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Bridges Subjected to Dynamic Loading

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Abstract

This chapter reviews the analysis of problems of highway and rail bridge dynamic response to moving traffic loads. Bridge vibrations analyses comprise solution of many interdisciplinary problems. During the two last centuries, these problems have been studied by theoretical, numerical and experimental way by many investigators. Therefore, the present chapter contains only the basic approaches for solving the complex problem of bridges subjected to dynamic loading.

Keywords: bridge structures, dynamic response of the bridge, FEM, bridge natural frequencies, moving load on bridges, full-scale bridge testing, bridges dynamic loading tests and monitoring, spectral analysis

1. Introduction

Analysis of the effects of moving loads on bridge structures was motivated by the development of rail transport in the two last centuries, which necessitated the construction of many bridges. The busy bridge transport operation eventually resulted in failures, e.g., the collapse of *Chester rail bridge* in 1947, *Takoma highway bridge* in 1940, etc. The first theoretical studies of dynamic bridge response, idealized as an elastic beam of finite length with a moving mass point, were presented in 1849 by Willis [1] and Stokes [2] and later in 1896 by Zimmerman [3]. The moving of massless force across a beam was analyzed by Krylov [4] and Timoshenko [5], who also simultaneously solved the problem of force moving across a mass-beam at constant speed. The total knowledge of the problem from that period was summarized by Inglis [6]. Nowadays, similar bridges problems are solved by numerical finite element methods via modal co-ordinate analysis of structures subjected to moving loads, e.g., Rao [7]. While the problem of rail bridge vibration has been investigated intensively since the second half of the last century, serious attempts to solve the problems of highway bridge vibration date from

the middle of this century. The first report on this problem was published in 1931 by the American Society of Civil Engineers—ASCE [8] after which significant advances were made using analogue and digital computers, see also Biggs et al. [9], Looney [10], Huang and Valetsos [11], Tung et al. [12], Chaallal and Shahawi [13]. Czechoslovakia (till 1993), relevant studies were performed by Koloušek [14], Frýba [15], Baťa [16], Benčat [17], and others.

In the majority of studies, the bridge is considered as a one-dimensional beam, for which the differential equations of motion have been solved by numerical methods. The application of advanced calculation methods (finite element methods—FEM and other relevant numerical methods) enables two- and three-dimensional simulation models of bridge vibration to be solved. In the 1970s, Ting et al. [18] proposed an algorithm for solving this problem, based on an integral formulation of bridge vibration, which took into account the relations of kinematic bond of the *moving vehicle-and-bridge interaction*. The application of the above-mentioned methods in many cases was successful and led to the introduction of design standards and methods of assessing bridge structures. From a historical point of view, these solutions represent a gradual development in the understanding of bridge dynamic response, due to moving vehicles and their interactions.

The solution of bridge service life and reliability problems, as influenced by bridge vibration, which is mostly of random character, contributes to the complexity of this problem. However, in spite of all the complications of bridge vibration and the numerous parameters incorporated in regulations and standards in many countries, the *natural frequencies* and corresponding *modes of vibration*, the *dynamic coefficient* (dynamic increase of stress or deformation) and the *damping* are the basic bridge vibration characteristics, which can be verified by in situ experimental tests and monitoring.

Presently for *evaluation of dynamic response and projected parameters, new and existing bridges are utilizing numerical and experimental bridges dynamic analysis. Full-scale bridges dynamic testing and monitoring* give relevant information for projecting and assessment of real bridge behaviors [16–23]. This information consists of observed quantities obtained by experimental tests, theoretical analysis, and numerical computation and their comparison. Nowadays, the important role in the control of the bridge structures with bridge dynamic parameters (relative change of eigen-frequencies, damping parameters, fatigue parameters, vibration effective amplitudes value in time histories, etc.) plays *monitoring* of the structural parameters during normal bridge traffic on. Some results from *bridge forced vibration tests* (vibration is artificially induced (e.g., during the *dynamic loading tests—DLT*, etc.)) and also from *bridge monitoring ambient vibration tests* (input excitation is not under the control of the test engineer) are also used.

2. Theoretical and numerical approach

2.1. Simply supported beam subjected to a moving constant force

The simplest calculation model of bridge vibration is based on a simply supported elastic beam with a mass moving across the beam at constant velocity. The moving mass is assumed

negligible compared with the mass of the beam. This basic case of dynamic bridge response was solved by authors, e.g., [4, 6, 13, 15] and others. The vibration caused by a force moving across an elastic Bernoulli-Euler's beam (**Figure 1**) with viscous damping, is described by the equation

$$EJ \frac{\partial^4 v(x_1 t)}{\partial x^4} + \mu \frac{\partial^2 v(x_1 t)}{\partial t^2} + 2\mu\omega_b \frac{\partial v(x_1 t)}{\partial t} = \delta(x - ct)F \quad (1)$$

with appropriate boundary and initial conditions.

The state of oscillation may be expressed as follows:

$$v(x_1 t) = v_0 \sum_{j=1}^{\infty} \frac{1}{j^2[j^2(j^2 - \alpha^2) + 4\alpha^2\beta^2]} [j^2(j^2 - \alpha^2)\sin j\omega t - \frac{j\alpha[j^2(j^2 - \alpha^2) - 2\beta^2]}{(j^4 - \beta^2)^{\frac{1}{2}}} e^{-\omega_b t} \sin \omega'_{(j)} t - 2j\alpha\beta(\cos j\omega t - e^{-\omega_b t} \cos \omega'_{(j)} t)] \sin \frac{j\pi x}{l} \quad (2)$$

where

$$v_0 = \frac{Fl^3}{48EJ} \approx \frac{2F}{\mu l \omega_{(1)}^2} = \frac{2Fl^3}{\pi^4 EJ} \quad (3)$$

is the static deflection at mid-span of the beam. The *circular frequency of the damped beam vibration*, with light damping, is

$$\omega'_{(j)} = \omega_{(j)}^2 - \omega_b^2 \quad (4)$$

and with heavy damping is

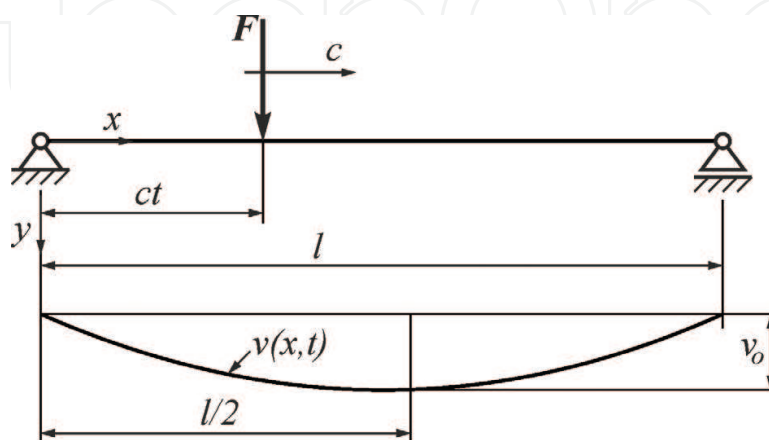


Figure 1. Simple beam subjected to a moving load.

$$\omega'_{(j)} = \omega_b^2 - \omega_{(j)}^2 \quad (5)$$

Parameters α and β are defined as

$$\alpha = \frac{\omega}{\omega_1} = \frac{c}{2f_{(1)}l} = \frac{T_{(1)}}{2T} = \frac{cl}{\pi} \left(\frac{\mu}{EJ} \right)^{\frac{1}{2}} = \frac{c}{c_{cr}} \quad (6)$$

$$\beta = \frac{\omega_b}{\omega_{(1)}} = \frac{\omega_b l^2}{\pi^2} \left(\frac{\mu}{EJ} \right)^{\frac{1}{2}} = \frac{\vartheta}{2\pi} \quad (7)$$

The *circular frequency* of the j th mode of vibration of a simply supported beam is denoted by

$$\omega_{(j)}^2 = \frac{j^4 \pi^4 EJ}{l^4 \mu} \quad (8)$$

the corresponding *natural frequency* by

$$f_{(j)} = \frac{\omega_{(j)}}{2\pi} = \frac{j^2 \pi}{2l^2} \left(\frac{EJ}{\mu} \right)^{\frac{1}{2}} \quad (9)$$

and the *circular frequency* by

$$\omega = \frac{\pi c}{l} \quad (10)$$

The solution of Eq. (2) has been analyzed by Fryba [15], with regard to parameters α and β . The maximum dynamic deflection corresponds to values $\alpha \approx 0.5$ – 0.7 . For large values of α , deflection tends to zero, while for small values of α , deflection is practically equal to the static deflection. The critical velocities defined as

$$c_{cr} = 2f_{(1)}l = \frac{\pi}{l} \left(\frac{EJ}{\mu} \right)^{\frac{1}{2}} \quad (11)$$

are too high for practical cases. The critical velocity for the first natural frequency of steel bridges is

$$c_{cr} = 2f_{(1)}l \approx 2l \frac{10^3}{4l} = 500 \text{ m/s} = 1800 \text{ km/h} \quad (12)$$

