Chapter 10

Fuzzy Fault Detection Filter Design for One Class of Takagi-Sugeno Systems

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Additional information is available at the end of the chapter

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Abstract

The constrained unitary formalism to fuzzy fault detection filter synthesis for one class of nonlinear systems, representable by continuous-time Takagi-Sugeno fuzzy models, is presented in the chapter. In particular, a way to produce the special set of matrix parameters of the fuzzy filter is proposed to obtain the desired H_∞ norm properties of the filter transfer function matrix. The significance of the treatment in relation to the systems under influence of actuator faults is analyzed in this context, and relations to corresponding setting of singular values of filters are discussed.

Keywords: multiple models, continuous-time Takagi-Sugeno fuzzy models, fuzzy fault detection filters, fuzzy state observers

1. Introduction

Since the work of Hou and Patton [1], there has been much interest in the design of fault residuals for linear systems that use H_∞/H_- optimization principle in transfer function matrix of fault detection filter designed to scale up fault detection punctuality and high sensitivity to faults [2]. While retaining these features, a novel class of fault detection filters are proposed in [3, 4], preserving the unitary implementation of the fault detection filter transfer function matrix and receipting residual signal directional properties. However, the use of this methodology for Takagi-Sugeno (TS) fuzzy systems hits the boundaries of the working sectors and requires special adaptations.

Considering the properties of TS fuzzy models [5, 6], and some specifics in frequency characteristic evaluation of multiple model structures, the approach proposed in the chapter reformulates the H_∞ norm technique suitable in TS fuzzy fault detection filter design. The problem is solved via unitary modal technique when every linear TS fuzzy filter part is designed to have the same singular values of the transfer function matrix. Since working sector constraints may cause that the stable linear filter component cannot be obtained for a linear part in TS fuzzy model, to maintain H_∞ norm of the filter, the LQ modal control principle [7] is used for additional stabilization. Because additional stabilization aggravates directional properties of the applied linear part, in general, if additional stabilization is necessary, the residuals are only quasi-directional. It is immediately apparent that the formulated problem is related to forcing the singular values conditioned as state observer dynamics. The chosen model of the system is selected for this chapter to be sufficiently complex in illustration of all these specifics of synthesis.

Throughout the chapter, the following notations are used: $\boldsymbol{x}^T$ and $\boldsymbol{X}^T$ denote the transpose of the vector $\boldsymbol{x}$ and the matrix $\boldsymbol{X}$, respectively; for a square matrix $\boldsymbol{X} \geq 0$ means that $\boldsymbol{X}$ is a symmetric positive semi-definite matrix; the symbol $\boldsymbol{I}_n$ indicates the nth-order unit matrix; $\mathbb{R}$ denotes the set of real numbers; and $\mathbb{R}^n$ and $\mathbb{R}^{n\times r}$ refer to the set of all n-dimensional real vectors and $n \times r$ real matrices.

2. System description

The considered class of the Takagi-Sugeno dynamic systems with additive faults is described as the following:

$$\dot{\boldsymbol{q}}(t) = \sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\theta}(t))\big(\boldsymbol{A}_i\boldsymbol{q}(t) + \boldsymbol{B}_i\boldsymbol{u}(t) + \boldsymbol{F}_i\boldsymbol{f}(t)\big) \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) \tag{2}$$

where $\boldsymbol{q}(t) \in \mathbb{R}^n$, $\boldsymbol{u}(t) \in \mathbb{R}^r$, and $\boldsymbol{y}(t) \in \mathbb{R}^m$ stand for state, control input, and measurable output, respectively; $\boldsymbol{f}(t) \in \mathbb{R}^p$ is an additive fault vector; $\boldsymbol{A}_i \in \mathbb{R}^{n\times n}$, $\boldsymbol{B}_i \in \mathbb{R}^{n\times r}$, $\boldsymbol{F}_i \in \mathbb{R}^{n\times p}$, $\boldsymbol{C} \in \mathbb{R}^{m\times n}$, and $m = p$ and the matrix products $\boldsymbol{V}_i = \boldsymbol{C}\boldsymbol{F}_i$ and $\boldsymbol{V}_i \in \mathbb{R}^{m\times m}$ are regular matrices for all i.

The variables $\theta_j(t)$ and $j = 1, 2, \dots, o$, bound with the sector TS model, span the o-dimensional vector of premise variables:

$$\boldsymbol{\theta}(t) = [\theta_1(t) \;\; \theta_2(t) \;\; \cdots \;\; \theta_o(t)] \tag{3}$$

and [8]

$$\sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\theta}(t)) = 1 \tag{4}$$

where $\mathrm{h}_i(\boldsymbol{\theta}(t))$, $i = 1, 2, \dots, s$ is the set of normalized membership function. It is supposed that the measurable premise variables, the nonlinear sectors, and the normalized membership

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functions are chosen in such a way that the pairs $(\boldsymbol{A}_i, \boldsymbol{B}_i)$ are controllable and the pairs $(\boldsymbol{A}_i, \boldsymbol{C})$ are observable for all i.

3. Basic preliminaries from linear systems

Let the state-space description of a linear continuous-time dynamic systems take the form with equivalent meanings and dimensions as they are described in Section 2. The nature of the characterization of expected solutions to the system [(5), (6)] is given by the following results.

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{F}\boldsymbol{f}(t) \tag{5}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) \tag{6}$$

Definition 1 [9, 10] *If* $\boldsymbol{A}$ *has no imaginary eigenvalues, the* H_∞ *norm of the system transfer function matrix*

$$\boldsymbol{G}(s) = \boldsymbol{C}(s\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{B} \tag{7}$$

is

$$\|\boldsymbol{G}(s)\|_\infty = \sup_{\omega \in R} \sigma_1(\boldsymbol{G}(j\omega)) = \sup_{\omega \in R} \sqrt{\varepsilon_1(\boldsymbol{G}^*(j\omega)\boldsymbol{G}(j\omega))} \tag{8}$$

while the kth singular value σ_h of the complex matrix $\boldsymbol{G}(j\omega)$ is the nonnegative square root of the kth largest eigenvalue ε_k of $\boldsymbol{G}^*(j\omega)\boldsymbol{G}(j\omega)$, $\boldsymbol{G}^*(j\omega)$ is the adjoint of $\boldsymbol{G}(j\omega)$, and σ_1 is the largest singular value. The singular values of the transfer function matrix $\boldsymbol{G}(s)$ are evaluated on the imaginary axis, and it is assumed that the singular values are ordered such that $\sigma_k \geq \sigma_{k+1}, \quad k = 1, 2, \ldots, n-1$.

To apply in design methodology, the following result from [4] is quoted.

Lemma 1 *If* $m = p$ *and* $\boldsymbol{V} = \boldsymbol{C}\boldsymbol{F}$ *are regular matrices, then the system matrix factorization can be realized such that*

$$\boldsymbol{C} = [\,\boldsymbol{V} \quad \boldsymbol{0}\,]\boldsymbol{T}, \quad \boldsymbol{T}\boldsymbol{F} = \begin{bmatrix} \boldsymbol{I}_m \\ \boldsymbol{0} \end{bmatrix} \tag{9}$$

and the transform matrix $\boldsymbol{T} \in \mathbb{R}^{n \times n}$ takes the form

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{V}^{-1}\boldsymbol{C} \\ \boldsymbol{F}^\perp \end{bmatrix}, \tag{10}$$

where $\boldsymbol{V}^{-1}\boldsymbol{C} \in \mathbb{R}^{m \times n}$, $\boldsymbol{F}^\perp \in \mathbb{R}^{(n-m) \times n}$, and $\boldsymbol{F}^\perp$ are the left orthogonal complements to $\boldsymbol{F}$.

The idea of the following condition was derived originally as an approximation in the frequency domain for the fault transfer function matrix reflecting Eqs. (5) and (6) from [12]. Here, it is demonstrated that it can be simply adapted for fault residual filter design.

