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Optical Glass: A High-Tech Base Material as Key Enabler for Photonics

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Abstract

Optical glass is the base material for the fabrication of spherical lenses, aspheres, prisms, beam splitters, optical fibers, axicons, or other optical components. The photonic industry relies on such components and so on optical glass. Photonics is a key enabling technology for many market segments and applications. The requirements for optical glass are the highest transmission and tight tolerances of not only the optical properties such as refraction and dispersion but also the mechanical properties such as sufficient size and low stress content. In order to achieve the above mentioned specification, a sophisticated melting technology, hot forming processes, annealing procedure, and measurement devices are required. This chapter discusses the most relevant information of these processes.

Keywords: optical glass, photonic, index of refraction, abbe number, refractometer, homogeneity

1. Introduction

Photonics is a key enabling technology for many market segments and applications. This becomes visible by looking on current trends like [1–3]:

- Augmented and virtual reality for consumer and even more relevant for industrial applications.
- Assisting systems in automotive as rear/side view cameras, LED/laser lighting, lidar, night vision support, head-up displays, etc.

- Industry 4.0 with connected individual productions and substitution of humans in mechanical and electric production lines by robots requires exact three-dimensional imaging, object recognition, bar code scanning, quality inspection, distance control, and absolute optical measurements.
- Continuous development of displays with higher resolutions or trends like “internet of things” needs lithographic production lines with accurate positioning that relies on interferometric measurement and recording devices with precisely imaging objectives.
- Enlarged usage of laser technology for cutting, engraving, or welding.
- Use of cameras for security and defense applications or in drones.
- Three-dimensional printing requires also three-dimensional imaging.

Due to these trends, the overall optics and photonics market will grow significantly above 600 billion USD until 2020 (see **Figure 1**). Taking their relevance on further products and services into account, its impact is even bigger.

All these photonic products have in common that they rely on optical glass. Optical glass is the raw material for the fabrication of spherical lenses, aspheres, prisms, beam splitters, optical fibers, axicons, or other optical components (see **Figure 2**).

Optical engineers use components out of various optical glasses to optimize their designs concerning resolution, aberrations, stray light, etc. Such applications take advantage of the essential combination of material features of optical glass like high light transmission, large variety but also precise light deflection (index of refraction and dispersion), high uniformity of light deflection, and sufficient environmental resistance.

Today, most of the optical glasses are melted in continuous tank technology. In order to melt optical glass, it is essential to keep the process parameter like composition and temperatures within tight tolerances and further control the optical features like refractive index and Abbe

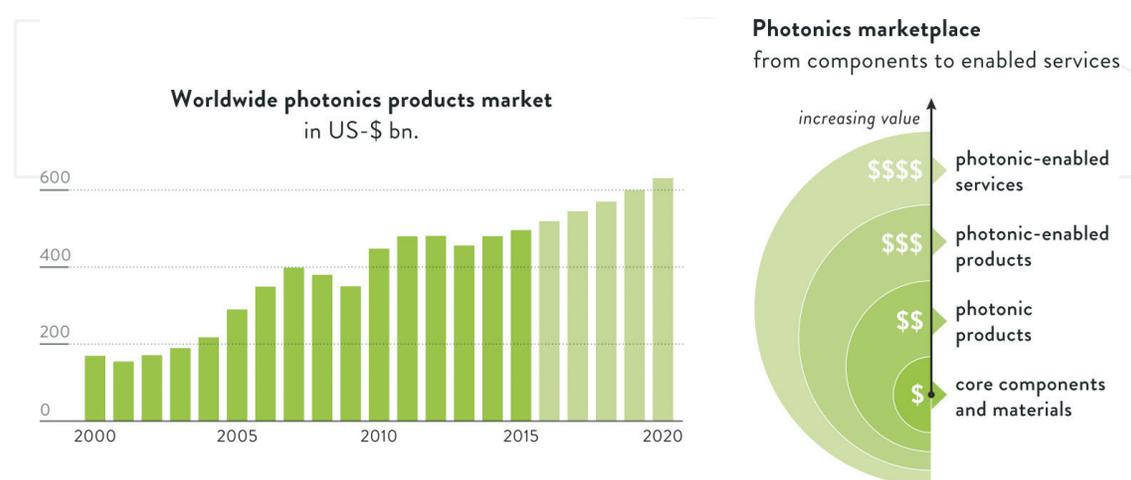


Figure 1. Development of the global market size of photonic products (left). The impact on the global economy is due to products and services that rely on photonic products significantly bigger as indicated by the right diagram [4].



Figure 2. Fabricated optical components out of raw strips and blocks of optical glass (left) [5]. Profile of a digital single-lens reflex camera (Leica S) showing several components made of optical glass (right) [6].

number in adequate loops. Right after the casting, an annealing lehr cools down the glass to avoid cracks due to mechanical stress caused by temperature gradient between inner and outer parts of the glass. This process is called coarse annealing. The composition of the glass defines the first three digits of the refractive index, but even simple optical designs require more precise refraction values. The velocity of the annealing process has an influence on the internal structure of the glass matrix and so on the optical properties. Therefore, an additional fine annealing procedure defines the final optical features. This is necessary because applications like interferometric measurements are sensitive to variations in the sixth digit of the index of refraction. While the melting of the optical glass takes in the order of magnitude of 1 day, the fine annealing procedure could take from several days (small dimensions) up to several months (large dimensions). Cold processing steps like sawing, cutting, grinding, lapping, and polishing convert the fine annealed raw glass into the required optical components to enable the optical functionalities as described above.