The results of the theoretical analysis given in this paragraph are applicable for large-span rail and highway bridges. These bridges have very low values of the first natural frequencies and the vehicle mass is negligible compared with the bridge mass, across which they are moving. Since damping of large-span bridges is light, the dynamic displacement may be calculated from

$$v_{(x_1t)} \approx v_o \sum_{j=1}^{\infty} \sin \frac{j\pi x}{l} \times \frac{1}{j^2(j^2 - \alpha^2)} \left(\sin j\omega t - \frac{\alpha}{j} e^{\omega_b t} \sin \omega_{(j)} t \right) \quad (13)$$

Eq. (13) can be simplified for low vehicle speed, $\alpha \ll 1$, into the form

$$v_{(x_1t)} \approx v_o \sum_{j=1}^{\infty} \frac{1}{j^4} \sin \frac{j\pi x}{l} \sin j\omega t \quad (14)$$

Eq. (14) represents the influence line of beam deflection at point x , expressed as a Fourier series. Since the terms for $j > 1$ are negligible, for practical applications, it is sufficient to take into account only the first term of the series

$$v_{(x_1t)} \approx v_o \sin \omega t \sin \frac{\pi x}{l} \quad (15)$$

2.2. Moving harmonic force

In the first half of twentieth century, dynamic bridges response analyses were focused mainly on studies regarding to the rail bridge vibration caused by steam traction. It has been the subject of much research (e.g., [6, 7, 14]). Inglis [7] modeled the so-called “hammer blows,” due to unbalanced weights on the driving wheels of a locomotive, by a sinusoidal alternating force moving at a constant velocity across a beam. Expressing the time variation of the concentrated force by $F(t) = Q \sin \Omega t$, and considering only the first mode of beam response, the dynamic deflection in the region of resonance is given by

$$v_{(x_1t)} = v_o \frac{Q \omega_{(1)} \cos \omega_{(1)} t}{2F \omega^2 - \omega_b^2} \left[\omega (\cos \omega t - \exp^{-\omega_b t}) - \omega_b \sin \omega t \right] \sin \frac{\pi x}{l} \quad (16)$$

where $\omega_{(1)} = \Omega$ and v_o is as defined by Eq. (3).

The *dynamic coefficient* is often defined as the ratio of the maximum dynamic deflection to the static deflection at the mid-span of the beam (**Figure 2**).

$$\delta = \frac{\max v(l/2, t)}{v_o} \quad (17)$$

The dependence of the dynamic coefficient on speed is sometimes called the *resonance curve*. The dynamic coefficient attains its maximum at resonance, e.g., when $\omega_{(1)} = \Omega$ and is given by

$$\delta = 1 + \frac{Q}{2F} \frac{\omega_{(1)}}{\omega^2 - \omega_b^2} (\omega e^{-\omega_b^{1/2} c} + \omega_b) = 1 + \frac{Q}{F} \Delta \quad (18)$$

where Δ , after substitution of the speed and damping parameters α and β from Eqs. (6) and (7) is expressed as

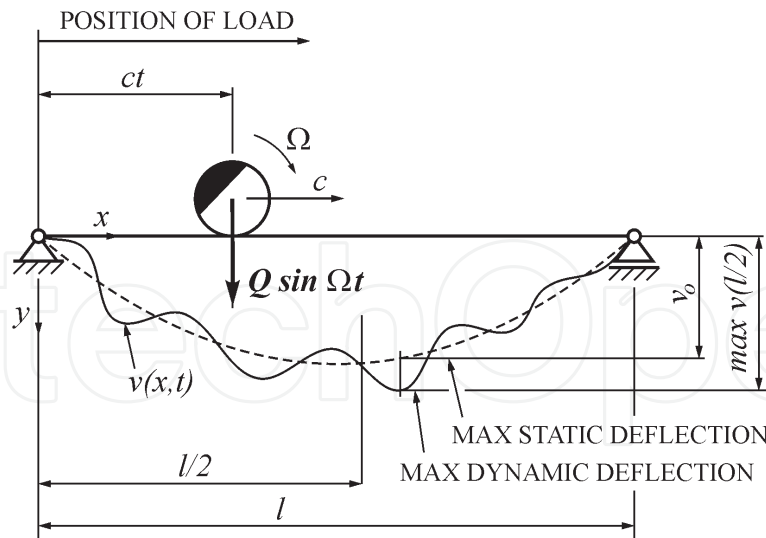


Figure 2. Mid-span deflection produced by a constant moving harmonic load.

$$\Delta = \frac{1}{2(\alpha^2 + \beta^2)} (\alpha e^{-\pi\beta/2\alpha} + \beta) \quad (19)$$

The *dynamic deformation* (or stress) *increment* may be defined as an alternative to Eq. (17), e.g., EMPA (Swiss Federal Laboratories for Material Testing and Research), for experimental tests of bridges, defined the *dynamic increment* ϕ [19]:

$$\phi = \frac{v_{\text{dyn}} - v_{\text{stat}}}{v_{\text{stat}}} \quad (20)$$

where v_{dyn} is the peak value of the bridge displacement measured during a passage of the test vehicle across the bridge and v_{stat} is the peak value of the bridge deflection observed under static loading caused by the same vehicle.

Application of the theoretical analysis of bridge vibration caused by a moving harmonic force is presently not of practical significance, due to the decline in the use of steam engines. The given knowledge, however, serves to explain the bridge vibration concepts which developed from the literature of that period. The parameter that was expressed as the *dynamic coefficient* (δ) and its dependence on moving vehicle speed is still one of the most important parameters characterizing bridge stiffness.

From the previous sections, it follows that *moving vehicle* on a bridge generates deflection and stresses in the bridge structure that are greater than those generated by the same vehicle applied statically. In general, the *dynamics amplification* (DA) is defined by

$$DA = \frac{R_{\text{dyn}} - R_{\text{stat}}}{R_{\text{stat}}} \quad (21)$$

where R_{dyn} and R_{stat} are maximum *dynamic and static response* (deflection, stresses, etc.) of the bridge, see also Eq. (20). Therefore, dynamic response can be calculated as

$$R_{dyn} = DAF \times R_{stat} \quad (22)$$

where DAF is the *dynamic amplification factor* given by

$$DAF = 1 + DA \quad (23)$$

In addition to specifications of *most codes*, the dynamic effects of vehicles on bridges are considered by multiplying the static live loads by a *dynamic load factor* ($DLF = \delta$) *greater than one*; DAF—there are many ways of interpreting this simple definition of the DAF from test data, see also, e.g., [19, 24].

2.3. Massless beam subjected to a moving load

The problem of a beam loaded by a moving load with negligible mass has been discussed in Section 2.1. The other extreme problem of negligible mass beam, subjected to a moving load of finite mass, was solved in [2, 3]. Consider a simply supported beam with span l and negligible mass, traversed by a load F with mass $m = F/g$, moving with a constant velocity c (**Figure 3**). Since the instantaneous position of the mass and the beam deformation are defined by a vertical displacement $v(a)$ at point “a,” where the force is situated, the system has one degree of freedom.

The total acting force $Y(a)$ consists of mass gravity $F = mg$ and inertia force $-md^2v(a)/dt^2$, which depends on the vertical acceleration at point $a = ct$, i.e.,

$$Y(a) = mg - m \frac{d^2v(a)}{dt^2} \quad (24)$$

The static deflection caused by force F is given by Eq. (3) and hence the approximate solution for the dynamic coefficient δ , given by Zimmerman [3], is as follows

$$\delta = 1 + \frac{16v_0c^2}{gl^2} \left(1 + \frac{40v_0c^2}{gl^2} \right) \quad (25)$$

From Eq. (25), it is evident that magnitude of δ decreases with increasing span l . Large-span bridge structures are heavy and their mass cannot be neglected compared with the mass of moving vehicles. However, the real dynamic action of vehicles moving across short-span bridges is not reliably described by Eqs. (24) and (25). The effect of the moving mass is fairly

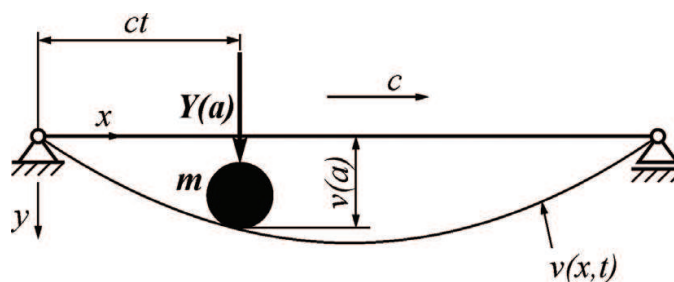


Figure 3. Massless beam with a moving mass.

small compared to that of other factors which produce high dynamic stresses in such bridges. For example, in short-span rail bridges, impact effects of flat wheels, rail joints, etc., predominate over those of the moving load. Thus, a vehicle cannot be represented adequately by a single moving point mass, even for short-span bridges.

2.4. Beam subjected to a moving system with two degrees of freedom and two axle

The need to quantify dynamic bridge response induced by moving vehicles has led to the development of improved but more complex physical models. The use of modern computers and advanced numerical methods enables satisfactory solutions of such problems to be obtained.