Theorem 1 *A linear fault detection filter to the system [(5), (6)] is stable and unitary if for regular* $\boldsymbol{V} = \boldsymbol{CF}$ *and a given positive scalar* $s_o \in \mathbb{R}$ *the square transfer function matrix* $\boldsymbol{G}_r(s)$ *of the fault detection filter satisfies the conditions*

$$P(s) = \det(s\boldsymbol{I}_n - (\boldsymbol{A} - \boldsymbol{JC})) = (s + s_o)^m P_o(s), \tag{11}$$

$$\sigma_1 = \sigma_2 = \cdots = \sigma_m, \qquad \lim_{\omega \to 0} \sigma_h = s_o, \tag{12}$$

$$\boldsymbol{G}_r(0) = \mathrm{diag}\left[s_o^{-1} \quad s_o^{-1} \quad \cdots \quad s_o^{-1} \right], \tag{13}$$

$$\boldsymbol{G}_r(s) = \boldsymbol{V}^{-1}\boldsymbol{C}(s\boldsymbol{I}_n - (\boldsymbol{A} - \boldsymbol{JC}))^{-1}\boldsymbol{F} = (s + s_o)^{-1}\boldsymbol{I}_m, \tag{14}$$

$$\boldsymbol{A}_o = \boldsymbol{TAT}^{-1} = \begin{bmatrix} \boldsymbol{A}_{o11} & \boldsymbol{A}_{o12} \\ \boldsymbol{A}_{o21} & \boldsymbol{A}_{o22} \end{bmatrix}, \tag{15}$$

$$\boldsymbol{J} = \boldsymbol{T}^{-1}\boldsymbol{L}^o\boldsymbol{V}^{-1}, \qquad \boldsymbol{L}^o = \begin{bmatrix} s_o\boldsymbol{I}_m + \boldsymbol{A}_{o11} \\ \boldsymbol{A}_{o21} \end{bmatrix}, \tag{16}$$

where $\boldsymbol{J} \in \mathbb{R}^{n\times r}$ is the residual filter gain matrix, σ_1 is the maximal singular value of $\boldsymbol{G}_r(s)$, the polynomial $P_o(s)$ of order $(n - m)$ is stable, and $\boldsymbol{G}_r(0) \in \mathbb{R}^{m\times m}$.

Proof. Considering the fault transfer function matrix of dimension $m \times m$ as

$$\boldsymbol{G}_f(s) = \boldsymbol{C}(s\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{F} \tag{17}$$

and then regrouping terms using Eqs. (9) and (10), it yields immediately the expressions

$$\boldsymbol{G}_f(s) = \boldsymbol{CT}^{-1}\boldsymbol{T}(s\boldsymbol{I}_n - \boldsymbol{A})^{-1}\boldsymbol{T}^{-1}\boldsymbol{TF} = \boldsymbol{CT}^{-1}\left(\left(s\boldsymbol{I}_n - \boldsymbol{TAT}^{-1}\right)^{-1}\boldsymbol{TF}, \right. \tag{18}$$

$$\boldsymbol{G}_f(s) = [\,\boldsymbol{V} \quad \boldsymbol{0}\,](s\boldsymbol{I}_n - \boldsymbol{A}_o)^{-1}\begin{bmatrix} \boldsymbol{I}_p \\ \boldsymbol{0} \end{bmatrix}, \tag{19}$$

respectively, where $\boldsymbol{A}_o$ is given in Eq. (15).

Specifying the following matrix product $\boldsymbol{A}^o = \boldsymbol{TMV}^{-1}\boldsymbol{CT}^{-1}$, where $\boldsymbol{M} \in \mathbb{R}^{n\times m}$ is a real matrix, it yields

$$\boldsymbol{A}^o = \boldsymbol{TMV}^{-1}\boldsymbol{CT}^{-1} = \begin{bmatrix} \boldsymbol{V}^{-1}\boldsymbol{C} \\ \boldsymbol{F}^{\perp} \end{bmatrix}\boldsymbol{MV}^{-1}[\,\boldsymbol{V} \quad \boldsymbol{0}\,] = \begin{bmatrix} \boldsymbol{V}^{-1}\boldsymbol{CM} & \boldsymbol{0} \\ \boldsymbol{F}^{\perp}\boldsymbol{M} & \boldsymbol{0} \end{bmatrix} \tag{20}$$

and, with the block matrix structure of Eqs. (15) and (21), it can be defined as

$$\Delta\boldsymbol{A}_o = \boldsymbol{A}_o - \boldsymbol{A}^o = \begin{bmatrix} \boldsymbol{A}_{o11} - \boldsymbol{V}^{-1}\boldsymbol{CM} & \boldsymbol{A}_{o12} \\ \boldsymbol{A}_{o21} - \boldsymbol{F}^{\perp}\boldsymbol{M} & \boldsymbol{A}_{o22} \end{bmatrix}. \tag{21}$$

Presetting

$$\boldsymbol{A}_{o11} - \boldsymbol{V}^{-1}\boldsymbol{C}\boldsymbol{M} = -s_o\boldsymbol{I}_m, \qquad \boldsymbol{A}_{o21} - \boldsymbol{F}^{\perp}\boldsymbol{M} = \boldsymbol{0}, \tag{22}$$

where $s_o \in I\!R$ is a prescribed positive real value. The plus sign is introduced for the purposes that come to light in the stability ensuing development of the observer system matrix.

Then,

$$\Delta\boldsymbol{A}_o = \begin{bmatrix} -s_o\boldsymbol{I}_m & \boldsymbol{A}_{o12} \\ \boldsymbol{0} & \boldsymbol{A}_{o22} \end{bmatrix} \tag{23}$$

and it is evident that $\Delta\boldsymbol{A}_o$ is stable if $\boldsymbol{A}_{o22}$ is Hurwitz, denoting here that

$$P_o(s) = \det(s\boldsymbol{I}_{n-m} - \boldsymbol{A}_{o22}). \tag{24}$$

Rewriting the set of Eq. (22) to admit a stable solution

$$\begin{bmatrix} s_o\boldsymbol{I}_m + \boldsymbol{A}_{o11} \\ \boldsymbol{A}_{o21} \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}^{-1}\boldsymbol{C} \\ \boldsymbol{F}^{\perp} \end{bmatrix}\boldsymbol{M} = \boldsymbol{T}\boldsymbol{M} = \boldsymbol{T}\boldsymbol{T}^{-1}\boldsymbol{L}^o = \boldsymbol{L}^o, \tag{25}$$

where

$$\boldsymbol{M} = \boldsymbol{T}^{-1}\boldsymbol{L}^o, \tag{26}$$

then Eqs. (20) and (21) must satisfy the following conditions:

$$\boldsymbol{A}^o = \boldsymbol{T}\boldsymbol{M}\boldsymbol{V}^{-1}\boldsymbol{C}\boldsymbol{T}^{-1} = \boldsymbol{T}\boldsymbol{J}\boldsymbol{C}\boldsymbol{T}^{-1}, \tag{27}$$

$$\Delta\mathbf{A}_o = \mathbf{A}_o - \mathbf{A}^o = \mathbf{T}(\mathbf{A} - \mathbf{J}\mathbf{C})\mathbf{T}^{-1} = \mathbf{T}\mathbf{A}_e\mathbf{T}^{-1}. \tag{28}$$

Therefore, the observer system matrix $\boldsymbol{A}_e$ takes the form

$$\boldsymbol{A}_e = \boldsymbol{A} - \boldsymbol{J}\boldsymbol{C} = \boldsymbol{A} - \boldsymbol{M}\boldsymbol{V}^{-1}\boldsymbol{C} \tag{29}$$

and

$$\boldsymbol{J} = \boldsymbol{M}\boldsymbol{V}^{-1} = \boldsymbol{T}^{-1}\boldsymbol{L}^o\boldsymbol{V}^{-1} \tag{30}$$

implies Eq. (16).

