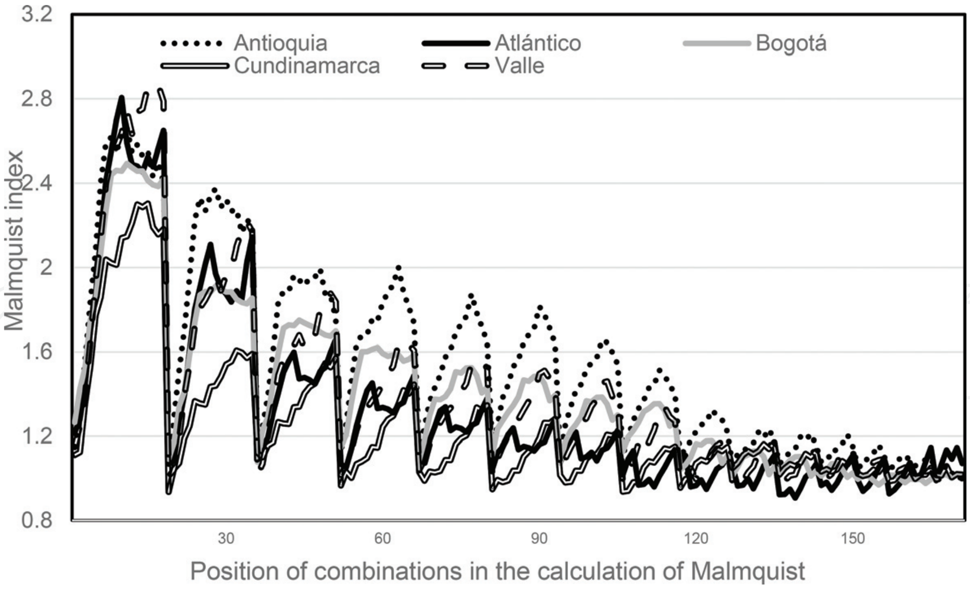

Figure 4. Malmquist indexes for all the combinations of periods.

End

End

End

End

In order to facilitate understanding of **Figure 4**, **Table 5** shows the equivalences between position of combinations and the pairs of periods involved.

Note the importance to consider a historical Malmquist over classical Malmquist. For example, in **Figure 4**, if time period 1–11 is considered (position 10 for this combination in horizontal axis according with **Table 5**) it is clear that Atlántico shows higher change of efficiency ($M_{1,11}$) than Cundinamarca and Antioquia. However, this figure shows that if the Malmquist index were calculated with periods 7–8 (position 94 for this combination in horizontal axis according with **Table 5**) or with periods 2–19 (position 35), Atlántico would begin to show less MI than Antioquia. While position for the combination increases, the MI of Atlántico decreases. For example, note that MI of Antioquia is better than the others with periods 6–19 (position 93) or periods 8–19 (position 116), in fact, after position 40 Antioquia achieves important levels of Malmquist.

The discussion above illustrates the disadvantage of considering MI only with 2 periods when DMUs have historical data and when historical efficiency is required. So, if historical efficiency is needed, MI is too weak, because It treats only two periods and there is not exist a measure that shows the changes of efficiency more than two periods and does not take account all possible changes between periods. For example, MI would be an unfair measure for DMU Antioquia if only periods 1–10 were considered (position 9), when it is evident that in **Figure 4**

Position	**1**	**2**	**…**	**18**	**19**	**20**	**…**	**35**	**36**	**…**	**51**	**52**
Periods involved	1–2	1–3	…	1–19	2–3	2–4	…	2–19	3–4	…	3–19	4–5
Position	…	66	67	…	80	81	…	93	94	…	105	106
Periods involved	…	4–19	5–6	…	5–19	6–7	…	6–19	7–8	…	7–19	8–9
Position	…	116	117	…	126	127	…	135	136	…	143	144
Periods involved	…	8–19	9–10	…	9–19	10–11	…	10–19	11–12	…	11–19	12–13
Position	…	150	151	…	156	157	158	…	161	162	…	165
Periods involved	…	12–19	13–14	…	13–19	14–15	14–16	…	14–19	15–16	…	15–19
Position	166	167	168	169	170	171						
Periods involved	16–17	16–18	16–19	17–18	17–19	18–19						

Table 5. Equivalences between position of combinations and two periods involved.

Antioquia is much stronger than the others. For that, historical Malmquist presented in this chapter possesses more information that explains all the history of DMUs.