If the vehicle mass as well as the bridge mass is taken into account, the problem is more complicated than the problems analyzed in Sections 2.1 pending 2.3. The actual problem is described by the differential equation

$$EJ \frac{\partial^4 v(x_1 t)}{\partial x^4} + \mu \frac{\partial^2 v(x_1 t)}{\partial t^2} + 2\mu\omega_b \frac{\partial v(x_1 t)}{\partial t} = \delta(x - ct) \left[F - m \frac{d^2 v(ct, t)}{dt^2} \right] \quad (26)$$

The right-hand side of Eq. (26) expresses the motion of the force F with mass m , including the inertia effect. This problem has been discussed by [7, 9, 11–13] and others. Ting et al. [18] proposed a solution which takes into account a kinematic bond of the vehicle-bridge system. Many other solutions of this problem are published in contemporary works. These cannot be described in the context of this chapter. Therefore, only the basic formulation of the problem has been introduced to identify the parameters which influence the *bridge-vehicle system vibration*. The specific characteristics of the kinematic bond of rail bridges and highway bridges should be taken into account. A vehicle is a complex mechanical system. For the purpose of axle load calculation, it can be represented by a plane model consisting of mass points, material planes, and connecting elements. It is possible to idealize the physical model of a vehicle as a one-, two- or multi-axle system, with or without damping (**Figure 4**).

The bridge is modeled as a simply supported Bernoulli-Euler beam, with a continuously distributed mass, or as a discrete system with n -lumped masses. The surface of the beam may be assumed perfectly smooth or to have irregularities. The beam stiffness can be assumed constant or variable, based on a layered system with variable stiffness in each of the elastic layers mainly for application to rail bridges (**Figure 5**). The real behavior of the *bridge-vehicle system* can be described, more or less successfully, with the combination of physical vehicle and bridge models shown in **Figures 4** and **5**. Satisfactory results have been obtained using the vehicle models in **Figure 4(A)–(C)** for bridges with spans $l > 30$ m. The two- or multi-axle models of the vehicle system are more appropriate for short-span bridge investigation. The following simplifying assumptions are made in relation to the physical models:

- the load remains in contact with the surface of the bridge;
- the vehicle speed is constant;
- the bridge and vehicle damping is proportional to the velocity of vibration (viscous damping);

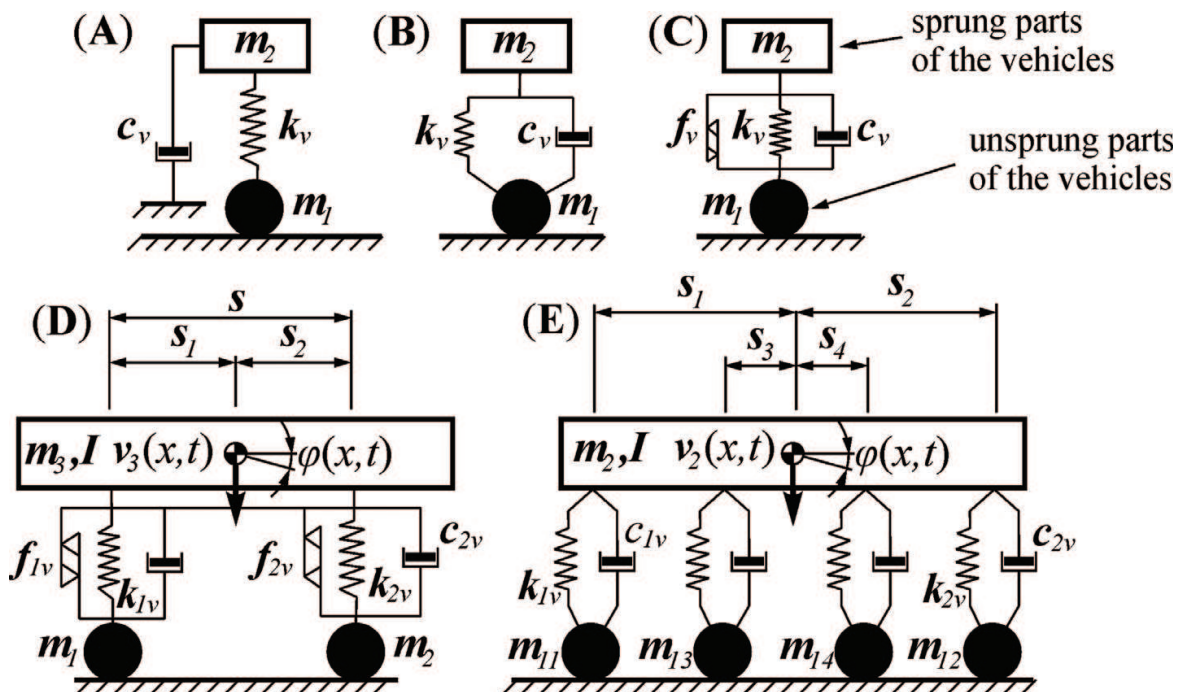


Figure 4. Physical models of a vehicle.

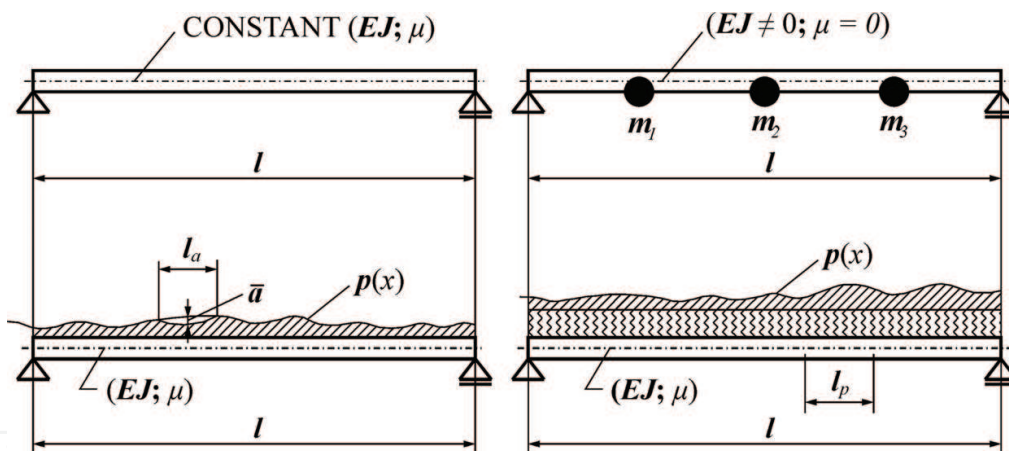


Figure 5. Physical models of bridge.

- the springing and damping of the tires are not taken into account (highway bridges);
- variable stiffness of elastic layers is taken into account for steel railway bridges (sleeper spacing effect).

The mathematical formulation of the problem of synchronous *bridge-vehicle system* vibration, taking into account the above simplifying assumptions, leads to the set of three simultaneous differential equations with variable coefficients (because of variable stiffness of elastic layers and track irregularities) describing, respectively, the vertical displacements of sprung and unsprung masses and beam vibration. The set of differential equations may be solved by numerical integration utilizing a digital computer with relevant software package.

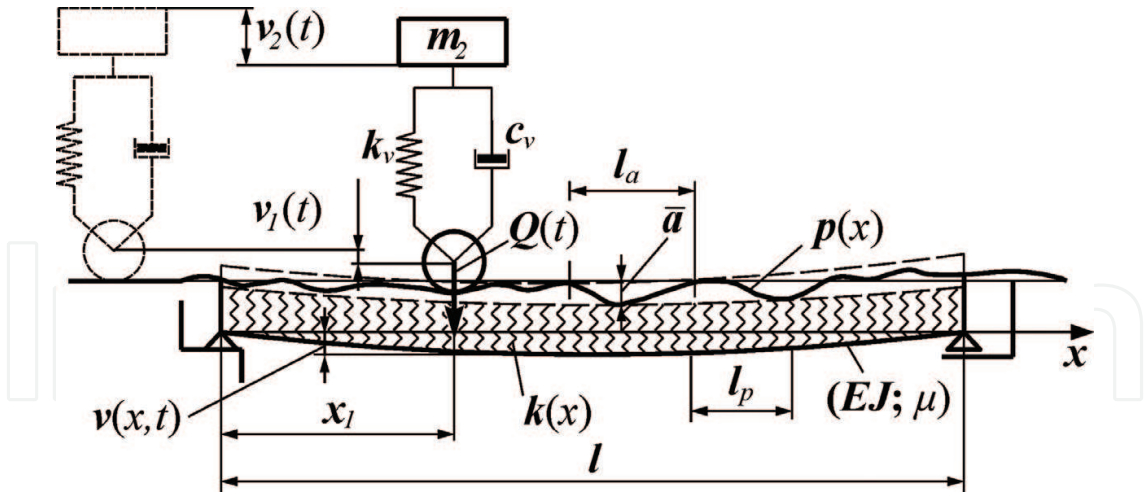


Figure 6. Model of a beam with an elastic layer and irregularities subjected to a moving system with two degrees of freedom and force $Q(t)$.

Consider the physical model of a rail bridge (**Figure 6**) with the following assumptions [15]:

1. The moving vehicle is idealized by a system with two degrees of freedom. An unsprung mass m_1 is in direct contact with the beam; m_2 denotes the sprung parts of the vehicle and the total weight of the vehicle is

$$F = F_1 + F_2 = g(m_1 + m_2) \quad (27)$$

The coordinate of the contact point is $x_1 = ct$, because of the constant speed c along the beam.