Regarding the transfer function matrix $\boldsymbol{G}_e(s)$ of the state error estimate as follows

$$\boldsymbol{G}_e(s) = \boldsymbol{C}(s\boldsymbol{I}_n - \boldsymbol{A}_e)^{-1}\boldsymbol{F}, \tag{31}$$

then with Eq. (29), it is

$$G_e(s) = CT^{-1}\left(sI_n - TA_eT^{-1}\right)^{-1}TF = [\,V \quad 0\,](sI_n - \Delta A_o)^{-1}\begin{bmatrix} I_p \\ 0 \end{bmatrix}. \tag{32}$$

Since

$$sI_n - \Delta A_o = \begin{bmatrix} (s+s_o)I_m & -A_{o12} \\ 0 & sI_{n-m} - A_{o22} \end{bmatrix}, \tag{33}$$

$$(sI_n - \Delta A_o)^{-1} = \begin{bmatrix} (s+s_o)^{-1}I_m & (s+s_o)^{-1}A_{o12}(sI_{n-m} - A_{o22})^{-1} \\ 0 & (sI_{n-m} - A_{o22})^{-1} \end{bmatrix}, \tag{34}$$

Substituting Eq. (34) into Eq. (32), it can obtain

$$G_e(s) = V(s+s_o)^{-1}I_m = \frac{V}{s+s_o}. \tag{35}$$

Thus, defining the fault detection filter transfer function matrix as $G_r(s) = V^{-1}G_e(s)$, then

$$G_r(s) = V^{-1}G_e(s) = (s+s_o)^{-1}I_m \tag{36}$$

and Eq. (36) implies Eq. (14). This concludes the proof.

Corollary 1 *Evidently, writing the fault residual vector as*

$$r(t) = V^{-1}Ce(t) = V^{-1}C\left(q(t) - q_e(t)\right), \tag{37}$$

where

$$e(t) = q(t) - q_e(t) \tag{38}$$

and $r(t) \in \mathbb{R}^m$ is the vector of residual signals, then based on the following observer structure

$$\dot{q}_e(t) = Aq_e(t) + Bu(t) + JC\left(q(t) - q_e(t)\right), \tag{39}$$

$$y_e(t) = Cq_e(t), \tag{40}$$

the autonomous observer error equation is

$$\dot{e}(t) = (A - JC)e(t), \tag{41}$$

where $q_e(t) \in \mathbb{R}^n$ is the observer state, $y_e(t) \in \mathbb{R}^m$ is the estimated system output, and $J \in \mathbb{R}^{n\times m}$ is the observer gain matrix; the fault detection filter (37), (39) is stable and unitary if for given positive scalar $s_o \in \mathbb{R}$ and the Hurwitz matrix A_{o22} the conditions (15) and (16) are satisfied.

Practically, with understanding Eq. (30), the observer sensor subsystem for the fault detection filter can be designed as follows:

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$$\boldsymbol{e}_z(t) = \boldsymbol{z}(t) - \boldsymbol{z}_e(t) = \boldsymbol{V}^{-1}\boldsymbol{C}\big(\boldsymbol{q}(t) - \boldsymbol{q}_e(t)\big) \tag{42}$$

and, consequently, it yields

$$\dot{\boldsymbol{q}}_e(t) = \boldsymbol{A}\boldsymbol{q}_e(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{M}\boldsymbol{C}\big(\boldsymbol{q}(t) - \boldsymbol{q}_e(t)\big). \tag{43}$$

Another option is to design the observer sensor subsystem so that $\boldsymbol{V} = \boldsymbol{I}_m$.

With existence of the system parameter transformation, the above structures really mean that the subset of transformed state variables whose dynamics is explicitly affected by the additive fault $\boldsymbol{f}(t)$ and the second one, whose dynamics is not affected explicitly by the fault $\boldsymbol{f}(t)$, exists.

Remark 1 *It is important to note the fact that the eigenvalues of* $\boldsymbol{A}$ *and of* $\boldsymbol{A}_o$ *are the same whenever* $\boldsymbol{A}_o$ *is related to* $\boldsymbol{A}$ *as* $\boldsymbol{A}_o = \boldsymbol{T}\boldsymbol{A}\boldsymbol{T}^{-1}$ *for any invertible* $\boldsymbol{T}$ [11]. *But this does not mean that if eigenvalues of the matrix* $\boldsymbol{A}_o$ *are stable then eigenvalues of the matrix* $\boldsymbol{A}_{o22}$ *are also stable. Thus, as well as for a stable system, it can lead to an unstable matrix* $\boldsymbol{A}_{o22}$, *and any additional stabilization is required.*

To apply the above results, it is necessary to be able to design fault residual filter if an unstable $\boldsymbol{A}_{o22}$ results such that $\boldsymbol{A}_e$ be stable without loss of unitarity.

Lemma 2 [7, 12] *To change signs of unstable eigenvalues of the system matrix* $\boldsymbol{A}$, *the gain matrix* $\boldsymbol{K} \in \mathbb{R}^{n\times r}$ *of the state feedback additive stabilization*

$$\boldsymbol{u}(t) = -\boldsymbol{K}\boldsymbol{q}(t) \tag{44}$$

is a solution of the continuous-time algebraic Riccati equation (CARE)

$$\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{P} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P} + \boldsymbol{Q} = \boldsymbol{0}, \tag{45}$$

where the matrix $\boldsymbol{Q} \in \mathbb{R}^{n\times n}$ is null matrix and $\boldsymbol{R} \in \mathbb{R}^{r\times r}$ and $\boldsymbol{R} = \boldsymbol{R}^T > 0$ are positive definite symmetric matrices.

Then, $\boldsymbol{K}$ *is given as*

$$\boldsymbol{K} = \boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}. \tag{46}$$

It is in that form that is able to be exploit for specific properties of the problem in TS fuzzy fault detection filter design.

In view of the above, these results hold for continuous-time linear systems, and, in principle, Theorem 1 gives a practical method to design unitary fault residual filters for the given linear system. Similar results are obtained for unitary TS fuzzy fault detection filter design in the following section.

4. TS fuzzy fault detection filters

Using the same set of membership functions, the fuzzy fault detection filter is built on the TS fuzzy observer

$$\dot{\boldsymbol{q}}_e(t) = \sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\theta}(t))\big(\boldsymbol{A}_i\boldsymbol{q}(t) + \boldsymbol{B}_i\boldsymbol{u}_i(t) + \boldsymbol{J}_i\boldsymbol{C}\big(\boldsymbol{q}(t) - \boldsymbol{q}_e(t)\big)\big) \tag{47}$$

$$\boldsymbol{y}_e(t) = \boldsymbol{C}\boldsymbol{q}_e(t) \tag{48}$$

where $\boldsymbol{q}_e(t) \in \mathbb{R}^n$ is the observer state vector, $\boldsymbol{y}_e(t) \in \mathbb{R}^m$ is the estimated system output vector, and $\boldsymbol{J}_i \in \mathbb{R}^{n\times m}$ and $i = 1, 2, \ldots, s$ are the sets of the observer gain matrices. Additionally, the output vector of the residual TS fuzzy filter is defined as

$$\boldsymbol{r}(t) = \sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\theta}(t))\boldsymbol{r}_i(t) = \sum_{i=1}^{s} \mathrm{h}_i(\boldsymbol{\theta}(t))\boldsymbol{V}_i^{-1}\boldsymbol{C}\boldsymbol{e}(t) \tag{49}$$

$$\boldsymbol{r}_i(t) = \boldsymbol{V}_i^{-1}\boldsymbol{C}\boldsymbol{e}(t) \tag{50}$$

$$\boldsymbol{e}(t) = \boldsymbol{q}(t) - \boldsymbol{q}_e(t) \tag{51}$$

where $\boldsymbol{r}(t), \boldsymbol{r}_i(t) \in \mathbb{R}^m$, $\boldsymbol{V}_i \in \mathbb{R}^{m\times m}$. Evidently, $\boldsymbol{V}_i = \boldsymbol{C}\boldsymbol{F}_i$ has to be a regular matrix for all i.