4.4. Forecast Malmquist

There are two advantages of historical Malmquist over classical Malmquist. First, historical Malmquist is capable of calculating, in a single measure, the change of efficiency taking into account all the possible combinations of periods. Second, historical Malmquist is capable of calculating forecasts in changes of efficiencies, because it works with time series in its input and output variables.

In **Table 6** the historical Malmquist are calculated with Eq. (16) and they are ranked from high to low. This table also shows the M_h with forecasts in periods 20 and 21.

Table 6 shows temporal efficiencies obtained by forecast, it can be very useful in empresarial organizations. Note that the DMUs Antioquia and Bogotá present the best temporal index. **Table 7** shows what the expected MI can be for the change of periods 20 and 21, although they do not yet observed. Right of table the ranking of forecast Malmquist is presented, so the ranking of DMUs. In this forecast it is notable that Valle could increase in its participation while Bogotá could decrease in its participation. This analysis would be very important for performance in empresarial organizations, because it allows, through the techniques presented, to anticipate the results according the historical trend of data time series in input and output variables of DMUs. The difference between results in **Tables 6** and **7** is that in **Table 6** all possible combinations of periods have been used, while in **Table 7**, only two periods have been used, 20 and 21.

Forecast in Malmquist also helps to establish the instant efficiency, for example, in **Table 8** the efficiencies of periods 20 and 21 can be observed under deterministic output oriented CCR DEA with constant return to scale (CCR-CRS output oriented), see [10]. It can be expected a good participation in ranking of efficiency of Valle in instant periods 20 and 21, although they do not yet observed. So a good recommendation to Valle is that if It wish to obtain the first

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DMU	*Mh*	*Mh* with forecast including periods 20 and 21
Antioquia	1.500	1.473
Bogotá	1.360	1.338
Valle	1.340	1.336
Atlántico	1.268	1.270
Cundinamarca	1.203	1.209

Table 6. *Mh* indexes for 5 industrial centers.

DMU	Forecast Malmquist 20–21	DMU ranking	Forecast Malmquist 20–21 ranking
Antioquia	1.007	Antioquia	1.007
Atlántico	1.006	Atlántico	1.006
Bogotá	0.914	Valle	1.005
Cundinamarca	0.997	Cundinamarca	0.997
Valle	1.005	Bogotá	0.914

Table 7. Expected Malmquist according forecast of periods 20 and 21.

DMU	Forecast efficiencies for period 20	Forecast efficiencies for period 21
Antioquia	1	1
Atlántico	1	1
Bogotá	1	1
Cundinamarca	1	1
Valle	1.054	1.052

Table 8. Forecast efficiencies for periods 20 and 21.

position in DEA ranking, It must ensure the increase in the output levels and decrease in the input levels. Because otherwise, plus the fact unexpected increases in the input levels and decrease in the output levels are presented, Valle would have not a good classification in specific periods 20 and 21.

4.5. Analysis of catch-up and frontier effects in historical Malmquist

A great advantage that MI offers is that frontier shift and catch-up effects can be separated. In this analysis these effects have been separated for all the possible combinations of periods. In **Figure 5** the frontier effect is shown and **Figure 6** shows the catch-up effect. Note that frontier effect in **Figure 5** has a great similarity with MI in **Figure 4** with minor differences. The following code helps building the **Figure 5**.

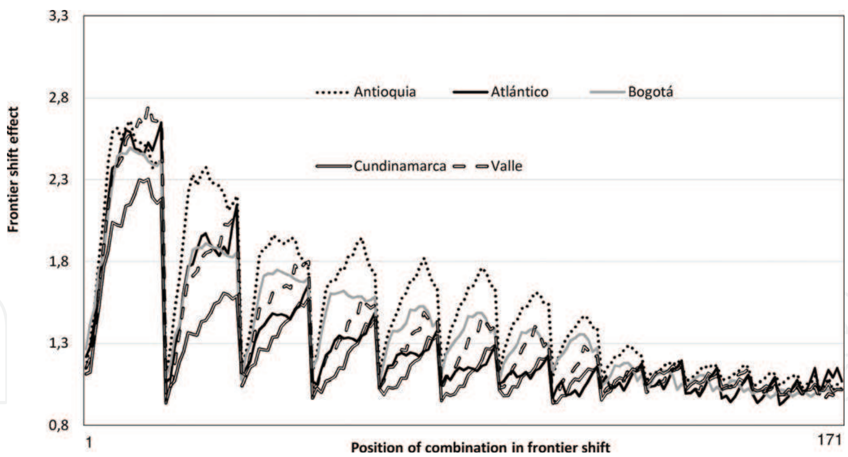

Figure 5. Frontier shift effect after it has been separated from MI.