2. The unsprung mass is acted upon only by harmonic force.
3. The top surface of the beam is covered with an elastic layer of variable stiffness $k(x)$.
4. Track irregularities are assumed to vary harmonically along the bridge span as

$$p(x) = \frac{1}{2}\bar{a}\left(1 - \cos 2\pi x/l_a\right) \quad (28)$$

where $\bar{a}$ is the maximum depth of track unevenness and l_a is the length of track irregularity. The equations of motion of the synchronous *vehicle-bridge system* within the interval $0 \leq t$ can be written

$$-m_2 \frac{d^2 v_2(t)}{dt^2} - k_v [v_2(t) - v_1(t)] - c_v \left[\frac{dv_2(t)}{dt} - \frac{dv_1(t)}{dt} \right] = 0 \quad (29)$$

$$\begin{aligned} F + Q(t) - m_1 \frac{d^2 v_1(t)}{dt^2} + k_v [v_2(t) - v_1(t)] \\ + c_v \left[\frac{dv_2(t)}{dt} - \frac{dv_1(t)}{dt} \right] - R(t) = 0 \end{aligned} \quad (30)$$

$$EJ \frac{\partial^4 v(x_1 t)}{\partial x^4} + \mu \frac{\partial^2 v(x_1 t)}{\partial t^2} + 2\mu\omega_b \frac{\partial v(x_1 t)}{\partial t} = \bar{\varepsilon} \delta(x - x_1) R(t) \quad (31)$$

where

$$R(t) = k(x_1) [v_1(t) - \bar{\varepsilon} v_{(x_1, t)} - p(x_1)] \geq 0 \quad (32)$$

is the interactive force by which the moving system acts on a beam at the point of contact x_1 , and

$$\bar{\varepsilon} = \begin{cases} 1 & \text{for } 0 \leq x_1 \leq 1 \\ 0 & \text{for } x_1 < 0; x_1 > 1. \end{cases}$$

Eqs. (29)–(31) should satisfy the boundary conditions of a simply supported beam as well as the appropriate initial conditions. These equations provide a very general statement of the problem of vibrations excited by a system of masses moving along a beam. Simpler sets of differential equations, which describe dynamic bridge response with sufficient accuracy, can be derived with simplifying assumptions. These equations have been solved numerically in [15].

Various individual bridge and vehicle parameters, as well as the interaction between them, were included in the theoretical analysis of the bridge dynamic response. The parameters and effects considered were: *the vehicle speed, the frequency parameter of unsprung and sprung masses, variable stiffness of elastic layers, the ratio between the weights of the vehicle and the beam, the ratio between the weights of the unsprung and sprung parts of the vehicle, the beam damping, the vehicle spring damping, the initial conditions and others.* Many contributions and solutions of this problem can be found in the literature on bridge vibrations. Wen [25] was the first author to solve this problem with application to highway bridges. In [15], the influence of the individual bridge parameters and two-axle systems on the dynamic response of steel rail bridges is also analyzed.

Rail bridges—remarks. The theoretical results have been verified by experimental tests on more than 50 rail bridges in Slovak Republic and Czech Republic and also in former Czechoslovakia (Research Rail Institute, Prague; Department of Structural Mechanics, University of Transport and Communications (UTC) Žilina; University of Žilina (1993–2017) and others).

The following conclusions can be made from the results:

1. (a) for large-span bridges with spans over 30 m, it is appropriate to consider the physical vehicle model as a moving system with two degrees of freedom (see **Figure 4(A)–(C)**); (b) for short-span bridges with spans less than 30 m, it is necessary to idealize the vehicle as a two-axle or multi-axle system.
2. The greatest influence on the dynamic increment of deflection or stress (ϕ , DA) is the vehicle speed.
3. It is necessary to include in the theoretical calculations the influence of the cross beams, uniform sleeper spacing and other regular unevenness that enlarges the local peaks in the dynamic coefficient (δ)-velocity diagram.

4. The dynamic effect of railway vehicles increases approximately in proportion to the frequency of sprung masses and the vehicle weight.
5. The dynamic stresses in short-span railway bridges are affected primarily by the impact resulting from track or wheel irregularities (rail joints, flat wheels, etc.)
6. For short-span rail bridges, the effects of sprung and unsprung vehicle masses that have been set in vibration prior to crossing the bridge are important.
7. The periodic irregularities (sleeper effects) when multi-axle vehicle systems cross the bridge can cause their vibration with resonance, especially at velocities of 100–200 km/h.

Figure 7 shows a comparison of the computed and measured deflections at mid-span of a bridge, and the dynamic coefficients δ at different locomotive speeds.

Highway bridge—remarks. The preceding discussion was directed primarily toward railway bridge vibration. Highway bridge vibration analysis should incorporate the specific features which are associated with *highway bridge structures* and *vehicle construction*, which result in different *interaction of the bridge-vehicle system*. In the case of highway bridges, the load bearing system of modern bridge structures consists mainly of prismatic and non-prismatic beams of box, open or partly closed cross section. In the majority of cases, the bridge structure approximates to the typical linear structure model. At the formulation stage of the physical model of the *bridge-vehicle system*, it is necessary to take into account the effect of variable stiffness of the roadway, which may be replaced by the effect of track irregularities. The real bridge, as well as vehicle response, can be described adequately by the physical model of the vehicle-bridge system shown in **Figure 8**. Theoretically, the problem of forced vibration of a system consisting of a moving vehicle and a bridge structure (**Figure 8**) can be described generally by operator relations, e.g. [15]

$$\begin{aligned} L_1[\{r_q(t)\}, \{r_s(t)\}, h_q(t), \{v(x_q, z_q, t)\}, a_q(t)] &= 0 \\ L_2[\{v(x, z, t)\}, \{r_q(t)\}, h_q(t), a_q(t), \{v(x_q, z_q, t)\}] &= 0 \end{aligned} \quad (33)$$

where L_1 and L_2 are linear or non-linear operators; $\{r_q(t)\}$ and $\{r_s(t)\}$ are displacement vectors of vehicle elements, conditioned by upper and lower links; $\{v(x, z, t)\}$ is a vector of bridge

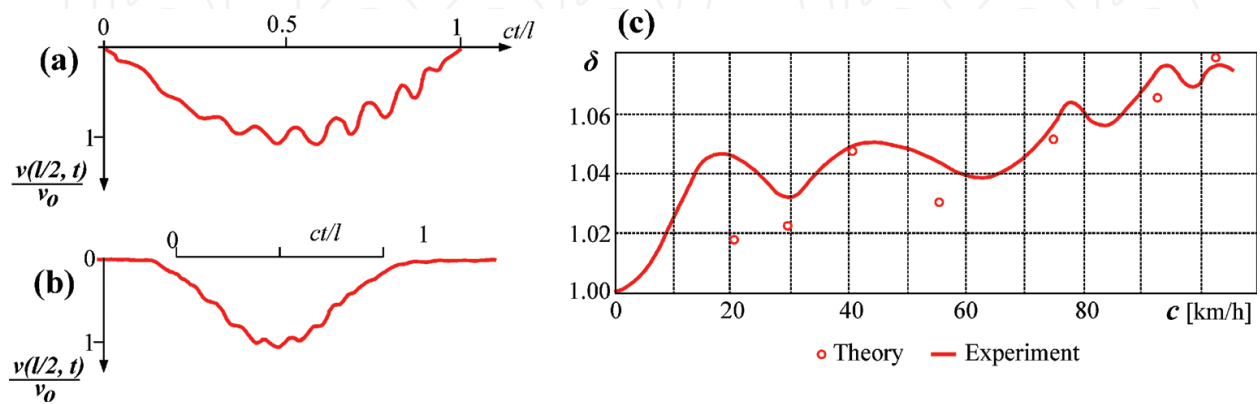


Figure 7. Deflection at the center of beam with span $l = 34.8$ m, traversed by an electric locomotive E 469 at speed $c = 40.7$ km/h [15]: (a) theory, (b) experiment (c) theoretical and experimental dependence of the dynamic coefficient δ .

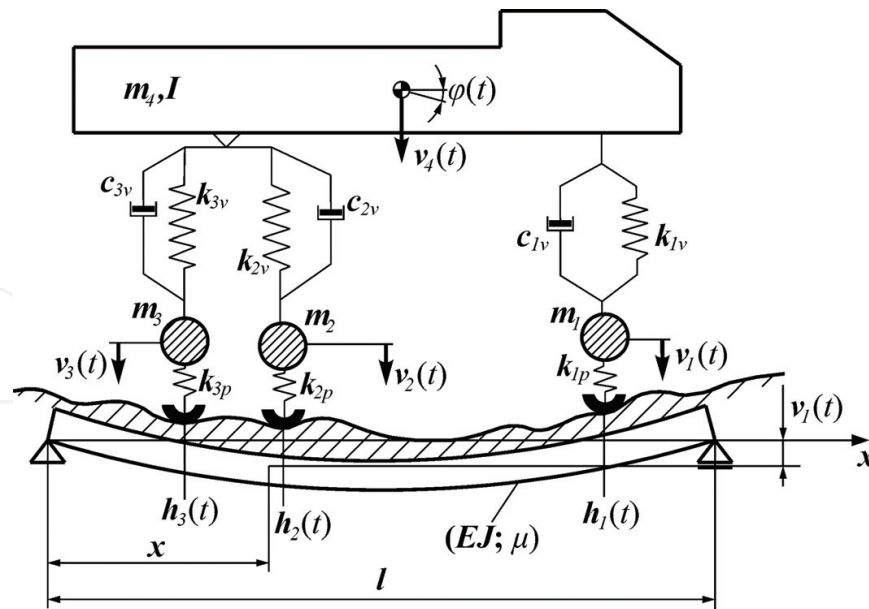


Figure 8. Physical model of a system consisting of a bridge structure and a moving vehicle.

structure displacements at the points with coordinates x, z at time t ; $\{v(x_q, z_q, t)\}$ is a vector of bridge structure displacements at the q th lower link between the vehicle and the bridge structure; $a_q(t)$ is the law defining the vehicle movement along the longitudinal bridge axis; $h_q(t)$ is the function describing the irregularities of the road surface; and x_q, z_q are the coordinates of the lower link between the vehicle and the bridge structure. Eq. (33) by the differential operators, together with boundary and initial conditions, defines the motion of a system consisting of a moving vehicle and a bridge structure.