Formally, the following result can be simply derived.

Theorem 2 *A TS fuzzy fault detection filter to the system [(1), (2)] is stable and unitary if for the set of regular matrices $\boldsymbol{V}_i = \boldsymbol{C}\boldsymbol{F}_i$ and $i = 1, 2, \ldots, s$, and a given positive scalar $s_o \in \mathbb{R}$ every square transfer function matrix $\boldsymbol{G}_{ri}(s)$ of the fault detection filter satisfies for all i the conditions*

$$\sigma_1 = \sigma_2 = \cdots = \sigma_m, \qquad \lim_{\omega\to 0} \sigma_h = s_o, \tag{52}$$

$$\boldsymbol{G}_{ri}(0) = diag\big[\, s_o^{-1} \quad s_o^{-1} \quad \cdots \quad s_o^{-1} \,\big], \tag{53}$$

$$\boldsymbol{G}_{ri}(s) = \boldsymbol{V}_i^{-1}\boldsymbol{C}\big(s\boldsymbol{I}_n - (\boldsymbol{A}_i - \boldsymbol{J}_i\boldsymbol{C})\big)^{-1}\boldsymbol{F}_i = (s + s_o)^{-1}\boldsymbol{I}_m, \tag{54}$$

while

$$\boldsymbol{T}_i = \begin{bmatrix} \boldsymbol{V}_i^{-1}\boldsymbol{C} \\ \boldsymbol{F}_i^{\perp} \end{bmatrix}, \tag{55}$$

$$\boldsymbol{A}_{oi} = \boldsymbol{T}_i\boldsymbol{A}_i\boldsymbol{T}_i^{-1} = \begin{bmatrix} \boldsymbol{A}_{o11i} & \boldsymbol{A}_{o12i} \\ \boldsymbol{A}_{o21i} & \boldsymbol{A}_{o22i} \end{bmatrix}, \tag{56}$$

$$\boldsymbol{J}_i = \boldsymbol{T}_i^{-1}\boldsymbol{L}_i^o\boldsymbol{V}_i^{-1}, \qquad \boldsymbol{L}_i^o = \begin{bmatrix} s_o\boldsymbol{I}_m + \boldsymbol{A}_{o11i} \\ \boldsymbol{A}_{o21i} \end{bmatrix}, \tag{57}$$

$$P_i(s) = \det\left(s\boldsymbol{I}_n - \left(\boldsymbol{A}_i - \boldsymbol{J}_i\boldsymbol{C}\right)\right) = (s + s_o)^m P_{oi}(s), \tag{58}$$

$$P_{oi}(s) = det(s\boldsymbol{I}_{n-m} - \boldsymbol{A}_{o22i}). \tag{59}$$

$\boldsymbol{J}_i \in \mathbb{R}^{n\times r}$ is the residual filter gain matrix, σ_1 is the maximal singular value of $\boldsymbol{G}_{ri}(s)$, the polynomial $P_{oi}(s)$ of order $(n-m)$ is stable, and $\boldsymbol{G}_{ri}(0) \in \mathbb{R}^{m\times m}$ and $\boldsymbol{F}_i^{\perp} \in \mathbb{R}^{(n-m)\times n}$ are left orthogonal complements to the fault input matrix $\boldsymbol{F}_i$.

Proof. Because every sub-model in Eq. (47) is described by linear equations, Eqs. (15) and (16) imply directly the conditions (56) and (57), and Eq. (58) is given by Eq. (11). This concludes the proof.

Corollary 2 *In practice, an additive fault typically enters through a matrix $\boldsymbol{F}$ that does not depend on the sectoral boundaries defining the TS model. In this case, the synthesis is substantially simplified because $\boldsymbol{V}$ is a constant matrix, and so it yields*

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{V}^{-1}\boldsymbol{C} \\ \boldsymbol{F}^{\perp} \end{bmatrix}, \tag{60}$$

$$\boldsymbol{A}_{oi} = \boldsymbol{T}\boldsymbol{A}_i\boldsymbol{T}^{-1} = \begin{bmatrix} \boldsymbol{A}_{o11i} & \boldsymbol{A}_{o12i} \\ \boldsymbol{A}_{o21i} & \boldsymbol{A}_{o22i} \end{bmatrix}, \tag{61}$$

$$\boldsymbol{J}_i = \boldsymbol{T}^{-1}\boldsymbol{L}_i^o\boldsymbol{V}^{-1}, \qquad \boldsymbol{L}_i^o = \begin{bmatrix} s_o\boldsymbol{I}_m + \boldsymbol{A}_{o11i} \\ \boldsymbol{A}_{o21i} \end{bmatrix}. \tag{62}$$

Corollary 3 *Since, independently on i, the condition (52) is satisfied ($\sigma_1 = \sigma_2 = \cdots = \sigma_m$), all sub-filter transfer function matrices have the same H_∞ norm, i.e.,*

$$\|\boldsymbol{G}_{ri}(s)\|_\infty = \|\boldsymbol{G}_{ro}(s)\|_\infty \quad for \;\; all \;\; i. \tag{63}$$

Moreover, considering that $\sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t)) = 1$, then

$$\|\boldsymbol{G}_r(s)\|_\infty = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(t))\|\boldsymbol{G}_{ri}(s)\|_\infty = \|\boldsymbol{G}_{ro}(s)\|_\infty \tag{64}$$

That is, the H_∞ norm of the transfer function matrix of such defined TS fuzzy fault detection filter is independent on the system working point. Of course, this cannot be said about the dynamics of the time response of the sub-filter components.

Moreover, $\boldsymbol{G}_{ri}(0)$ implies that all residual components of TS fuzzy fault detection filter have the same directional properties, which ensure unitary properties of the filter.

Remark 2 *Sectoral boundaries may cause a matrix $\boldsymbol{A}_i$ to be such, when transformed using $\boldsymbol{T}_i$ that $\boldsymbol{A}_{o22i}$ will not be Hurwitz matrix. Because the transfer function matrix of the corresponding filter linear component in this case is unstable, maintaining the unitary property requires changes in the signs of the unstable eigenvalues of the associated $\boldsymbol{A}_{ei}^{\circ} = \boldsymbol{A}_i - \boldsymbol{J}_i\boldsymbol{C}$.*

Applying the duality principle and inserting the additive observer gain component $\boldsymbol{K}_{si}^T$ obtained as a solution of the Riccati equation (45) for $\boldsymbol{A}_{ei}^{\circ T}$, according to the scheme given in Lemma 2, the observer gain matrix is changed as

$$\boldsymbol{J}_i^{\circ} = \boldsymbol{J}_i + \boldsymbol{K}_{si}^T, \quad \boldsymbol{A}_{ei}^{\circ} = \boldsymbol{A}_i - \boldsymbol{J}_i^{\circ}\boldsymbol{C}. \tag{65}$$

This additive stabilization results that the consequential characteristic polynomial, taking also the form

$$P_i(s) = \det(s\boldsymbol{I}_n - \boldsymbol{A}_{ei}) = (s + s_o)^m P_{oi}(s), \tag{66}$$

is stable since $P_{oi}(s)$ is now stable.

The price for such an additional stabilization is that if j signs are changing in eigenvalues of $\boldsymbol{A}_{o22i}$ to obtain the stable $\boldsymbol{A}_{o22i}$, also j eigenvalues s_o of $\boldsymbol{G}_{ri}(0)$ change their signs and the resulting matrix $\boldsymbol{G}_{ri}(0)$ will not be diagonal. According to Eq. (8), this does not result in a change in H_∞ norm, but such filter component will arrive at the unitary directional residual properties.