This chapter provides a brief overview about the variety of optical glasses and their production procedure including melting, coarse, and fine annealing. Further, the author introduces the definition of the Abbe number. This value describes the dispersion and defines together with the refractive index the optical position. The Abbe diagram depicts the optical position of various optical glasses. This diagram explains the naming conventions of optical glasses. Finally, the measurement techniques in an industrial environment for the main properties of optical glasses are explained.

2. What is optical glass?

Glass has many unique properties. For each application mentioned in this book, the supplying industry breeds special glass types with adapted features to serve the requirements of their application. The most obvious feature of optical glass for a human eye is the high transparency as indicated in **Figure 3**. Compared to window glass, the optical path in optical glass is more than 30 times longer for achieving the same transmission, which is a huge difference.

LIGHT TRANSMISSION OF GLASS

How thick can different glass types be so that 1% of the emitted light is still transmitted?

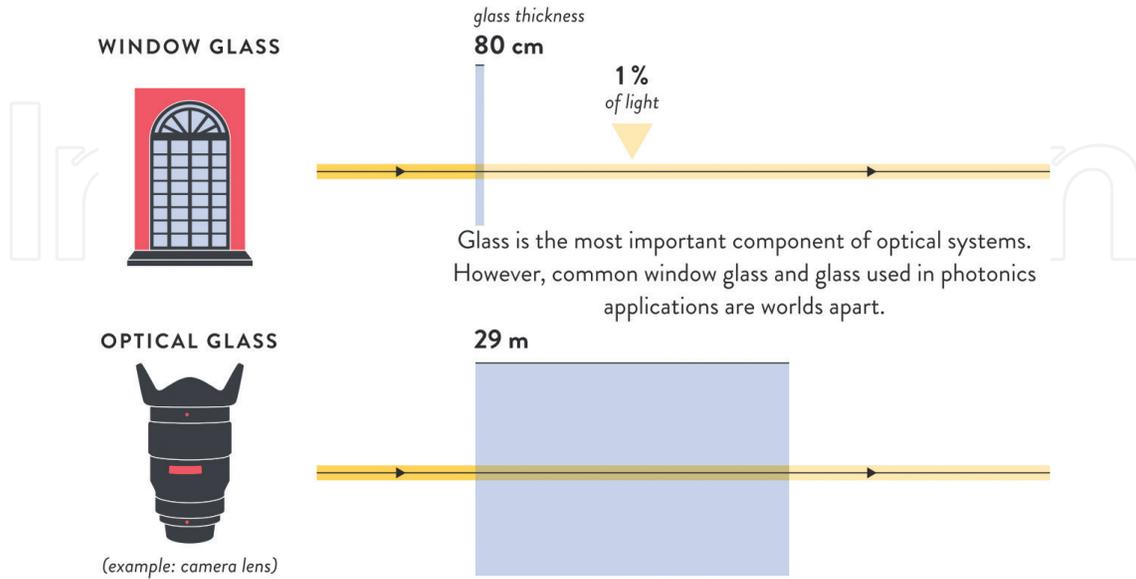


Figure 3. Comparison of the light transmission through window glass and optical glass [4].

In order to achieve such high grades of transmission, the requirements on the purity of the ingredients, the haze level, the number of bubbles, and the inclusions are significantly stronger than for window glass. Figure 4 shows the internal transmittance over the visible spectrum and the near-infrared regime.