Research into highway bridges has not been systematic, either in Europe or elsewhere, and in some countries, it has been limited mainly to random tests of extraordinary bridge structures, before being put into operation. This has led to a variety of methods for calculating dynamic effects in individual countries, particularly in the provisions concerning dynamic coefficients (δ , DA, DLA) in respective standards or bridge regulations. At the present time, much information is available on the forced vibration of highway bridges, which should be taken into account during the formulation of the vehicle-bridge physical model. The results of theoretical analysis and parametric studies of highway bridge response, as well as the results of experimental bridge investigations, performed by the relevant research divisions of the UTC Žilina or University of Žilina (UZ Žilina) on more than 60 highway and road bridges, are now discussed in detail. The dynamic bridge response is influenced primarily by:

1. As for railway bridges, vehicle speed has the greatest influence on the dynamic increments of stress and deflection of highway bridges.
2. The first mode natural frequency of vibration of the bridge in the vertical plane (bending) and the natural frequency of vibration of the sprung vehicle mass in the vertical direction. The theoretical dynamic coefficient δ (DLA) and also experimental δ_{obs} (DAF) are maximum when $\omega_{(1)} \approx \omega_v$. It was noted also that the influence of frequency ratio $\omega_{(1)}/\omega_v$ diminishes with increasing mass ratio (m_v/m_{br}).

3. The vehicle vibration at the moment the vehicle enters the bridge, since the vehicle's energy of vibration is the primary source of the dynamic bridge response. The vertical amplitude of the vehicle vibration is decisive. The initial angular amplitude of the vehicle's sprung mass vibration can be neglected in the analysis.
4. The character of the road irregularities (joints, potholes, inserted hinges, frozen snow, etc.).

The effects of the damping of the vehicles and bridges, as well as the ratio of the sprung vehicle mass to the bridge mass, are not significant for *long-span bridges*. However, they are a significant influence on *short-span bridges vibration*. It was confirmed by the theoretical analysis and the experimental tests that the curve expressing the dependence of the dynamic coefficients on the vehicle speed is not a smooth curve but has many local projections and branching points [26].

3. Natural frequencies and modes of bridge vibration

Calculation of the natural frequencies and corresponding modes of vibration forms a basis for the determination of the dynamic characteristics of bridge vibration. At present, bridges, especially of larger spans, are complicated space structural systems, comprising many elements which interact with one another, e.g., continuous beams, framed structures, arch construction, suspended structures, and others. Principally, three different simulation models of the bridge can be used: the *discrete model*, the *model with continuously distributed mass* and *models formulated by the FEM*. Advanced numerical methods FEM have been widely developed for practical application in this field, and they are described in the technical literature and available as computer software. Therefore, only two calculation methods are shortly described in this section. The theoretical determination of the natural frequencies and modes of vibration of such structures is fairly difficult in most cases and their verification is advised by experimental measurements.

3.1. Multi-degree of freedom systems

The equations of motion of a multi-degree of freedom system take the form

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \quad (34)$$

where $\mathbf{f}$ is a column matrix of applied forces and $\mathbf{u}$ is a column matrix of displacement components. Both $\mathbf{f}$ and $\mathbf{u}$ correspond to the same set of points on the structure and the same directions at these points. $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the inertia, damping, and stiffness matrices corresponding to the displacement components $\mathbf{u}$. If the applied forces and damping forces are absent, Eq. (34) becomes

$$[\mathbf{K} - \omega^2 \mathbf{M}]\{\boldsymbol{\phi}\} = 0 \quad (35)$$

if the motion is assumed to be harmonic, that is, $\mathbf{u} = \boldsymbol{\phi} \exp\{i\omega t\}$. This equation is a *linear eigen problem* similar to *n-DOF* system. The eigenvalues, ω_i^2 , represent the squares of the natural frequencies and the eigenvectors, ϕ_i , represent the shapes of the corresponding modes of free

vibration. An eigenvector is arbitrary to the extent that a scalar multiple of it is also a solution of Eq. (35). It is convenient to choose this multiplier in such a way that ϕ has some desirable property. Such *eigen vectors* are called *normalized eigenvectors*.

3.2. Numerical procedure application in bridge structure dynamic analysis

To avoid creating complicated and sophisticated numerical models involving extensive assumptions in modeling (boundary and initial conditions, mechanisms of bridge flexibility and energy dissipation, inertia, etc.), it is useful to develop an appropriate model with realistic prediction of their dynamic response upon the comparison of the experimental results and theoretical predictions. This enables also the realistic and optimal economical designs. Nowadays, very popular and useful numerical method for engineering analysis is finite element method. FEM is a numerical procedure for obtaining solutions to many of the problems encountered in civil and structure engineering. Numerical solutions of the bridge dynamic analysis problems in many cases need experimental verification in situ, e.g., [20, 27, 29, 30, 31].

To create relevant analytical models with real dynamic bridge structure with input parameters, it is useful to apply experimental modal analysis (EMA) which provides mainly *structure natural modes engine frequencies and damping parameters* of the tested bridge structure [31]. For such type of bridge dynamic tests performance in most cases, the real bridge service conditions are too restrictive for performance such type bridge tests. In these cases, operational modal analysis (OMA) procedure is applicable, which enables to perform bridge dynamic testing and also bridge health monitoring measurements without interrupting bridge service. A well-presented review of bridge testing methods explaining their conditions, advantages and limitations was presented by Salawu and Williams [27].

The bridge dynamic analysis programs are commonly available and computational problems are not complicated to solve. A lot of FEM software packages are used in this field mainly for structures modal analysis and dynamic response of bridges (ANSYS Civil FEM Bridge, BRASS, BRIDGES, BridgeSoft, BRIDGADES (ABAQUS), ADINA, DYNASOLV, LUSAS, etc.).

4. Dynamic loading tests of bridges and monitoring

In situ dynamic testing of bridges gives very useful information for numerical modeling and assessment of real bridge dynamic parameters and service conditions. In many countries, the requirement of putting the bridges into operation is the execution of *bridge static and dynamic loading tests*, which aim is to *prove* and *confirm* the projected parameters (standards criteria, serviceability, safety limit states, etc.) of tested bridge structures according to technical standards, e.g., in Slovakia by standard STN—Slovak Technical Standards [28]. Results from *static or dynamic test* enable to calibrate a bridge analytical model and can be utilized as basic data for a bridge health monitoring program and for other sophisticated calculations of the bridge dynamic response (seismic, fatigue, etc.).

4.1. Test procedures

In this section, bridges dynamic test procedure is shortly described. Bridges are tested according to the rules of the *dynamic loading test* (DLT) [28]. Excitation of *highway bridges* are commonly due to the passage of single, fully loaded, *multi-axles lorries*. The testing vehicles' gross weight usually lies near the legal limit which is defined by standards and regulations. In the case of *railway bridges*, locomotives are used. Also *normal traffic flow* is used for both highway and railway bridges.

For the expected *dynamic bridge response* caused by well-defined individual testing vehicles, *dynamic calculations* are carried out before the bridge dynamic tests. The testing vehicle is driven with a constant speed (in each measurement travel) along the bridge and respectively in the same direction or in both directions. The tests begin with a vehicle speed of $c = 5$ km/h, which is increased after each passage in steps of 5 km/h, up to the maximum achievable speed [22, 26]. If a *static test* with the used testing vehicle is not performed before the *dynamic tests*, the bridge deflection caused by vehicle traveling at a speed $c = 5$ km/h can be considered to present the static deflection w_s with sufficient accuracy (e.g., via filtering signals).

In the case of *highway bridges*, the tests on the undisturbed bridges pavement are also repeated with a plank or *standard obstacles* placed across highway pavement, **Figure 9(a)**. The cross section of the standard obstacle (length 5000 mm) is a cylindrical sector of height 60 mm and chord length 500 mm [28]. During the tests of highway bridges pay load, tires and tire pressure are kept the same; it means that the vehicle dynamic properties remain approximately constant.

Pulse forces produced by the ignition of *pulse rocket engines* (PRE) during DLT are also used mainly on large bridge structures. *Harmonically variable forces* produced by *vibration exciters* and the free vibrations of the bridge are also applied.

This type of the DLT so-called proof-loading test is performed for checking if the construction of the bridge has been constructed according to the design project. These tests (DLT) comprise the evaluation of the *dynamic loading allowance* (DLA) $\rightarrow$ from standard = dynamic coefficients



Figure 9. Testing load: (a) lorries traveling over the obstacles; (b) the PRE applied mainly on large bridge structure.