5. Illustrative example

The three-tank system is described by the set of Eqs. [13, 14] as

$$\frac{dq_1(t)}{dt} = \frac{u_1(t)}{F_1} - \frac{\alpha_1 \mathrm{sign}\big(q_1(t) - q_2(t)\big)\sqrt{2g|q_1(t) - q_2(t)|}}{F_1 \sum_{i=1}^{3} \lambda_i q_i(t)} \sum_{i=1}^{3} \lambda_i q_i(t),$$

$$\begin{aligned}\frac{dq_2(t)}{dt} &= \frac{u_2(t)}{F_2} - \frac{\alpha_2\sqrt{2gq_2(t)}}{F_2 q_2(t)} q_2(t) + \frac{\alpha_1 \mathrm{sign}\big(q_1(t) - q_2(t)\big)\sqrt{2g|q_1(t) - q_2(t)|}}{F_1 \sum_{i=1}^{3} \lambda_i q_i(t)} \sum_{i=1}^{3} \lambda_i q_i(t) \\ &+ \frac{\alpha_3 \mathrm{sign}\big(q_3(t) - q_2(t)\big)\sqrt{2g|q_3(t) - q_2(t)|}}{F_3 \sum_{i=1}^{3} \eta_i q_i(t)} \sum_{i=1}^{3} \eta_i q_i(t),\end{aligned}$$

$$\frac{dq_3(t)}{dt} = \frac{u_3(t)}{F_3} - \frac{\alpha_3 \mathrm{sign}\big(q_3(t) - q_2(t)\big)\sqrt{2g|q_3(t) - q_2(t)|}}{F_3 \sum_{i=1}^{3} \eta_i q_i(t)} \sum_{i=1}^{3} \eta_i q_i(t),$$

$$y_k(t) = F_k q_k(t), \quad k = 1, 2, 3,$$

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where the measured output variables $y_k(t)$ are water levels in tanks $q_k(t)\ \ [m],\ \ k = 1, 2, 3$ and the incoming flows are considered as the inputs variables $u_k(t)\ \ [m^3/s],\ \ k = 1, 2, 3$; the bounds of the state and input variables are

$$q_1^{max} = q_3^{max} = 1.00\ \ [m],\qquad q_2^{max} = 0.90\ \ [m],\qquad u_{1,2,3}^{min} = 0\ \ [m^3/s],$$
$$q_1^{min} = q_3^{min} = 0.02\ \ [m],\qquad q_2^{min} = 0.01\ \ [m],\qquad u_{1,2,3}^{max} = 0.005\ \ [m^3/s].$$

$\lambda_k, \eta_k \in \mathbb{R}$ are positive scalars and $\mathrm{sign}(\cdot)$ is the sign function.

The model parameters of the system are considered as:

g -the gravitational acceleration $9.80665\,[\mathrm{m/s^2}]$,

F_k -the (same) section of tanks $0.25\,[\mathrm{m^2}]$,

α_1 -the equivalent section of the pipe between the first and second tank $6.5 \times 10^{-4}\,[\mathrm{m^2}]$,

α_3 -the equivalent section of the pipe between the third and second tank $6.5 \times 10^{-4}\,[\mathrm{m^2}]$,

α_2 -the equivalent section of the outlet pipe from the second tank $6.5 \times 10^{-3}\,[\mathrm{m^2}]$,

Minimizing the number of premise variables and excluding switching modes in controller work, the premise variables are chosen as follows

$$\theta_1(t) = \frac{\alpha_1 \mathrm{sign}(q_1(t) - q_2(t))\sqrt{2g|q_1(t) - q_2(t)|}}{F_1 \sum_{i=1}^{3} \lambda_i q_i(t)},$$

$$\theta_2(t) = \frac{\alpha_2\sqrt{2g q_2(t)}}{F_2 q_2(t)} = \frac{\alpha_2}{F_2}\sqrt{\frac{2g}{q_2(t)}},$$

$$\theta_3(t) = \frac{\alpha_3 \mathrm{sign}(q_3(t) - q_2(t))\sqrt{2g|q_3(t) - q_2(t)|}}{F_3 \sum_{i=1}^{3} \eta_i q_i(t)}.$$

Computed from the input variable bounds, the sector bounds of the premise variables imply the numbering:

$$\begin{aligned}
&i = 1 \leftarrow (\theta_1^{max}, \theta_2^{max}, \theta_3^{max}), && i = 2 \leftarrow (\theta_1^{max}, \theta_2^{max}, \theta_3^{min}),\\
&i = 3 \leftarrow (\theta_1^{max}, \theta_2^{min}, \theta_3^{max}), && i = 4 \leftarrow (\theta_1^{max}, \theta_2^{min}, \theta_3^{min}),\\
&i = 5 \leftarrow (\theta_1^{min}, \theta_2^{max}, \theta_3^{max}), && i = 6 \leftarrow (\theta_1^{min}, \theta_2^{max}, \theta_3^{min}),\\
&i = 7 \leftarrow (\theta_1^{min}, \theta_2^{min}, \theta_3^{max}), && i = 8 \leftarrow (\theta_1^{min}, \theta_2^{min}, \theta_3^{min}),
\end{aligned}$$

which is used in the system state matrix construction

$$\boldsymbol{A}_i = \begin{bmatrix} -\lambda_1\theta_1^i & -\lambda_2\theta_1^i & -\lambda_3\theta_1^i \\ \lambda_1\theta_1^i + \eta_1\theta_3^i & \lambda_2\theta_1^i + \eta_2\theta_3^i - \theta_2^i & \lambda_3\theta_1^i + \eta_3\theta_3^i \\ -\eta_1\theta_3^i & -\eta_2\theta_3^i & -\eta_3\theta_3^i \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} F_1^{-1} & 0 & 0 \\ 0 & F_2^{-1} & 0 \\ 0 & 0 & F_3^{-1} \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} F_1 & 0 & 0 \\ 0 & F_2 & 0 \\ 0 & 0 & F_3 \end{bmatrix}$$

and prescribed, moreover, that the matrix $\boldsymbol{C}$ is given in such a way that the product $\boldsymbol{CB}$ is the identity matrix. This regularizes the residual design conditions if $\boldsymbol{B}$ and $\boldsymbol{C}$ are diagonal matrices.

The sector functions are trapezoidal, and the membership functions are constructed as product of three sector functions with the same ordering as $\boldsymbol{A}_i$.

The set of real scalars, λ_k, η_k, and $k = 1, 2, 3$, is interactively optimized under limitations that all couples $(\boldsymbol{A}_i, \boldsymbol{B})$ and $(\boldsymbol{A}_i, \boldsymbol{C})$ are controllable and observable for the given set of indices i, where

$$\begin{aligned} \lambda_1 &= 0.1992, & \lambda_2 &= 0.6894, & \lambda_3 &= 0.1618, \\ \eta_1 &= 0.6891, & \eta_2 &= 0.3646, & \eta_3 &= 0.0569. \end{aligned}$$

Consequently, the TS model matrix parameters are

$$\boldsymbol{A}_1 = \begin{bmatrix} -0.0163 & -0.0563 & -0.0132 \\ 0.1225 & -1.0392 & 0.0220 \\ -0.1062 & -0.0562 & -0.0088 \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} -0.0163 & -0.0563 & -0.0132 \\ -0.0054 & -1.1069 & 0.0114 \\ 0.0217 & 0.0115 & 0.0018 \end{bmatrix},$$

$$\boldsymbol{A}_3 = \begin{bmatrix} -0.0163 & -0.0563 & -0.0132 \\ 0.1225 & -0.0089 & 0.0220 \\ -0.1062 & -0.0562 & -0.0088 \end{bmatrix}, \quad \boldsymbol{A}_4 = \begin{bmatrix} -0.0163 & -0.0563 & -0.0132 \\ -0.0054 & -0.0766 & 0.0114 \\ 0.0217 & 0.0115 & 0.0018 \end{bmatrix},$$

$$\boldsymbol{A}_5 = \begin{bmatrix} 0.0034 & 0.0119 & 0.0028 \\ 0.1028 & -1.1073 & 0.0060 \\ -0.1062 & -0.0562 & -0.0088 \end{bmatrix}, \quad \boldsymbol{A}_6 = \begin{bmatrix} 0.0034 & 0.0119 & 0.0028 \\ -0.0251 & -1.1750 & -0.0046 \\ 0.0217 & 0.0115 & 0.0018 \end{bmatrix},$$

$$\boldsymbol{A}_7 = \begin{bmatrix} 0.0034 & 0.0119 & 0.0028 \\ 0.1028 & -0.0771 & 0.0060 \\ -0.1062 & -0.0562 & -0.0088 \end{bmatrix}, \quad \boldsymbol{A}_8 = \begin{bmatrix} 0.0034 & 0.0119 & 0.0028 \\ -0.0251 & -0.1447 & -0.0046 \\ 0.0217 & 0.0115 & 0.0018 \end{bmatrix}.$$

$$\boldsymbol{B} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}, \qquad \boldsymbol{C} = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}.$$