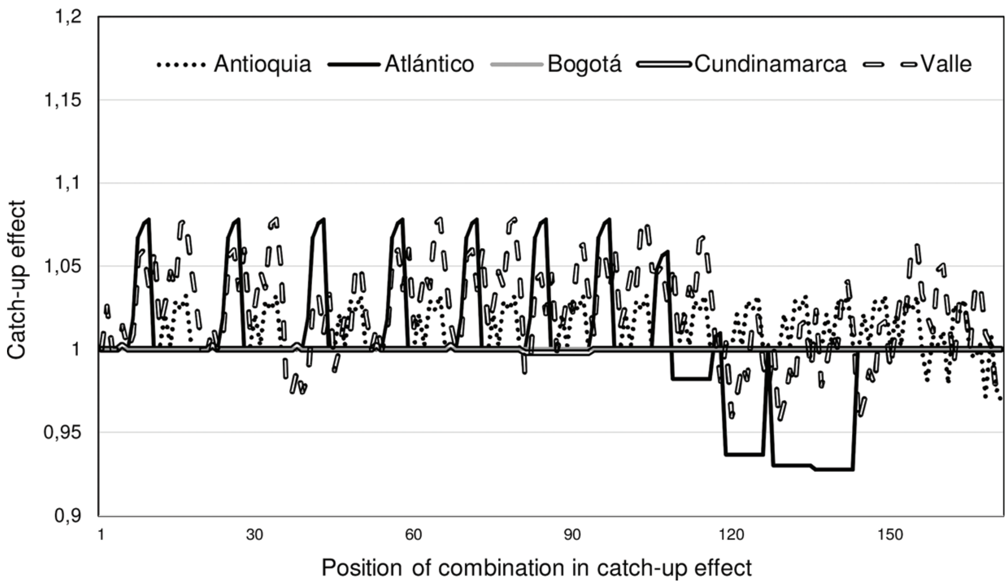

Figure 6. Catch-up effect after it has been separated from MI.

For *DMU =1 to 5 # for DMUs Antioquia, Atlántico, Bogotá, Cundinamarca and Valle*

For *i=1 to 18*

For *j=2 to 19*

If *(j > i)* ***then***

Run model Eq. (12) and obtain frontier-shift

Else

End

```
        End
    End
End
```

The following code helps building the **Figure 6**.

```
For DMU =1 to 5 # for DMUs Antioquia, Atlántico, Bogotá, Cundinamarca and Valle
    For i=1 to 18
        For j=2 to 19
            If (j > i) then
                Run model Eq. (11) and obtain Catch-up effect
            Else
            End
        End
    End
End
```

In Malmquist theory, if catch-up effect 1, it will indicate progress in relative efficiency from period 1 to 2, if catch-up effect 1 and catch-up effect 1 respectively indicate no change and regress in efficiency. If frontier shift effect is bigger than 1, it will indicate progress in the frontier technology around DMU observed from period 1 to 2. Note, in **Figure 6**, that Atlántico and Valle present regress in efficiency after position 110 and 118 respectively, in Atlántico the regress in efficiency is after combinations of periods 8-12, 8–13, …, 12–13. Antioquia also presents regress in efficiency in higher positions.

Applications of historical Malmquist can be presented in industry, economy and supply chain. In general, when there are time series and it is necessary to monitor and to control the changes of efficiency using historical periods.

5. Conclusions

A new model of efficiency change over time has been presented in this chapter based in [28]. That idea arise from explorations of temporal DEA in literature and it was established that there is no new techniques and methodologies, with the exception of Malmquist index and window analysis. The new temporal DEA has been presented like historical Malmquist and its primary advantage is that it accepts time series in input and output variables. Hence, it inherits the advantage of time series technique like use them to make forecast.

A great advantage of new temporal DEA is that it offers an historical information in terms of time, that is a disadvantage of classical Malmquist index that evaluates change of efficiency only in two periods. In this chapter it has been shown that the use of two periods of time to calculate the change of efficiency it is unfair in terms of history of DMUs.

With historical Malmquist it is possible to separate the catch-up and frontier shift effects for all the possible combinations of periods including the next no observed periods, through forecast. This allows to know the trend to progress in relative efficiency, regress in efficiency, progress in the frontier technology and trend to regress in the frontier technology. This will also allow that operational decisions can be taken before efficiency decreases.

Historical Malmquist allows to work both stochastic input and stochastic output variables, because it accepts time series in both cases, in contrast with classical MI that allows deterministic values and it is not common that a DEA model allows to work with probability functions in both input and output.

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