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Advances in Parameter Estimation and Performance Improvement in Adaptive Control

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1. Introduction

In most adaptive control algorithms, parameter estimate errors are not guaranteed to converge to zero. This lack of convergence adversely affects the global performance of the algorithms. The effect is more pronounced in control problems where the desired reference setpoint or trajectory depends on the system's unknown parameters. This paper presents a parameter estimation routine that allows exact reconstruction of the unknown parameters in finite-time provided a given excitation condition is satisfied. The robustness of the routine to an unknown bounded disturbance or modelling error is also shown.

To enhance the applicability of the finite-time (FT) identification procedure in practical situations, a novel adaptive compensator that (almost) recover the performance of the FT identifier is developed. The compensator guarantees exponential convergence of the parameter estimation error at a rate dictated by the closed-loop system's excitation. It was shown how the adaptive compensator can be used to improve upon existing adaptive controllers. The modification provided guarantees exponential stability of the parametric equilibrium provided the given PE condition is satisfied. Otherwise, the original system's closed-loop properties are preserved.

The results are independent of the control structure employed. The true parameter value is obtained without requiring the measurement or computation of the velocity state vector. Moreover, the technique provides a direct solution to the problem of removing auxiliary perturbation signals when parameter convergence is achieved. The effectiveness of the proposed methods is illustrated with simulation examples.

There are two major approaches to online parameter identification of nonlinear systems. The first is the identification of parameters as a part of state observer while the second deals with parameter identification as a part of controller. In the first approach, the observer is designed to provide state derivatives information and the parameters are estimated via estimation methods such as least squares method [19] and dynamic inversion [6]. The second trend of parameter identification is much more widespread, as it allows identification of systems with unstable dynamics. Algorithms in this area include parameter identification methods based on variable structure theory [22, 23] and those based on the notion of passivity [13].

In the conventional adaptive control algorithms, the focus is on the tracking of a given reference trajectory and in most cases parameter estimation errors are not guaranteed to

converge to zero due to a lack of excitation [10]. Parameter convergence is an important issue as it enhances the overall stability and robustness properties of the closed-loop adaptive systems [14]. Moreover, there are control problems whereby the reference trajectory is not known *a priori* but depends on the unknown parameters of the system dynamics. For example, in adaptive extremum seeking control problems, the desired target is the operating setpoint that optimizes an uncertain cost function [8, 21].

Assuming the satisfaction of appropriate excitation conditions, asymptotic and exponential parameter convergence results are available for both linear and nonlinear systems. Some lower bounds which depends (nonlinearly) on the adaptation gain and the level of excitation in the system have been provided for some specific control and estimation algorithms [11, 17, 20]. However, it is not always easy to characterize the convergence rate.

Since the performance of any adaptive extremum seeking control is dictated by the efficiency of its parameter adaptation procedure. This chapter presents a parameter estimation scheme that allows exact reconstruction of the unknown parameters in finite-time provided a given persistence of excitation (PE) condition is satisfied. The true parameter estimate is recovered at any time instant the excitation condition is satisfied. This condition requires the integral of a filtered regressor matrix to be invertible. The finite-time (FT) identification procedure assumes the state of the system $x(\cdot)$ is accessible for measurement but does not require the measurement or computation of the velocity state vector $\dot{x}(\cdot)$. The robustness of the estimation routine to bounded unknown disturbances or modeling errors is also examined. It is shown that the parameter estimation error can be rendered arbitrarily small for a sufficiently large filter gain.

A common approach to ensuring a PE condition in adaptive control is to introduce a perturbation signal as the reference input or to add it to the target setpoint or trajectory. The downside of this approach is that a constant PE deteriorates the desired tracking or regulation performance. Aside from the recent results on intelligent excitation signal design [3, 4], the standard approach has been to introduce such PE signal and remove it when the parameters are *assumed* to have converged. The fact that one has perfect knowledge of the convergence time in the proposed framework allows for a direct and immediate removal of the added PE signal. The result on finite-time identification has been published in [2].

The main drawback of the finite-time identification algorithm is the requirement to check the invertibility of a matrix online and compute the inverse matrix when appropriate. To avoid these concerns and enhance the applicability of the FT method in practical situations, the procedure was employed to develop a novel adaptive compensator that (almost) recover the performance of the FT identifier. The compensator guarantees exponential convergence of the parameter estimation error at a rate dictated by the closed-loop system's excitation. It was shown how the adaptive compensator can be used to improve upon existing adaptive controllers. The modification provided guarantees exponential stability of the parametric equilibrium provided the given PE condition is satisfied. Otherwise, the original system's closed-loop properties are preserved.

2. Problem Description and Assumptions

The system considered is the following nonlinear parameter affine system

$$\dot{x} = f(x, u) + g(x, u)\theta \quad (1)$$

where $x \in \mathbb{R}^{n_x}$ is the state and $u \in \mathbb{R}^{n_u}$ is the control input. The vector $\theta \in \mathbb{R}^{n_\theta}$ is the unknown parameter vector whose entries may represent physically meaningful unknown model parameters or could be associated with any finite set of universal basis functions. It is assumed that θ is uniquely identifiable and lie within an initially known compact set Θ^0 . The n_x -dimensional vector $f(x, u)$ and the $(n_x \times n_\theta)$ -dimensional matrix $g(x, u)$ are bounded and continuous in their arguments. System (1) encompasses the special class of linear systems,

$$\begin{aligned} f(x, u) &= A_0 x + B_0 u \\ g(x, u) &= [A_1 x + B_1 u, A_2 x + B_2 u, \dots, A_{n_\theta} x + B_{n_\theta} u], \end{aligned}$$

where A_i and B_i for $i = 0 \dots n_\theta$ are known matrices possibly time varying.

Assumption 2.1 The following assumptions are made about system (1).

1. The state of the system $x(\cdot)$ is assumed to be accessible for measurement.
2. There is a known bounded control law $u = \alpha(\cdot)$ and a bounded parameter update law $\dot{\hat{\theta}}$ that achieves a primary control objective.

The control objective can be to (robustly) stabilize the plant and/or to force the output to track a reference signal. Depending on the structure of the system (1), adaptive control design methods are available in the literature [12, 16].

For any given bounded control and parameter update law, the aim of this chapter is to provide the true estimates of the plant parameters in finite-time while preserving the properties of the controlled closed-loop system.

3. Finite-time Parameter Identification

Let \hat{x} denote the state predictor for (1), the dynamics of the state predictor is designed as

$$\dot{\hat{x}} = f(x, u) + g(x, u)\hat{\theta} + k_w(t)e + w\dot{\hat{\theta}}, \quad (2)$$

where $\hat{\theta}$ is a parameter estimate generated via any update law $\dot{\hat{\theta}}$, $k_w > 0$ is a design matrix, $e = x - \hat{x}$ is the prediction error and w is the output of the filter

$$\dot{w} = g(x, u) - k_w w, \quad w(t_0) = 0. \quad (3)$$

Denoting the parameter estimation error as $\tilde{\theta} = \theta - \hat{\theta}$, it follows from (1) and (2) that

$$\dot{e} = g(x, u)\tilde{\theta} - k_w e - w\dot{\hat{\theta}}. \quad (4)$$

The use of the filter matrix w in the above development provides direct information about parameter estimation error $\tilde{\theta}$ without requiring a knowledge of the velocity vector \dot{x} . This is achieved by defining the auxiliary variable

$$\eta = e - w\tilde{\theta} \quad (5)$$

with η , in view of (3, 4), generated from

$$\dot{\eta} = -k_w \eta, \quad \eta(t_0) = e(t_0). \quad (6)$$

Based on the dynamics (2), (3) and (6), the main result is given by the following theorem.