Since the orthogonal complement to a square matrix does not exist, three fault detection filters can be considered for single actuator fault detection. To illustrate the design procedure, the TS fuzzy fault detection filter for the pair ($\boldsymbol{C}_{23}$, $\boldsymbol{B}_{23}$) is considered, i.e.,

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$$\boldsymbol{C} \Leftarrow \boldsymbol{C}_{23} = \begin{bmatrix} 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}, \boldsymbol{F} \Leftarrow \boldsymbol{B}_{23} = \begin{bmatrix} 0 & 0 \\ 4 & 0 \\ 0 & 4 \end{bmatrix},$$

with the derived parameters

$$\boldsymbol{V} = \boldsymbol{CF} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{V}^{-1}\boldsymbol{C} = \begin{bmatrix} 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}, \quad \boldsymbol{F}^{\perp} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{T} = \begin{bmatrix} 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \\ 1 & 0 & 0 \end{bmatrix}.$$

Note that in this case all $\boldsymbol{A}_i$ with index higher than 4 lead to an unstable structure of $\overset{\circ}{\boldsymbol{A}}_{o22i}$ and the resulting observer matrices $\boldsymbol{A}_{ei}$ need to be additionally stabilized, applying the principle given in Lemma 2.

Applying Eq. (56), the following structure of $\boldsymbol{A}_{o1}$ for the initial matrix $\boldsymbol{A}_1$ is computed:

$$\boldsymbol{A}_{o1} = \begin{bmatrix} -1.0392 & 0.0220 & 0.0306 \\ -0.0562 & -0.0088 & -0.0266 \\ -0.2250 & -0.0528 & -0.0163 \end{bmatrix}, \quad \begin{matrix} \boldsymbol{A}_{o111} = \begin{bmatrix} -1.0392 & 0.0220 \\ -0.0562 & -0.0088 \end{bmatrix}, & \boldsymbol{A}_{o121} = \begin{bmatrix} 0.0306 \\ -0.0266 \end{bmatrix}, \\ \boldsymbol{A}_{o211} = \begin{bmatrix} -0.2250 & -0.0528 \end{bmatrix}, & \boldsymbol{A}_{o221} = \begin{bmatrix} -0.0163 \end{bmatrix}, \end{matrix}$$

and $\boldsymbol{A}_{o221} = -0.0163$ implies that the associated TS fuzzy fault detection filter linear component can be designed directly.

Choosing $s_o = 5$, it is resulting from Eqs. (57) and (58) that

$$\overset{\circ}{\boldsymbol{L}}_1 = \begin{bmatrix} 3.9608 & 0.0220 \\ -0.0562 & 4.9912 \\ -0.2250 & -0.0528 \end{bmatrix}, \boldsymbol{J}_1 = \begin{bmatrix} -0.2250 & -0.0528 \\ 15.8432 & 0.0879 \\ -0.2248 & 19.9649 \end{bmatrix}, \boldsymbol{A}_{e1} = \begin{bmatrix} -0.0163 & 0 & 0 \\ 0.1225 & -5.0 & 0 \\ -0.1062 & 0 & -5.0 \end{bmatrix},$$

where the eigenvalue spectrum of $\boldsymbol{A}_{e1}$ and the steady-state value of the TS fuzzy fault detection filter transfer function matrix $\boldsymbol{G}_{r1}(0)$ are

$$\rho(\boldsymbol{A}_{e1}) = \{ -0.0163 \quad -5.0 \quad -5.0 \}, \quad \boldsymbol{G}_{r1}(0) = -\boldsymbol{V}^{-1}\boldsymbol{C}\boldsymbol{A}_{e1}^{-1}\boldsymbol{F} = \begin{bmatrix} 0.2 & \\ & 0.2 \end{bmatrix},$$

respectively. It is evident that all diagonal elements of $\boldsymbol{G}_{r1}(0)$ take the value $s_o^{-1} = 0.2$. The same structure of $\boldsymbol{G}_{r*}(0)$ is obtained solving with $\boldsymbol{A}_l$ for $l = 1, 2, 3, 4$.

Analogously, designing for the matrix $\boldsymbol{A}_5$, it can be seen that

$$\boldsymbol{A}_{o5} = \begin{bmatrix} -1.1073 & 0.0060 & 0.0257 \\ -0.0562 & -0.0088 & -0.0266 \\ 0.0475 & 0.0111 & 0.0034 \end{bmatrix}, \quad \begin{matrix} \boldsymbol{A}_{o511} = \begin{bmatrix} -1.1073 & 0.0060 \\ -0.0562 & -0.0088 \end{bmatrix}, & \boldsymbol{A}_{o512} = \begin{bmatrix} 0.0257 \\ -0.0266 \end{bmatrix}, \\ \boldsymbol{A}_{o521} = \begin{bmatrix} 0.0475 & 0.0111 \end{bmatrix}, & \boldsymbol{A}_{o522} = \begin{bmatrix} 0.0034 \end{bmatrix}. \end{matrix}$$

Since $A_{o222} = 0.0034$, evidently, the associated TS fuzzy fault detection filter linear component with the unitary transfer function matrix has to be stabilized additively.

Solving also for $s_o = 5$, then

$$\overset{\circ}{L}_5 = \begin{bmatrix} 3.8927 & 0.0060 \\ -0.0562 & 4.9912 \\ 0.0475 & 0.0111 \end{bmatrix}, J_5 = \begin{bmatrix} 0.0475 & 0.0111 \\ 15.5707 & 0.0239 \\ -0.2248 & 19.9649 \end{bmatrix}, A_{e5} = \begin{bmatrix} 0.0034 & 0 & 0 \\ 0.1028 & -5.0 & 0 \\ -0.1062 & 0 & -5.0 \end{bmatrix}.$$

It is evident that matrix F_{e5} is not Hurwitz and has to be additively stabilized.

Thus, defining the weighting matrices of appropriate dimensions as

$$Q = 0, \quad S = 0, \quad R = VV^T = I_2$$

and solving the dual LQ control problem to change the sign of unstable eigenvalue of F_{e5} using the MATLAB function $K_{s5} = care\left(F_{e2}^T, Q, R, S, I_3\right)$, then

$$K_{s5}^T = \begin{bmatrix} 0.6456 & -0.6671 \\ 0.0133 & -0.0137 \\ -0.0137 & 0.0142 \end{bmatrix}, \quad A_{es5} = A_{e5} - K_{s5}^T C = \begin{bmatrix} 0.0034 & -0.1614 & 0.1668 \\ 0.1028 & -5.0033 & 0.0034 \\ -0.1062 & 0.0034 & -5.0035 \end{bmatrix}.$$

It can be easily verified that

$$\rho(A_{es5}) = \{\, -0.0034 \quad -5.0 \quad -5.0 \,\},$$

$$G_{r5}(0) = -V^{-1} C A_{es5}^{-1} F = \begin{bmatrix} -0.0066 & -0.1999 \\ -0.1999 & 0.0066 \end{bmatrix}, \quad \rho(G_{r5}(0)) = \{\, -0.2000 \quad 0.2000 \,\}.$$

while, evidently, $G_{r5}(0)$ is not diagonal and the eigenvalues of $G_{r5}(0)$ are $\pm 0.2 = \pm s_0^{-1}$.