δ calculated by designer and *dynamic amplification factor* (DAF—from DLT), greater than 1 (the amount by which the static effects are increased by bridge-vehicle interaction contribution).

Note 1 In relevant standards of many countries, it is defined how to obtain coefficient DLA (= δ) in a normalized way (e.g., Canada, France, Germany, India, Spain, Switzerland-UK, USA, and former Czechoslovakia, etc.).

Note 2 Nowadays for Slovak Republic (CEN member), from 1.5.2006, it is mandatory to applying new European Standard. For the chapter content, it is actual Eurocode 1—Action on Structures, Part 2: Traffic loads on bridges (EN 1991–2).

Note 3 In this EN, for road bridges, the dynamic amplification was included into the *load models* (fatigue accepted), although established for a medium pavement quality and pneumatic vehicle suspension, which depends on various parameters and on the action effect under consideration. Therefore, *it cannot be represented by a unique factor*. For example, in former Slovak Standard (till 2006)—STN 73 6203, *Load actions on bridges* for calculation of δ = DLA was used a formula in unnumbered format

$$\delta = \frac{1}{0.95 - (1.4I)^{-0.6}}$$

Note 4 For railway bridges, the dynamic amplification was accepted and dynamic factor Φ (= DLA = δ) is possible to calculate according to the given algorithm (EN 1991–2, Section 6).

4.2. DLT data acquisition and recording

In this section, *data acquisition and recording* (DAR) processes during a bridge DLT are shortly described. More detailed DAR processes descriptions are in [23, 26, 29]. Dynamic deflections are measured by *pick-ups* at the characteristic points of the bridge, which is normally at the mid-span. The bridge structure dynamic response at these points, in the horizontal and vertical directions, is then recorded in the form of time histories signal. Deflections $w(t)$ are also measured at additional points along the super-structure. Except for dynamic deflections, other relevant parameters are measured: speed of the loading vehicles, magnitude and time history of excitation forces, temperature of the structure and ambient air, wind velocity, etc.

Instrumentation: During standard dynamic tests of bridges (**Figure 10(a)**), *inductive displacement transducers*—IDT are mounted at the bridge parapet or bottom of the bridge structure, which are used to monitor displacement amplitudes time histories. In these cases, recorded displacement amplitudes time histories contains both static and dynamic components of the bridge dynamic response. The measured baseline is given by an invar wire (max 30 m), strained between the measuring points of the structure and a fixed reference point under the bridge structure. The application IDT enables extracting the static component from displacement time histories $w(t)$ by using filtering techniques. This procedure is applied for DAF calculation. When the measured structure cross section is situated over water (e.g., river, lake, bay, etc.), the IDT are usually replaced by *accelerometers* or *velocity-type transducers*, **Figure 10(b)**, or *strain gauges*, **Figure 10(c)**, (measuring of strain amplitudes time history contains both static and dynamic components of the bridge response due to moving load) with relevant hardware



Figure 10. Examples of bridge DLT instrumentation: (a) inductive displacement transducers (IDT); (b) accelerometers set up; (c) strain sensor (d) charge amplifiers devices; (e) accelerometer installation process to bridge bottom for DLT.

components (amplifiers, cables, wireless technique, etc.), **Figure 10(d)**. The view of accelerometers installation on bridge bottom for DLT is showed in **Figure 10(e)**.

The signals from the used pick-ups are amplified and filtered by the signal amplifiers and low-band pass filters, and then recorded by portable notebook with relevant software and hardware facilities in test *measuring station* (MS). Scheme and view of the equipment in MS used in situ tests are plotted in **Figure 11**. During DLT, the signals transmission from measurement

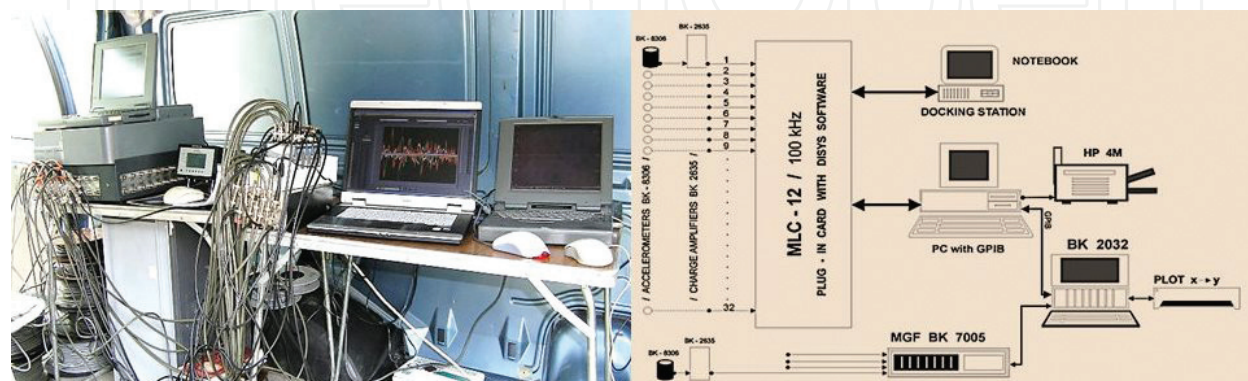


Figure 11. View of the equipment in MS with its scheme mounting used during the bridge dynamic tests.

devices to the recording technique at the MS by special low noise cables were usually used. The application of the *wireless sensor network* (WSN) platform for DLT or distributed measurement application, e.g., bridge structural health monitoring, is possible to eliminate the need for costly and work-intensive wiring measuring technique. The WSN platform simplifies remote monitoring applications and delivers low-power, reliable measurement nodes that feature local control capabilities.

The application of the *quick-setup* WSN enables to implement a stand-alone remote monitoring system or easily connect with measuring PC and control systems (e.g., NI WSN, BK PULSE WSN). **Figure 12** shows examples of equipment set for experimental measurement data wireless transmissions by NI WSN modules with portable PC layout and scheme.

The final experimental analysis is usually carried out in the laboratory. The bridge vibrations induced by the lorries crossing the bridge during the DLT with different velocities are analyzed in order to quantify and compare the different dynamic effects on the bridge structure. The analysis of the time histories of vibration recorded, when lorries crossed the viaduct bridge, is processed with the following operations:

- double integration of the accelerations to displacements and evaluation of their maxima and RMS values. The displacements maximum value is used for DAF calculations;
- offset and linear trends removal;
- digital filtering with a low-pass Butterworth filter with a cut frequency, e.g., of 150 Hz and with a high-pass Butterworth filter with a cut frequency of 0.5 Hz; and
- maximum and RMS values of acceleration amplitudes evaluation.

Also from the bridge dynamic response and free vibration measured time histories, can be obtained:

- frequencies of one or more vibration modes of the loaded and unloaded bridge;
- the natural vibration damping parameters, dominant in free decay;

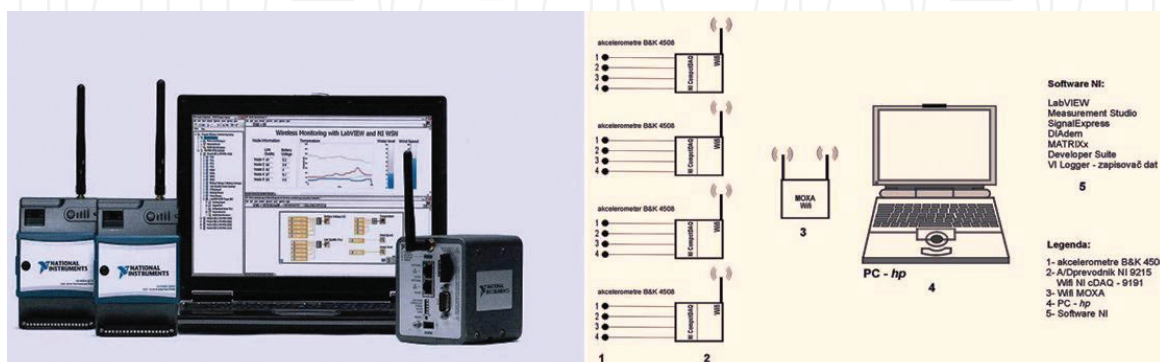


Figure 12. The experimental measurement data wireless transmissions by NI WSN modules with portable PC layout and scheme.

- percentage of vibration critical damping via the 3 dB band with method and curve fitting techniques;
- DAF and its dependence on vehicle speed;
- dynamic and static deflections as well as stresses magnitude in certain important parts of the bridge structure; and
- time history of vibration amplitudes and their classification by classification method (e.g., rain-flow classification method).

4.3. Bridges' dynamic parameters monitoring

Long-term bridges observation is discussed in literature, e.g., [29–37]. Some dynamic methods used by other authors [33], were applied to correlate relative changes of material, frequencies, and damping with carrying capacity. It was found that used monitoring techniques gave an early indication of incipient deterioration. The main scope of monitoring tests was to evaluate mainly the *relative change of well-defined natural frequencies or the corresponding damping and the RMS value of the displacements amplitude* of the bridge vibration due to traffic loading. The monitoring technique based on measurement of the bridge vibration time history due to regular traffic is not focused to give detailed bridge information but for making decision if more detailed bridge assessment methods should be used. The sophisticated bridge monitoring was introduced e.g. on the *Akashi Kaikyo* bridge in Japan, (**Figure 13**) completed 1998. At that time, it was the largest and longest suspension bridge in the world. Bridge is a 3-span 2-hinged bridge with steel-truss-stiffened girders located near Kobe City.