Theorem 3.1 Let $Q \in \mathbb{R}^{n_\theta \times n_\theta}$ and $C \in \mathbb{R}^{n_\theta}$ be generated from the following dynamics:

$$\dot{Q} = w^T w, \quad Q(t_0) = 0 \quad (7a)$$

$$\dot{C} = w^T (w\hat{\theta} + e - \eta), \quad C(t_0) = 0 \quad (7b)$$

Suppose there exists a time t_c and a constant $c_1 > 0$ such that $Q(t_c)$ is invertible i.e.

$$Q(t_c) = \int_{t_0}^{t_c} w^T(\tau)w(\tau) d\tau \succ c_1 I, \quad (8)$$

then

$$\theta = Q(t)^{-1}C(t) \quad \text{for all } t \geq t_c. \quad (9)$$

Proof: The result can be easily shown by noting that

$$Q(t)\theta = \int_{t_0}^t w^T(\tau)w(\tau) [\hat{\theta}(\tau) + \tilde{\theta}(\tau)] d\tau. \quad (10)$$

Using the fact that $w\tilde{\theta} = e - \eta$, it follows from (10) that

$$\theta = Q(t)^{-1} \int_{t_0}^t \dot{C}(\tau) d\tau = Q(t)^{-1}C(t) \quad (11)$$

and (11) holds for all $t \geq t_c$ since $Q(t) \succeq Q(t_c)$. ■

The result in theorem 3.1 is independent of the control u and parameter identifier $\hat{\theta}$ structure used for the state prediction (eqn 2). Moreover, the result holds if a nominal estimate θ^0 of the unknown parameter (no parameter adaptation) is employed in the estimation routine. In this case, $\hat{\theta}$ is replaced with θ^0 and the last part of the state predictor (2) is dropped ($\dot{\hat{\theta}} = 0$).

Let

$$\theta^c \triangleq Q(t_c)^{-1}C(t_c) \quad (12)$$

The finite-time (FT) identifier is given by

$$\hat{\theta}^c(t) = \begin{cases} \hat{\theta}(t), & \text{if } t < t_c \\ \theta^c, & \text{if } t \geq t_c. \end{cases} \quad (13)$$

The piecewise continuous function (13) can be approximated by a smooth approximation using the logistic functions

$$\psi_1 \triangleq \frac{\hat{\theta}(t)}{2} \left(1 - \tanh \nu_1(t - t_c) \right) = \frac{\hat{\theta}(t)}{1 + \exp^{2\nu_1(t-t_c)}} \quad (14a)$$

$$\psi_2 \triangleq \frac{\theta^c}{2} \left(1 + \tanh \nu_2(t - t_c) \right) = \frac{\theta^c}{1 + \exp^{-2\nu_2(t-t_c)}} \quad (14b)$$

$$\hat{\theta}^{\tilde{c}} = \psi_1 + \psi_2 \quad (14c)$$

where larger ν_1, ν_2 correspond to a sharper transition at $t = t_c$ and $\lim_{(\nu_1, \nu_2) \rightarrow \infty} \hat{\theta}^{\tilde{c}} = \hat{\theta}^c$. An example of such approximation is depicted in Figure 1 where the function

$$z(t) = \begin{cases} 6 + t^{0.3} & \text{if } t < 5 \\ 4 & \text{otherwise} \end{cases}$$

is approximated by (14) with $\nu_1 = \nu_2 = 5$.

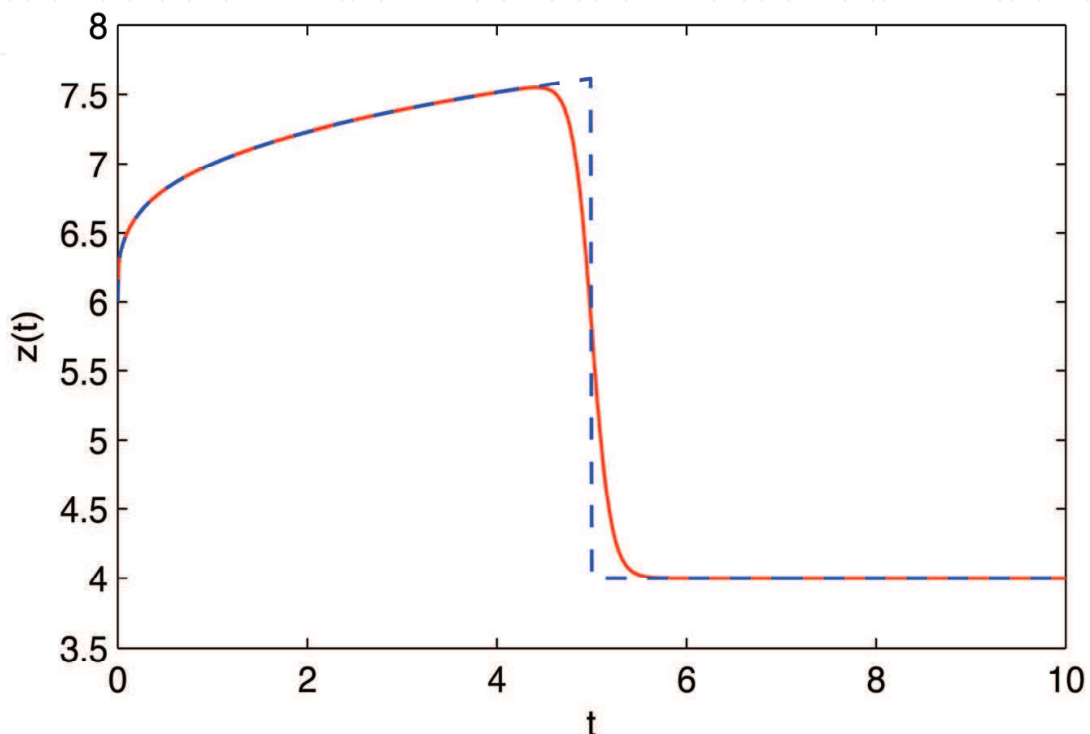


Figure 1. Approximation of a piecewise continuous function. The function $z(t)$ is given by the full line. Its approximation is given by the dotted line

The invertibility condition (8) is equivalent to the standard persistence of excitation (PE) condition required for parameter convergence in adaptive control. The condition (8) is satisfied if the regressor matrix g is PE. To show this, consider the filter dynamic (3), from which it follows that

$$w(t) = \int_{t_0}^t \exp^{-k_w(t-\tau)} g(\tau) d\tau = \frac{1}{s + k_w} [g(t)]. \quad (15)$$

Since $g(t)$ is PE by assumption and the transfer function $\frac{1}{s+k_w}$ is stable, minimum phase and strictly proper, we know that $w(t)$ is PE [18]. Hence, there exists t_c and a c_1 for which (8) is satisfied. The superiority of the above design lies in the fact that the true parameter value can be computed at any time instant t_c the regressor matrix becomes positive definite and subsequently stop the parameter adaptation mechanism.

The procedure in theorem 42 involves solving matrix valued ordinary differential equations (3, 7) and checking the invertibility of $Q(t)$ online. For computational considerations, the

invertibility condition (8) can be efficiently tested by checking the determinant of $Q(t)$ online. Theoretically, the matrix is invertible at any time $\det(Q(t))$ becomes positive definite. The determinant of $Q(t)$ (which is a polynomial function) can be queried at pre-scheduled times or by propagating it online starting from a zero initial condition. One way of doing this is to include a scalar differential equation for the derivative of $\det(Q(t))$ as follows [7]:

$$\frac{d}{dt}\det(Q) = \text{Trace}(\text{Adjugate}(Q) w^T w), \quad \det(Q(t_0)) = 0 \quad (16)$$

where $\text{Adjugate}(Q)$, admittedly not a light numerical task, is also a polynomial function of the elements of Q .