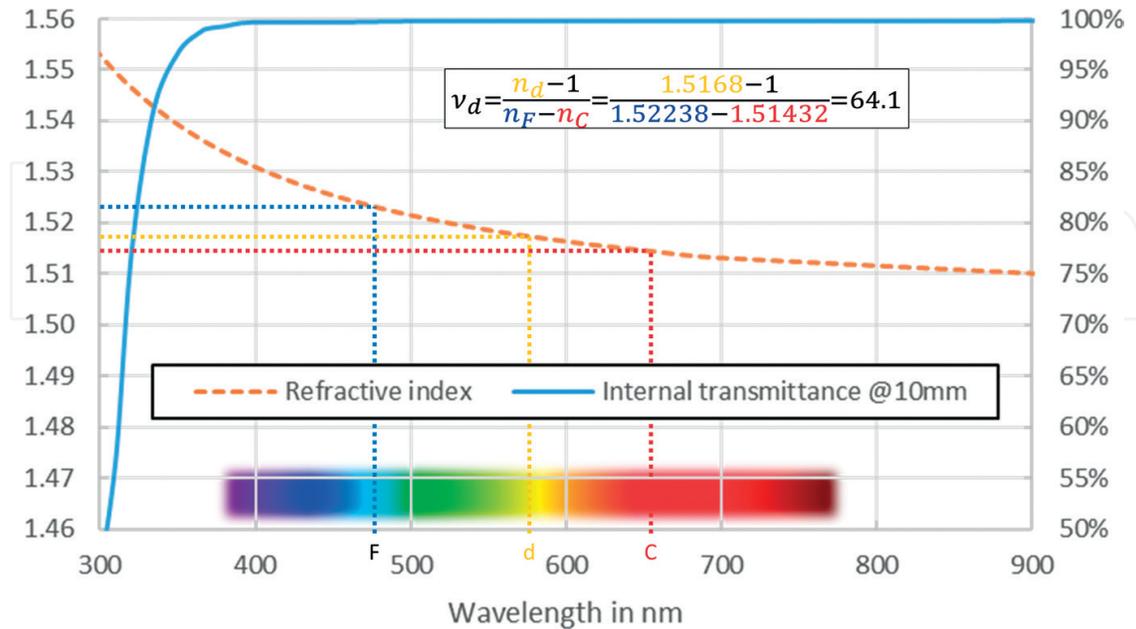


Figure 4. Spectral development of the internal transmittance and refractive index of an optical glass (e.g., SCHOTT N-BK7®). (Data taken from [7]).

In contrast to the transmission, the optical position of an optical glass is not obvious for a human eye. Hereby, the refractive index at a specific wavelength and the Abbe number describing the dispersion define the optical position.

The graphic in **Figure 4** shows the decrease of the refractive index starting from the ultraviolet over the visible spectrum to the near-infrared regime. Dispersion is the name of this spectral refractive index development. The Abbe number v_d as defined in the upper box of **Figure 4** is a measure for the dispersion. The value n_d at the d-line (587.5618 nm) is typically the reference for the refractive index. These two values n_d and v_d define the optical position and exhibit the main distinctive features of optical glass types.

This dispersion is one of the main reasons why we need optical glasses and high sophisticated lens systems for the photonic products at all. If a single lens focuses a blue light ray (e.g., the F-line at 486.1327 nm) and a red light ray (e.g., the C-line at 656.2725 nm), both rays experience different deflections due to the varying refractive index. Therefore, the focus position of both colors differs. If an optical designer combines a flint and a crown glass lens in a proper way, the designer achieves that the focus of the blue and the red ray overlaps. This doublet is an achromatic system. Unfortunately, the focus position of other colors still varies. Therefore, a further chromatic correction and other aberrations require a complex multi-lens design as depicted in **Figures 5** and **2** (right) [8]. Such lens system design relies on a broad portfolio of optical glass that spread widely in their optical position.

Figure 6 shows a n_d-v_d diagram, also called Abbe diagram. This diagram maps the different optical glass types by using the refractive index and Abbe number as coordinates. The left part of the diagram corresponding to high Abbe numbers contains the crown glasses indicated with the letter "K" in the end. The right part of the diagram corresponding to low Abbe numbers contains the flint glasses as indicated with the letter "F" in the end. Besides the rough differentiation between crowns and flints, the map shows further areas of similar chemical composition, e.g., the region of barium flints with the label "BAF" or the area of lanthanum crowns with the label "LAK" [9]. According to their position in the diagram, the glasses get their labels, e.g., F2 is located in the "F" regime. The number at the very end of the glass-type label is without any further information and counts of the developments in the relevant area (seldom followed by a letter indicating a new version). The prefix "N-" indicates that no lead and arsenic are contained in the glass [8]. Then, the prefix "P-" types are special

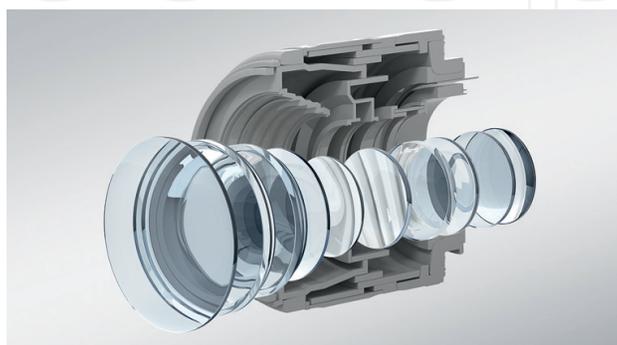


Figure 5. Exploded view on a lens system consisting of 10 different lenses made of different optical glass types. This depicts the complexity of such setups [5].

