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# Quantum Non-Demolition Measurement of Photons

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## Abstract

According to Heisenberg's uncertainty principle, measurement of a quantum observable introduces noise to this observable and thus limits the available precision of measurement. Quantum non-demolition measurements are designed to circumvent this limitation and have been demonstrated in detecting the photon flux of classical light beam. Quantum non-demolition measurement of a single photon is the ultimate goal because it is of great interest in fundamental physics and also a powerful tool for applications in quantum information processing. This chapter presents a brief introduction of the history and a review of the progress in quantum non-demolition measurement of light. In particular, a detailed description is presented for two works toward cavity-free schemes of quantum non-demolition measurement of single photons. Afterward, an outlook of the future in this direction is given.

**Keywords:** QND measurement, single photon, four-wave mixing, Rabi oscillation

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## 1. What is quantum non-demolition measurement?

Measurement of observables is at the very heart of quantum measurement. In the classical macroscopic world, measurement of a classical object can be conducted without introducing perturbation to the detected object. Repeating measurement of a classical object can improve the precision to arbitrarily accurate. Counterintuitively, the measurement of an observable of a quantum object cannot be arbitrarily precise in the microscopic world according to the well-known Heisenberg's uncertainty principle [1], which roots in the wave nature of quantum mechanics. For non-commuting operators,  $A$  and  $B$ , described as physical quantities in the quantum formalism, a very precise measurement of  $A$ , resulting in a very small uncertainty  $\Delta A$ , will be associated with a large value of uncertainty,  $\Delta B$ , in  $B$ . Measuring a quantum object will inevitably cause perturbation in the measured object. This perturbation due to measurement is called as the "measurement back action." This quantum back action, in turn, enlarges uncertainty of the observables. As a result, it limits the available precision in a series of repeated measurements. Then a natural question is what is the limitation of sensitivity in measurement set by quantum mechanics.

In response to this question, Braginsky and Vorontsov introduced in the 1970s the concept of “quantum non-demolition measurement” (QND) to evade the unwanted quantum back action in measurement [2]. Through studying the detectable minimum force on a quantum oscillator, they concluded that “Nondestructive recording of the  $n$ -quantum state of an oscillator is possible in principle.” Their measurement strategy opened a door for circumventing the issue of back action in quantum measurement. Thorne, Drever, Caves, Zimmermann, Sandberg, Unruh, and others developed the concept of QND measurement further [3–5]. The key point in the QND measurement is to keep the back-action noise confined to the unwanted observable quadrature, without being coupled back onto the quantity to be measured.

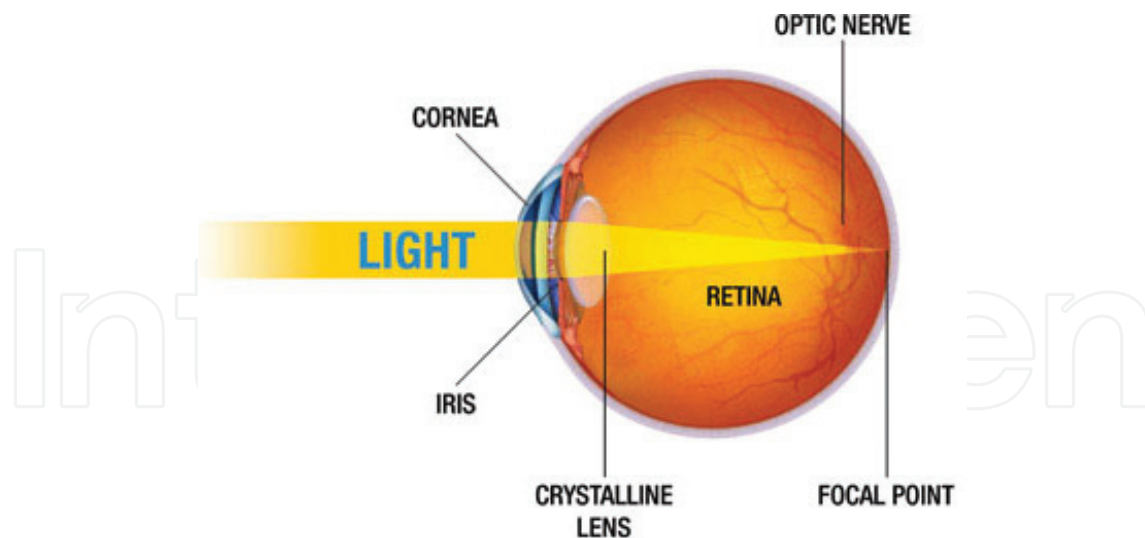
Although a great number of efforts have been made in various systems, quantum optics is particularly well suited for implementing QND measurement. The reason is threefold: (1) there are optical sources with very good quality; (2) photon detectors can be extremely sensitive, even being able to detect a single photon; and (3) a quantum system can be initialized with very high accuracy. The photon number and phase are two complementary observables of quantum light. They are associated with non-commuting operators. It means that QND measurement of photon number of a quantum field will inevitably add quantum noise to the phase quadrature. If only, in principle, the photon number of field remains unchanged during measurement, the measurement is QND. Of course, the real implementation of experiment may be imperfect, and this imperfection can cause noise to the variable of interest.

