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Global Feed-forward Adaptive Fuzzy Control of Uncertain MIMO Nonlinear Systems

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1. Abstract

This study proposes a novel adaptive control approach using a feedforward Takagi-Sugeno (TS) fuzzy approximator for a class of highly unknown multi-input multi-output (MIMO) nonlinear plants. First of all, the design concept, namely, *feedforward fuzzy approximator* (*FFA*) *based control*, is introduced to compensate the unknown feedforward terms required during steady state via a forward TS fuzzy system which takes the desired commands as the input variables. Different from the traditional fuzzy approximation approaches, this scheme allows easier implementation and drops the boundedness assumption on fuzzy universal approximation errors. Furthermore, the controller is synthesized to assure either the disturbance attenuation or the attenuation of both disturbances and estimated fuzzy parameter errors or globally asymptotic stable tracking. In addition, all the stability is guaranteed from a feasible gain solution of the derived linear matrix inequality (LMI). Meanwhile, the highly uncertain holonomic constrained systems are taken as applications with either guaranteed robust tracking performances or asymptotic stability in a global sense. It is demonstrated that the proposed adaptive control is easily and straightforwardly extended to the robust TS FFA-based motion/force tracking controller. Finally, two planar robots transporting a common object is taken as an application example to show the expected performance. The comparison between the proposed and traditional adaptive fuzzy control schemes is also performed in numerical simulations.
Keywords: Adaptive control; Takagi-Sugeno (TS) fuzzy system; holonomic systems; motion/force control.

2. Introduction

In recent years, plenty of adaptive fuzzy control methods (Wang & Mendel, 1992)-(Alata et al., 2001) have been proposed to deal with the control problem of poorly modeled plants. All these researches are based on the fuzzy universal approximator (first proposed by Wang & Mendel, 1992), which is properly adjusted to compensate the uncertainties as close as possible. Due to the use of states as the inputs of the fuzzy system, we call this approach as the *state-feedback fuzzy approximator* (*SFA*) *based control*. In details, this methodology can be further classified into two types: i) Mamdani fuzzy approximator (Wang & Mendel, 1992;

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Chen et al., 1996; Lee & Tomizuka, 2000; Lin & Chen, 2002); and ii) Takagi-Sugeno (TS) fuzzy approximator (Ying, 1998; Tsay et al., 1999; Chen & Wong, 2000; Alata et al., 2001). The first type approach constructs the consequent part only via tunable fuzzy sets, but a good enough approximation usually requires a large number of fuzzy rules. In contrast, the TS SFA-based controller uses the linear/nonlinear combination of states in consequent part such that fewer rules are required. Without loss of generality, the configuration of these controllers is shown in Fig. 1. The SFA-based control contains the following disadvantages: i) numerous fuzzy rules and tuning parameters are required, especially for multivariable systems; ii) the fuzzy approximation error is assumed *a priori* to be upper bounded although the bound depends on state variables; and iii) the consequent part of TS fuzzy approximator will become complex for dealing with multivariable nonlinear systems, i.e., needing a complicated consequent part.

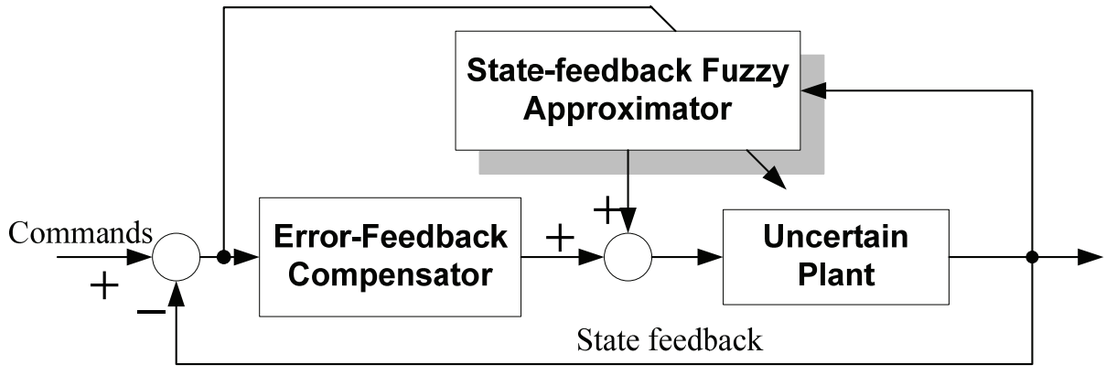

Figure 1. Configuration of SFA-based adaptive controller

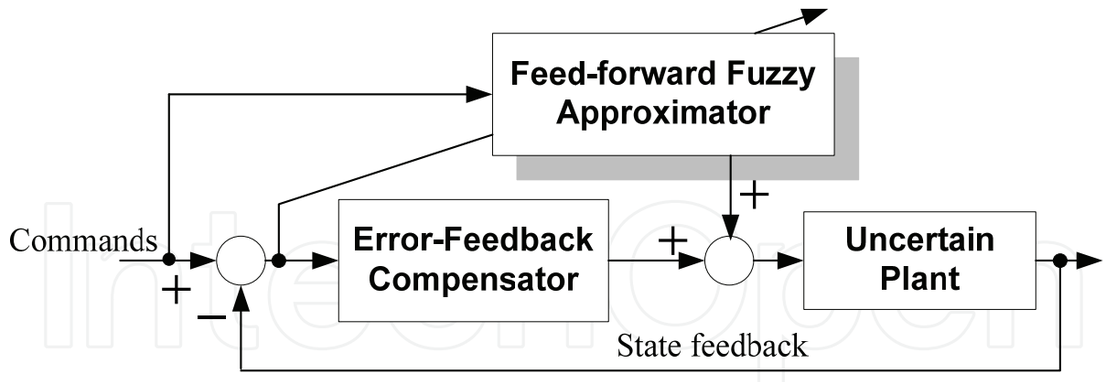

Figure 2. Configuration of FFA-based adaptive controller

To remove the above limitations, this study introduces the *feed-forward fuzzy approximator* (*FFA*) *based control* which takes the desired commands as the premise variables of fuzzy rules and approximately compensates an unknown feed-forward term required during steady state (note that the configuration is illustrated in Fig. 2). At the first glance, the SFA and FFA based control methods have a common adaptive learning concept, that is the feedback-error is used for tuning parameters of the compensator. But, a closer investigation reveals the differences on: i) the type of training signals, ii) the process of taming dynamic uncertainties; and iii) the

type of error feedback terms. Especially, compared to SFA-based approaches (shown in Fig. 1), the FFA-based adaptive controller needs a nonlinear damping term. However, omitting feedback information in the fuzzy approximator leads to a less complex implementation (i.e., a simpler architecture compared to traditional SFA-based controllers). Furthermore, the fuzzy approximation error of FFA is always bounded, such that the synthesized controller assures global stability. In addition, the number of fuzzy rules can be further reduced by using a TS-type FFA. In other words, the FFA-based adaptive controller has better advantages than the SFA-based adaptive controller.

To demonstrate the high application potential of the FFA-based adaptive control method to complicated and high-dimension systems, the FFA-based motion/force tracking controller is constructed for holonomic mechanical systems with an environmental constraint (McClamroch & Wang, 1988) or a set of closed kinematic chains (Tarn et al., 1987; Li et al., 1989). Holonomic systems represent numerous industrial plants — two for example, are constrained robots and cooperative multi-robot systems. From the pioneering work (McClamroch & Wang, 1988), a reduced-state-based approach is utilized in most researches (Tarn et al., 1987; Li et al., 1989; Wang et al., 1997). When considering parametric uncertainties, adaptive control schemes were introduced in (Jean & Fu, 1993; Liu et al., 1997; Yu & Lloyd, 1997; Zhu & Schutter, 1999). Unfortunately, the reduced-state-based approach usually has a force tracking residual error proportional to estimated parameter errors. Thus, a high gain force feedback or acceleration feedback is needed (e.g., Jean & Fu, 1993; Yu & Lloyd, 1997). An alternative hybrid motion/force control stated in (Yuan, 1997) has assured both motion and force tracking errors to be zero. To deal with unstructured uncertainties, several robust control strategies (Chiu et al., 2004; Zhen & Goldenberg, 1996; Gueaieb et al., 2003) provide asymptotic motion tracking and an ultimate bounded force error. In contrast to discontinuous control laws, the works (Chang & Chen, 2000; Lian et al., 2002) apply adaptive fuzzy control to compensate unmodeled uncertainties and achieve H^{∞} tracking performance. However, their applications are limited due to high computation load arising from the numerous fuzzy rules and tuning parameters. All these points motivate the further research on improving the control of holonomic systems by using the FFA-based control.

As a result, the proposed adaptive controller is no longer with the disadvantages of the traditional SFA-based adaptive controllers mentioned above. In detail, the stability is guaranteed in a rigorous analysis via Lyapunov's method. The attenuation of both disturbances and estimated fuzzy parameter errors is achieved in an L_2 -gain sense, while the LMI techniques (Boyd et al., 1994) are used to simplify the gain design. If applying the sliding mode control, the controlled system can further achieve asymptotic stability of tracking errors. Notice that the proposed approach assures global stability for controlling general MIMO uncertain systems in a straightforward manner. Compared to the mainly relative works (Chang & Chen, 2000; Lian et al., 2002), the proposed scheme achieves both robust motion and force tracking control (but the work (Lian et al., 2002) does not) for more general holonomic systems. Meanwhile, the scheme has a novel architecture which can be easily implemented.

The remainder of this chapter is organized as follows. First, the TS FFA-based adaptive control method is introduced in Sec. 3. Then, the proposed control method is modified to motion/force tracking controller for holonomic constrained systems in Sec. 4. Section 5 shows the simulation results of controlling a cooperative multi-robot system transporting a common object. Finally, some concluding remarks are made in Sec. 6.