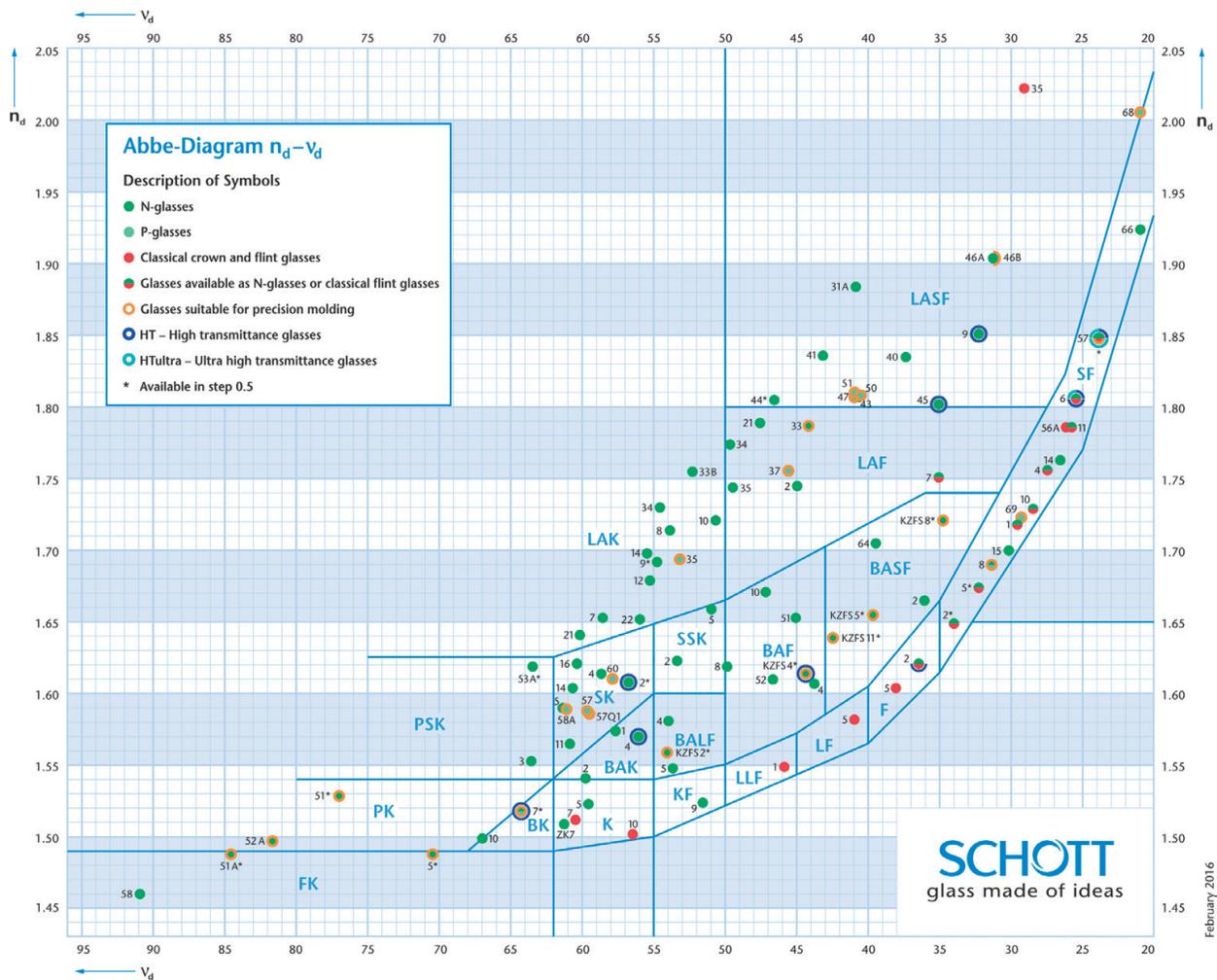


Figure 6. The $n_d - v_d$ diagram, also called Abbe diagram, displaying the optical position of the various glass types [11].

glasses that are environment friendly as well but additionally show a low transformation temperature to enable precise molding process [10]. Finally, the suffix “HT” or “HTUltra” defines special versions of the glass type with high or even ultrahigh transmittance. All explanations are valid for the optical glass manufacturer SCHOTT as a reference but are in general transferable to other manufactures as well.

3. Production of optical glass

In earlier days, optical glass manufacturer filled pots with the ingredients of the optical glass composition, melted the raw material, reduced the bubble content by refining processes, mixed the liquid composition, casted the glass, and filled up the pot again [12]. The state-of-the-art method is to melt glass in a continuous process in a tank production. Figure 7 shows a sketch of such a tank. Compared to other glass industries mentioned in this book, the optical glass production is rather tiny. Seldom, the overall volume inside such tank exceeds more than 5 tons.