Throughout this chapter, we focus on the measurement of light according to the principle of quantum optics. In particular, we introduce the measurement of photon number of a light beam. In the conventional “direct” measurement, the light is absorbed. Therefore, the measurement completely changes the observable of photon number and causes a very large back action onto the light beam. In a QND measurement of photon number, it is required that the amount of photon number is *measured without changing*. Of course, the measurement still adds perturbation to the light. However, the perturbation is only confined to the phase of the photon but is not added to the photon flux of interest in measurement. In a restricted mathematical language, the condition for QND measurement is that  $\langle A_s \rangle_i = \langle A_s \rangle_{i+1}$  and  $\langle \Delta A_s \rangle_i = \langle \Delta A_s \rangle_{i+1}$  for two successive detections of observable  $A_s$ .

## 2. Classical measurement by absorbing photons

In the classical world, measurement of light always absorbs photons and then gets energy from them. In this way, the photon carried by a light beam disappears and is destroyed completely. This type of photon detector includes eyes, photoelectric converter, semiconductor photon detector, superconducting photon detector, and so on.

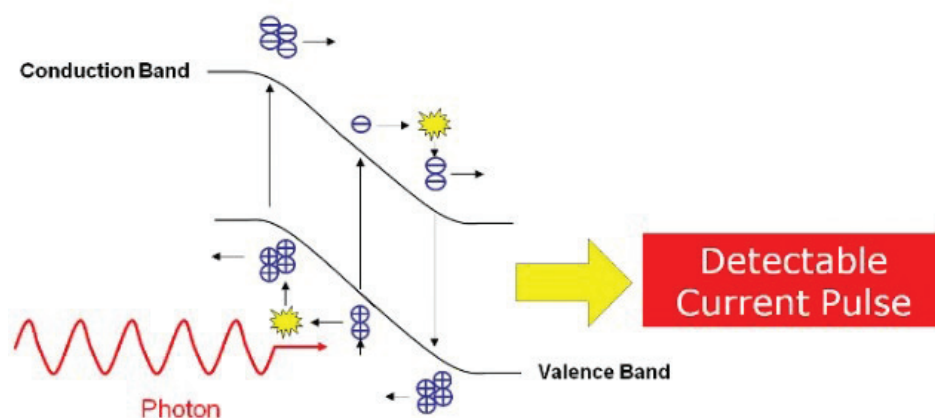
Eyes are photon detectors we use most often (**Figure 1**). It converts the energy of light into electric current and stimulates the nerve. Photons of light enter the eye through the cornea, that is the clear front “window” of the eye. Then light is bent by the cornea, passes freely through the pupil, the opening in the center of the iris, the eye’s natural crystalline lens, and then is focused into a sharp point on the retina. The retina is responsible for capturing all of the light rays, processing them into light impulses through millions of tiny eye nerve endings, and then



**Figure 1.** Sketch for seeing photons with eyes (from [www.nkcf.org](http://www.nkcf.org)).

converting these light impulses to signals which can be recognized by the optic nerve. In doing so, eyes convert light into bioelectric signals.

Semiconductor photon detector is a sensitive man-made photodetector, which is made by using semiconductor materials. Two principal classes of semiconductor photodetectors are in common use: thermal detectors and photoelectric detectors. Thermal detectors convert photon energy into heat. Most thermal detectors are rather inefficient and relatively slow. Therefore, photoelectric detectors are widely used for optics. The operation of photoelectric detectors is based on the photoeffect. Similar to eyes, the detector absorbs photons from light, generating electronic current pulse which can be measured. The semiconductor photon detector is the most used photodetector in industry. The most common semiconductor-based devices are single-photon avalanche diode (SPAD) detectors and can reach sensitivity at the single photon level. The SPAD detector is reversely biased above the avalanche breakdown voltage in the Geiger mode. When a photon is captured by this SPAD detector, the absorbed photon generates an electron-hole pair which causes a self-sustaining avalanche, rapidly generating a measurable current pulse (**Figure 2**).



**Figure 2.** Schematic diagram for semiconductor photon detectors (from [www.single-photon.com](http://www.single-photon.com)).

Superconducting nanowires have been used to detect single photons. It exploits a different principle in comparison with eyes and semiconductor photon detectors. It is designed in this way [6, 7]: a patterned superconducting nanowire is cooled below the transition temperature of the superconducting material. The superconducting nanowire is biased by an external current slightly smaller than the critical current at the operating temperature. When a single photon hits the nanowire, it creates a transient normal spot in the resistive state. As a result of loss of superconductivity, a nonzero voltage is induced between two terminals of the nanowire. Measuring this induced voltage can tell the arrival of the single photon. To date, superconducting single photon detectors have achieved a detection efficiency of more than 90% [8, 9].

The abovementioned are three representatives of photon detectors. All of them destroy photons in signals.

### 3. Measuring light intensity without absorption

QND measurement of light needs to keep the quantum average of the observable and its uncertainty unchanged after detection. In general quantum measurement, the observable of a signal system,  $A_s$ , is measured by detecting the change of observable,  $A_m$ , of a “meter” system. The concept can be explained by describing the measurement as a joint Hamiltonian [10]

$$H = H_s + H_M + H_I, \quad (1)$$

where  $H_s$  is the unperturbed Hamiltonian of the signal system to be measured,  $H_M$  is that of the meter system, and  $H_I$  describes the way in which the meter measures the signal. The motion for  $A_s$  and  $A_M$  under measurement is

$$-i\hbar \frac{dA_s}{dt} = [H_s, A_s] + [H_I, A_s], \quad (2)$$

$$-i\hbar \frac{dA_M}{dt} = [H_s, A_M] + [H_I, A_M]. \quad (3)$$

QND measurement requires (i)  $[H_s, A_s] = 0$ , which is normally satisfied; (ii)  $[H_I, A_s] = 0$ ; and (iii)  $[H_I, A_M] \neq 0$ . The second condition guarantees that the back action is isolated from  $A_s$ . The third one implies that a measurement can induce change in the meter system.