3. TS FFA-based Adaptive Fuzzy Control

3.1 FFA-based Compensation Concept

Without loss of generality, let us consider an n-th order multivariable nonlinear system

$$G(\underline{x}(t))x^{(n)}(t) = f(\underline{x}(t)) + u(t) + w(t) \tag{1}$$

where $n \geq 2$; $x \in R^m$ is a part of the state vector $\underline{x}$ defined as $\underline{x}(t) = [x^T(t) \quad \dot{x}^T(t) \quad \cdots \quad (x^{(n-1)}(t))^T]^T \in R^{nm}$; $f(\underline{x}(t)) \in R^m$ is an unknown nonlinear function which satisfies $f(\underline{x}_d(t)) \in L_\infty$ for an appropriate bounded desired tracking command $\underline{x}_d(t) = [x_d^T(t) \quad \dot{x}_d^T(t) \quad \cdots \quad (x_d^{(n-1)}(t))^T]^T$; $G(\underline{x}(t)) \in R^{m \times m}$ is an unknown positive-definite symmetric matrix which satisfies $G(\underline{x}_d(t))$, $\dot{G}(\underline{x}_d(t)) \in L_\infty$; $u(t) \in R^m$ is the control input; and $w(t) \in R^m$ is an external disturbance assumed to be bounded. Clearly, if the terms $f(\underline{x}(t))$ and $G(\underline{x}(t))$ are exactly known and no disturbance exists, we are able to apply the feedback linearization concept and set the control law as

$$u = -f(\underline{x}) + G(\underline{x})\dot{q}_a(t) + \frac{1}{2}\dot{G}(\underline{x})s + Ks \tag{2}$$

where the notations are given as $e(t) = x_d(t) - x(t)$, $s(t) = q_a(t) - x^{(n-1)}(t)$, $q_a(t) = x_d^{(n-1)}(t) + \Lambda_{n-1}e^{(n-2)}(t) + \cdots + \Lambda_2\dot{e}(t) + \Lambda_1 e(t)$; $\Lambda_v \in R^{m \times m}$, for $v = 1, 2, ..., (n-1)$, is a positive-definite diagonal matrix; and $K \in R^{m \times m}$ is a symmetric positive-definite matrix. This renders to the error dynamics $G(\underline{x})\dot{s} = -\frac{1}{2}\dot{G}(\underline{x})s - Ks - w(t)$, which is exponentially stable once there is no disturbance. However, the state feedback term $u_b = -f(\underline{x}) + G(\underline{x})\dot{q}_a(t) + \frac{1}{2}\dot{G}(\underline{x})s$ is often poorly understood such that the fuzzy approximator is considered to realize the ideal control law (2) in conventional SFA-based control methods. Nevertheless, when the tracking goal is achieved, terms $f(\underline{x}(t))$ and $G(\underline{x}(t))$ accordingly converge to functions $f(\underline{x}_d(t))$ and $G(\underline{x}_d(t))$. The state feedback term u_b converges to

$$u_f = -f(\underline{x}_d) + G(\underline{x}_d)x_d^{(n)} \tag{3}$$

which is only dependent on the pre-planned desired command $\underline{x}_d$. In other words, the state feedback control law becomes a feedforward compensation law during steady state. Therefore, different to traditional works (Wang & Mendel, 1992)-(Alata et al., 2001), here we use the universal fuzzy approximator to closely obtain the feed-forward compensation law (3), while the effect of omitting transient dynamics is compensated by error feedback. Since the pre-planned desired commands would be taken as the inputs of the fuzzy approximator, the so-called *feed-forward fuzzy approximator* (FFA) arises. By this way, we assume that there exist positive constants $\psi_1, ..., \psi_p$ and positive-semidefinite symmetric matrices Ψ_s, Ψ_e such that the error between $u_b(\underline{x})$ and $u_f(\underline{x}_d)$ is shaped by

$$s^T(u_b(\underline{x}) - u_f(\underline{x}_d)) \leq \sum_{\kappa=1}^{p} \psi_\kappa \left\| e_o \right\|^{2\kappa} \left\| s \right\|^2 + s^T \Psi_s s + e_o^T \Psi_e e_o \tag{4}$$

with the tracking error $e_o = \underline{x} - \underline{x}_d$. Then the design idea can be realized by combining both FFA and error-feedback based compensations later. Note that the above inequality is often held for most physical systems, such as robotic systems, dc motors, etc... Moreover, $p = 1$ is often held. The similar property as (4) for nonlinear systems can be found in (Sadegh & Horowitz, 1990; Chiu et al., 2006; Chiu, 2006).

From the definition of u_f in (3), the TS-type FFA consists of the following rules:

Rule l: If $z_1(t)$ is $\mathbf{X}_1^l$ and ... and $z_h(t)$ is $\mathbf{X}_h^l$. Then

$$\hat{u}_{fi} = \theta_{0i}^l + \theta_{1i}^l \chi(\underline{x}_d), l = 1, 2, ..., r \tag{5}$$

where $z_1(t)$, ..., $z_h(t)$ are the premise variables composed of the desired commands $x_d(t)$, $\dot{x}_d(t)$, ..., $x_d^{(n-1)}(t)$ since $u_f(\underline{x}_d(t))$ is functional of $\underline{x}_d(t)$; $l = 1$, 2, ..., r with r denoting the total number of rules; $\mathbf{X}_1^l$, ..., $\mathbf{X}_h^l$ are proper fuzzy sets determined by the known behavior of the desired signals; $\hat{u}_{fi}$ is the i-th element of approximation of u_f; $\chi \in R^g$ is a basis vector functional of $\underline{x}_d(t)$ to be chosen from the nonlinearity of u_f; $\theta_{0i}^l \in R$ and $\theta_{1i}^l \in R^{1\times g}$ are fuzzy parameters. Using the singleton fuzzifier, product fuzzy inference and weighted average defuzzifier, the inferred output of the fuzzy system (5) is

$$\hat{u}_{fi}(z_d, \Theta_{u_{fi}}) = \xi^T(z_d)\Theta_{u_{fi}}\bar{\chi}(z_d) \tag{6}$$

where $z_d(t) \equiv [z_1(t) \quad z_2(t) \quad ... \quad z_h(t)]^T$; $\Theta_{u_{fi}} \equiv [\theta_{u_{fi}}^1 \quad \theta_{u_{fi}}^2 \quad ... \quad \theta_{u_{fi}}^r]^T \in R^{r\times(g+1)}$ with $\theta_{u_{fi}}^l = [\theta_{0i}^l \quad \theta_{1i}^l]^T \in R^{g+1}$; $\bar{\chi} = [1 \quad \chi^T]^T \in R^{g+1}$; and $\xi(z_d(t)) \equiv [\xi_1 \quad \xi_2 \quad ... \quad \xi_r]^T \in R^r$ is a fuzzy basis function vector consisting of $\xi_l(z_d(t)) = \mu_l(z_d(t))/\sum_{l=1}^r \mu_l(z_d(t))$ with $\mu_l(z_d(t)) = \prod_{\zeta=1}^h \mathbf{X}_\zeta^l(z_\zeta(t)) \geq 0$ for all l. Note that the form of (6) is a TS type of fuzzy representation. When we let $\chi = 0$, the fuzzy system (5) is reduced to the special case with a Mamdani fuzzy representation, i.e., $\hat{u}_{fi} = \xi^T(z_d)\Theta_{u_{fi}}$ for $\Theta_{u_{fi}} \in R^r$ and $\bar{\chi} = 1$. Based on the above fuzzy approximator (6), the overall approximation of u_f is obtained as

$$\hat{u}_f(z_d(t), \Theta_{u_f}) = \left[\hat{u}_{fi}(z_d(t), \Theta_{u_{fi}})\right]_{m\times 1} = Y_d(z_d(t))\Theta_{u_f}\bar{\chi} \tag{7}$$

where $\Theta_{u_f} = [\Theta_{u_{f1}}^T \quad \Theta_{u_{f2}}^T \quad \cdots \quad \Theta_{u_{fm}}^T]^T \in R^{mr\times(g+1)}$; and $Y_d =$ block-diag $\{\xi^T, ..., \xi^T\} \in R^{m\times mr}$ is a regression matrix. From the observation on (7), if Θ_{u_f} is bounded, then $\hat{u}_f \in L_\infty$ for all t (due to $Y_d(z_d(t)) \in L_\infty$ and $\bar{\chi} \in L_\infty$ for all bounded $\underline{x}_d(t)$). In light of this, we limit the tunable fuzzy parameter Θ_{u_f} to a specified region

$$\Omega_{\theta_u} \equiv \left\{\Theta_{u_f} \in R^{mr\times(g+1)} \middle| \mathrm{tr}(\Theta_{u_f}^T\Theta_{u_f}) \leq \bar{\theta}_u, \bar{\theta}_u > 0\right\}$$

with an adjustable parameter $\bar{\theta}_u$. Meanwhile, an appropriate projection algorithm will be applied later to keep the tuned fuzzy parameters within the bounded region. Inside the

specified set, there exists an optimal approximation parameter $\Theta_{u_f}^*$ defined as (for U_z is a discussed space of z_d)

$$\Theta_{u_f}^* \equiv \text{argmin}_{\Theta_{u_f} \in \Omega_{\theta_u}} \left(\sup_{z_d \in U_z} \left| u_f - \hat{u}_f(z_d, \Theta_{u_f}) \right| \right)$$

which leads to the minimum approximation error for u_f. This means that the minimum approximation error is

$$W_{u_f} = u_f(z_d) - Y_d(z_d(t))\Theta_{u_f}^* \bar{\chi} . \tag{8}$$

Note that if the parametric constraint is removed, the optimal approximation parameter $\Theta_{u_f}^*$ is still upper bounded (cf. Wang & Mendel, 1992). Due to $\hat{u}_f(z_d, \Theta_{u_f}^*) \in L_\infty$ and $u_f(\underline{x}_d) \in L_\infty$, it is reasonably concluded that W_{u_f} is upper bounded for all t. Moreover, based on the universal approximation theorem (Wang & Mendel, 1992), $\left\| W_{u_f} \right\|$ can be arbitrarily small. In addition, special characteristics of the feedforward fuzzy approximator are summarized below.

Next, according to the FFA (7) and the bounded fashion of $u_b(\underline{x}) - u_f(\underline{x}_d)$ as (4), the overall controller with an adaptively tuned FFA is given as follows:

$$u = \hat{u}_f(z_d, \Theta_{u_f}) + \sum_{\kappa=1}^{p} \psi_\kappa \left\| e_o \right\|^{2\kappa} s + Ks \tag{9}$$

$$\dot{\Theta}_{u_f} = \begin{cases} \gamma_0 Y_d^T s \bar{\chi}^T - \gamma_0 c_u(\Theta_{u_f}) \dfrac{\text{tr}(\bar{\chi} s^T Y_d \Theta_{u_f})}{\text{tr}(\Theta_{u_f}^T \Theta_{u_f})} \Theta_{u_f}, & \text{if } (c_u(\Theta_{u_f}) \geq 0 \text{ and} \\ & \text{tr}(\bar{\chi} s^T Y_d \Theta_{u_f}) > 0) \\ \gamma_0 E_d^T s \bar{\chi}^T, & \text{otherwise.} \end{cases} \tag{10}$$

where $\gamma_0 > 0$; $c_u(\Theta_{u_f}) = (\text{ tr } (\Theta_{u_f}^T \Theta_{u_f}) - \bar{\theta}_u + \varepsilon_u)/\varepsilon_u$ with $c_u(\Theta_{u_f}(t_0)) < 0$ and $\bar{\theta}_u > \varepsilon_u > 0$. Note that the above update law is an application of the smooth projection algorithm developed in the work (Pomet & Praly, 1992). The update law assures the following properties: (a) $\text{tr } (\Theta_{u_f}^T \Theta_{u_f}) \leq \bar{\theta}_u$ for all $t \geq t_0$ and (b) $\gamma_0 \text{ tr}(\bar{\chi} s^T Y_d \tilde{\Theta}_{u_f}) - \text{tr } (\dot{\Theta}_{u_f}^T \tilde{\Theta}_{u_f}) \leq 0$ for $\tilde{\Theta}_{u_f} = \Theta_{u_f}^* - \Theta_{u_f}$. Then, the controller (9) results in the overall error system