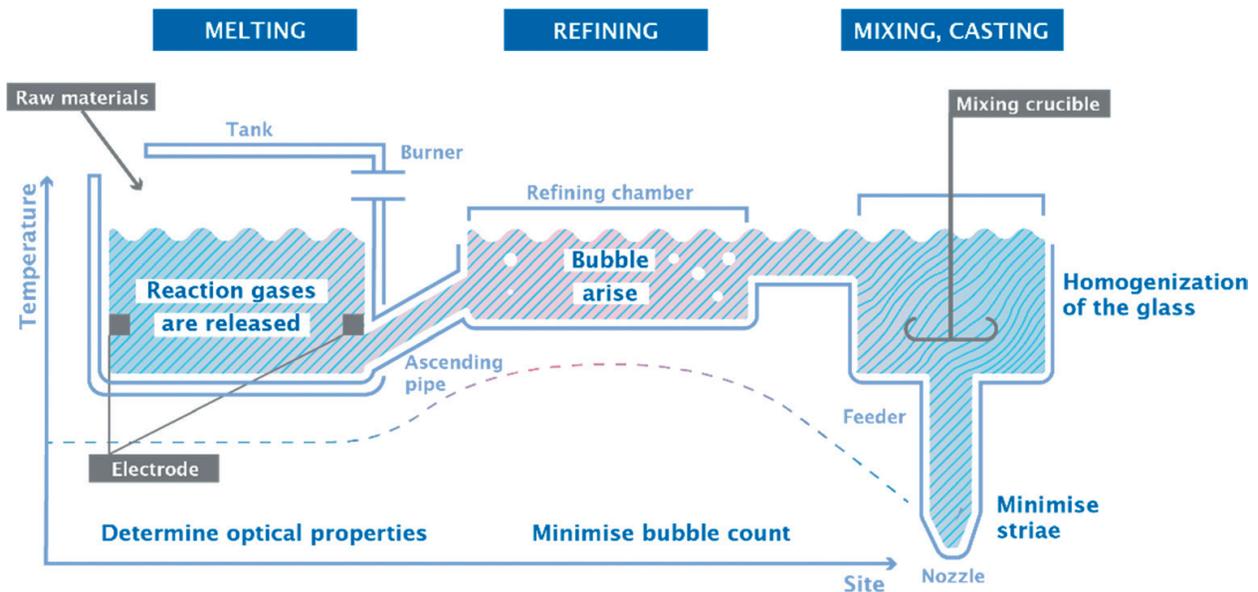


Figure 7. Sketch of a melting tank for optical glasses including the spatial temperature profile. The overall time consumption from the raw material melting to the casting takes several hours (with courtesy of SCHOTT).

A glass manufacturer feeds continuously the ingredients into the melting chamber. Gas burners and electrodes heat up and finally melt the ingredients. The picture in **Figure 8** shows some still solid raw material on the liquid surface in the melting chamber.

The melted material contains some bubbles due to residual air inside the raw material and due to chemical reactions between the ingredients. Just driven by convection, the liquefied material flows into the neighboring refining chamber. The increased temperature in the refining chamber leads to growing gas bubbles and so to a larger buoyancy. Additionally, the higher temperature, the reduced viscosity of the melt supports this upthrust, and the gas bubbles vanish. Afterward, the melted material flows into the mixing chamber. A mechanical stirrer homogenizes the melt by rotational motion. The decreased temperature in the mixing chamber and feeder increases the

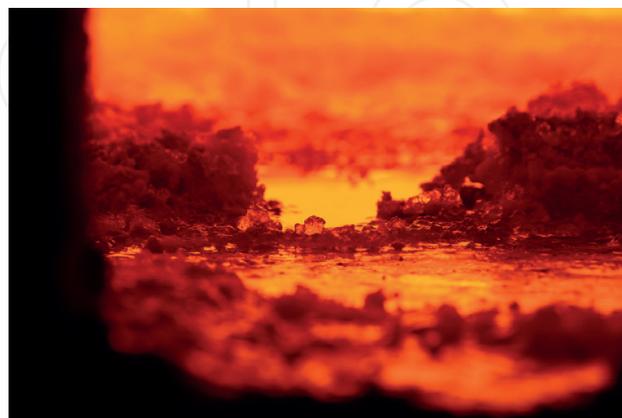


Figure 8. View inside the melting chamber with some still solid raw material on the surface of the melted material (with courtesy of SCHOTT).

viscosity of the melt in order to enable a proper hot forming during the casting. Typically, glass manufacturer produces an endless strip (width ~160 mm, height ~40 mm); sometimes the glass manufacturer uses molds to produce one block after another (length ~200 mm, width ~200 mm, height ~200 mm). **Figure 9** shows the hot forming process of a strip production. The glass is not yet frozen and still glowing red due to black body radiation.

As glass has a rather low heat conductivity ($\sim 1 \text{ W}/(\text{m} \times \text{K})$), a fast cooling process results in a high value of stress. The outer part is already frozen, but the inner part of a strip is still liquid. So, the volume change during the freezing of the inner part cannot be compensated by the already solid outer part. If this stress exceeds a certain threshold, some cracks or breakage occurs. With increasing thickness of a strip, this risk of damage rises. Therefore, a controlled cooling process is necessary for optical glass. An annealing lehr of several meter lengths after the casting minimizes the risk of damage. At the hot end of the annealing lehr has a similar temperature as the feeder and at the other end a few hundred degrees. **Figure 10** shows a view through such an annealing lehr with an endless strip of optical glass inside.