It is quite straightforward to get the cross-Kerr effect in mind for QND measurement of photon flux,  $n_s = A_s^\dagger A_s$ , of light beam [10, 11]. The Hamiltonian describing the cross-Kerr interaction is as follows:

$$H_I = \chi A_s^\dagger A_s A_M^\dagger A_M, \quad (4)$$

where  $\chi$  is the strength of nonlinear interaction. Obviously,  $[H_I, A_s] = 0$  is met.

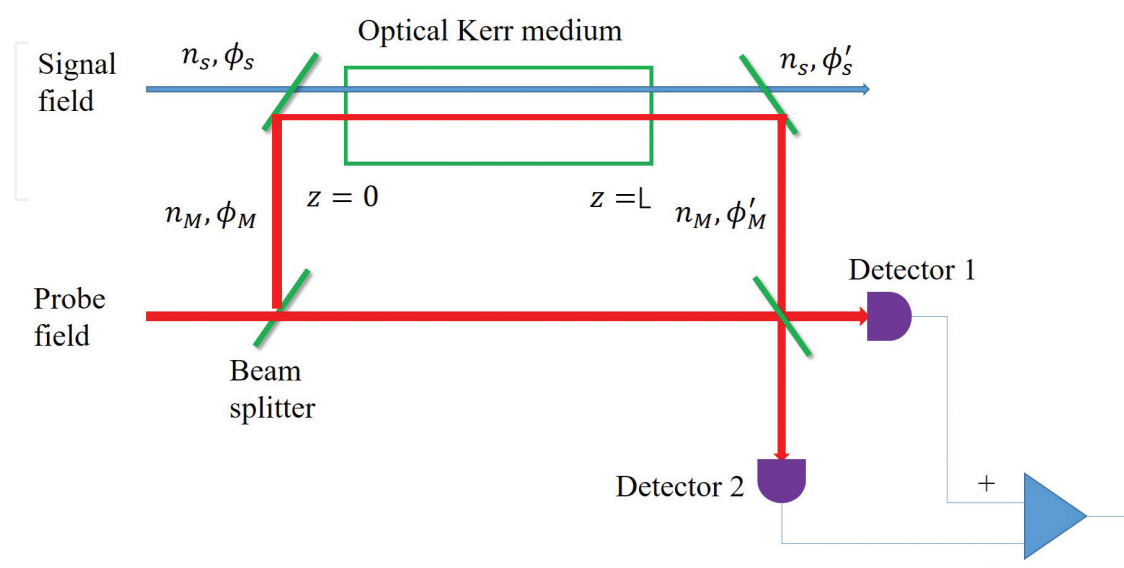
The condition  $[H_I, A_M] \neq 0$  holds if the phase of probe field is measured. The intuitive picture of QND measurement of photon flux,  $n_s$ , with the cross-Kerr effect can be well explained in **Figure 3**. The signal and probe laser fields co-propagate in a Kerr nonlinear medium with

length  $L$ . Due to the cross-Kerr optical nonlinearity, the refractive index of medium is dependent on the intensity,  $I_s \propto n_s$ , of the signal field. Its change is proportional to  $n_s$  and subsequently causes a phase shift,  $\Delta\phi_M = \phi'_M - \phi_M$ , to the probe field. Obviously, this phase shift  $\Delta\phi_M$  is proportional to the photon number of signal field. Measuring  $\Delta\phi_M$  can determine the intensity of the signal field without absorbing its photon.

The concept of QND measurement based on the cross-Kerr effect has been demonstrated in experiments for classical light including many photons [12]. However, QND measurement at the single photon level is still a challenging problem. The difficulty is twofold. Technically, the nonlinearity of normal materials is too weak to induce a large phase shift per photon. Although the cross-Kerr nonlinearity can be improved by orders by using atom system, typically, a single photon can only cause an mrad scale phase shift [13]. It is worth noting two recent experiments in cross-phase modulation [14, 15], which demonstrated the  $\pi$  phase shift at the single photon level via the cross-Kerr nonlinearity of atoms. At first sight, the methods may be able to apply to QND measurement of single photons. Actually, they are yet to meet the criteria of QND measurement.

In the first work [14], by storing a single photon in a cloud of Rydberg atoms, Tiarks et al. achieved a  $\pi$  phase shift imprinted onto a probe field including only 0.9 photon. However, the efficiency of storing and retrieving signal photon is very low, that is only 0.2. The signal photon suffers a big loss and has a small possibility to survive after inducing the phase shift. In this, this scheme cannot be used for QND measurement of single photons.