$$G(\underline{x})\dot{s} = -\frac{1}{2}\dot{G}(\underline{x})s - \sum_{\kappa=1}^{p} \psi_\kappa \left\| e_o \right\|^{2\kappa} s - Ks + Y_d \tilde{\Theta}_{u_f} \bar{\chi} + \Delta u + w_a(t) \tag{11}$$

$$\dot{\underline{e}} = \begin{bmatrix} 0 & I_m & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & I_m \\ -\Lambda_1 & \cdots & -\Lambda_{n-2} & -\Lambda_{n-1} \end{bmatrix} \begin{bmatrix} e \\ \vdots \\ e^{(n-3)} \\ e^{(n-2)} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I_m \end{bmatrix} s$$

$$\equiv \Lambda \underline{e} + Bs \tag{12}$$

where $\tilde{\Theta}_{u_f} = \Theta^*_{u_f} - \Theta_{u_f}$; $w_a(t) = W_{u_f} - w(t)$; $\Delta u = u_b(\underline{x}) - u_f(\underline{x}_d)$; the definition of W_{u_f} as (8) has been used; $\underline{e} = [e^T \quad \dot{e}^T \quad \cdots \quad (e^{(n-2)})^T]^T \in R^{m(n-1)}$; $\Lambda \in R^{m(n-1)\times m(n-1)}$ and $B \in R^{m(n-1)\times m}$ are defined from the above associated components. Since the error system (11) is only perturbed by the bounded approximation error $w_a(t)$, the globally uniform ultimate bound of e_o is assured straightforwardly. The detailed stability analysis will be carried out in the next subsection.

3.2 Robustness Design

To further enhance the robustness of the controlled system, three modified FFA-based adaptive controllers are developed in this subsection. First, the robust gain design is performed here. Let us consider the Lyapunov function candidate

$$V_1(t) = \frac{1}{2} s^T G(\underline{x}) s + \underline{e}^T P \underline{e} + \frac{1}{2\gamma_0} \mathrm{tr}(\tilde{\Theta}_{u_f}^T \tilde{\Theta}_{u_f}) \tag{13}$$

with a positive-definite symmetric matrix P . The time derivative of V along the error dynamics (11) and (12) is

$$\begin{aligned}
\dot{V}_1 &= -s^T K s + \underline{e}^T (\Lambda^T P + P\Lambda)\underline{e} + s^T B^T P \underline{e} + \underline{e}^T P B s + s^T \Delta u + s^T w_a \\
&\quad - \sum_{\kappa=1}^{p} \psi_\kappa \left\| e_o \right\|^{2\kappa} s^T s + \mathrm{tr}\left(s^T Y_d \tilde{\Theta}_{u_f} \bar{\chi} \right) - \frac{1}{\gamma_0} \mathrm{tr}(\dot{\Theta}_{u_f}^T \tilde{\Theta}_{u_f}) \\
&\le -s^T (K - \Psi_s) s + \underline{e}^T (\Lambda^T P + P\Lambda)\underline{e} + s^T B^T P \underline{e} + \underline{e}^T P B s \\
&\quad + e_o^T \Psi_e e_o + s^T w_a
\end{aligned}$$

where the facts $\mathrm{tr}(s^T Y_d \tilde{\Theta}_{u_f} \bar{\chi}) = \mathrm{tr}(\bar{\chi} s^T Y_d \tilde{\Theta}_{u_f})$, $\mathrm{tr}(\dot{\Theta}_{u_f}^T \tilde{\Theta}_{u_f}) \ge \gamma_0 \mathrm{tr}(\bar{\chi} s^T Y_d \tilde{\Theta}_{u_f})$ and the inequality (4) have been applied. Furthermore, if the expressions $s = [-B^T \Lambda \quad I_m] e_o$ and $\underline{e} = [I_{m(n-1)} \quad 0_{m(n-1)\times m}] e_o$ are applied, $\dot{V}$ satisfies

$$\dot{V}_1 \le e_o^T \left(\begin{bmatrix} H - \Lambda^T B K_r B^T \Lambda & PB + \Lambda^T B K_r \\ B^T P + K_r B^T \Lambda & -K_r \end{bmatrix} + \Psi_e \right) e_o + \frac{1}{\rho_1^2} \left\| w_a(t) \right\|_2^2 \tag{14}$$

where $H = \Lambda^T P + P\Lambda - \Lambda^T B B^T P - PBB^T \Lambda$ and $K_r = K - \Psi_s - \frac{\rho_1^2}{4}$. Therefore, the robust control result is summarized in the following theorem.

***Theorem 1*:** Consider the highly unknown system (1) using the TS FFA-based adaptive fuzzy controller (9) with the update law (10). If there exist symmetric positive-definite matrices P , K satisfying the following LMI problem

$$\begin{aligned}
&\textit{Given } \rho_1 > 0, \Lambda_v > 0, Q \ge 0 \\
&\textit{subject to } P, K > 0 \\
&\begin{bmatrix} H - \Lambda^T B K_r B^T \Lambda & PB + \Lambda^T B K_r \\ B^T P + K_r B^T \Lambda & -K_r \end{bmatrix} + \Psi_e + Q \le 0
\end{aligned} \tag{15}$$

then the closed-loop error system has the following properties: (i) all error signals and fuzzy parameters are bounded; (ii) the H^∞ tracking performance criterion

$$\int_{t_0}^{t_f} e_o^T(t)Qe_o(t)dt \le V_1(t_0) + \frac{1}{\rho_1^2}\int_{t_0}^{t_f} \|w_a(t)\|_2^2 dt \tag{16}$$

is assured; and (iii) if $w_a(t) \in L_2$, then e_o asymptotically converges to zero in a global manner.

***Proof*:** From the inequality (14), a feasible solution of the LMI (15) yields

$$\dot{V} \le -e_o^T Q e_o + \frac{1}{\rho^2}\|w_a(t)\|_2^2 . \tag{17}$$

Since $V_1 > 0$ and $\dot{V}_1$ is negative semidefinite outside the compact set $\{\|e_o\| \| \|e_o\| \le \frac{1}{\eta_0\rho_1}\|w_a\| < \infty\}$, for $\eta_0 = \lambda_{\min}(Q)\}$, we have $\underline{e}, s \in L_\infty$ and $\tilde{\Theta}_{u_f} \in L_\infty$. As a result, $\dot{\underline{e}}, \dot{s} \in L_\infty$ is assured from the boundedness of all terms on right-hand side of (11) and (12). In turn, $e_o, \dot{e}_o \in L_\infty$.

Moreover, by integrating the inequality (17), the H^∞ tracking performance criterion (16) is assured. In other words, the disturbance $w_a(t)$ is attenuated to a prescribed level $1/\rho_1$. Also, $e_o \in L_2$ if $w_a(t)$ is L_2 integrable. Due to the fact that $e_o, \dot{e}_o \in L_\infty$ and $e_o \in L_2$, the result $\lim_{t\to\infty} e_o(t) = 0$ is concluded by Barbalat's lemma. In addition, since the augmented disturbance $w_a(t)$ is naturally bounded, all the stability is in a global sense. ▩

Furthermore, to avoid an unexpected transient response due to poor fuzzy approximation, the attenuation of fuzzy parameter errors is taken into consideration below.

***Theorem 2*:** Consider the highly unknown system (1) using the TS FFA-based adaptive fuzzy controller

$$u = \hat{u}_f(z_d, \Theta_{u_f}) + (\sum_{\kappa=1}^{p} \psi_\kappa \|e_o\|^{2\kappa} + \frac{\rho_2^2}{4}\|\bar{\chi}\|^2 \|Y_{2d}\|)s + Ks \tag{18}$$

with $\rho_2 > 0$, $Y_{2d} = Y_d Y_d^T = \text{diag}\{\xi^T\xi, ..., \xi^T\xi\} \in R^{m\times m}$, and the update law (10). If there exist symmetric positive-definite matrices P, K satisfying the LMI problem (15), then the closed-loop error system achieves the H^∞ tracking performance criterion

$$\int_{t_0}^{t_f} e_o^T(t)Qe_o(t)dt \le V_2(t_0) + \int_{t_0}^{t_f} (\frac{1}{\rho_1^2}\|w_a(t)\|_2^2 + \frac{1}{\rho_2^2}\text{tr}(\tilde{\Theta}_{u_f}^T(t)\tilde{\Theta}_{u_f}(t)))dt \tag{19}$$

where $V_2(t_0)$ is a quadratic term dependent on the initial values of tracking errors; and $1/\rho_2 > 0$ is a prescribed attenuation level for the fuzzy parametric error $\tilde{\Theta}_{u_f}$.

***Proof*:** Consider the Lyapunov function candidate

$$V_2(t) = \frac{1}{2}s^T G(\underline{x})s + \underline{e}^T P \underline{e}$$

with symmetric positive-definite matrices $G(\underline{x})$ and P. Similar to the proof in Thm. 1, the feasibility of the LMI (15) and the control law (18) of u lead to

$$\dot{V}_2 \le -e_o^T Q e_o + \frac{1}{\rho_1^2}\|w_a\|_2^2 + \mathrm{tr}\left(s^T Y_d \tilde{\Theta}_{u_f}\bar{\chi}\right) - \frac{\rho_2^2}{4}\|\bar{\chi}\|^2\|Y_{2d}\|s^T s$$

From the property

$$\mathrm{tr}(s^T Y_d \tilde{\Theta}_{u_f}\bar{\chi}) \le \frac{\rho_2^2}{4}\mathrm{tr}(\bar{\chi}^T\bar{\chi}s^T Y_{2d}s) + \frac{1}{\rho_2^2}\mathrm{tr}(\tilde{\Theta}_{u_f}^T\tilde{\Theta}_{u_f})$$

$\dot{V}_2$ further satisfies

$$\dot{V}_2 \le -e_o^T Q e_o + \frac{1}{\rho_1^2}\|w_a\|_2^2 + \frac{1}{\rho_2^2}\mathrm{tr}(\tilde{\Theta}_{u_f}^T\tilde{\Theta}_{u_f}).$$

Integrating both sides of the above inequality, the closed-loop system guarantees the robust performance criterion (19). The gain ρ_2 is the adjustable attenuation level of fuzzy parametric errors. In addition, the boundedness of the error system is assured from the same argument in Thm. 1. ▩

From the observation on w_a, the boundedness has been assured from the bounded fuzzy approximation output (7) and error (8) in a global sense. This implies that there exists a conservative upper bound of w_a to be a constant η such that $\eta \ge \max_{i=1,\ldots,m}\{\sup_t |w_{ai}(t)|\}$ (where w_{ai} denotes the i-th element of the vector w_a). Then we are able to give an asymptotic stable result as below.