After the coarse annealing in the lehr, the glass manufacturer breaks or saws the glass strip into manageable length depending on the final application. Actually, the annealing rate has a significant impact on the optical position of the glass. Controlling the chemical composition tightly is mandatory to hit the target values of the refractive index and the Abbe number. The order of magnitude of the accuracy required is 10^{-4} to 10^{-6} in the refractive index depending on the application. By keeping the chemical composition constant, the glass manufacturer can control the refractive index within an accuracy of 10^{-3} to 10^{-4} . The annealing velocity influences the internal glass structure and so the optical features. The fine adjustment of the refractive index takes place in the so-called fine annealing. Therefore, ovens heat up each piece of glass again. At a target temperature around the glass-type specific transformation temperature, the stress inside the glass relaxes. By cooling the glass with a constant rate, the glass manufacturer can control the refractive index with the required precision of 10^{-4} and 10^{-6} . **Figure 11** shows the influence of the annealing rate on the optical position. The red cross in the center of the diagram corresponds to the target value that is mentioned, e.g., in a catalog or a data sheet [5, 9]. The figure also contains the preferred tolerance steps for refractive index and Abbe number from ISO 12123 that specifies raw optical glass (bluish boxes) [14].

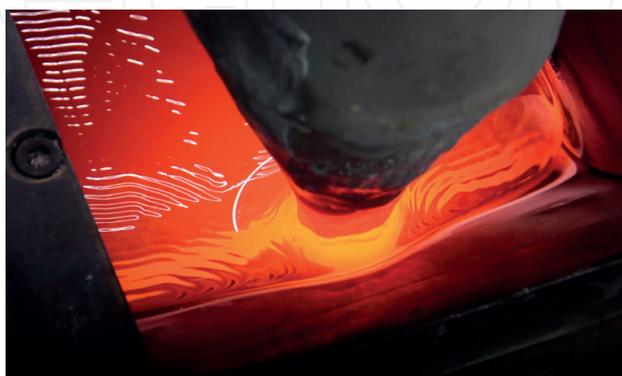


Figure 9. Hot forming process to produce an endless strip of width 160 mm and height 40 mm. The still liquid glass is glowing due to the black body radiation [13].



Figure 10. View through an annealing lehr of roughly 12 m length that cools down the endless strip slowly to avoid stress and cracks. This procedure takes several hours (with courtesy of SCHOTT).

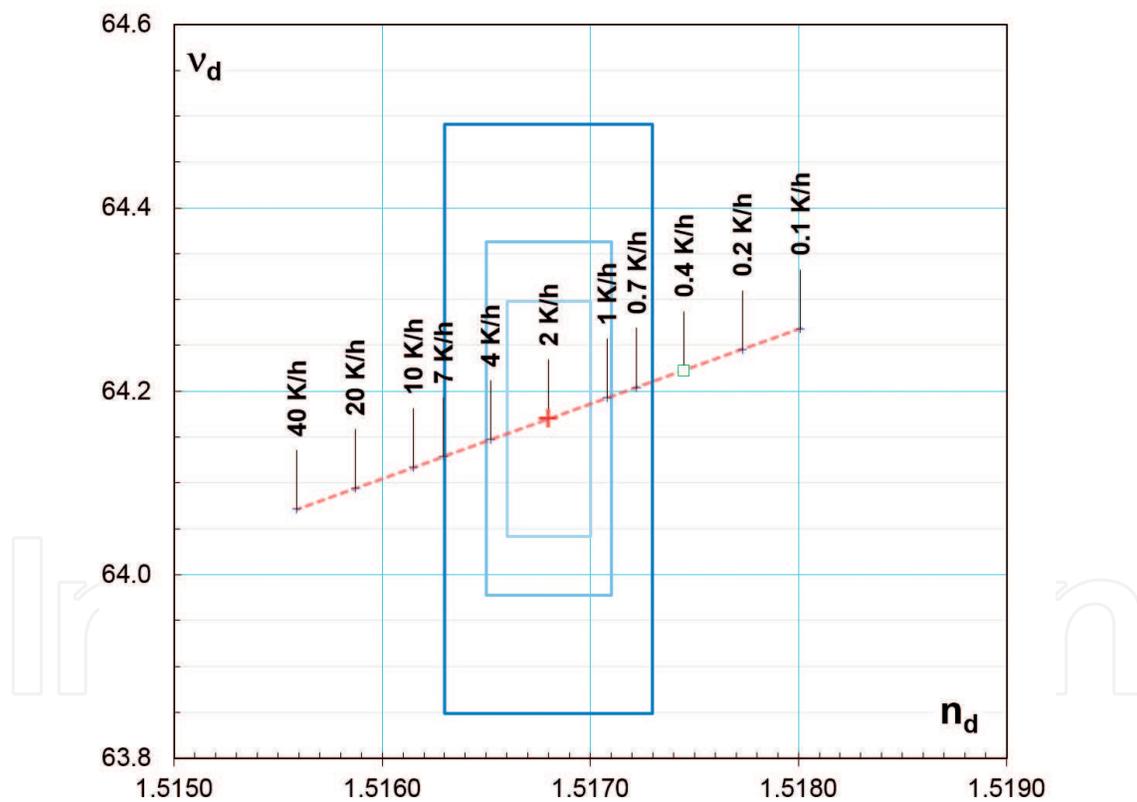


Figure 11. Close-up of $v_d - n_d$ diagram that indicates the required precision and the impact of the annealing rate on the optical position. Data shows SCHOTT N-BK7® based on data taken from [15].