3.1 Absence of PE

If the PE condition (8) is not satisfied, a given controller and the corresponding parameter estimation scheme preserve the system established closed-loop properties. When a bounded controller that is robust with respect to input $(\tilde{\theta}, \dot{\tilde{\theta}})$ is known, it can be shown that the state prediction error e tends to zero as $t \rightarrow \infty$. An example of such robust controller is an input-to-state stable (iss) controller [12].

Theorem 3.2 Suppose the design parameter k_w in (2) is replaced with $k_w(t) = k_{w_1} + k_{w_2}(t)$, $k_{w_1} > \frac{1}{4}I$, and $k_{w_2}(t) = \frac{1}{4}g\gamma g^T$. Then the state predictor (2) and the parameter update law

$$\dot{\tilde{\theta}} = \gamma(w^T + g^T)e \quad (17)$$

with $\gamma = \gamma^T > 0$, a design constant matrix, guarantee that

1. $(e, \eta, \tilde{\theta}) \in \mathcal{L}_\infty$ and $(e, \eta) \rightarrow 0$ as $t \rightarrow \infty$.
2. $\lim_{t \rightarrow \infty} \tilde{\theta}(t) = \bar{\theta}$, a constant.

Proof:

1. Consider a Lyapunov function

$$V = \frac{1}{2} \left(e^T e + \tilde{\theta}^T \gamma^{-1} \tilde{\theta} + \eta^T \eta \right) \quad (18)$$

It follows from equations (4), (5), (6) and (17) that

$$\dot{V} = -e^T k_{w_1} e - e^T \left(\frac{1}{4} g \gamma g^T + w \gamma w^T + w \gamma g^T \right) e - \tilde{\theta}^T w^T w \tilde{\theta} - \tilde{\theta}^T w^T \eta - \eta^T k_w(t) \eta \quad (19)$$

$$\leq -e^T k_{w_1} e - \lambda_{\min}(\gamma) \|w^T e + \frac{1}{2} g^T e\|^2 - \eta^T k_{w_3} \eta - \|w \tilde{\theta} + \frac{1}{2} \eta\|^2 \quad (20)$$

$$\leq -(e^T k_{w_1} e + \eta^T k_{w_3} \eta). \quad (21)$$

where $k_{w_3} = k_{w_1} - \frac{1}{4}$. This implies uniform boundedness of $(\eta, e, \tilde{\theta})$ as well as global asymptotic convergence of (η, e) to zero. Hence, it follows from (5) that $\lim_{t \rightarrow \infty} w \tilde{\theta} = 0$.

2. This can be shown by noting from (17) that $\tilde{\theta}(t) = \tilde{\theta}(t_0) - \gamma \int_{t_0}^t (w^T + g^T) e d\sigma$. Since $g(\cdot)$ and e are bounded signals and $e \rightarrow 0$, the integral term exists and it is finite. ■

4. Robustness Property

In this section, the robustness of the finite-time identifier to unknown bounded disturbances or modeling errors is demonstrated. Consider a perturbation of

(1):

$$\dot{x} = f(x, u) + g(x, u)\theta + \vartheta(t, x, \theta) \quad (22)$$

where $\vartheta(\cdot)$ is a disturbance or modeling error term that satisfies $\|\vartheta(t)\| \leq M_\vartheta(t) < \infty$. If the PE condition (8) is satisfied and the disturbance term is known, the true unknown parameter vector is given by

$$\theta_\vartheta^c \triangleq \theta = Q(t)^{-1} \int_{t_0}^t w^T(\tau) [w(\tau)\hat{\theta}(\tau) + e(\tau) - \eta_\vartheta(\tau)] d\tau, \quad \text{for all } t \geq t_c, \quad (23)$$

with $e = x - \hat{x}$ and the signals \hat{x} , w , $\eta_\vartheta = e - w\theta$ generated from (2), (3) and

$$\dot{\eta}_\vartheta = -k_w \eta_\vartheta + \vartheta(\cdot), \quad \eta_\vartheta(t_0) = e(t_0) \quad (24)$$

respectively.

Since $\vartheta(\cdot)$ is unknown, we provide a bound on the parameter identification error $\tilde{\theta}^c = \theta_\vartheta^c - \theta^c$ when (6) is used instead of (24). Considering (9) and (23), it follows that

$$\tilde{\theta}^c = Q(t)^{-1} \int_{t_0}^t w^T(\tau) (-\eta_\vartheta(\tau) + \eta(\tau)) d\tau \quad (25)$$

$$= -Q(t)^{-1} \int_{t_0}^t w^T(\tau) \tilde{\eta}(\tau) d\tau. \quad (26)$$

where $\tilde{\eta} = \eta_\vartheta - \eta$ is the output of

$$\dot{\tilde{\eta}} = -k_w \tilde{\eta} + \vartheta(\cdot), \quad \tilde{\eta}(t_0) = 0. \quad (27)$$

Since $k_w \geq k_{w1} > 0$, it follows that

$$\|\tilde{\eta}(t)\| \leq \frac{M_\vartheta}{k_{w1}} \quad (28)$$

and hence

$$\|\tilde{\theta}^c(t)\| \leq \|Q(t)^{-1}\| \left\{ \frac{\bar{w} M_\vartheta (t - t_0)}{k_{w1}} \right\}, \quad \text{for all } t \geq t_c. \quad (29)$$

where $\bar{w} = \max_{\sigma \in [t_0, t]} \|w^T(\sigma)\|$.

This implies that the identification error can be rendered arbitrarily small by choosing a sufficiently large filter gain k_{w1} . In addition, if the disturbance term ϑ and the system satisfies some given properties, then asymptotic convergence can be achieved as stated in the following theorem.

Theorem 4.1 Suppose $\vartheta \in \mathcal{L}_p$, for $p = 1$ or 2 and $\lim_{t \rightarrow \infty} \lambda_{\min}(Q) = \infty$, then $\tilde{\theta}^c \rightarrow 0$ asymptotically with time.

To proof this theorem, we need the following lemma

Lemma 4.2 [5]: Consider the system

$$\dot{x}(t) = Ax(t) + u(t) \quad (30)$$

Suppose the equilibrium state $x_e = 0$ of the homogeneous equation is exponentially stable,

1. if $u \in L_p$ for $1 < p < \infty$, then $x \in L_p$ and
2. if $u \in L_p$ for $p = 1$ or 2 , then $x \rightarrow 0$ as $t \rightarrow \infty$.