Alternatively, Liu et al. used a double- $\Lambda$  system to induce a giant cross-Kerr nonlinearity to achieve the  $\pi$  phase shift per photon [15]. In their configuration, the signal and the probe fields, each including eight photons, share a common ground state, while they couple to their individual dark states created by other two control fields. As a result, a giant cross-Kerr nonlinearity between them is created. A  $\pi$  cross-phase shift is induced at the single photon level. However, the reported scheme is still classical but has yet to reach the quantum regime



**Figure 3.** Configuration for the QND measurement of the signal photon number via cross-Kerr nonlinearity [10].



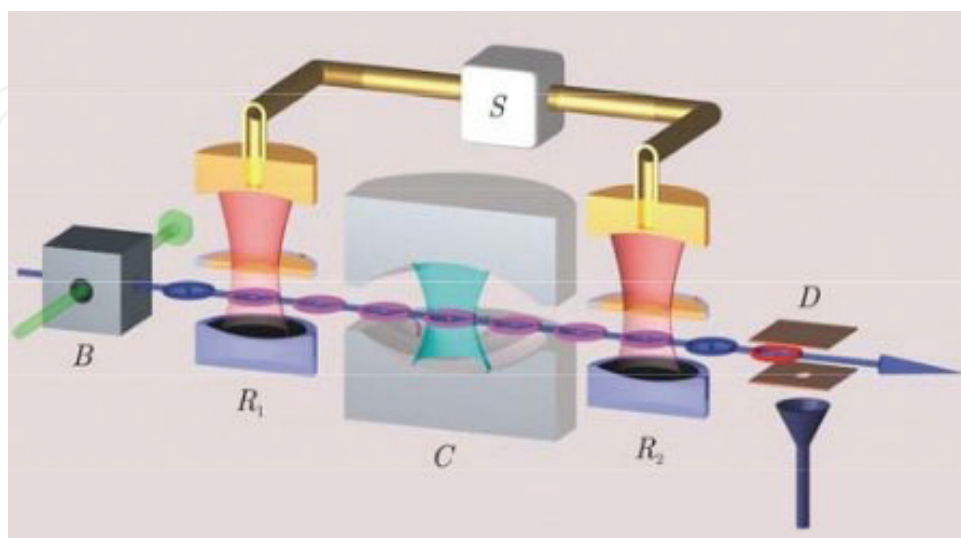
for detecting a single photon in a QND way. There are two prerequisites in this scheme. First, to ensure the atoms are transparent, the probe field needs to be known. Other than the optimal phase, the absorption is considerable. But this phase is unknown for a signal photon to be detected. It means a large loss for the signal photon to be detected. Second, the phase shift is obtained in the steady state where the probe field is classically treated as a constant field. This is not the case for a single photon as it is a quantum field. Therefore, it is hard to do genuine QND measurement at the single photon level.

At the fundamental level, the cross-Kerr-based QND measurement is found invalid when a continuous spatiotemporal multimode model [16] or a finite response time [17–19] is considered. In this sense, although many important progresses have been achieved, QND detection of a moving single photon still needs proposals.

#### 4. Non-demolition measurement of photons with cavities

With the progress of cavity electrodynamics, in particular the ultrastrong coupling between a microwave cavity and an artificial atom, QND measurement of single mw photons have been realized via qubit-photon CNOT gate [20], ac Stark effect [21–23], and the intrinsic phase shift in Rabi oscillation [24]. Photon blockade has been demonstrated as a new effect to implement QND measurement of a single optical photon trapped in a high-quality optical cavity [25].

The first breakthrough of QND measurement of single photons was accomplished by Haroche et al. exploiting the intrinsic  $\pi$  phase shift after a full Rabi oscillation of an atom [24]. The principle can be understood using the schematic diagram as shown in **Figure 4**. The atom is first prepared in Rydberg state with the ground state  $|g\rangle$ , the excited state  $|e\rangle$ , and an auxiliary state  $|i\rangle$  by B. R1 and R2 conduct the Ramsey interferometer measurement. R1 drives the Rydberg atom into a superposition state of  $C_g|g\rangle + C_i|i\rangle$ . The mw cavity C induces a phase shift dependent on the photon number in it. It is off resonance with  $|g\rangle \leftrightarrow |i\rangle$ , but on resonance



**Figure 4.** Schematic diagram for QND measurement of a single microwave photon via the intrinsic phase shift of a full Rabi oscillation [24].

with  $|g\rangle \leftrightarrow |e\rangle$ . It is designed to cause a full Rabi oscillation if the cavity includes one photon and results in a  $\pi$  phase shift to  $|g\rangle$  yielding  $-C_g|g\rangle + C_i|i\rangle$ . While in the empty-cavity case, the atomic state is unchanged. In short, the atomic coherent changes its phase by  $\pi$  if there is one photon in C. R2 mixes the atomic state again, probing after C the superposition phase shift. The final atomic population can be detected with a state-selective detector. The probability of finding the atom in  $|g\rangle$  is a cosine function of the phase shift and thus gives information about the phase shift. In this way, Haroche et al. implemented the QND measurement of a single mw photon.

## 5. Cavity-free schemes for non-demolition measurement of single photons

The concept of QND measurement and its realization in measuring classical light intensity have been introduced earlier. QND measurement of single photons is the ultimate goal. Single “static” photon in cavity has been detected nondestructively. Measuring “moving” single photons without destroying it is still far to be achieved. Two important progresses toward this direction are presented in the following.