***Theorem 3*:** Consider the highly unknown system (1) using the TS FFA-based adaptive fuzzy controller

$$u = \hat{u}_f(z_d, \Theta_{u_f}) + \sum_{\kappa=1}^{p}\psi_\kappa\|e_o\|^{2\kappa}s + Ks + \eta\,\mathrm{sign}(s) \tag{20}$$

with $\mathrm{sign}(s) \equiv [\mathrm{sign}(s_1) \ \cdots \ \mathrm{sign}(s_p)]$ for s_i being the i-th element of vector s and the update law (10). If there exist symmetric positive-definite matrices P, K satisfying the following LMI problem (15) for given $\rho_1 = 0$, then the tracking error asymptotically converges to zero in a global sense.

***Proof*:** Consider the Lyapunov function candidate (13) again. Analogous to the proof of Thm. 1, the feasibility of the LMI (15) with $\rho_1 = 0$ and the control law (20) yield

$$\begin{aligned}\dot{V}_1 &\le -e_o^T Q e_o + s^T w_a - \eta s^T \mathrm{sign}(s)\\ &\le -e_o^T Q e_o + \sum_{i=1}^{m}|s_i||w_{ai}| - \eta\sum_{i=1}^{m}|s_i|\\ &\le -e_o^T Q e_o\end{aligned}$$

where the upper boundedness of w_a has been used. Due to $V_1 > 0$ and $\dot{V}_1 \le 0$, we are able to conclude the tracking error $\underline{e}$ will asymptotically converge to zero as $t \to \infty$. ▩

***Remark 1*:** The proposed feedforward fuzzy system (5) has four important characteristics — (a) the premise variables only consist of desired commands such that some fuzzy inference steps (e.g., calculation of $\Upsilon_d(z_d(t))$) can be performed off-line; (b) an assumption on the bounded approximation error is not needed; (c) due to the naturally bounded approximation error W_{u_f}, the total number of fuzzy rules can be flexibly reduced if a large approximation error is acceptable; and (d) TS-type fuzzy rules provide more flexible approximation by using fewer rules. Therefore, the feedforward fuzzy approximator allows less computation and the synthesized controller has simpler implementation along with a globally stable manner. ∎

4. Application on Holonomic Systems

4.1 Model Descriptions of Holonomic Systems

Consider a non-redundant holonomic system with a generalized coordinate $q \in R^m$ and the holonomic constraint $\phi(q) = 0$ and $A(q)\dot{q} = 0$, where $\phi : R^m \mapsto R^p$ and $A(q) = \frac{\partial \phi(q)}{\partial q}$. Without loss of generality, we assume that the system is operated away from any singularity with the exactly known function $\phi(q) \in C^2$. From investigation on well-known holonomic systems, different model descriptions exist due to the two kinds of constraints — an environmental constraint and a set of closed kinematic chains. Nevertheless, the model's general form is able to be formulated into a fully actuated system with a constraint. Referring to (Chiu et al., 2006), the general model of a holonomic system is written as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + \tau_d(t) = B_g\tau_g + A^T\lambda_g \tag{21}$$

where $M(q)$, $C(q,\dot{q})\dot{q}$, $g(q)$ are the inertia matrix, Coriolis/centripetal force, gravitational force, respectively (which are continuous and assumed to be poorly known); $\tau_d(t)$ is a bounded external disturbance; $\tau_g \in R^m$ is an applied force; $B_g(q)$ is an invertible input matrix; and $\lambda_g \in R^p$ physically presents a reaction force for an environmental constraint or an internal force for a set of closed kinematic chains.

Since the motion is subject to a p-dimensional constraint, the configuration space of the holonomic system is left with $(m-p)$ degrees of freedom. From the implicit function theorem (McClamroch & Wang, 1988), we find a partition of q as $q = [q_1^T \quad q_2^T]^T$ for $q_1 \in R^{m-p}$, $q_2 \in R^p$, such that the generalized coordinate q_2 is expressed in terms of the independent coordinate q_1 as $q_2 = \Omega(q_1)$ with a nonlinear mapping function Ω. Due to the nonsingularity assumption, the terms $\frac{\partial \Omega}{\partial q_1}$ and $\frac{\partial^2 \Omega}{\partial q_1^2}$ are bounded in the work space. The generalized displacement and velocity can be expressed in terms of the independent coordinates $q_1, \dot{q}_1$ as

$$q = [q_1^T \quad (\Omega(q_1))^T]^T \tag{22}$$

$$\dot{q} = \begin{bmatrix} I_{n-m} \\ \dfrac{\partial \Omega(q_1)}{\partial q_1} \end{bmatrix} \dot{q}_1 \equiv J\dot{q}_1. \tag{23}$$

From above equations, the constraint of velocity $A(q)\dot{q} = 0$ leads to $A(q_1)J(q_1)\dot{q}_1 = 0$. Notice that here we use $A(q_1)$ to denote $A(q_1, \Omega(q_1))$ for brevity. In other words, $A(q_1)J(q_1) = 0$ since $A(q_1)J(q_1)$ is full column-rank and $\dot{q}_1$ is an independent coordinate (see (McClamroch & Wang, 1988)). Thus, there exists a reduced dynamics for the holonomic system (21). Due to the velocity transformation (23), the generalized acceleration satisfies $\ddot{q} = J\ddot{q}_1 + \dot{J}\dot{q}_1$. The motion equation (21) is further represented by the independent coordinates $q_1, \dot{q}_1, \ddot{q}_1$ as

$$M(q_1)J\ddot{q}_1 + \overline{C}(q_1, \dot{q}_1)\dot{q}_1 + g(q_1) + \tau_d(t) = B_g(q_1)\tau_g + A^T\lambda_g \tag{24}$$

where $\overline{C} = M\dot{J} + CJ$. According to the fact $A(q_1)J(q_1) = 0$, a reduced dynamics (McClamroch & Wang, 1988) is obtained after multiplying J^T on both sides of (24):

$$\underline{M}(q_1)\ddot{q}_1 + \underline{C}(q_1, \dot{q}_1)\dot{q}_1 + \underline{g}(q_1) + \underline{\tau}_d(q_1, t) = J^T B_g \tau_g \tag{25}$$

with $\underline{M} = J^T M J$; $\underline{C} = J^T \overline{C}$; $\underline{g} = J^T g$; and $\underline{\tau}_d = J^T \tau_d$. From the dynamics (25), some useful properties are addressed below.

Property 1: For the partition $I_m = [E_1 | E_2]$ with $E_1 = [I_{m-p} \quad 0_{(m-p)\times p}]^T \in R^{m\times(m-p)}$ and $E_2 = [0_{p\times(m-p)} \; I_p]^T \in R^{m\times p}$, the velocity transformation matrix J satisfies $J^T E_1 = I_{m-p}$.

Property 2: From the existence of $\Omega(\cdot)$ and the implicit function theorem, A_2 is invertible.

Property 3: The matrix $\underline{M}$ is symmetric and positive-definite while $\underline{M}^{-1} \in L_\infty$.

Property 4: Matrix $(\underline{M} - 2\underline{C})$ is skew-symmetric (cf. McClamroch & Wang, 1988), i.e., $\zeta^T(\underline{M} - 2\underline{C})\zeta = 0$, $\forall \zeta \in R^{m-p}$.

4.2 FFA-Based Adaptive Motion/Force Control

For holonomic systems, the control objective is to track a desired motion trajectory $q_{1d}(t) \in C^2$ while maintaining force λ_g at a desired $\lambda_{gd}(t)$. Inspired by pure motion tracking, some notations are defined as

$$e_m = q_{1d} - q_1, \; e_m \in R^{m-p};$$

$$q_a = \Lambda_m e_m + \dot{q}_{1d}, \; q_a \in R^{m-p};$$

$$s = q_a - \dot{q}_1, \; s \in R^{m-p}; \tag{26}$$

where e_m, q_a, s are the motion error, auxiliary signal vector, error signal, respectively; and $\Lambda_m \in R^{(m-p)\times(m-p)}$ is a symmetric positive-definite matrix. If the system satisfies $\lim_{t\to\infty} s(t) = 0$, then position and velocity tracking errors $e_m, \dot{e}_m$ exponentially converge to zero. In other

words, the motion tracking problem is transformed to the problem of stabilizing $s(t)$. On the other hand, a force tracking error and force error filter are accordingly defined as

$$\tilde{\lambda} = \lambda_{gd} - \lambda_g \in R^p \tag{27}$$

$$\dot{e}_\lambda + \eta_1 e_\lambda = \eta_2 \tilde{\lambda}, \text{ with } \eta_1, \eta_2 > 0 . \tag{28}$$

Then, the reduced-state based scheme is to drive the motion trajectory into the stable subspace while the contact force is separately controlled maintaining a zero e_λ.

In order to derive the adaptive fuzzy controller, the error dynamics of s along the motion equation (24) is written as

$$\begin{aligned} MJ\dot{s} &= MJ\dot{q}_a - MJ\ddot{q}_1 \\ &= -\overline{C}s + f + \tau_d - A^T\lambda_g - B_g\tau_g \end{aligned} \tag{29}$$

where $f = M(q_1)J(q_1)\dot{q}_a + \overline{C}(q_1,\dot{q}_1)q_a + g(q_1) \in R^m$. By traditional SFA-based control, we usually require to take q_1, $\dot{q}_1$, q_{1d}, $\dot{q}_{1d}$, $\ddot{q}_{1d}$ as the premise variables, such that a large computational load exists on the controller processor. To avoid this situation, the FFA-based control method is used to provide the feed-forward compensation term $f_d(q_{1d},\dot{q}_{1d},\ddot{q}_{1d}) = M(q_{1d})J(q_{1d})\ddot{q}_{1d} + \overline{C}(q_{1d},\dot{q}_{1d})\dot{q}_{1d} + g(q_{1d})$. Since f_d is independent to state variables, $f_d(\cdot)$ is a much simpler function than $f(\cdot)$. If the effect of omitting the error $f - f_d$ can be coped with by feedback of tracking error, the concept of using the forward compensation f_d is feasible. According to the FFA-based control in the above section, we closely approximate and compensate the forward term $f_d(\cdot)$ by a TS fuzzy system with the singleton fuzzifier and product inference. Then the fuzzy inferred output is

$$\hat{f}_d(z_d(t),\Theta_{f_d}) = Y_d(z_d(t))\Theta_{f_d}\overline{\chi} \tag{30}$$

where $z_d(t)$, $Y_d(z_d(t))$, and $\overline{\chi}$ have the same definition as (7) being functional of $q_{1d}(t),\dot{q}_{1d}(t),\ddot{q}_{1d}(t)$; and $\Theta_{f_d} \in R^{mr\times(g+1)}$ is a fuzzy tuning parametric vector in the consequent part of rules, with r denoting the total number of rules. For the FFA (30), there exists an optimal approximation parameter

$$\Theta^*_{f_d} \equiv \text{argmin}_{\Theta_{f_d}\in\Omega_{\theta fd}}\left\{\sup_{z_d\in U_z}\left|f_d - \hat{f}_d(z_d(t),\Theta_{f_d})\right|\right\}$$

in an appropriate parametric constraint region $\Omega_{\theta_{fd}}$, which provides the most accurate approximation with the minimum error:

$$W_{f_d} = f_d - Y_d(z_d(t))\Theta^*_{f_d}\overline{\chi} . \tag{31}$$

From the observation on the right-hand side of the above equation, the fuzzy approximation error W_{f_d} is upper bounded for $t \geq 0$ from $f_d \in L_\infty$ and $\hat{f}_d \in L_\infty$.