Proof of theorem 4.1. It follows from Lemma 4.2.2 that $\tilde{\eta} \rightarrow 0$ as $t \rightarrow \infty$ and therefore $\lim_{t \rightarrow \infty} \int_{t_0}^t w^T(\tau) \tilde{\eta}(\tau) d\tau$ is finite. So

$$\lim_{t \rightarrow \infty} \tilde{\theta}^c = \lim_{t \rightarrow \infty} \left\{ Q(t)^{-1} \int_{t_0}^t w^T(\tau) \tilde{\eta}(\tau) d\tau \right\} = 0. \quad (31)$$

5. Dither Signal Design

The problem of tracking a reference signal is usually considered in the study of parameter convergence and in most cases, the reference signal is required to provide sufficient excitation for the closed-loop system. To this end, the reference signal $y_r(t) \in \mathbb{R}^r$ is appended with a bounded excitation signal $d(t)$ as

$$y_{rd}(t) = y_r(t) + d(t) \quad (32)$$

where the auxiliary signal $d(t)$ is chosen as a linear combination of sinusoidal functions with \hbar distinct frequencies:

$$d(t) := \sum_{k=1}^{\hbar} a_k(t) \sin(\omega_k t) = \mathcal{A}(t) \zeta(t) \quad (33)$$

where

$$\mathcal{A}(t) = \begin{bmatrix} a_{11} & \cdots & a_{1\hbar} \\ \vdots & & \vdots \\ a_{r1} & \cdots & a_{r\hbar} \end{bmatrix}$$

is the signal amplitude matrix and

$$\zeta(t) = [\sin \omega_1 t \ \dots \ \sin \omega_{\hbar} t]^T, \ \omega_i \neq \omega_j \text{ for } i \neq j$$

is the corresponding sinusoidal function vector.

For this approach, it is sufficient to design the perturbation signal such that the regressor matrix g is PE. There are very few results on the design of persistently exciting (PE) input signals for nonlinear systems. By converting the closed-loop PE condition to a sufficient richness (SR) condition on the reference signal, attempts have been made to provide verifiable conditions for parameter convergence in some classes of nonlinear systems [3, 1, 14, 15].

5.1 Dither Signal Removal

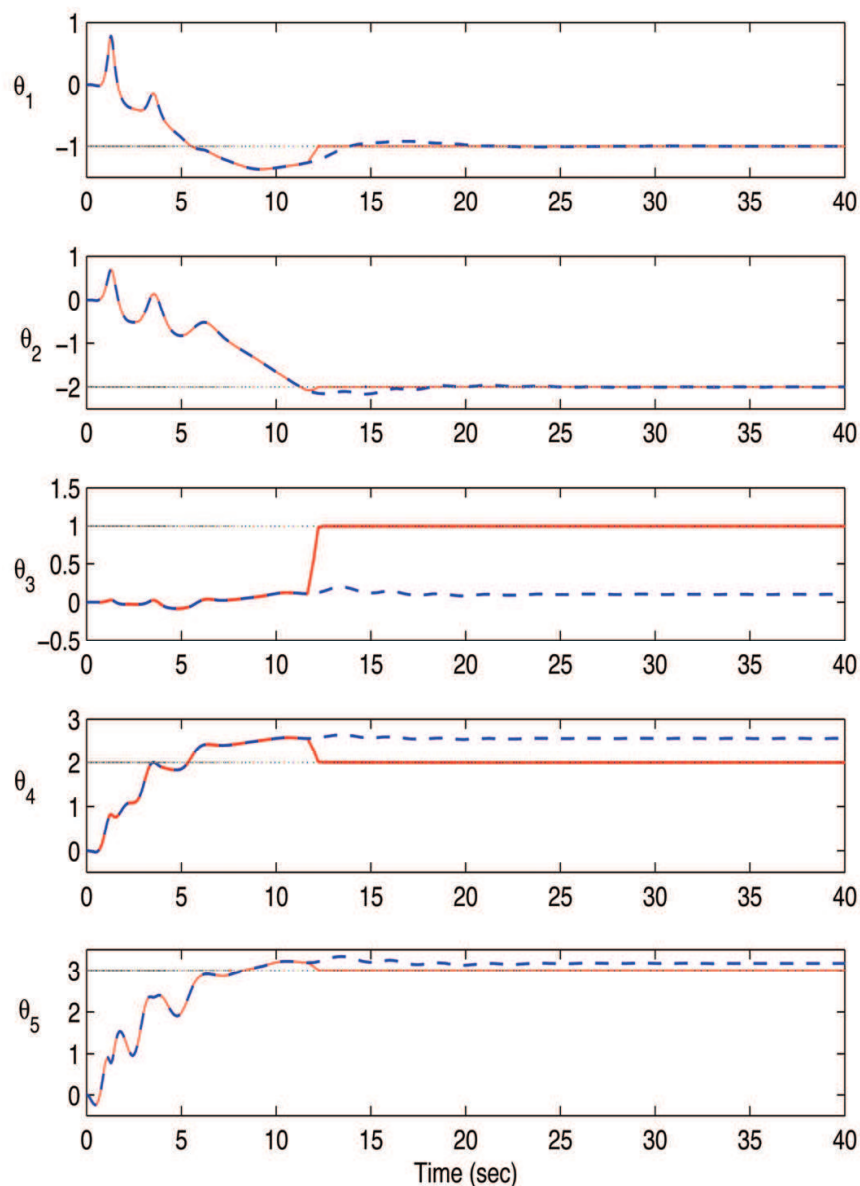


Figure 2. Trajectories of parameter estimates. Solid(-) : FT estimates $\hat{\theta}^c$ dashed(--): standard estimates $\hat{\theta}$ [15]; dashdot(-.): actual value

Let $\mathcal{H} \leq (\bar{h} \times r)$ denotes the number of distinct elements in the dither amplitude matrix $\mathcal{A}(t)$ and let $a \in \mathbb{R}^{\mathcal{H}}$ be a vector of these distinct coefficients. The amplitude of the excitation signal is specified as

$$a(t) = \begin{cases} a, & \text{if } t < t_c \\ 0, & \text{otherwise} \end{cases} \quad (34)$$

or approximated by

$$a(t) \approx \frac{a}{1 + \exp^{2\nu(t-t_c)}} \quad (35)$$

where equality holds in the limit as $\nu \rightarrow \infty$.

6. Simulation Examples

6.1 Example 1

We consider the following nonlinear system in parametric strict feedback form [15]:

$$\begin{aligned}\dot{x}_1 &= x_2 + \theta_1 x_1 \\ \dot{x}_2 &= x_3 + \theta_2 x_1 \\ \dot{x}_3 &= \theta_3 x_1^3 + \theta_4 x_2 + \theta_5 x_3 + (1 + x_1^2)u \\ y &= x_1,\end{aligned}\tag{36}$$

where $\theta^T = [\theta_1, \dots, \theta_5]$ are unknown parameters. Using an adaptive backstep-ping design, the control and parameter update law presented in [15] were used for the simulation. The pair stabilize the plant and ensure that the output y tracks a reference signal $y_r(t)$ asymptotically. For simulation purposes, parameter values are set to $\theta^T = [-1, -2, 1, 2, 3]$ as in [15] and the reference signal is $y_r = 1$, which is sufficiently rich of order one. The simulation results for zero initial conditions are shown in Figure 2. Based on the convergence analysis procedure in [15], all the parameter estimates cannot converge to their true values for this choice of constant reference. As confirmed in Fig. 2, only θ_1 and θ_2 estimates are accurate. However, following the proposed estimation technique and implementing the FT identifier (14), we obtain the exact parameter estimates at $t = 17\text{sec}$. This example demonstrates that, with the proposed estimation routine, it is possible to identify parameters using perturbation or reference signals that would otherwise not provide sufficient excitation for standard adaptation methods.

6.2 Example 2

To corroborate the superiority of the developed procedure, we demonstrate the robustness of the developed procedure by considering system (36) with added exogenous disturbances as follows:

$$\begin{aligned}\dot{x}_1 &= x_2 + \theta_1 x_1 + [1 \ 0]\vartheta \\ \dot{x}_2 &= x_3 + \theta_2 x_1 + [1 \ x_1]\vartheta \\ \dot{x}_3 &= \theta_3 x_1^3 + \theta_4 x_2 + \theta_5 x_3 + (1 + x_1^2)u + [0 \ 1]\vartheta \\ y &= x_1,\end{aligned}\tag{37}$$

where $\vartheta = [0.1 \sin(2\pi t/5)]$ and the tracking signal remains a constant $y_r = 1$.