### 5.1. QND measurement via Rabi-type photon-photon interaction

As mentioned earlier, although the optical cross-Kerr effect has been proposed for implementing intensity QND measurement of light, detection of light at the single photon level in a QND way is still a challenging task. In the cross-Kerr-based proposals [10], the signal photon changes the refractive index  $n_I$  of medium. The change of  $n_I$  causes a phase shift of the co-propagating probe photon. The interaction between the signal and probe photons is “Ising” type. Its application for single-photon QND measurement is questionable at the fundamental level [16–19]. A “Rabi” type photon-photon interaction created from four-wave mixing (FWM) was proposed for a photon-photon controlled quantum phase gate [24]. The proposal treated the moving fields as a single mode and suggested equal group velocity for both the signal and probe pulses. The work did not circumvent the issues raised in [16–19]. Instead, Xia and his coworker studied this type of photon-photon interaction for QND measurement of a single photon taking into account the quantum nonlocality [26]. In the proposal, the four-wave mixing occurs in an optical nonlinear medium. One of the light modes in four-wave mixing is a strong coherent laser. This coherent laser is used to coherent pump the nonlinear process and perform an effective three-wave mixing process involving the signal mode,  $a_s$ , the probe mode,  $a_p$ , and an auxiliary mode,  $a_a$ . The Hamiltonian describing the interaction among these three modes takes the form

$$H_I = \frac{g(E_c)}{2} a_a a_p^\dagger a_s^\dagger + \frac{g(E_c)}{2} a_a^\dagger a_p a_s, \quad (5)$$

where  $g(E_c)$  indicates the nonlinear coupling strength that can be tuned by the intensity of the pump field  $E_c$ .

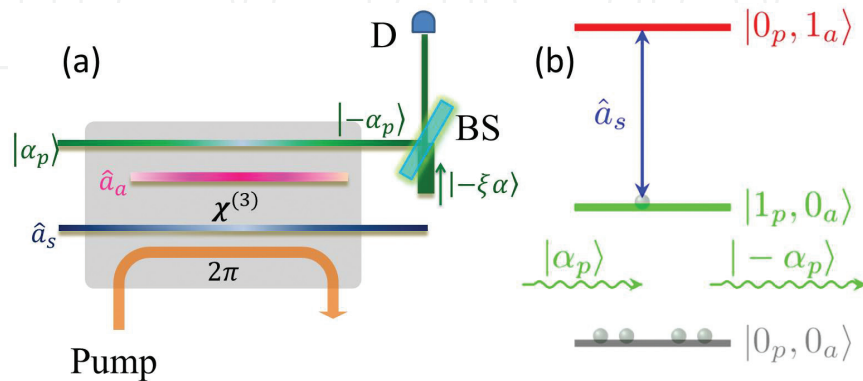
To induce a Rabi-type interaction, the auxiliary mode is initially in a vacuum state. The signal field has at most one photon. The probe field is assumed to be weak that, to a good approximation, it can be considered as the superposition state of  $|\alpha_p\rangle \approx |0_p\rangle + \alpha_p|1_p\rangle$  with  $\alpha_p \ll 1$ .



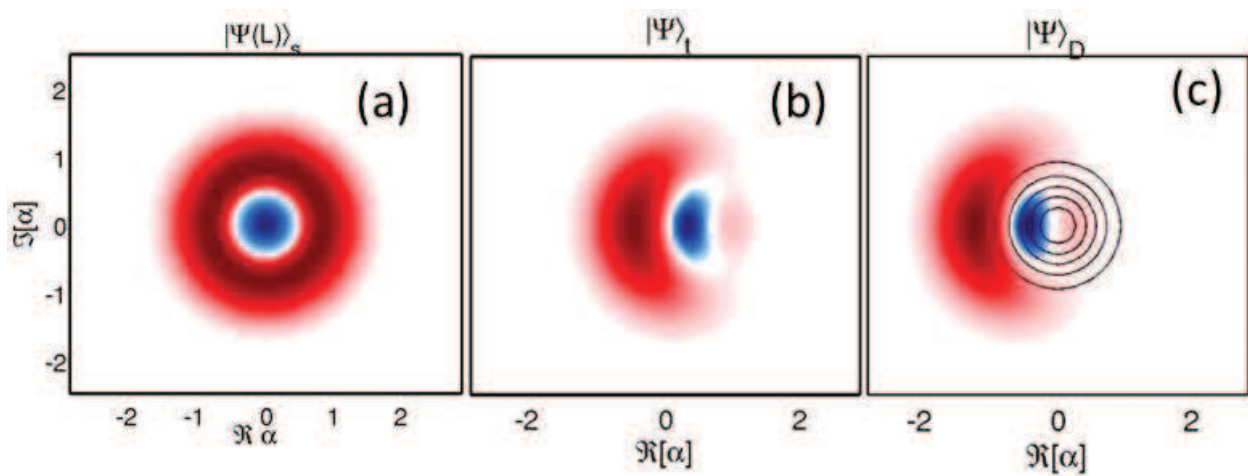
Here, the probe field is truncated up to  $|1_p\rangle$ . Focusing on the space spanned by the associated state of the probe and auxiliary modes, as shown in **Figure 5(a)**, these two modes form a ladder-type quantum system. The ground state is  $|0_p, 0_a\rangle$ , and the first and second excited states are  $|1_p, 0_a\rangle$  and  $|0_p, 1_a\rangle$ , respectively. The incoming signal photon will drive the transition between  $|1_p, 0_a\rangle$  and  $|0_p, 1_a\rangle$ . This photon-driven transition between photonic states is a photonic counterpart of atomic Rabi oscillation. For a weak probe field  $|\alpha_p\rangle$ , the initial state is  $|0_p, 0_a\rangle + \alpha_p|1_p, 0_a\rangle$ . Similar to the Rabi oscillation in atoms, the state  $|1_p, 0_a\rangle$  will suffer a  $\pi$  phase shift after a full Rabi oscillation. As a result, the probe field passing through the medium becomes  $|0_p\rangle - \alpha_p|1_p\rangle \approx |-\alpha_p\rangle$ . Effectively, the probe field is shifted by phase of  $\pi$ . The concept is depicted in **Figure 5(b)**. Such full Rabi oscillation can be conducted by controlling the pump field intensity or the length of nonlinear medium.