Next, the overall controller is synthesized in the following. Based on the TS FFA-based fuzzy system (30), the overall control law is set in the form:

$$\tau_g = B_g^{-1}[Y_d\Theta_{f_d}\bar{\chi} + E_1(Ks+\tau_a) - A^T(\lambda_{gd} + k_f e_\lambda)] \tag{32}$$

where $k_f > 0$ is a force feedback gain; $K \in R^{(m-p)\times(m-p)}$ is a symmetric positive definite matrix; τ_a is an auxiliary input designed later; the definition of s and e_λ is given in (26) and (28), respectively. Meanwhile, the fuzzy parameter Θ_{f_d} is adaptively adjusted by

$$\dot{\Theta}_{f_d} = \begin{cases} \gamma Y_d^T J s \bar{\chi}^T - c(\Theta_{f_d}) \dfrac{\mathrm{tr}(\bar{\chi} s^T J^T Y_d \Theta_{f_d})}{\mathrm{tr}(\Theta_{f_d}^T \Theta_{f_d})} \Theta_{f_d}, & \text{if } (c(\Theta_{f_d}) \geq 0 \text{ and} \\ & \mathrm{tr}(\bar{\chi} s^T J^T Y_d \Theta_{u_f}) > 0) \\ \gamma Y_d^T J s \bar{\chi}^T, & \text{otherwise} \end{cases} \tag{33}$$

with $c(\Theta_{f_d}(t_0)) < 0$, where $c(\Theta_{f_d}) = (\mathrm{tr}(\Theta_{f_d}^T \Theta_{f_d}) - \bar{\theta}_{f_d} + \varepsilon_f)/\varepsilon_f$ is a projection criterion function with a tunable parameter ε_f satisfying $\bar{\theta}_{f_d} > \varepsilon_f > 0$; and $\gamma > 0$ is an adaptation gain.

Furthermore, substituting the control law (32) into the dynamic equation (29) renders the closed-loop error dynamics:

$$MJ\dot{s} = -\bar{C}s - E_1(Ks+\tau_a) + Y_d\tilde{\Theta}_{f_d}\bar{\chi} + \Delta f + w + A^T(\tilde{\lambda} + k_f e_\lambda) \tag{34}$$

where $\Delta f \equiv f - f_d$; $w \equiv W_{f_d} + \tau_d$; and the definition of approximation error W_{f_d} in (31) and $\tilde{\lambda}$ in (27) have been applied. To analyze the convergence of motion and force tracking separately, we further multiply J^T on both sides of (34), which leads to the motion tracking error dynamics:

$$\underline{M}\dot{s} = -\underline{C}s - Ks + J^T Y_d \tilde{\Theta}_{f_d}\bar{\chi} + J^T\Delta f + \underline{w} + \tau_a, \tag{35}$$

where Property 1 ($J^T E_1 = I_{n-m}$) and the fact, $J^T(q_1)A^T(q_1) = 0$, have been applied; and $\underline{w} \equiv J^T w$. Then, replacing $\dot{s}$ of (34) by (35) and multiplying $A_2^{-T}E_2^T$ on both sides of (34), we obtain the force tracking error as follows:

$$\begin{aligned} \tilde{\lambda} + k_f e_\lambda &= A_2^{-T}E_2^T\Big(MJ\underline{M}^{-1}(-\underline{C}s - Ks + J^T Y_d\tilde{\Theta}_{f_d}\bar{\chi} \\ &\quad + J^T\Delta f + \underline{w} + \tau_a) + \bar{C}s - \Delta f - Y_d\tilde{\Theta}_{f_d}\bar{\chi} - w\Big) \\ &\equiv \varpi(e_m, s, \tilde{\Theta}_{f_d}, w, t) \end{aligned} \tag{36}$$

where Property 2 ($A_2^{-T} \in L_\infty$) and the fact, $E_2^T E_1 = 0$, have been applied above. It is a worthwhile note that the perturbed term Δf in (35) arises from the use of the feed-forward fuzzy compensation. Nevertheless, the term Δf is upper bounded by motion tracking errors in the following fashion:

$$s^T J^T\Delta f \leq s^T(\Psi_s + \frac{1}{2\kappa^2}I_{n-m})s + \|\Lambda_m e_m\|^2 s^T\Psi_{se}s + e_m^T(\Psi_e + \frac{\kappa^2}{2}\Psi_J)e_m \tag{37}$$

where there exist an intermediate parameter $\kappa > 0$ and symmetric positive semidefinite matrices $\Psi_s, \Psi_{se}, \Psi_e, \Psi_J$ dependent on the desired motion trajectory, control parameter Λ_m, and system parameters. This boundedness is assured for all well-known holonomic mechanical systems (cf. Appendix of (Chiu et al., 2006)).

Now, the main results of the FFA-based adaptive control of holonomic systems are stated as follows.

Theorem 4: Consider the holonomic system (21) using the TS FFA-based adaptive controller (32) tuned by the update law (33). If the auxiliary input is set as

$$\tau_a = 2P_m e_m + \left\|\Lambda_m e_m\right\|^2 \Psi_{se} s + \frac{\rho^2}{4}\bar{\chi}^T\bar{\chi} J^T Y_d Y_d^T J s \tag{38}$$

and there exist κ, K, P_m satisfying the following LMI problem

$$\begin{aligned} &\textit{Given } \rho, \Lambda_m > 0 \text{ and } Q = \begin{bmatrix} Q_{11} & Q_{21}^T \\ Q_{21} & Q_{22} \end{bmatrix} > 0 \\ &\text{subject to } \kappa, K, P_m > 0 \\ &\begin{bmatrix} K_p - Q_{11} & \Lambda_m^T K_a^T - Q_{21}^T & \kappa\Psi_J^{1/2} \\ K_a\Lambda_m - Q_{21} & K_a - Q_{22} & 0 \\ \kappa\Psi_J^{1/2} & 0 & 2I_{n-m} \end{bmatrix} \geq 0 \end{aligned} \tag{39}$$

with $K_a = K - (\frac{\rho^2}{4} + \frac{1}{2\kappa^2})I_{n-m} - \Psi_s$ and $K_p = \Lambda_m^T K_a \Lambda_m + \Lambda_m^T P_m + P_m \Lambda_m - \Psi_e$, then (a) error signals e_m, $\dot{e}_m$, e_λ, $\tilde{\lambda}$ and fuzzy parameter Θ_{f_d} are bounded; (b) error vectors e_m, s, $\tilde{\lambda}$ have globally uniform ultimate bounds being proportional to the inversion of control gains; and (c) the closed-loop system is guaranteed with the robust motion tracking performance

$$\int_{t_0}^{t_f} e_a^T Q e_a dt \leq V_s(t_0) + \frac{1}{\rho^2}\int_{t_0}^{t_f} (\left\|\underline{w}(t)\right\|^2 + \mathrm{tr}(\tilde{\Theta}_{f_d}^T(t)\tilde{\Theta}_{f_d}(t)))dt \tag{40}$$

for $e_a = [e_m^T \quad \dot{e}_m^T]^T$ and a nonnegative constant $V_s(t_0)$.

Proof: First, we prove the claim (a). Consider the Lyapunov function candidate

$$V = \frac{1}{2}s^T \underline{M} s + e_m^T P_m e_m + \frac{1}{2\gamma}\mathrm{tr}\left(\tilde{\Theta}_{f_d}^T \tilde{\Theta}_{f_d}\right)$$

with a proper symmetric positive-definite matrix P_m. Along the error dynamics (35) and the fact $\dot{e}_m = -\Lambda_m e_m + s$, the time derivative of V is written as follows:

$$\begin{aligned} \dot{V} &= \frac{1}{2}s^T(\dot{\underline{M}} - 2\underline{C})s - s^T K s - e_m^T(\Lambda_m^T P_m + P_m\Lambda_m)e_m - \left\|\Lambda_m e_m\right\|^2 s^T\Psi_{se}s - \frac{\rho^2}{4}\bar{\chi}^T\bar{\chi}s^T J^T Y_d Y_d^T J s \\ &\quad + s^T J^T Y_d \tilde{\Theta}_{f_d}\bar{\chi} - \frac{1}{\gamma}\mathrm{tr}\left(\dot{\Theta}_{f_d}^T\tilde{\Theta}_{f_d}\right) + s^T J^T \Delta f + s^T \underline{w} \\ &\leq -s^T K s - e_m^T(\Lambda_m^T P_m + P_m\Lambda_m)e_m - \left\|\Lambda_m e_m\right\|^2 s^T\Psi_{se}s + s^T J^T\Delta f + s^T\underline{w} \end{aligned}$$

where the definition of τ_a, Property 4, and the update law (33) have been applied; and the above inequality is ensured by the property of the update law (i.e., $s^T J^T \Upsilon_d \tilde{\Theta}_{f_d} \bar{\chi} - \frac{1}{\gamma} \mathrm{tr}(\dot{\Theta}_{f_d}^T \tilde{\Theta}_{f_d}) \le 0$). Due to the boundedness of Δf as the fashion (37), we further obtain

$$\dot{V} \le -s^T K_a s - e_m^T \Upsilon e_m + \frac{1}{\rho^2} \|\underline{w}\|^2 \tag{41}$$

where $K_a = K - (\frac{\rho^2}{4} + \frac{1}{2\kappa^2}) I_{n-m} - \Psi_s$; $\Upsilon = \Lambda_m^T P_m + P_m \Lambda_m - \Psi_e - \frac{\kappa^2}{2} \Psi_J$; and $\rho > 0$. Then, applying the expressions $s = [\Lambda_m \quad I_{n-m}] e_a$ and $e_m = [I_{n-m} \quad 0_{n-m}] e_a$, the inequality (41) is rewritten as

$$\dot{V} \le -e_a^T \left(\begin{bmatrix} \Lambda_m^T K_a \Lambda_m + \Upsilon & \Lambda_m^T K_a \\ K_a \Lambda_m & K_a \end{bmatrix} - Q \right) e_a$$

$$-e_a^T Q e_a + \frac{1}{\rho^2} \|\underline{w}\|^2$$

Thus, if the LMI (39) has a feasible solution, then the following $\dot{V}$ holds

$$\dot{V} \le -\alpha_0 \|e_a\|^2 + \frac{1}{\rho^2} \|\underline{w}\|^2 \tag{42}$$

with $\alpha_0 = \lambda_{\min}(Q)$. Since V is positive-definite and $\dot{V}$ satisfies the inequality (42), we can conclude that $s, e_m, \dot{e}_m \in L_\infty$ and $\tilde{\Theta}_{f_d}, \Theta_{f_d} \in L_\infty$. As a result, $\dot{s}, \ddot{e}_m \in L_\infty$ is assured based on the boundedness of all terms on right-hand side of (35). On the other hand, taking the force filter (28) into Eq. (36) yields that the force tracking error is expressed in the form:

$$\tilde{\lambda} = \left(1 - \frac{k_f \eta_2}{D + \eta_1 + k_f \eta_2} \right) \varpi(e_m, s, \tilde{\Theta}_{f_d}, w, t) \tag{43}$$

where D is a differential operator. Since $\frac{k_f \eta_2}{D + \eta_1 + k_f \eta_2}$ is a stable filter and all signals $e_m, s, \tilde{\Theta}_{f_d}, w$ are bounded, the bounded $\varpi(\cdot)$ implies the boundedness of $\tilde{\lambda}$ and e_λ . Note that since the boundedness assumption on the fuzzy approximation error W_{f_d} is not utilized here, this proof is achieved in a global sense.