The simulation result, Figure 3, shows convergence of the estimate vector to a small neighbourhood of θ under finite-time identifier with filter gain $k_w = 1$ while no full parameter convergence is achieved with the standard identifier. The parameter estimation error $\tilde{\theta}(t)$ is depicted in Figure 4 for different values of the filter gain k_w . The switching time for the simulation is selected as the time for which the condition number of Q becomes less than 20. It is noted that the time at which switching from standard adaptive estimate to FT estimate occurs increases as the filter gain increases. The convergence performance improves as k_w increases, however, no significant improvement is observed as the gain is increased beyond 0.5.

7. Performance Improvement in Adaptive Control via Finite-time Identification Procedure

This section demonstrates how the finite-time identification procedure presented in section 3 can be employed to improve the overall performance (both transient and steady state) of adaptive control systems in a very appealing manner. First, we develop an adaptive compensator which guarantees exponential convergence of the estimation error provided the integral of a filtered regressor matrix is positive definite. The approach does not involve online checking of matrix invertibility and computation of matrix inverse nor switching between parameter estimation methods. The convergence rate of the parameter estimator is directly proportional to the adaptation gain and a measure of the system's excitation. The adaptive compensator is then combined with existing adaptive controllers to guarantee exponential stability of the closed-loop system.

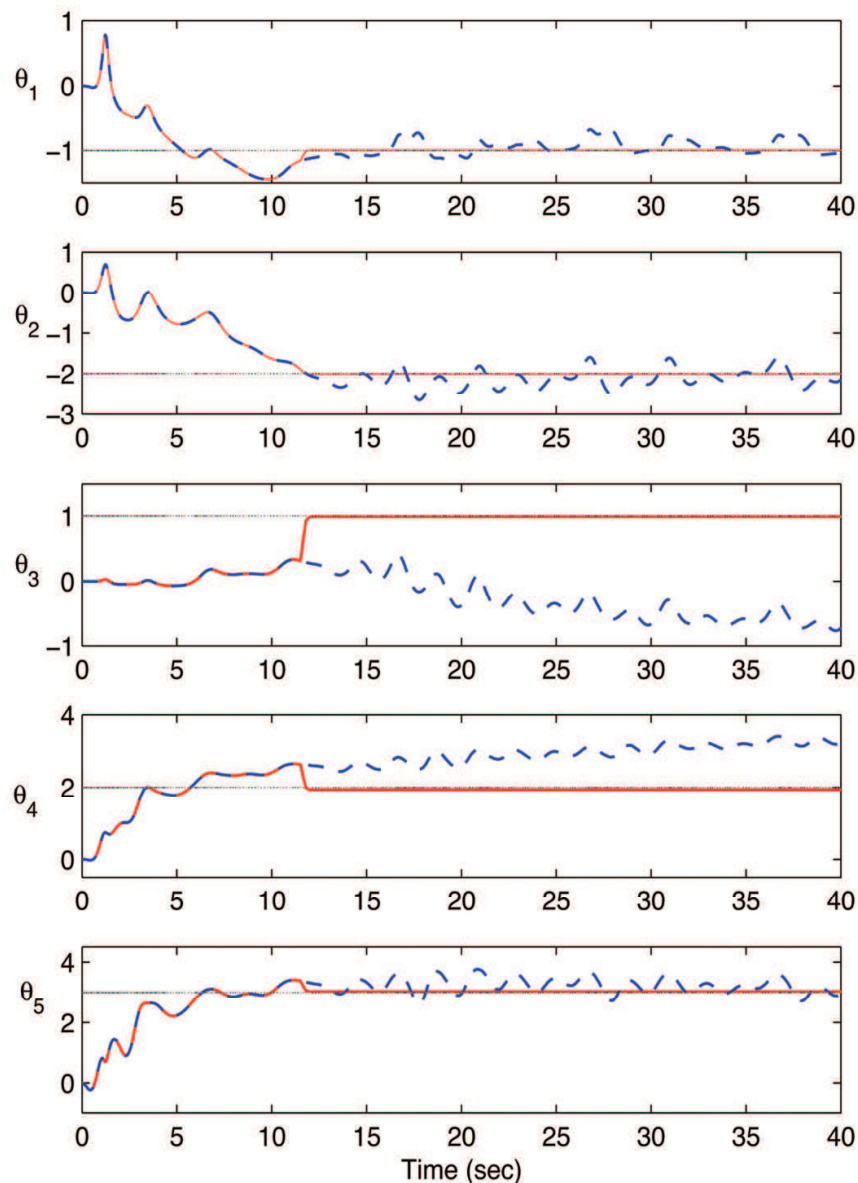


Figure 3. Trajectories of parameter estimates. Solid(-) : FT estimates for the system with additive disturbance $\hat{\theta}^c$, dashed(--): standard estimates $\hat{\theta}$ [15]; dashdot(-.): actual value

8. Adaptive Compensation Design

Consider the nonlinear system 1 satisfying assumption 2.1 and the state predictor

$$\dot{\hat{x}} = f(x, u) + g(x, u) \theta^0 + k_w(x - \hat{x}) \quad (38)$$

where $k_w > 0$ and θ^0 is the nominal initial estimate of θ . If we define the auxiliary variable

$$\eta = x - \hat{x} - w(\theta - \theta^0) \quad (39)$$

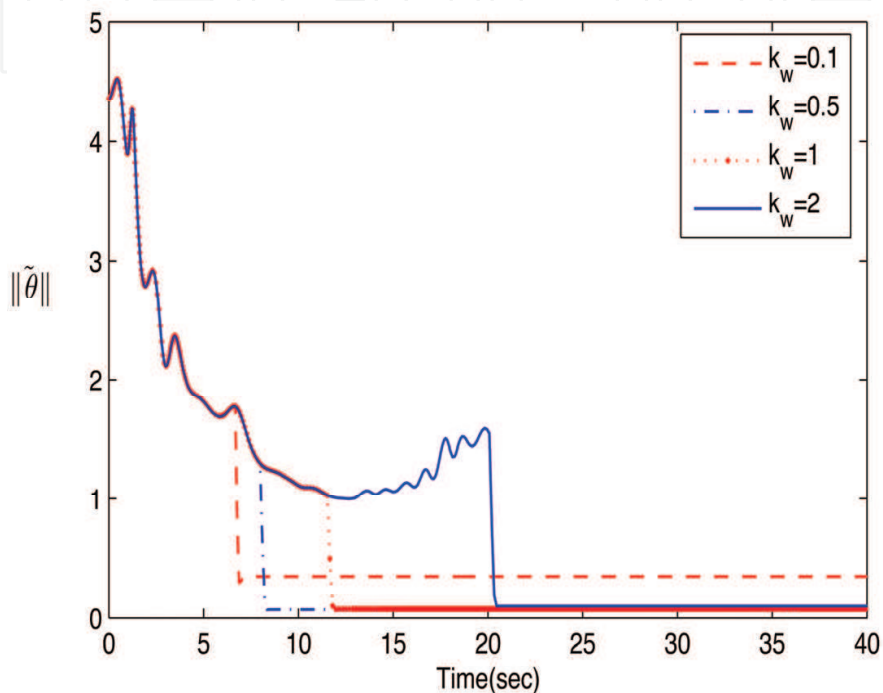


Figure 4. Parameter estimation error for different filter gains k_w and select the filter dynamic as

$$\dot{w} = g(x, u) - k_w w, \quad w(t_0) = 0 \quad (40)$$

then η is generated by

$$\dot{\eta} = -k_w \eta, \quad \eta(t_0) = e(t_0). \quad (41)$$

Based on (38) to (41), our novel adaptive compensation result is given in the following theorem.