To determine the phase shift of the probe field, a strong local bias is overlapped on the transmitted probe field via a highly reflective beam splitter. By properly choosing the bias field, the transmitted probe field presented to the detector is displaced by  $|-\alpha_p\rangle$ , yielding  $| -2\alpha_p\rangle$  in the presence of a single signal photon or  $|0\rangle$  in the absence of signal field. Simply observing the photon “click” on the single-photon detector can determine whether a single signal photon passes through the medium without destroying it. This accomplishes the QND measurement of a single signal photon. Of course, this measurement will cause disturbance in the phase of signal field. However, the photon flux is concerned, and the noise added to the phase quadrature is not unwanted.

To evaluate the performance of the QND measurement, only one investigates the response of system to the initial case of a single signal photon input,  $|1_s\rangle$ , and a weak probe field,  $|\alpha_p\rangle$ . Numerical simulation of corresponding quantum Langevin equation shows the transmitted signal and probe fields, and the displaced field presented to the detector for the input  $|1_s\rangle$  and  $|\alpha_p\rangle$ , as shown in **Figure 6**. It is found that the transmitted signal field keeps its initial state with a very high fidelity, while the transmitted probe field on the detector, shifted by a phase of  $\pi$  due to the presence of signal photon, can be well distinguished from the transmission without phase shift in the absence of signal photon.

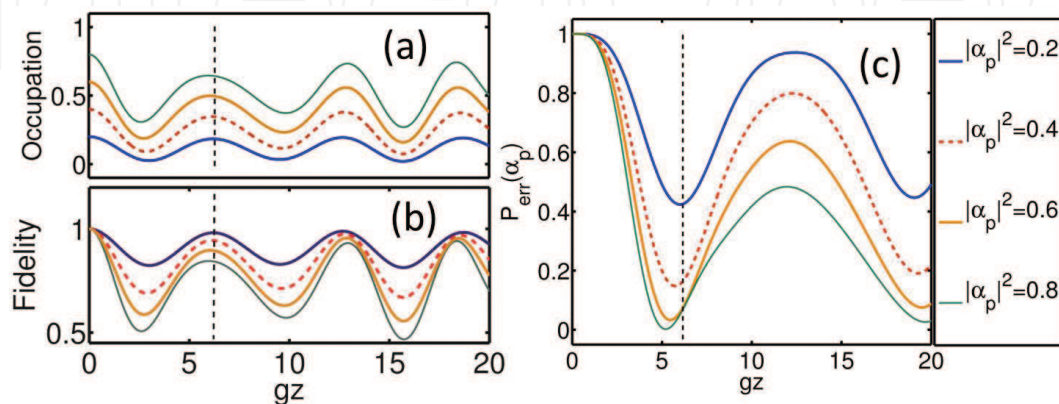


**Figure 5.** Schematic for detection of a single moving photon. (a) Configuration for QND detection of a single moving photon via four-wave mixing in a nonlinear medium. (b) Level diagram describing the interaction between the signal, auxiliary, and probe photons [26].



**Figure 6.** Wigner functions of the transmitted and detected states for a probe field with  $|\alpha_p|^2 = 0.6$ . In (a) [(b)] transmitted signal (probe) state after interacting (a Full Rabi oscillation) for the length of the media; (c) detected state of probe field presented to detector. The concentric circles show the Wigner function contours of the detection field in the absence of signal input [26].

In the presence of a single signal photon, the field presented to the detector is  $|-2\alpha_p\rangle$ . In this case, even an ideal photon detector can have a “dark count,” that is, no detection, because the state  $|-2\alpha_p\rangle$  includes a small occupation in vacuum state  $|0\rangle$ . This dark count causes error in detection of signal photon. The resulted error probability is given by  $P_{err}(\alpha_p) \approx e^{-4|\alpha_p|^2}$ . It decreases exponentially as the intensity of probe field increases. However, the fidelity of transmitted signal field decreases as well. Therefore, a weak probe field is preferable for achieving a high fidelity, while a relative strong probe field is required to reduce the detection error. An optimal trade-off is  $|\alpha_p|^2 = 0.6$ , yielding  $P_{err} = 0.09$  and a fidelity of 0.9 (**Figure 7**). To reduce the error probability and improve the fidelity, a cascade configuration is needed. In such configuration, the transmitted signal field of the former QND measurement is fed into the latter. The transmitted probe field is detected in each measurement. For an N-cascade configuration, the error probability decreases exponentially as a function of N, but the fidelity



**Figure 7.** Evolution of the occupation (a), the fidelity (b) and the detection error probability (c) for different probe field,  $|\alpha_p|^2$ . The black dashed lines at  $gz = 2\pi$  are the guides to eye [26].

decreases linearly. A four-cascade detection unit can already achieve  $\frac{F^4}{P_{err}^4} > 23.75$  for a very weak probe field of  $|\alpha_p|^2 = 0.2$ .