The green square depicts a piece of glass, which was annealed with a cooling rate of 0.4 K/h. Obviously, the optical position is not within the accepted maximum tolerance range (dark blue box). This piece of glass could be annealed again (reversible process) with a higher annealing rate along the annealing line inside the tolerance range [12]. So, the refractive index

and Abbe number are fine-tuned by the annealing process. In this example, an annealing with 2 K/h would adapt the optical position to the target value. The printed annealing line is constant for a specific glass type but differs significantly from one glass type to another. Unfortunately, there are boundaries to the annealing rate. Below a glass-type specific annealing rate, the piece of glass tends to crystallize, which would lead to significant haze. Above a certain threshold, which depends on the glass type and the smallest dimension, stress birefringence gets significant. In general, glass is an isotropic media so there is no preferred direction inside the glass system. Nevertheless, if the glass is annealed with a high cooling rate, the inner and the outer parts of a glass blank experience a different temperature gradient (low heat conductivity $\sim 1 \text{ W}/(\text{m} \times \text{K})$). This leads to mechanical stress. This mechanical stress leads to a preferred direction and so to a refractive index that depends on the polarization orientation of the transmitted light. This effect is called stress-induced birefringence or in short stress birefringence. Besides the mechanical stress, the spatially different temperature rates also influence the regional refractive index (see **Figure 11**). The homogeneity is the feature that summarizes the result of stress birefringence and regional refractive index variation. An interferometer measures the grade of the homogeneity. Therefore, a plane wave travels through the plane-parallel polished glass blank and overlaps afterward with an undisturbed plane wave. The space-resolved intensity distribution depicts the two-dimensional wave front distortion in false color illustration caused by the glass blank (see, e.g., **Figure 12**).

The maximum difference of the wave front distortion divided by the blank thickness (peak-to-valley value $PV = 0.47 \times 10^{-6}$) defines the homogeneity grade. The tightest homogeneity tolerance in ISO 12123 is a peak-to-valley value below 10^{-6} , which is roughly four times smaller than the width of the green square in **Figure 11**. Especially for larger dimensions, this requires a very sophisticated technology of tight controlling of the chemistry and the hot forming process to avoid bubbles, inclusion, or striae. Further, experienced knowledge of the fine annealing procedure to reduce stress birefringence and regional refractive index variation to a minimum is necessary.

Typically, a V-block refractometer measures the optical position of the optical glass in order to monitor an accurate production [17]. **Figure 13** schematically illustrates the underlying principle.

The glass manufacturer prepares out of the produced and annealed glass a cuboidal sample with dimensions of about $20 \times 20 \times 5 \text{ mm}^3$. This sample fits in a V-shaped glass block with a precisely known refractive index $n_{V\text{-block}}$. An immersion oil between the V-block and the sample decreases the surface quality requirements to the scale of 1 mm in flatness. The glass manufacturer measures the deflection angle θ from optical axis of a light ray that propagates through the setup as depicted in **Figure 13**. The refractive index of the sample n_{sample} for the color of the light ray (wavelength λ) results to [18]

$$n_{\text{sample}}^2 = n_{V\text{-block}}^2 - n_{\text{air}} \cdot \sin(\theta) \cdot \sqrt{n_{V\text{-block}}^2 - n_{\text{air}} \cdot \sin^2(\theta)} \quad (1)$$

with the refractive index of the surrounding air n_{air} . Although the accuracy of this method is lower than for the minimum angle deviation method [19], the V-block refractometer is the much faster and cheaper approach. Therefore, the V-block refractometer is the ideal tool to monitor the quality of an optical glass production economically.