Theorem 8.1 Let Q and C be generated from the following dynamics:

$$\dot{Q} = w^T w, \quad Q(t_0) = 0 \quad (42a)$$

$$\dot{C} = w^T (w \theta^0 + x - \hat{x} - \eta), \quad C(t_0) = 0 \quad (42b)$$

and let t_c be the time such that $Q(t_c) > 0$, then the adaptation law

$$\dot{\hat{\theta}} = \Gamma (C - Q \hat{\theta}), \quad \hat{\theta}(t_0) = \theta^0 \quad (43)$$

with $\Gamma = \Gamma^T \succ 0$ guarantees that $\|\tilde{\theta}\| = \|\theta - \hat{\theta}\|$ is non-increasing for $t_0 \leq t \leq t_c$ and converges to zero exponentially fast, starting from t_c . Moreover, the convergence rate is lower bounded by $\mathcal{E}(t) = \lambda_{\min}(\Gamma Q(t))$.

Proof: Consider a Lyapunov function

$$V_{\tilde{\theta}} = \frac{1}{2} \tilde{\theta}^T \tilde{\theta}, \quad (44)$$

it follows from (43) that

$$\dot{V}_{\tilde{\theta}}(t) = -\tilde{\theta}^T(t) \Gamma \left(C(t) - Q(t) \hat{\theta}(t) \right). \quad (45)$$

Since $w\theta = w\theta^0 + x - \hat{x} - \eta$ (from (39)), then

$$C(t) = \int_{t_0}^t \dot{C}(\tau) d\tau = \int_{t_0}^t w^T(\tau) w(\tau) d\tau \theta = Q(t) \theta \quad (46)$$

and equation (45) becomes

$$\dot{V}_{\tilde{\theta}}(t) = -\tilde{\theta}^T(t) \Gamma Q(t) \tilde{\theta}(t) \quad (47)$$

$$\leq -\mathcal{E}(t) V_{\tilde{\theta}}(t) \quad (48)$$

This implies non-increase of $\|\tilde{\theta}\|$ for $t \geq t_0$ and the exponential claim follows from the fact that $\Gamma Q(t) = \Gamma \int_{t_0}^t w(\tau)^T w(\tau) d\tau$ is positive definite for all $t \geq t_c$. The convergence rate is shown by noting that

$$\dot{V}_{\tilde{\theta}}(t) = -\tilde{\theta}^T(t) \Gamma \left(Q(t_c) + \int_{t_c}^t w(\tau)^T w(\tau) d\tau \right) \tilde{\theta}(t), \quad \forall t \geq t_c \quad (49)$$

$$\leq -\tilde{\theta}^T(t) \Gamma Q(t_c) \tilde{\theta}(t) \leq -\mathcal{E}(t_c) V(t) \quad (50)$$

which implies

$$\|\tilde{\theta}(t)\| \leq \exp^{-\mathcal{E}(t_c)(t-t_0)} \|\tilde{\theta}(t_0)\|, \quad \forall t \geq t_c \quad (51)$$

Both the FT identification (9) and the adaptive compensator (43) use the static relationship developed between the unknown parameter θ and some measurable matrix signals C , i.e., $Q\theta = C$. However, instead of computing the parameter values at a known finite-time by inverting matrix Q , the adaptive compensator is driven by the estimation error $C - Q\hat{\theta} = Q\tilde{\theta}$.

9. Incorporating Adaptive Compensator for Performance Improvement

It is assumed that the given control law u and stabilizing update law (herein denoted as $\dot{\hat{\theta}}^s$) result in closed-loop error system

$$\dot{Z} = AZ + \Phi^T \tilde{\theta}^s \quad (52a)$$

$$\dot{\tilde{\theta}}^s = -\Gamma\Phi Z \quad (52b)$$

where the matrix A is such that $A + A^T < -2k_A I < 0$, Φ is a bounded matrix function of the regressor vectors, $\tilde{\theta}^s = \theta - \hat{\theta}^s$ and $Z = [z_1, z_2, \dots, z_{n_x}]^T$ is a vector function of the tracking error with $z_1 = y - y^r$. This implies that the adaptive controller guarantees uniform boundedness of the estimation error $\tilde{\theta}^s$ and asymptotic convergence of the tracking error Z dynamics. Such adaptive controllers are very common in the literature. Examples include linearized control laws [16] and controllers designed via backstepping [12, 15].

Given the stabilizing adaptation law $\tilde{\theta}^s$, we propose the following update law which is a combination of the stabilizing update law (52b) and the adaptive compensator (43)

$$\dot{\hat{\theta}} = \Gamma (\Phi Z + C - Q\hat{\theta}). \quad (53)$$

Since $C(t) = Q(t)\theta$, the resulting error equations becomes

$$\begin{bmatrix} \dot{Z} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} A & \Phi^T \\ -\Gamma\Phi & -\Gamma Q \end{bmatrix} \begin{bmatrix} Z \\ \tilde{\theta} \end{bmatrix}. \quad (54)$$

Considering the Lyapunov function $V = \frac{1}{2} (z^T z + \tilde{\theta}^T \Gamma^{-1} \tilde{\theta})$ and differentiating along (54) we have

$$\dot{V} = \frac{1}{2} z^T (A + A^T) z - \tilde{\theta}^T Q \tilde{\theta} \leq -k_A z^T z - \tilde{\theta}^T Q \tilde{\theta} \quad (55)$$

Hence $\tilde{\theta} \rightarrow 0$ exponentially for $t \geq t_c$ and the initial asymptotic convergence of Z is strengthened to exponential convergence.

For feedback linearizable systems

$$\begin{aligned} \dot{x}_i &= x_{i+1} \quad 1 \leq i \leq n-1 \\ \dot{x}_n &= f_1(x) + f_2(x)u + \theta^T g_n(x) \\ y &= x_1 \end{aligned}$$

the PE condition $Q(t_c) \succ 0$ translates to a priori verifiable sufficient condition on the reference setpoint. It requires the rows of the regressor vector $g_n(x)$ to be linearly independent along a desired trajectory $x^r(t)$ on any finite interval $t \in [t_1, t_2]$, $t_1 < t_2 < \infty$. This condition is less restrictive than the one given in [9] for the same class of system. This is because the linear independence requirement herein is only required over a finite interval and it can be satisfied by a non-periodic reference trajectory while the asymptotic stability result in [9] relies on a T-periodic reference setpoint. Moreover exponential, rather than asymptotic stability of the parametric equilibrium is achieved.

10. Dither Signal Update

Perturbation signal is usually added to the desired reference setpoint or trajectory to guarantee the convergence of system parameters to their true values. To reduce the variability of the closed-loop system, the added PE signal must be systematically removed in a way that sustains parameter convergence.