The measured photon and the probe photon are “moving” pulse-shaped wavefunctions. The quantum Langevin equation describes the motion of system in the single mode regime, in which both the signal and the probe photons are treated as a single mode. In the real experiment, they are moving pulse including continuous spatiotemporal modes and can be confined in a one-dimensional (1D) waveguide. Therefore, a model accounting for the interaction of continuous spatiotemporal modes is required. The method developed by Fan et al. can model the interaction of the signal and probe photons in 1D real space [27]. In the Fan’s method, the photons are the wavefunctions of quantum fields propagating in 1D real space. The probability density of photon appearing at certain time (position) is the squared absolute value of wavefunctions. For the purpose of single-photon QND measurement, only one needs the fidelity and phase shift of a photon-pair input state  $|1_p, 1_s\rangle$  after propagating a certain distance. Starting from the vacuum auxiliary field, it can be excited during the propagation of the probe and signal fields. One can define an associate wavefunction  $\varnothing_{ps}(t; z_p, z_s)$  for the state  $|1_p, 1_s\rangle$ , and the wavefunction  $\varnothing_a(t; z_a)$  for the state  $|1_a\rangle$ . These wavefunctions imply that the photons  $|1_p\rangle$  and  $|1_s\rangle$  ( $|1_a\rangle$ ) appear(s) at  $z_p$  and  $z_s$  ( $z_a$ ) at time  $t$  with probability density of  $|\varnothing_{ps}(t; z_p, z_s)|^2$  ( $|\varnothing_a(t; z_a)|^2$ ). The nonlinear medium can be assumed to possess a spatial nonlocal response distribution with an interaction length of  $\sigma$  that  $f_g(z_a, z_p, z_s) = \frac{1}{\sqrt{\pi}\sigma^3} e^{-[(z_a - z_p)^2/2\sigma^2]} e^{-[(z_a - z_s)^2/2\sigma^2]}$ . Following Fan’s treatment, the evolution of the photonic wavefunctions is governed by the partial differential equations [27, 28]

$$\frac{\partial \varnothing_{ps}}{\partial t} = -v_p \frac{\partial \varnothing_{ps}}{\partial z_p} - v_s \frac{\partial \varnothing_{ps}}{\partial z_s} - \frac{ig_0}{2} \int_0^L f_g(z_a, z_p, z_s) \varnothing_a dz_a, \quad (6)$$

$$\frac{\partial \varnothing_a}{\partial t} = -v_a \frac{\partial \varnothing_a}{\partial z_p} - \frac{ig_0}{2} \int_0^L f_g(z_a, z_p, z_s) \varnothing_{ps} dz_p dz_s, \quad (7)$$

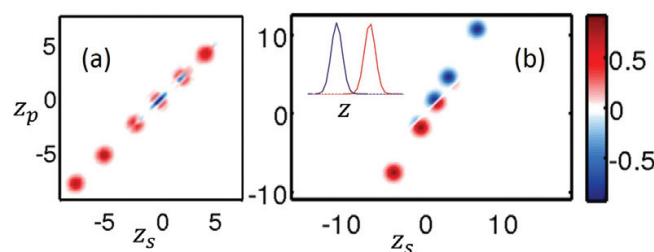
where  $g_0$  is the coupling amplitude,  $v_a(v_p, v_s)$  is the group velocity of the auxiliary (probe, signal) field in the 1D waveguide.  $g_0$  is not important because the coupling strength in experiment can be tuned via the pump laser intensity. The photon pulses are assumed to be long enough that the group velocity of each mode is constant in time, and the perfect phase and energy matching are satisfied.

Solving Eqs. (2) and (3)) can simulate the evolution of the fields in medium. Without loss of generality, a Gaussian input is applied. For a single-photon pulse which is a quantum field, the photon can appear everywhere within the pulse with a probability density determined by the wave packet. This is the nonlocal nature of a single photon pulse. When the probe and signal fields propagate at the same group velocity in the medium as previous schemes, they have no necessity to interact with each other. Actually, with a large probability, they propagate independently as they never meet each other. The signal photon couples the probe photon only if they appear at the same position. As a result, only the central part of  $\varnothing_{ps}$  reverses its sign, implying a

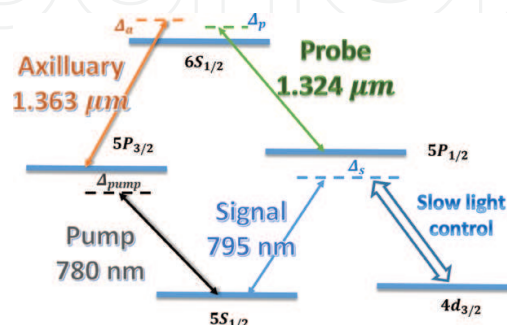
$\pi$  phase shift, see **Figure 8(a)**. To circumvent this issue raised by the nonlocality of single photon pulse, the probe field pulse is delayed with respect to the signal field pulse but propagates at a higher velocity. To do so, the signal mode can be slowed down via the electromagnetically induced transparency (EIT) technique. In such an arrangement, the probe field pulse scans over the signal field pulse. No matter where the probe and signal photons appear within the pulses, they will interact with each other once. It can be seen from **Figure 8(b)** that a  $\pi$  phase shift can be clearly induced after the probe pulse passes through the entire signal pulse. The fidelity is very high about unity. Another advantage of this arrangement over the former is that the phase shift will not change once the probe field passes the signal field, see **Figure 8(b)**.

By comparing two models, it can be seen that when the probe field has at most one photon, a unit fidelity for the transmitted signal mode is achieved. If the probe contains higher Fock states, then interaction with these high Fock states of probe mode prevents to achieve perfect non-demolition of the signal mode.

Rubidium vapor embedded in a hollow-core photonic crystal fiber [12] or a hollow antiresonant reflecting optical waveguide [29] can be a good experimental implementation for this QND measurement scheme. This setup, to a good approximation, can be modeled as a 1D nonlinear medium. The four-wave mixing can be effectively conducted using a diamond-level configuration as shown in **Figure 9**. The signal field can be slowed via EIT with the fifth level,  $4d_{3/2}$ .