Second, consider the claim (b). Since $\dot{V}$ is negative semidefinite outside the compact set $\{\|e_a\| \mid \|e_a\| < \frac{1}{\alpha_0 \rho} \|\underline{w}\|_\infty\}$ from the inequality (42), the tracking error $e_a(t)$ is globally uniformly ultimately bounded with convergence to a compact residual set. To find the uniformly ultimate bound, we rewrite (42) as

$$\dot{V} \le -\frac{\alpha_0}{\alpha_1} V + \frac{\alpha_0}{\alpha_1} \zeta(t)$$

where $\zeta = \frac{1}{2\gamma} \mathrm{tr}(\tilde{\Theta}_{f_d}^T \tilde{\Theta}_{f_d}) + \frac{\alpha_1}{\alpha_0 \rho^2} \|\underline{w}\|^2$ and $\alpha_1 = \sup_t \lambda_{\max}(M_a) > 0$ with $M_a = \frac{1}{2} [\Lambda_m \quad I_{n-m}]^T \underline{M} [\Lambda_m \quad I_{n-m}] + [I_{n-m} \quad 0_{n-m}]^T P_m [I_{n-m} \quad 0_{n-m}]$. Then, the solution of the above inequality leads to that the error trajectory of $e_a(t)$ is shaped by

$$\|e_a\| \le \sqrt{\tfrac{1}{\alpha_2} V(t_0)}\exp(-\tfrac{\alpha_0}{2\alpha_1}(t-t_0)) + \sqrt{\tfrac{1}{\alpha_2}[1-\exp(-\tfrac{\alpha_0}{\alpha_1}(t-t_0))]\sup_t \zeta(t)}$$

with $\alpha_2 = \inf_t \lambda_{\min}(M_a) > 0$. In other words, the uniform ultimate bound of $e_a(t)$ is

$$\|e_a\| \le \sqrt{\frac{1}{\alpha_2}\sup_t \zeta(t)} = \bar{\zeta}(\frac{1}{\gamma}\|\tilde{\Theta}_{f_d}\|_\infty, \frac{1}{\rho}\|\underline{w}\|_\infty)$$

which can be adjusted by tuning γ and ρ . Meanwhile, the residual force tracking error is adjusted by tuning η_1, η_2, k_f according to

$$\lim_{t\to\infty}\|\tilde{\lambda}(t)\| \cong \frac{\eta_1}{\eta_1 + k_f\eta_2}\bar{\varpi}(\bar{\zeta}, \|\tilde{\Theta}_{f_d}\|_\infty, \|w\|_\infty) \tag{44}$$

with a nonnegative constant $\bar{\varpi} = \sup_t \varpi(t)$ dependent on $\bar{\zeta}, \|\tilde{\Theta}_{f_d}(t)\|_\infty$, and $\|w(t)\|_\infty$.

Components \ Examples	Cooperative three-link robots		
	Mamdani SFA	Mamdani FFA	TS FFA
Approximated term	$f(\cdot)$	$f_d(\cdot)$	$f_d(\cdot)$
Premise Variables	q_1, $\dot{q}_1$ q_{1d}, $\dot{q}_{1d}$, $\ddot{q}_{1d}$	q_{1d}, $\dot{q}_{1d}$, $\ddot{q}_{1d}$	q_{1d}
Number of Premise Variables	15 (5×3)	9 (3×3)	3
Number of Rules $^{\Delta}$	32768 (2^{15})	512 (2^9)	8 (2^3)
Number of fuzzy Parameters	294912 (32768×9)	4608 (512×9)	720 ($8\times 9\times 10$)
Approximation errors	Assumedly Bounded *a priori*	Always Bounded	Always Bounded

$^{\Delta}$: each premise variable has two fuzzy sets.

Table 1. Comparisons between SFA and FFA Based Schemes

Third, we prove the claim (c). Consider an energy function $V_s = \frac{1}{2}s^T\underline{M}s + e_m^T P_m e_m$. Analogous to the proof of Theorem 2, a feasible solution of the LMI (39) leads to

$$\dot{V}_s \le -e_a^T Q e_a + s^T J^T Y_d \tilde{\Theta}_{f_d}\bar{\chi} - \frac{\rho^2}{4}\bar{\chi}^T\bar{\chi}s^T J^T Y_d Y_d^T Js + \frac{1}{\rho^2}\|\underline{w}\|^2$$

$$\le -e_a^T Q e_a + \frac{1}{\rho^2}(\|\underline{w}\|^2 + \mathrm{tr}(\tilde{\Theta}_{f_d}^T\tilde{\Theta}_{f_d}))$$

where the fact that $s^T J^T Y_d\tilde{\Theta}_{f_d}\bar{\chi} \le \frac{\rho^2}{4}\bar{\chi}^T\bar{\chi}s^T J^T Y_d Y_d^T Js + \frac{1}{\rho^2}\mathrm{tr}(\tilde{\Theta}_{f_d}^T\tilde{\Theta}_{f_d})$ has been applied. Therefore, integrating both sides of the above inequality, the robust performance (40) for the motion tracking objective is assured. ▩

***Remark 2*:** The comparison between SFA and FFA based controllers applied to typical holonomic systems is made in Table 1. From the work (Chang & Chen, 2000), the SFA-based

controller requires to take $q_1, \dot{q}_1, q_{1d}, \dot{q}_{1d}, \ddot{q}_{1d}$ as the premise variables. In contrast, the TS FFA-based controller only needs commands q_{1d} as the premise variable. The benefits of using the FFA-based controller (fewer rules and tuned parameters) are apparent. Moreover, the fuzzy approximation error of SFA-based controllers needs to be assumedly bounded *a priori*.

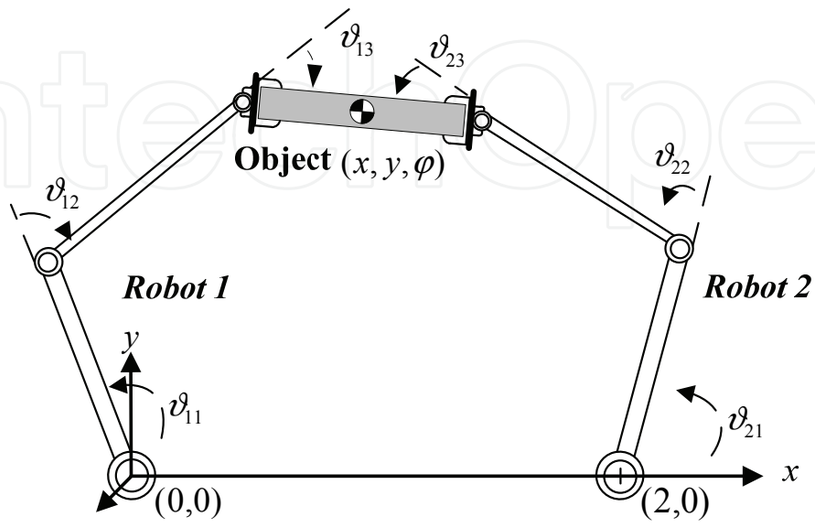

Figure 3. A two-link planer constrained robot manipulator

5. Simulation Example

To verify the theoretical derivations, we take a cooperative two-robot system transporting an object as an application example. This holonomic system is subject to a set of closed kinematic chains as illustration in Fig. 3. Two robots are identical in mass and length of links. The center of mass for each link is assumed at the end of each link. All the length of the first and second links l_1 , l_2 , and the held object are 1 M. The length of the third link is sufficiently short and is taken as a part of the object. Let (x , y , φ) denote the position and orientation of the held object. Let $\vartheta_{1\ell}$, $\vartheta_{2\ell}$ ($\ell = 1,2,3$) denote joint angles of two robots, respectively. The configuration coordinate of the system is thus denoted as $q_1 = [x \quad y \quad \varphi]^T$ and $q_2 = [\vartheta_{11} \quad \vartheta_{12} \quad \vartheta_{13} \quad \vartheta_{21} \quad \vartheta_{22} \quad \vartheta_{23}]^T$. Due to the fact that all the end-effectors are rigidly attached to the common object, the holonomic constraint $\phi(q) = [\phi_1^T(q) \quad \phi_2^T(q)]^T \in R^6$ consists of

$$\phi_1(q) = \begin{bmatrix} x - 0.5\cos\varphi \\ y - 0.5\sin\varphi \\ \varphi \end{bmatrix} - \psi_1 = 0$$

$$\phi_2(q) = \begin{bmatrix} x + 0.5\cos\varphi - 2 \\ y + 0.5\sin\varphi \\ \varphi + \pi \end{bmatrix} - \psi_2 = 0$$

$$\psi_j = \begin{bmatrix} \cos(\vartheta_{j1}) + \cos(\vartheta_{j1} + \vartheta_{j2}) \\ \sin(\vartheta_{j1}) + \sin(\vartheta_{j1} + \vartheta_{j2}) \\ \vartheta_{j1} + \vartheta_{j2} + \vartheta_{j3} \end{bmatrix}, \text{for } j = 1,2 .$$