Suppose the dither signal $d(t)$ is selected as a linear combination of sinusoidal functions as detailed in Section 5. Let a^0 be the vector of the selected dither amplitude and let $T > 0$ be the first instant for which $d(T) = 0$, the amplitude of the excitation signal is updated as follows:

$$a(t) = \begin{cases} a^0, & t \in [0, T) \\ \exp^{-\gamma \bar{\mathcal{E}} T} a(j-1)T, & t \in [jT, (j+1)T), \quad j \geq 1 \end{cases} \quad (56)$$

where the gain $\gamma > 0$ is a design parameter, $a(0) = a^0$ and

$$\begin{aligned} \mathcal{E}(0) &= 0, & \mathcal{E}(\tau) &= \lambda_{\min}(Q(\tau)) \\ \bar{\mathcal{E}} &= \max \{ \mathcal{E}(jT), \mathcal{E}((j-1)T) \}. \end{aligned}$$

It follows from (56) that the reference setpoint will be subject to PE with constant amplitude a^0 if $t \in [0, T)$. After which the trajectory of $a(t)$ will be dictated by the filtered regressor matrix Q . The amplitude vector $a(t)$ will start to decay exponentially when $Q(t)$ becomes positive definite. Note that parameter convergence will be achieved regardless of the value of the gain γ selected as the only requirement for convergence is $Q(t) \succ 0$.

Remark 10.1 The other major approach used in traditional adaptive control is parameter estimation based design. A well designed estimation based adaptive control method achieves modularity of the controller-identifier pair. For nonlinear systems, the controller module must possess strong parametric robustness properties while the identifier module must guarantee certain boundedness properties independent of the control module. Assuming the existence of a bounded controller that is robust with respect to $(\tilde{\theta}, \dot{\hat{\theta}})$, the adaptive compensator (43) serves as a suitable identifier for modular adaptive control design.

11. Simulation Example

To demonstrate the effectiveness of the adaptive compensator, we consider the example in Section 6 for both the nominal system (36) and the system under additive disturbance (37). The simulation is performed for the same reference setpoint $y_r = 1$, disturbance vector $\vartheta = [0.1 \sin(2\pi t/5), 0.2 \cos(\pi t)]^T$, parameter values $\theta = [-1, -2, 1, 2, 3]$ and zero initial conditions.

The adaptive controller presented in [15] is also used for the simulation. We modify the given stabilizing update law by adding the adaptive compensator (43) to it. The modification significantly improve upon the performance of the standard adaptation mechanism as shown in Figures 5 and 6. All the parameters converged to their values and we recover the performance of the finite-time identifier (14). Figures 7 and 8 depict the performance of the output and the input trajectories. While the transient behaviour of the

output and input trajectories is slightly improved for the nominal adaptive system, a significant improvement is obtained for the system subject to additive disturbances.

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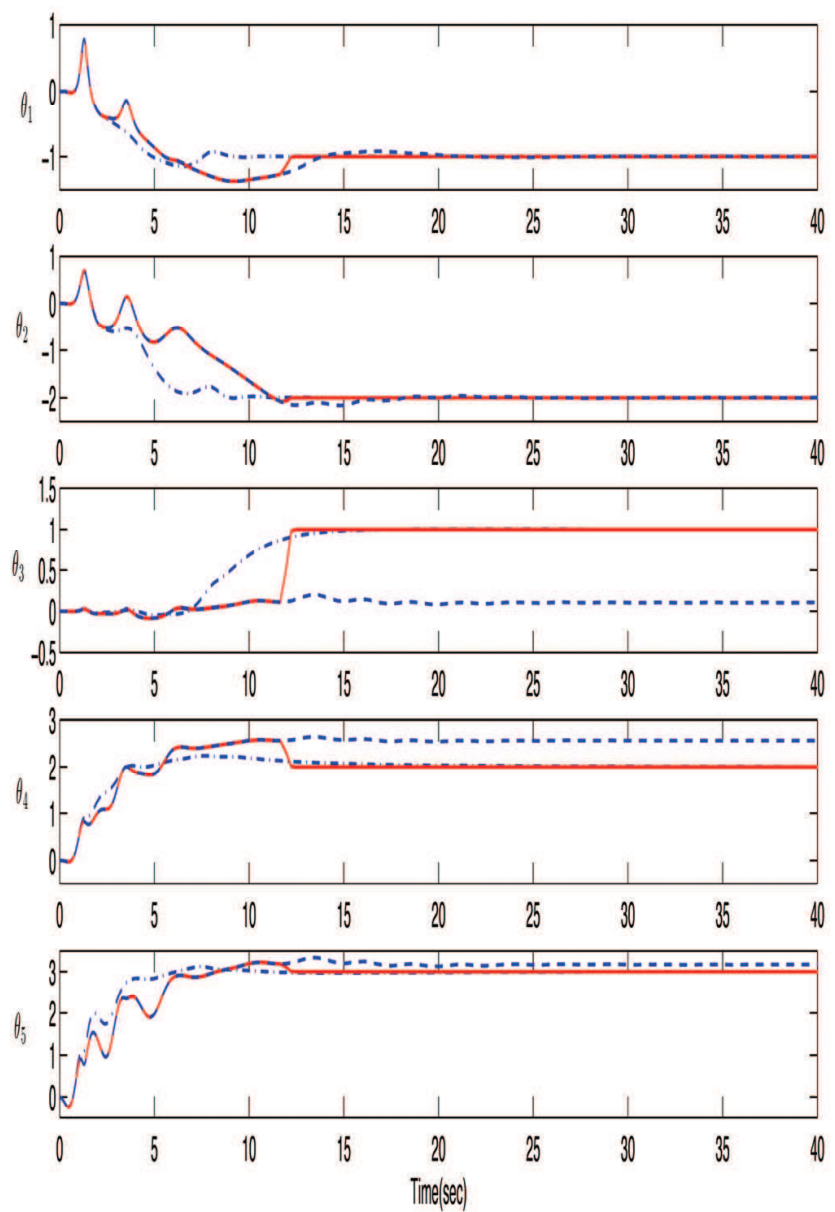


Figure 5. Trajectories of parameter estimates. Solid(-) : compensated estimates; dashdot(-.): FT estimates; dashed(--): standard estimates [15]