**Figure 8.** Evolution of the wave function  $\phi_{ps}$  for (a) the same propagating speeds  $v_p = v_s = 1$  and delay and (b) different speeds  $v_p > v_s$  and different delays [26].



**Figure 9.** Configuration for four-wave mixing realized in Rb atomic vapor in hollow waveguides. The signal field is slowed via EIT by a strong coupling between levels of  $5P_{1/2}$  and  $4d_{3/2}$ .

## 5.2. QND measurement with single emitters

Alternatively, Withaut et al. proposed another scheme for QND measurement of single photons by using a single V-type emitter coupling to a 1D waveguide [30]. The configuration is depicted in **Figure 10**.

A V-type three-level emitter strongly couples to one end of semi-infinite waveguide. The signal photon drives the transition between  $|g\rangle$  and  $|e\rangle$ . The coupling to the waveguide causes an external decay rate,  $\Gamma$ , of state  $|e\rangle$ . The metastable state  $|s\rangle$  is decoupled from the waveguide. The emitter is initially prepared in a superposition state of  $\alpha|g\rangle + \beta|e\rangle$  with  $\beta = \sqrt{1 - \alpha^2}$ . The reflection amplitude of a single-photon input is given by

$$t_\Delta = \frac{\Delta + i(\gamma - \Gamma)}{\Delta + i(\gamma + \Gamma)}, \quad (8)$$

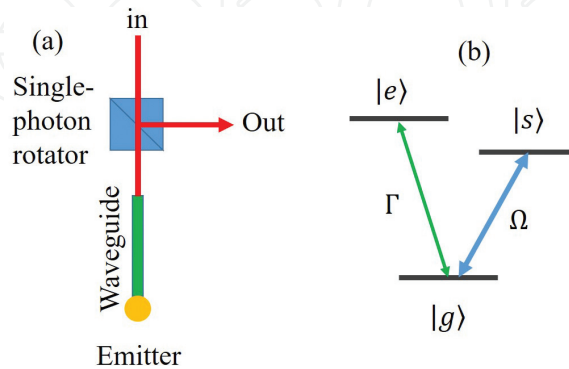
with  $\Delta$  is the detuning between the carrier frequency and the transition frequency between  $|g\rangle$  and  $|e\rangle$ .

A passing resonant photon then introduces a phase shift if and only if the emitter is in state  $|g\rangle$ . The transmission amplitude is given by  $t_\Delta = (\gamma - \Gamma)/(\gamma + \Gamma)$  for this on resonance input. When  $\Gamma \gg \gamma$ , a  $\pi$  phase shift is imprinted on the photon. Then another classical control pulse is applied to invert the state to  $-\beta|g\rangle + \alpha|e\rangle$ . The complete procedure thus realizes the mapping

$$1 \text{ signal photon} : |g\rangle \rightarrow (\beta^2 + t_\Delta \alpha^2)|g\rangle + \alpha\beta(1 - t_\Delta)|s\rangle, \quad (9)$$

$$0 \text{ signal photon} : |g\rangle \rightarrow |g\rangle. \quad (10)$$

Measuring the phase shift imprinted on an incident classical laser pulse can measure the state of emitter. The emitter in  $|s\rangle$  unambiguously reveals the presence of a signal single photon. This scheme is very unclear. They did not discuss how the phase of classical laser field can be shifted by an observable amount. It is also unclear how the single photon changes the state of emitter to be measured.



**Figure 10.** (a) Sketch of potential experimental setup for QND measurement of a single photon. The single-photon circulator is used to separate the input and output. (b) Level diagram of the emitter [30].



For simplicity, set  $v_a = v_p = 1$ . Without loss of generality, a Gaussian input,  $\varnothing_{ps}(t = 0; z_p, z_s) = (1/\sqrt{\pi\tau_p\tau_s})e^{-(z_p-z_{p,0})^2/2\tau_p^2}e^{-(z_s-z_{s,0})^2/2\tau_s^2}$  is applied, where  $z_{p,0}$  and  $z_{s,0}$  are the group delays of the probe and signal wavefunctions, respectively.

## 6. A possible bright future

QND measurement opens a door for precise measurement and versatile applications in photon-based quantum information processing. In principle, QND measurement enables repeated measurement of photon number,  $n$ , of a light beam. Because QND measurement does not disturb the photon number of light, it allows one to measure the photon number many times. This can surpass the standard quantum limit bounded by the “shot-noise” and allows to measure light with ultrahigh sensitivity. QND measurement down to the single photon level further enables potential application in quantum information processing. Remarkably, when a single signal photon can induce a  $\pi$  phase shift to another probe photon, the scheme for QND measurement essentially has the potential to implement a quantum controlled-phase gate between these two photonic modes. This kind of gate is a universal quantum gate for quantum computation. Another important application is to squeeze light via QND measurement. Although QND measurement has been well studied theoretically and has been realized in experiments, it is still questioned in its interpretation [31]. Monroe comments that photons can be independently generated once a signal photon is detected via absorption. He claims that the concept of QND measurement is confusing and should be demolished. However, his comments are also questionable. Squeezing light through QND measurement cannot be realized by simply generating photons according to the detection events. In summary, the concept of QND measurement applied to photons promises of great applications in quantum measurement. The progress approaching the single photon level may provide a simple router for implementing quantum information processing [32] or even quantum telescope [33].

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