Therefore, the Jacobian matrix $A(q)$ is consists of A_1^T = block-diag $\{A_{11}^T, A_{12}^T\}$ and A_2 = block-diag $\{A_{21}, A_{22}\}$ with:

$$A_{11}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.5\sin\varphi & -0.5\cos\varphi & 1 \end{bmatrix}$$

$$A_{12}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -0.5\sin\varphi & 0.5\cos\varphi & 1 \end{bmatrix}$$

$$A_{2j} = -\begin{bmatrix} -\sin(\vartheta_{j1}) - \sin(\vartheta_{j12}) & -\sin(\vartheta_{j12}) & 0 \\ \cos(\vartheta_{j1}) + \cos(\vartheta_{j12}) & \cos(\vartheta_{j12}) & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

where $\vartheta_{j12} = \vartheta_{j1} + \vartheta_{j2}$. The kinematic transformation matrix is written as $J = [I_3 \quad -(A_2^{-1}A_1)^T]^T$. In addition, the general dynamic model (21) is composed of M = block-diag $\{M_0, M_1, M_2\}$, C = block-diag$\{C_0, C_1, C_2\}$, $g = [g_0 \quad g_1 \quad g_2]^T$, $M_0 = \text{diag}\{m_o, m_o, I_o\}$, $C_0 = 0$, $g_0 = [0 \quad m_o g \quad 0]^T$,

$$M_j = \begin{bmatrix} a_{j1} + 2a_{j2}\cos(\vartheta_{j2}) + a_{j3} & (*) & (*) \\ a_{j2}\cos(\vartheta_{j2}) + a_{j3} & a_{j3} & (*) \\ a_{j4} & a_{j4} & a_{j4} \end{bmatrix}$$

$$C_j = \begin{bmatrix} -a_{j2}\sin(\vartheta_{j2})\dot{\vartheta}_{j2} & -a_{j2}\sin(\vartheta_{j2})\dot{\vartheta}_{j12} & 0 \\ a_{j2}\sin(\vartheta_{j2})\dot{\vartheta}_{j1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$g_j = \begin{bmatrix} (a_{j1}\cos(\vartheta_{j1}) + a_{j2}\cos(\vartheta_{j1} + \vartheta_{j2}))g/l_1 \\ a_{j2}\cos(\vartheta_{j1}+\vartheta_{j2})g/l_1 \\ 0 \end{bmatrix}$$

for $j = 1,2$, where (*) represents a symmetric term; $a_{j1} = (m_{j1} + m_{j2} + m_{j3})l_1^2$; $a_{j2} = (m_{j2} + m_{j3})l_1 l_2$; $a_{j3} = (m_{j2} + m_{j3})l_2^2 + I_{j3}$; $a_{j4} = I_{j3}$; and m_{j1}, m_{j2}, m_{j3}, I_{j3}, m_o, I_o are system parameters. The actual value of $(m_o, I_o, a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24})$ is set as (1, 0.25, 5, 3, 3.05, 0.05, 5, 3, 3.05, 0.05). According to the holonomic constraint $\phi(q) = 0$, we

can find $B_g = I_9$, $\tau_g = [(A_1^T \lambda_M)^T \quad (\tau + A_2^T \lambda_M)^T]^T$, and $\lambda_g = \lambda_I$, where $\tau = [\tau_1^T \quad \tau_2^T]^T \in R^6$ is the applied force for the two robots; λ_M denotes a motion-inducing force which has contribution to the motion of the object by $A_1^T \lambda_M$; and λ_I denotes an internal force which lies in a nontrivial null space $Z = \{\lambda_I \in R^m \mid A_1^T \lambda_I = 0\}$. Therefore, if the control input τ_g is designed according to Thm. 4, then the actual control input is calculated by

$$\tau = \tau_{g2} - A_2^T (A_1^T)^+ \tau_{g1}$$

where $\tau_{g1} \in R^3$, $\tau_{g2} \in R^6$ are partitioned components of τ_g (i.e. $\tau_g = [\tau_{g1}^T \quad \tau_{g2}^T]^T$); and $(A_1^T)^+ = A_1 (A_1^T A_1)^{-1}$ denotes the pseudo-inverse of A_1^T.

For this cooperative two-robot system, the control objective is to track desired trajectories for the object and internal force as

$$q_{1d}(t) = \begin{bmatrix} 1 + 0.25\cos(t) \\ 1 + 0.25\sin(t) \\ 0.25 \end{bmatrix},$$

$$\lambda_{gd1} = 40 \begin{bmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{bmatrix}, \lambda_{gd2} = 40 \begin{bmatrix} -\cos\varphi \\ -\sin\varphi \\ 0 \end{bmatrix}$$

where λ_{gd1} and λ_{gd2} represent the compressed force vector.

On the other hand, since the TS FFA has a general representation capability, we are able to properly choose the basis function such that fewer premise variables are used. According to the function $f_d(\cdot)$, the feed-forward TS FFA-based fuzzy system (30) is constructed with $\bar{\chi} = [1 \quad q_{1d1} \quad q_{1d2} \quad q_{1d3} \quad \dot{q}_{1d1}^2 \quad \dot{q}_{1d2}^2 \quad \dot{q}_{1d1}\dot{q}_{1d2} \quad \ddot{q}_{1d1} \quad \ddot{q}_{1d2} \quad \ddot{q}_{1d3}]^T \in R^{10}$ (where $q_{1d\ell}$ is the ℓ-th element of q_{1d}, for $\ell = 1,2,3$) and linguistic variables $q_{1d\ell}$, which accordingly are classified into two fuzzy sets. From the exactly known mean and varying region, the fuzzy sets are easily characterized by the following membership functions:

$$\begin{cases} \mu_{\mathbf{x}_{n\ell}}(q_{1d\ell}) = 1 - \mu_{\mathbf{x}_{s\ell}}(q_{1d\ell}) \\ \mu_{\mathbf{x}_{s\ell}}(q_{1d\ell}) = \exp(-2(q_{1d\ell} - 1)^2), \text{ for } \ell = 1,2 \end{cases}$$

$$\begin{cases} \mu_{\mathbf{x}_{n3}}(q_{1d3}) = 1 - \mu_{\mathbf{x}_{s3}}(q_{1d3}) \\ \mu_{\mathbf{x}_{s3}}(q_{1d3}) = \exp(-2(q_{1d3} + 0.25)^2), \text{ for } \ell = 3 \end{cases}$$

This results in the total number of fuzzy rules to be 8, i.e., $\Theta_{f_d} \in R^{72\times 10}$. When considering the special case with $\bar{\chi} = 1$ (i.e., Mamdani FFA), all of $q_{1d\ell}$, $\dot{q}_{1d\ell}$ and $\ddot{q}_{1d\ell}$ should be utilized as linguistic variables for an admissible approximation, which needs 512 fuzzy rules and 4608 tuning parameters. This implies that the proposed approach in this paper leads to less

numbers of fuzzy rules and tuning parameters. Furthermore, the fuzzy consequent parts are adjusted by (33), where $\gamma = 50$, $\bar{\theta}_{f_d} = 10^4$, $\varepsilon_f = 10$, and $\Theta_{f_d}(0) = [0]_{72\times 10}$.

Furthermore, the control parameters are chosen as: $\eta_1 = 0.1$, $\eta_2 = 20$, $k_f = 15$, $\Lambda_m =$ diag{10, 5, 5}, $\rho = 5$, and $Q = I_6$. Then, after choosing $\Psi_s =$ diag{40, 20, 10}, $\Psi_{se} = \Psi_e = 2I_3$, $\Psi_J = I_3$ for (37), the control gains are obtained as $\kappa = 7.9$, $K =$ diag{53.5, 45.2, 35.2}, and $P_m =$ diag{36.5, 51.7, 51.7}, by solving the LMI (39). In this simulation, the system begins at the position $q(0) = [1 \quad \frac{2-\sqrt{3}}{2} \quad 0 \quad \frac{\pi}{2} \quad -\frac{5\pi}{6} \quad \frac{\pi}{3} \quad \frac{\pi}{2} \quad \frac{5\pi}{6} \quad -\frac{\pi}{3}]^T$ and all have zero initial velocities, i.e., $\dot{q}(0) = 0$. The external disturbance is injected to the first joint of two robots as τ_d which is a square wave with amplitude 0.25 and frequency 0.5 Hz. According to Thm. 4, the simulation results of position and velocity tracking for the object are illustrated in Figs. 4 and 5, respectively. The internal force errors between the desired and actual internal force are shown in Fig. 6. The second joints of two robots are driven by torques illustrated in Fig. 7. For a comparison, a traditional SFA-based controller is also constructed and applied to the cooperative robots, where the Mamdani SFA takes $q_{1\ell}$, $\dot{q}_{1\ell}$, $q_{1d\ell}$, $\dot{q}_{1d\ell}$ and $\ddot{q}_{1d\ell}$ as the premise variables. Furthermore, the SFA-based control is set with the same initial conditions and feedback compensation part as the proposed controller but $\Psi_s = 0$, $\Psi_e = 0$, $\Psi_{se} = 0$, and $\Psi_J = 0$. Then, the position tracking results for using Mamdani SFA and TS FFA based control are made as a comparison given in Fig. 8. Obviously, the TS FFA-based controller leads to a smaller tracking error.

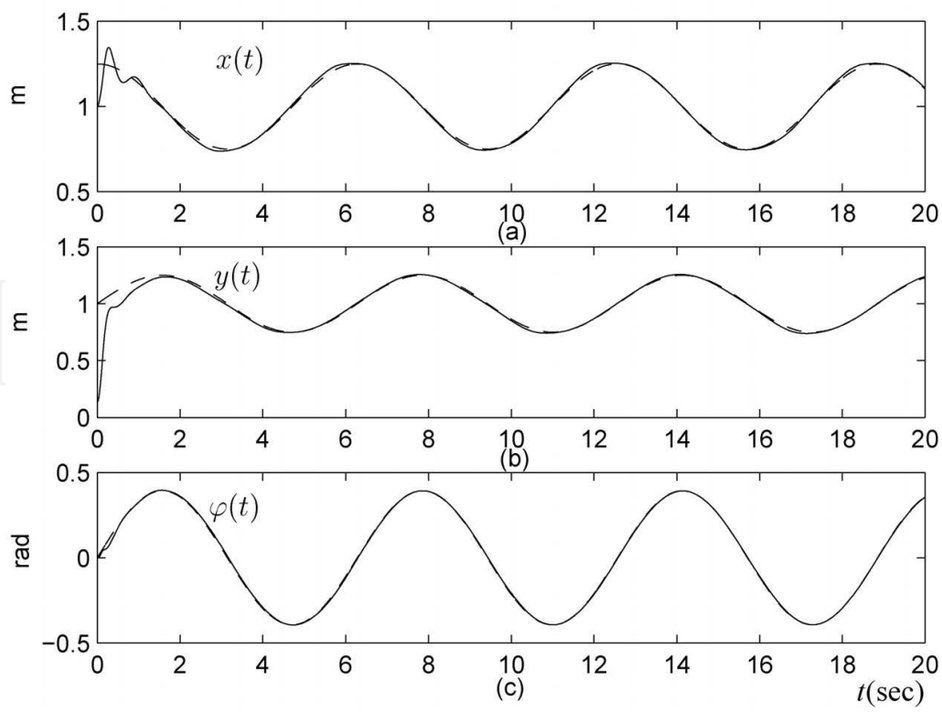

Figure 4. The position tracking results of the held object. (— object, - - reference)