Bridge has a impressive 1991-m center span between two main towers that rise 300.0 m above the sea level. The *Akashi Kaikyo* bridge, being easily affected by natural conditions and traffic means, requires high level of disaster prevention and bridge structure functionality with projected structure parameters. Therefore, to provide centralized control, traffic control and bridge structure and facility monitoring have been integrated into the Traffic Control Center. *There, information acquisition and processing are performed continuously 24 h a day, providing vital traffic and bridge structure information.*

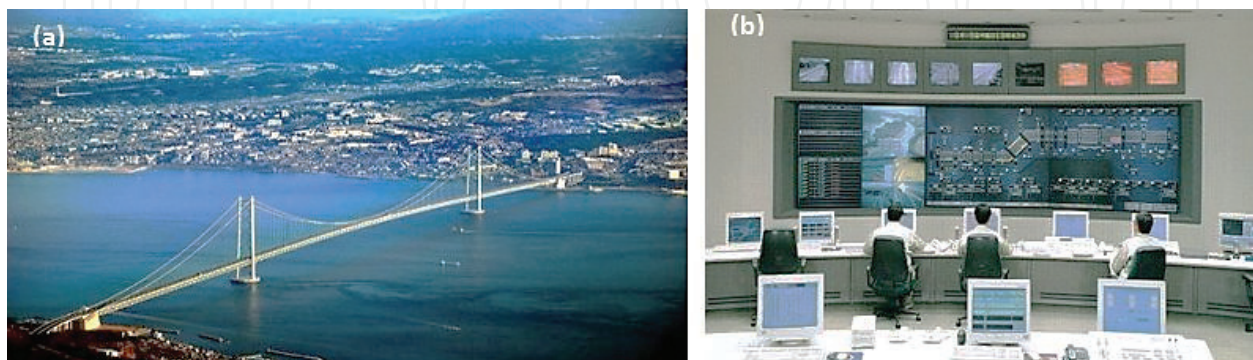


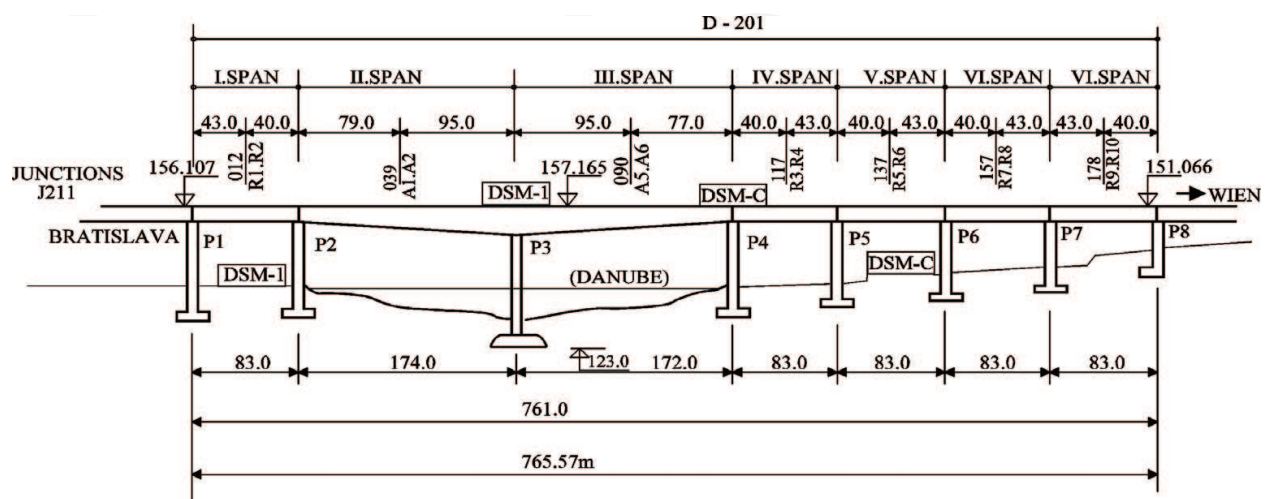
Figure 13. Akashi Kaikyo bridge: (a) look-out on the bridge; (b) 24 h a day monitoring center (source: *Kobe—Awaji—Naruto Expressway*. Honshu—Shikoku Bridge Authority 1998, advertising material).

5. Case study

The *dynamic loading test* and the following *dynamic monitoring* of the Lafranconi bridge over the Danube in Bratislava (Slovakia) are shortly described in this section [38–40]. The dynamic response behavior of a prestressed concrete, seven span highway bridge (761.0 m long) was examined via DLT according to standard [24] in 1990. Excitations of bridge structure were induced by the *passage of two fully loaded, multi-axles lorries* as well as by the *rocket engines*. Applied structural measurement technique was developed for in situ testing of the bridges. The DLT results enabled to identify bridge global dynamic characteristics of the bridge, e.g., *maximum and RMS of displacements amplitude, natural frequencies $f(j)$, mode shapes, DAF $\rightarrow \delta_{OBS}$ ($\delta_{OBS} = w_{max}/w_s$) and the structure amplitude damping parameter (ϑ)*. The obtained dynamic characteristics were compared with the numerical computed data [29] and standard prescriptions. For maximum and RMS displacements amplitude, and so on, see technical report [2].

5.1. Dynamic loading test of Lafranconi highway bridge over the Danube

The main bridge structure is composed of seven span continuous beams with one bridge frame pier (P3). The total length of the bridge was 761.0 m with spans 83.0 m + 174.0 m + 172.0 m + 4 × 83.0 m. The highway bridge consists of two independent bridges (left and right bridge) with three traffic lanes each (i.e., three in each bridge for one direction only) and sidewalks on both sides. The bridge's longitudinal section is shown in **Figure 14**. The bridge structure including multispan junctions, the test program, field measurements, and applied instrumentation are fully described in [29]. The vibration amplitudes were measured and recorded in 18 selected points. The measuring station for recording accelerometer signals (DSM-1) was situated on the top of the pier P3, **Figure 14**. The time history of vertical as well as horizontal vibration amplitudes have been registered by accelerometers in the second and the third span of the bridge. In the other bridge spans were applied inductive displacement transducer with working



range ± 40 mm. **Figure 14** shows the position of the accelerometers marked as A1, A2, A5, and A6 and transducers marked as R1–R10.

Output signals from the accelerometers were preamplified and recorded on two PC and four-channel portable tape FM recorders (BK-7005) and the signals from the inductive displacement transducers were recorded simultaneously by 12-channel portable tape recorder and FM recorder (BK-7005). The DAF have been determined by analyses of bridge amplitude vibration records from computer or tape recorders via relevant PC software pocket (*Disys*, 1990). The frequency response spectra (power spectrum $D_{xx}(f)$, power spectral densities $G_{xx}(f)$, cross power spectral densities $G_{xy}(f)$, etc.) have been obtained by PC spectral analysis programs and by coupled two-channel analyzer BK-2032 in the frequency range 0–10 Hz. Output signal in the form of power spectrums were recorded by digital recorder (BK-7400) and plotted by x – y plotter BK-2308. The bridge vibrations ambient-ability have been investigated by means of the correlation and spectral analysis by cross-correlation functions $R_{xy}(t)$ and coherence function $\gamma^2_{xy}(f)$. Examples of amplitude (a), and spectral (b) analyses results, DAF dependence on lorry velocities (c) and calculated and measured natural frequencies comparison (d) are depicted in **Figure 15**.

5.2. Bridge dynamic parameters monitoring

In this section, bridge monitoring process and results are shortly described. *Lafranconi* bridge over the *Danube* has been investigated by 24 h of bridge monitoring tests in the summer and