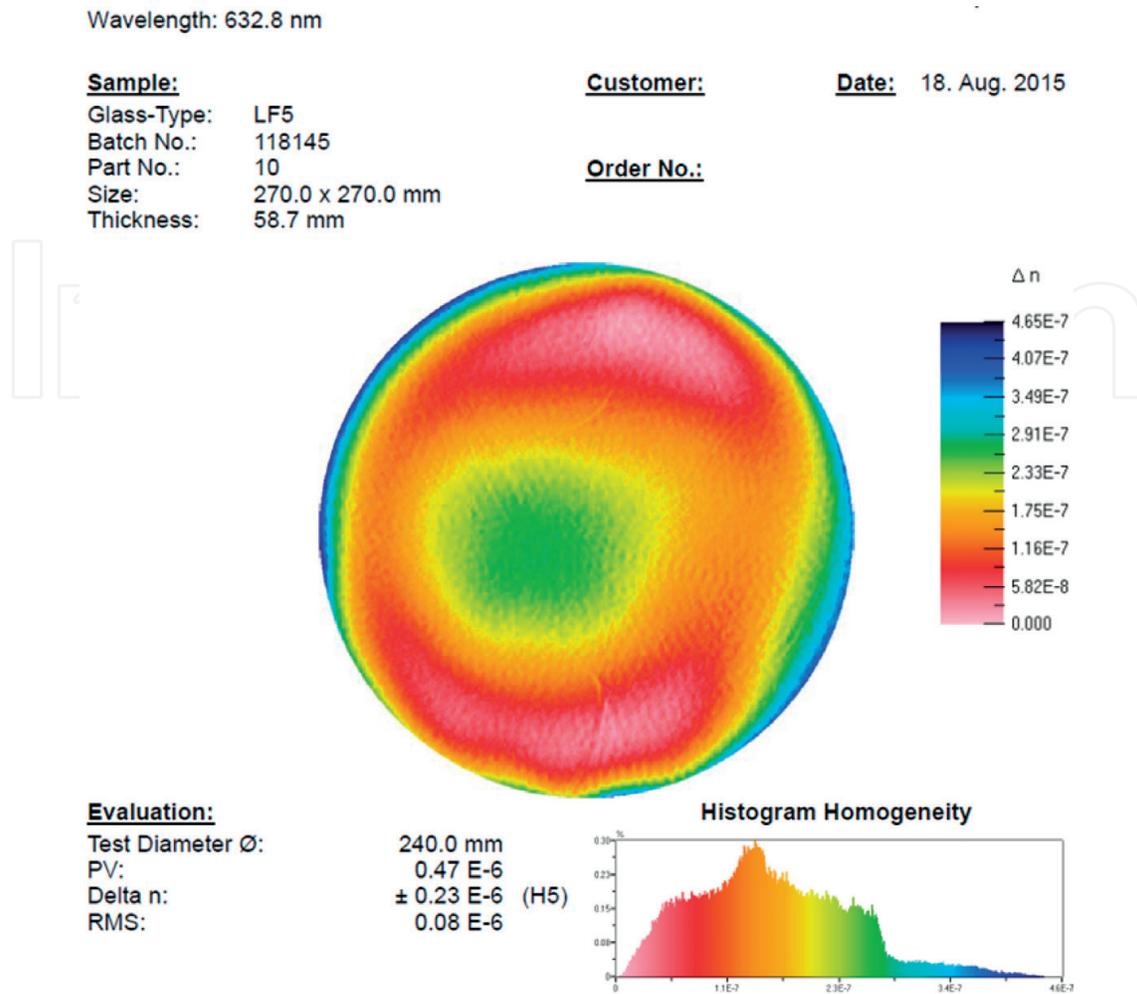


Figure 12. False color illustration of an interferometric wave front distortion of a round LF5 glass blank [16].

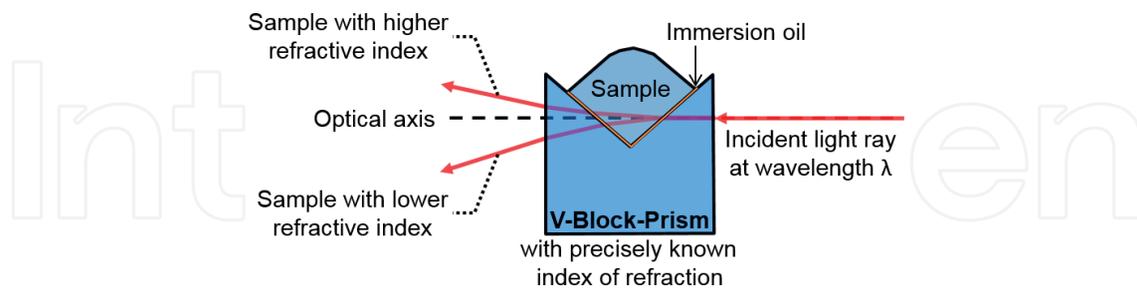


Figure 13. Sketch of the principal setup of a V-block refractometer [17].

Even though the manufacture of optical glass is volume wise, the smallest production of all the productions that are mentioned in this book, it has a significant impact on almost all industries. As indicated in the introduction due to a broad variety of market trends, the worth of this key enabling product is still increasing. On the other hand, only a few companies globally still master the rather complex and highly sophisticated production of optical glass that requires a lot of knowledge and experience [20].

4. Conclusion

Even though optical glass is a base material with quite some history, it will remain a relevant key enabler for the entire photonic industry. As the market for photonics is globally rising over almost all market segments, also the demand for optical glass will increase. High sophisticated optical systems require a broad variety of glass types with tight tolerance of the optical position and high transmission. Such tough requirements need an elaborated melting, annealing, and measurement technology.

Since the late nineteenth century, optical glass manufacturer develop optical glasses. A comparison of the $n_d - v_d$ diagrams of the various optical glass manufacturers shows rather similar portfolios. The optical glass development is limited, e.g., by the relation of refractive index and dispersion (Kramers-Kronig relation) [21]. Therefore, the author expects that the landscape of optical glasses will not change significantly in the future. However, a modified landscape is not necessary as the optical designers in general already have a sufficient portfolio of optical glass types to work with.

The various new market trends and segments like augmented or virtual reality, industry 4.0, autonomous driving, robotic and display development, laser material processing, and 3D printing will evolve quickly with slightly adapted requirements. The outlook for the overall optical glass landscape is therefore that the optical glass manufacturer will marginally improve their portfolio with segment-specific variants of their current portfolio concerning optical positions, low density, high transmittance, high chemical resistance, and extreme thermal behavior.

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