Note that the same structure of $G_{rl}(0)$ is obtained solving with the system matrices A_l and $l = 5, 6, 7, 8$ when additional stabilization is required. Evidently, elements of this set of TS fuzzy residual filter linear components are stable, non-unitary, and without directional residual properties. Nevertheless, these properties guarantee the same singular values of the linear transfer function matrix components; as follows the result of Definition 1, the TS fuzzy residual filter will have all the singular values the same. To document this, the singular value plot of the TS fuzzy fault detection filter, as well as of all its linear parts, is equal to that presented in **Figure 1**. With respect to the structure of the matrices B and C, the comparable results are obtainable for the matrix pairs (C_{12}, B_{12}) and (C_{13}, B_{13}).

The rest of gain matrices of the stable TS fault detection filter is as follows:

$$J_2 = \begin{bmatrix} -0.2250 & -0.0528 \\ 15.5725 & 0.0456 \\ 0.0459 & 20.0072 \end{bmatrix}, J_3 = \begin{bmatrix} -0.2250 & -0.0528 \\ 19.9643 & 0.0879 \\ -0.2248 & 19.9649 \end{bmatrix}, J_4 = \begin{bmatrix} -0.2250 & -0.0528 \\ 19.6935 & 0.0456 \\ 0.0459 & 20.0072 \end{bmatrix},$$

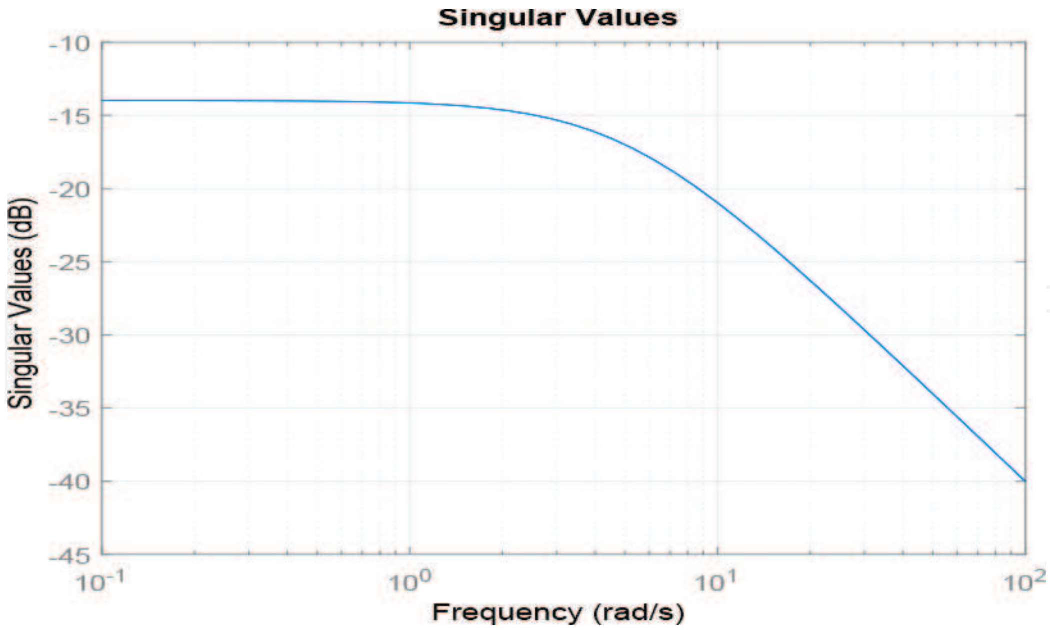

Figure 1. TS fuzzy fault detection filter singular value plot.

$$J_5 = \begin{bmatrix} 0.6930 & -0.6560 \\ 15.5840 & 0.0102 \\ -0.2385 & 19.9791 \end{bmatrix}, J_6 = \begin{bmatrix} -3.0809 & 2.7126 \\ 15.3157 & -0.0319 \\ 0.0324 & 20.0189 \end{bmatrix},$$

$$J_7 = \begin{bmatrix} 0.6930 & -0.6560 \\ 19.7051 & 0.0102 \\ -0.2385 & 19.9791 \end{bmatrix}, J_8 = \begin{bmatrix} -3.0809 & 2.7126 \\ 19.4367 & -0.0319 \\ 0.0324 & 20.0189 \end{bmatrix}.$$

Since the matrices A_i of the TS fuzzy system are not Hurwitz, the system in simulations is stabilized using the local-state feedback control laws, acting in the forced modes. Adapting the method presented in [14] to design the control law parameters, the local controller parameters are computed as

$$K_1 = \begin{bmatrix} 0.1780 & 0.0083 & -0.0150 \\ 0.0083 & -0.0701 & -0.0041 \\ -0.0150 & -0.0043 & 0.1798 \end{bmatrix}, K_2 = \begin{bmatrix} 0.1780 & -0.0079 & 0.0008 \\ -0.0075 & -0.0869 & 0.0028 \\ 0.0008 & 0.0027 & 0.1824 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 0.1780 & 0.0084 & -0.0150 \\ 0.0082 & 0.1842 & -0.0041 \\ -0.0150 & -0.0042 & 0.1798 \end{bmatrix}, K_4 = \begin{bmatrix} 0.1780 & -0.0078 & 0.0008 \\ -0.0076 & 0.1675 & 0.0027 \\ 0.0008 & 0.0028 & 0.1824 \end{bmatrix},$$

$$K_5 = \begin{bmatrix} 0.1829 & 0.0142 & -0.0131 \\ 0.0141 & -0.0870 & -0.0061 \\ -0.0130 & -0.0063 & 0.1798 \end{bmatrix}, K_6 = \begin{bmatrix} 0.1829 & -0.0020 & 0.0027 \\ -0.0017 & -0.1037 & 0.0008 \\ 0.0027 & 0.0007 & 0.1824 \end{bmatrix},$$

$$\boldsymbol{K}_7 = \begin{bmatrix} 0.1829 & 0.0143 & -0.0131 \\ 0.0140 & 0.1674 & -0.0061 \\ -0.0130 & -0.0063 & 0.1798 \end{bmatrix}, \boldsymbol{K}_8 = \begin{bmatrix} 0.1829 & -0.0019 & 0.0027 \\ -0.0018 & 0.1507 & 0.0007 \\ 0.0027 & 0.0007 & 0.1824 \end{bmatrix},$$

$$\boldsymbol{W}_1 = \begin{bmatrix} 0.1821 & 0.0224 & -0.0117 \\ -0.0223 & 0.1897 & -0.0096 \\ 0.0115 & 0.0098 & 0.1820 \end{bmatrix}, \boldsymbol{W}_2 = \begin{bmatrix} 0.1821 & 0.0062 & 0.0041 \\ -0.0061 & 0.1899 & -0.0001 \\ -0.0047 & -0.0002 & 0.1819 \end{bmatrix},$$

$$\boldsymbol{W}_3 = \begin{bmatrix} 0.1821 & 0.0225 & -0.0117 \\ -0.0224 & 0.1865 & -0.0096 \\ 0.0115 & 0.0098 & 0.1820 \end{bmatrix}, \boldsymbol{W}_4 = \begin{bmatrix} 0.1821 & 0.0063 & 0.0041 \\ -0.0062 & 0.1867 & -0.0001 \\ -0.0047 & -0.0001 & 0.1819 \end{bmatrix},$$

$$\boldsymbol{W}_5 = \begin{bmatrix} 0.1820 & 0.0112 & -0.0138 \\ -0.0116 & 0.1899 & -0.0076 \\ 0.0135 & 0.0077 & 0.1820 \end{bmatrix}, \boldsymbol{W}_6 = \begin{bmatrix} 0.1820 & -0.0049 & 0.0021 \\ 0.0046 & 0.1901 & 0.0019 \\ -0.0027 & -0.0022 & 0.1819 \end{bmatrix},$$

$$\boldsymbol{W}_7 = \begin{bmatrix} 0.1820 & 0.0114 & -0.0138 \\ -0.0117 & 0.1867 & -0.0076 \\ 0.0135 & 0.0078 & 0.1820 \end{bmatrix}, \boldsymbol{W}_8 = \begin{bmatrix} 0.1820 & -0.0048 & 0.0021 \\ 0.0045 & 0.1869 & 0.0019 \\ -0.0027 & -0.0021 & 0.1819 \end{bmatrix},$$

where

$$\boldsymbol{W}_i = -\left(\boldsymbol{C}(\boldsymbol{A}_i - \boldsymbol{B}\boldsymbol{K}_i)^{-1}\boldsymbol{B}\right)^{-1},$$

$$\boldsymbol{u}_i(t) = -\boldsymbol{K}_i\boldsymbol{q}(t) + \boldsymbol{W}_i\boldsymbol{w}_o,$$

while $\boldsymbol{w}_o \in \mathbb{R}^n$ is the vector of the desired steady-state system outputs.