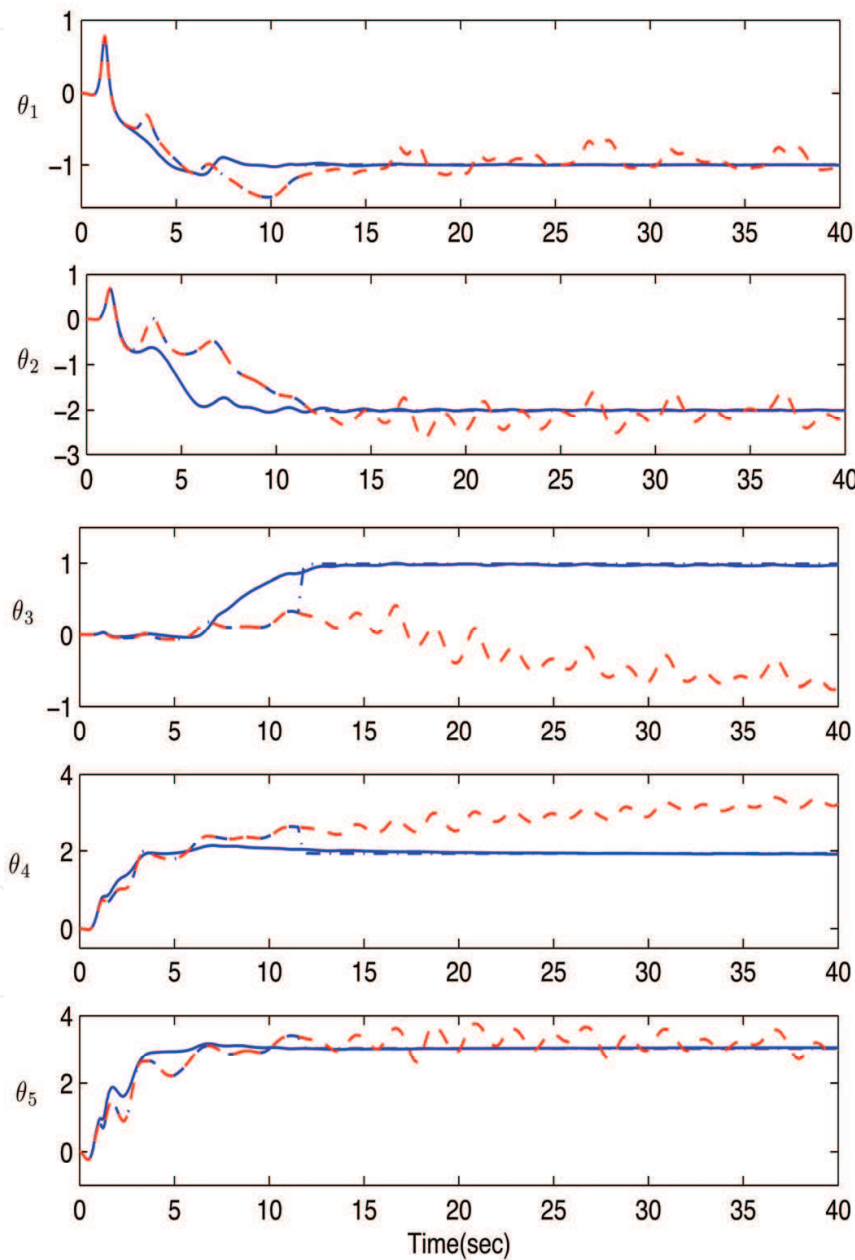


Figure 6. Trajectories of parameter estimates under additive disturbances. Solid(-): compensated estimates; dashdot(-.): FT estimates; dashed(--): standard estimates [15]

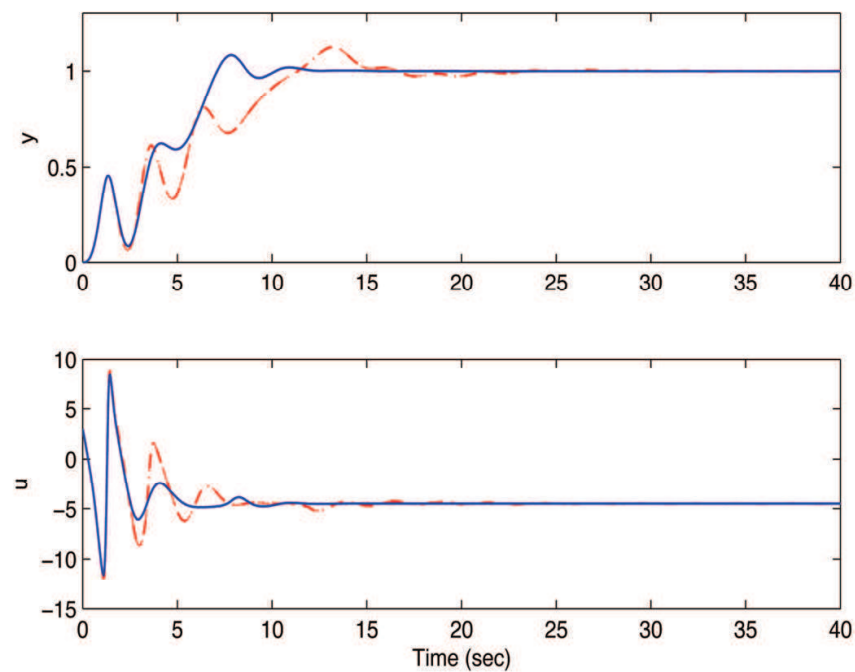


Figure 7. Trajectories of system's output and input for different adaptation laws. Solid(-): compensated estimates; dashdot(-.): FT estimates; dashed(--): standard estimates [15]

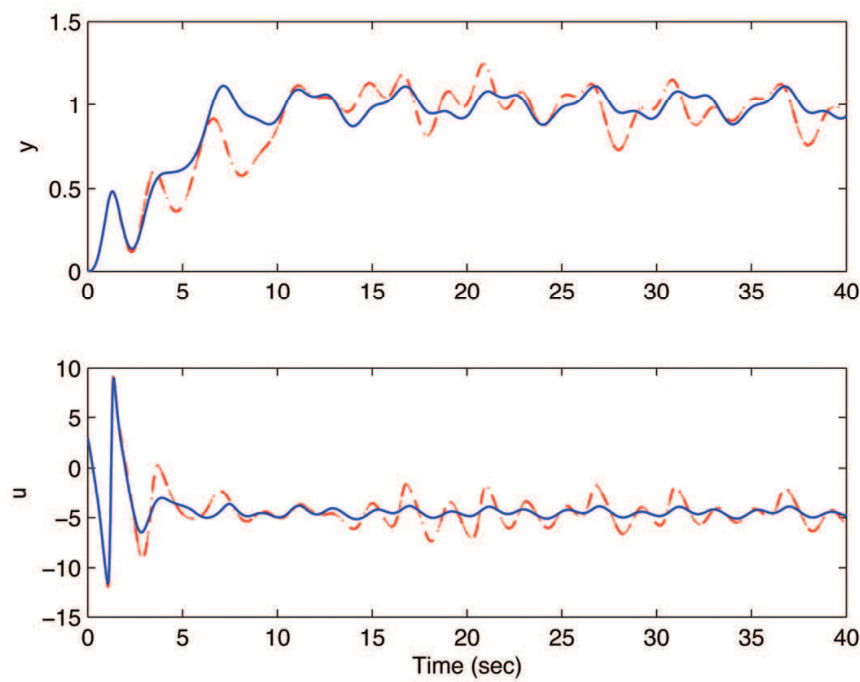


Figure 8. Trajectories of system's output and input under additive disturbances for different adaptation laws. Solid(-): compensated estimates; dashdot(-.): FT estimates; dashed(--): standard estimates [15]

12. Conclusions

The work presented in this chapter transcends beyond characterizing the parameter convergence rate. A method is presented for computing the exact parameter value at a finite-time selected according to the observed excitation in the system. A smooth transition from a standard estimate to the FT estimate is proposed. In the presence of unknown bounded disturbances, the FT identifier converges to a neighbourhood of the true value whose size is dictated by the choice of the filter gain. Moreover, the procedure preserves the system's established closed-loop properties whenever the required PE condition is not satisfied. We also demonstrate how the finite-time identification procedure can be used to improve the overall performance (both transient and steady state) of adaptive control systems in a very appealing manner. The adaptive compensator guarantees exponential convergence of the estimation error provided a given PE condition is satisfied. The convergence rate of the parameter estimator is directly proportional to the adaptation gain and a measure of the system's excitation. The adaptive compensator is then combined with existing adaptive controllers to guarantee exponential stability of the closed-loop system. The application reported in Section 9 is just an example, the adaptive compensator can easily be incorporated into other adaptive control algorithms.

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The objective of this book is to provide an up-to-date and state-of-the-art coverage of diverse aspects related to adaptive control theory, methodologies and applications. These include various robust techniques, performance enhancement techniques, techniques with less a-priori knowledge, nonlinear adaptive control techniques and intelligent adaptive techniques. There are several themes in this book which instance both the maturity and the novelty of the general adaptive control. Each chapter is introduced by a brief preamble providing the background and objectives of subject matter. The experiment results are presented in considerable detail in order to facilitate the comprehension of the theoretical development, as well as to increase sensitivity of applications in practical problems

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