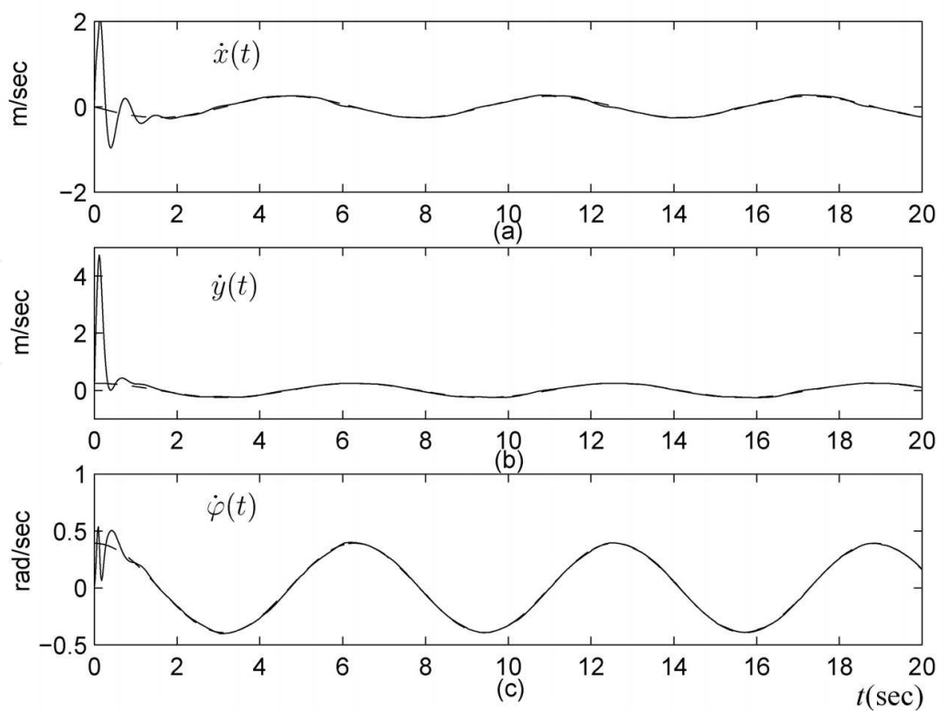

Figure 5. The velocity tracking results of the held object. (— object, - - reference)

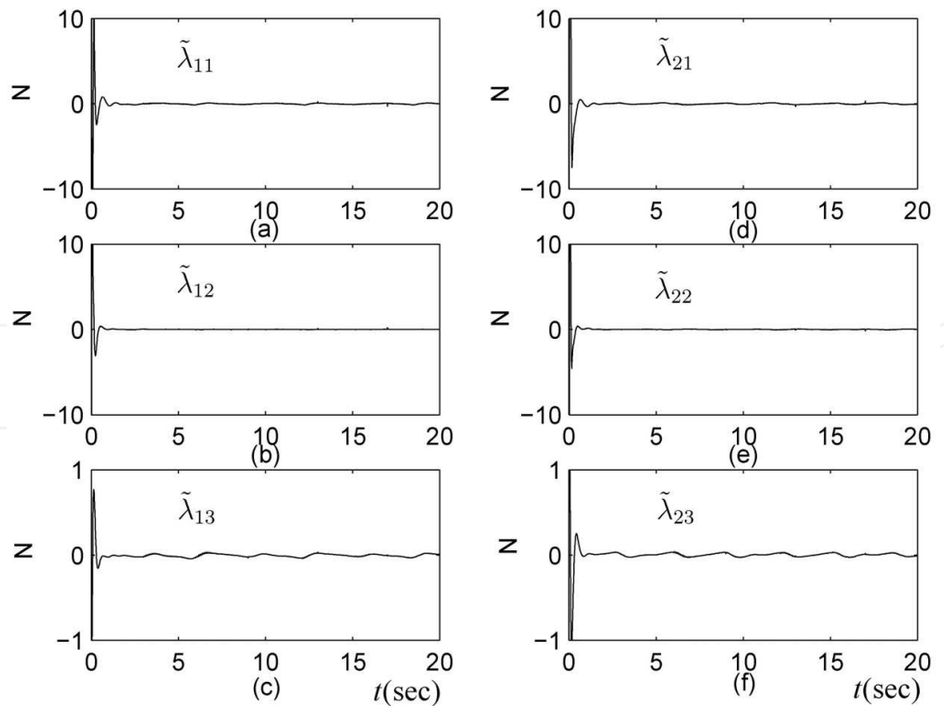

Figure 6. (a-c) Internal force tracking errors for Robot 1; and (d-f) internal force tracking errors for Robot 2

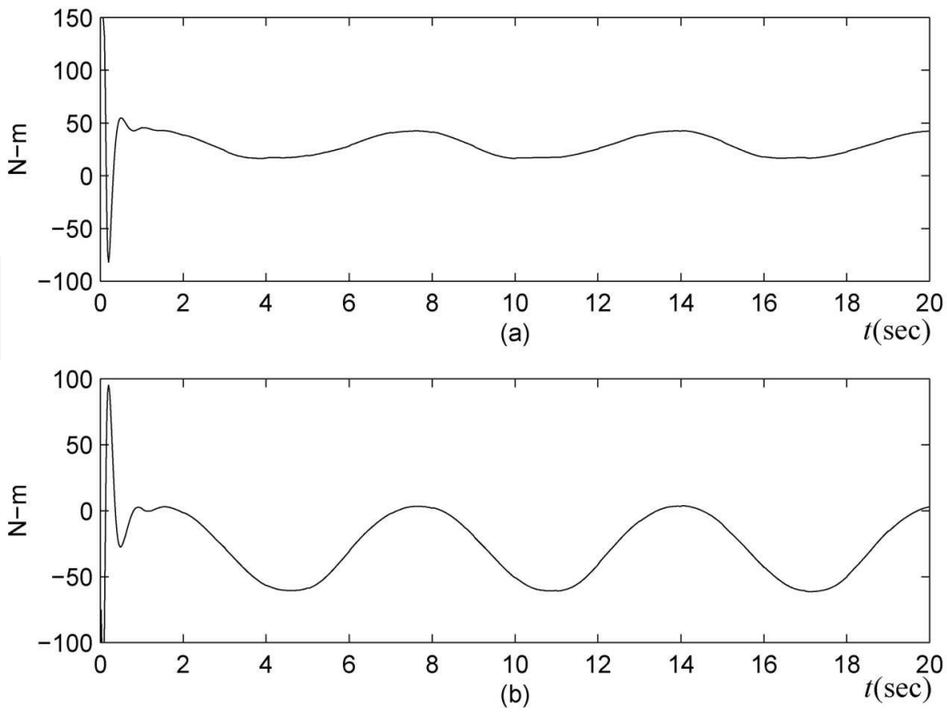

Figure 7. (a) Control input for the second joint of Robot 1; and (b) control input for the second joint of Robot 2

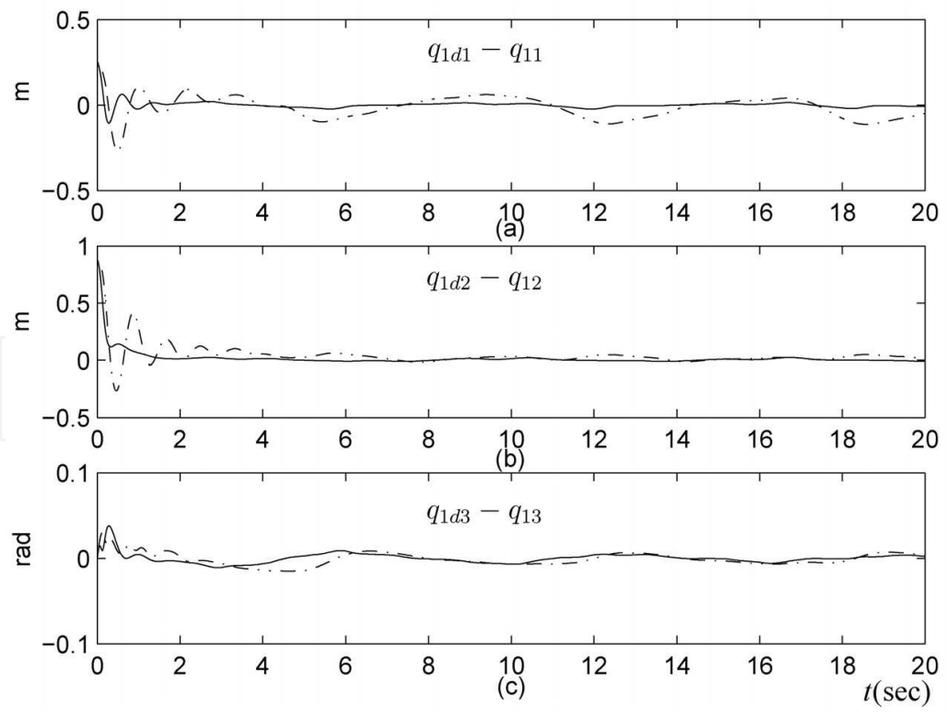

Figure 8. Comparison result of the position tracking errors of the held object. (— TS FFA, - - Mamdani SFA)

6. Conclusions

In this study, a novel TS FFA-based adaptive control scheme has been proposed and applied to motion/force tracking control of holonomic systems. By integrating the feed-forward fuzzy compensation and error-feedback concepts, the proposed FFA-based control concept avoids heavy computation load and achieves global control results. In detail, the FFA-based adaptive control has removed some disadvantages of traditional adaptive fuzzy control including the boundedness assumption on fuzzy approximation errors, a vast amount of rules and tuning parameters, and complicated implementation architecture. Based on an LMI technique and nonlinear damping error-feedback, the overall controlled uncertain system further assures either robust tracking performance or asymptotic convergence. In addition, the TS FFA-based adaptive controller can straightforwardly solve the control problem of complicated and high-dimension systems — holonomic systems. As a result, H^{∞} motion tracking performance is guaranteed with the attenuation of disturbances, approximation errors, and tuned fuzzy parameter errors. Meanwhile, the residual force tracking error is confined to a small value by adjusting control gains feasibly.

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The objective of this book is to provide an up-to-date and state-of-the-art coverage of diverse aspects related to adaptive control theory, methodologies and applications. These include various robust techniques, performance enhancement techniques, techniques with less a-priori knowledge, nonlinear adaptive control techniques and intelligent adaptive techniques. There are several themes in this book which instance both the maturity and the novelty of the general adaptive control. Each chapter is introduced by a brief preamble providing the background and objectives of subject matter. The experiment results are presented in considerable detail in order to facilitate the comprehension of the theoretical development, as well as to increase sensitivity of applications in practical problems

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