If necessary for any more complex system, PDS controller principle can be applied to stabilize the plant (see, e.g., authors' publications [15, 16] or other references [17, 18]).

To display simulations in the MATLAB and Simulink environment, the forced mode control is established with local controller parameter given as above for the system initial conditions $\boldsymbol{q}^T(0) = [0.2\ \ 0.3\ \ 0.2]$ and $\boldsymbol{w}_o^T = [0.6\ \ 0.5\ \ 0.4]$. Fault detection filter is constructed on the couple $(\boldsymbol{C}_{23}, \boldsymbol{B}_{23})$ and the set of matrices $\boldsymbol{A}_i$ and $i = 1, 2\ldots, 8$.

As the results, **Figure 2** shows the TS fuzzy system output responses, illustrating their asymptotic convergence to the steady states, and **Figure 3** presents the TS fuzzy fault detection filter response, reflecting a steplike 90% gain loss of the second actuator at the time instant $t = 60s$. These examples illustrate the power that can be invoked through the prescribed H_∞ norm properties.

It can verify that TS fuzzy fault detection filters created for the couple pairs $(\boldsymbol{C}_{12}, \boldsymbol{B}_{12})$ and $(\boldsymbol{C}_{13}, \boldsymbol{B}_{13})$ have similar properties as that defined for the couple $(\boldsymbol{C}_{23}, \boldsymbol{B}_{23})$. The difference is, for

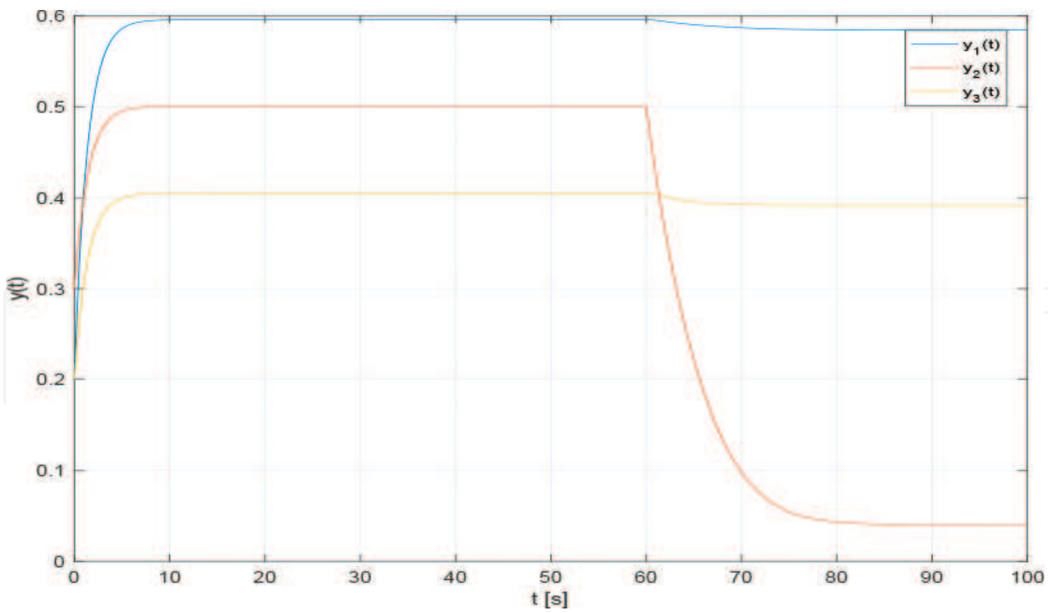

Figure 2. System output responses.

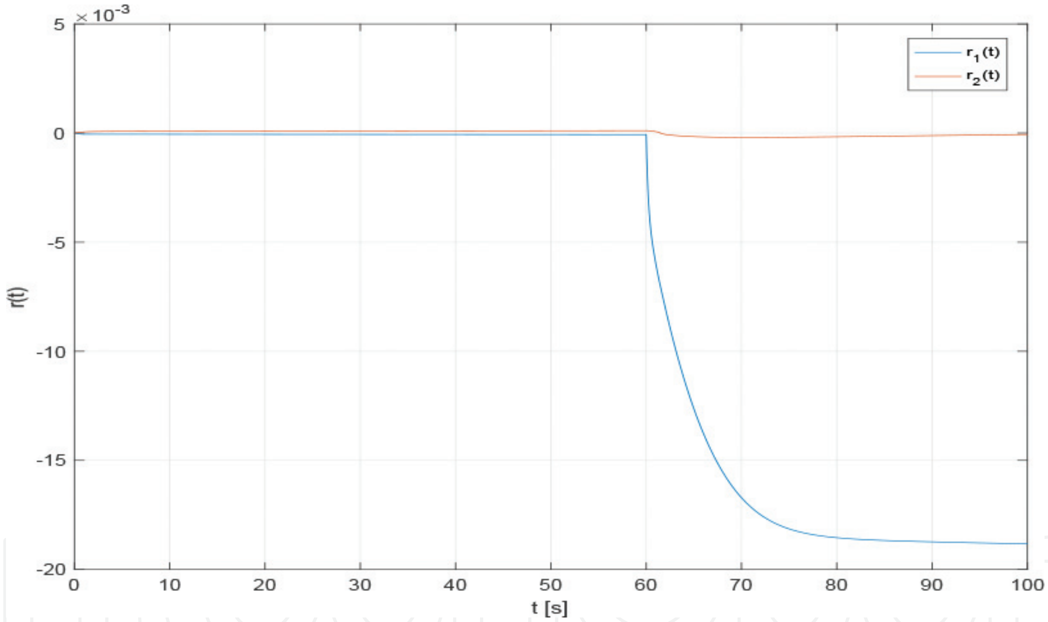

Figure 3. Residual signal responses.

example, that in the occurrence of a single fault of the second actuator the responses of TS fuzzy fault detection filter defined for the couple ($\boldsymbol{C}_{13}$, $\boldsymbol{B}_{13}$) naturally do not have directional properties, since the second column of $\boldsymbol{K}$ is not included in its construction.

As can be seen from the solution, the sector functions defined in this way cannot create a unitary TS fuzzy fault detection filter, but the obtained orthogonal properties of the residual signals are sufficient to detect and isolate actuator faults.

6. Concluding remarks

The problem of designing the TS fuzzy fault detection filters for highly nonlinear mechanical systems representable by the TS fuzzy model is considered, to achieve the desired filter H_∞ norm property in all working point belonging to the assigned work sectors. The proposed method exploits features offered in TS fuzzy system models to design TS fuzzy fault detection filters. The rules and formulation are developed to generate residual signals with quasi-directional properties and to make the TS filter transfer function matrix with prescribed H_∞ norm properties. By a convenient choose of the sector functions, this purpose is reached using a relative small number of membership functions. If unitary definition for TS fuzzy fault detection filters is satisfied, the design methodology provides new opportunities for fault detection and isolation rules in fault tolerant nonlinear control systems, their analysis, and optimization.

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