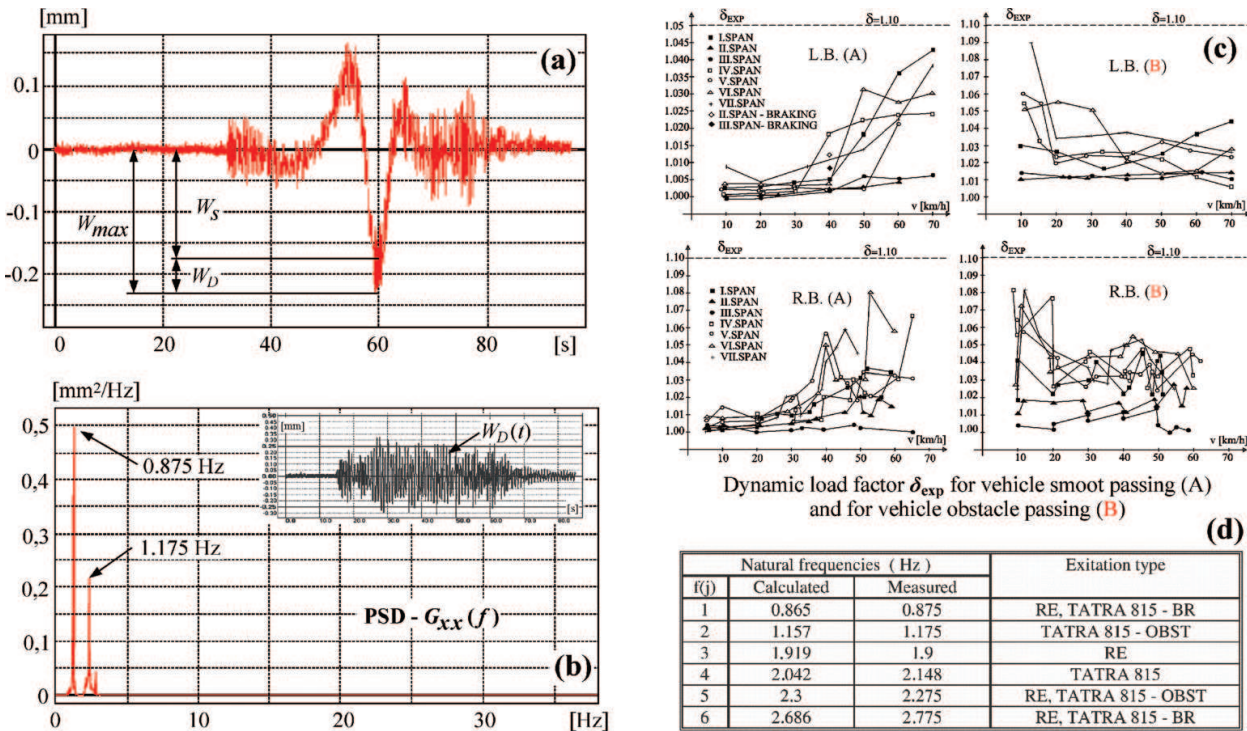


Figure 15. Examples of amplitude (a), and spectral (b) analyses results; DAF dependence on lorry velocities (c) and calculated and measured natural frequencies comparison (d).

the winter time during the years 1991–2001. The theoretical and experimental predictions of the bridge behavior and former DLT results are reported in [29, 38].

Bridge testing and experimental procedures: the bridge vibration amplitudes were measured and recorded in selected points of the second (174.0 m) and the third span (172.0 m) of the bridge. The time history of bending vertical amplitude vibration has been recorded by accelerometers at points marked as A1, A2, A5, and A6 (see also **Figure 14**) that were situated in the same position as during DLT [29]. Output signals from accelerometers were preamplified and recorded by portable computer (PC) with relevant software and hardware facilities for 24 h continuing test. The analysis of the experimental measured data has been carried out in the laboratory conditions. The records obtained in the bridge monitoring tests were investigated by using frequency analyzer BK-2034 and mentioned PC facilities. **Figure 16** shows power spectral density example of the monitoring test performed in August, 1994. The damping parameters were found by means of the 3 dB *bandwidth method* and *curve fitting techniques*. The amplitude analysis has been used to obtain RMS amplitude value of the bridge vibrations during the monitoring tests. As an example, **Figure 17** shows results in the form of *dominant frequency, damping* for lowest natural frequency in bending vibrations and RMS amplitude value from the monitoring period of years 1990–1997. A 2.7% change in frequency was observed during an year (summer-winter) but it is systematic from 1 year to the next year and is maybe due to changes in ambient temperature. The *frequencies measured at the same of the annular monitoring period have changes from year to year small and non-systematic* (coefficient of variation of about 0.01).

In comparison with the determined changes in structures natural frequency of about 30% corresponding to advanced failure observed in [31], it may be considered negligible. There are not systematic changes of structure damping but scattering of results are big [29]. These changes are maybe caused by changes in temperatures during the day; also, there are influences of changes in length of bridge which can modify support conditions and structural damping.

From monitoring results follow difference values of the *displacement RMS amplitude* measured in May 1991 in comparison with other measurements results. It was caused by both side motor traffic flows only on the left bridge. All the following monitoring measurements were performed in conditions of the one-side traffic flow on each of the both Lafranconi bridge. The changes of the *amplitude RMS value* are caused mainly by changes of the intensity of the regular motor traffic on the bridge.

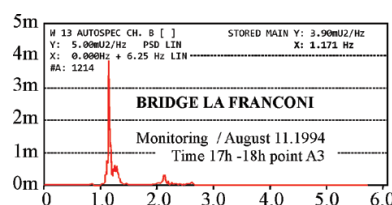


Figure 16. Power spectral density $G_{33}(f)$ of the bridge vibration displacement amplitude at point A3.

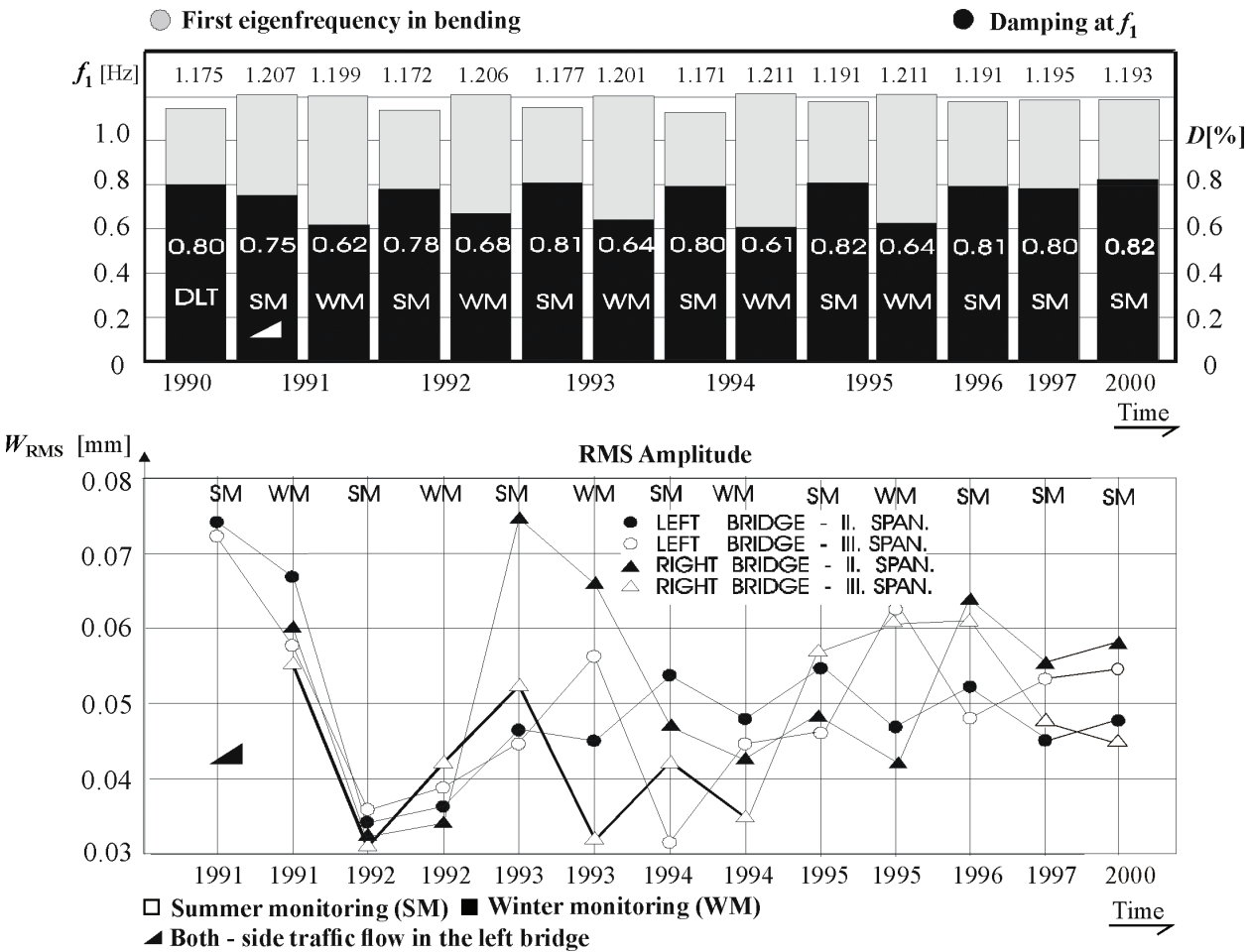


Figure 17. Changes in relative frequency, damping, and displacement RMS amplitude values during 1990–1997.

6. Conclusions

There has been a considerable amount of research conducted in the fields of bridge dynamics. From the analytical and experimental findings, the following conclusions arise:

1. Many experimental results have proved that the DAF is related to the bridges fundamental frequencies in the range 1–6 Hz. Most of the commercial lorries moving on bridges have fundamental frequencies in the range of 2–5 Hz, corresponding to the fundamental frequencies of roads and highway bridges. During the common performance of the bridges it results resonances effects.
2. Analytical and numerical models cannot reliably calculate the DAF for bridges with many specific boundaries, and initial and mechanical input parameters because of the difficulty to model them without experimental proving tests.
3. Many different formulas were suggested to evaluate the DAF from the experimental data obtained under testing vehicles (*DLT*) or regular traffic loading (*normal traffic flow*). More

researchers have used the ratio of maximum dynamic response over the maximum filtered response (e.g., deflections) as a definition of the DAF.

4. Inappropriate position of the pickups on a bridge cross section can give an unreliable experimental value of the DAF from bridge dynamic tests.
5. Full-scale testing under moving vehicle (*DLT, traffic flow*) loading is still the only economical and practical way to evaluate the *DAF* with reasonable certainty [27]. It is also suitable a reliable method for determining bridge structural dynamic properties and fully acceptable mainly for inspection purposes (even in cases of highway bridge dynamic investigation where *DLF* for highway bridges is not defined, e.g., in *Eurocodes*).

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