Derivation and Calculation of the Dynamics of Elastic Parallel Manipulators

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1. Introduction

Many algorithms for the modelling and calculation of the dynamics of rigid parallel manipulators already exist, and are based on two approaches: The Newton-Euler method and the Lagrangian principle. For the Newton-Euler method the dynamics equations are generated by the complete analysis of all forces and torques of each rigid body in the robot's structure (Featherstone & Orin, 2000, Spong & Vidyasagar, 1989). Therefore, the derivation of the equations of motion for complex systems becomes very complicated and laborious. However, due to the fact that all forces are explicitly regarded and analysed, this method supplies a very advanced understanding of the system's dynamics. The use of the Lagrangian principle is a much more elegant and efficient procedure. A scalar function called the Lagrangian is generated, and describes the entire kinetic, potential and dissipative energy of the system in generalized coordinates. For parallel manipulators, additional equations which describe the closed kinematic loop constraints, still have to be provided. The equation of motion for the parallel structure consists thus of the system of Lagrange and algebraic equations (DAE).
The Lagrangian method is very widely used in the area of parallel manipulators (Beyer, 1928, Kock, 2001). In particular, two procedures from this family are established here: Namely the Lagrangian equations of the first type (Kang & Mills, 2002, Miller & Clavel, 1992, Murray et al., 1994, Tsai, 1999) and the Lagrange-D'Alembert formulation (Nakamura, 1991, Nakamura & Ghodoussi, 1989, Park et al., 1999, Stachera, 2006a, Stachera, 2006b, Yiu et al., 2001). The use of generalized coordinates is employed in these procedures. Those being the coordinates of the active joints as well as an additional set of redundant coordinates of the passive non-actuated joints or end-effector coordinates. Active joints are the actuated joints of the machine. In the case of elastic manipulators a set of elastic degrees of freedom (DOF) will be introduced. In these generalized coordinates the energy function will be formulated. Additionally, the closed kinematic loop constraints of the parallel structure must also be considered. In the Lagrangian equations of the first type this is achieved by Lagrange multipliers. Contrary to these equations, for the Lagrange-D'Alembert formulation, the Jacobian matrices of the kinematic constraints parameterised by the non-redundant coordinates are used. The policy with the Jacobian matrix has the great advantage that the well known methods and techniques for the modelling of the manipulator's chain dynamics, which were already applied to serial elastic robots can be

used (Khalil & Gautier, 2000, Piedboeuf, 2001, Robinett et al., 2002). In this way, effects of friction, elasticities, etc. can be considered by the modelling of the dynamics without laborious modifications. This procedure provides compact equations of the manipulators dynamics, which is advantageous for system analysis and control design. The problem arises with the calculation of the direct dynamics of both presented approaches; it requires the inversion of the inertia matrix, which can be CPU-intensive for matrices of higher order and can thereby constitute a limitation in the real-time calculation for control purposes. The consideration of the manipulator's elasticities can introduce matrices of such high order. The partitioning of the dynamics equations into many groups and their calculation in parallel can reduce the computational effort. Approaches, which consider this problem can be found in the literature. A virtual spring approach has been proposed for this type of parallel processing (Wang & Gosselin, 2000, Wang et al., 2002). In this method, the modelling technique requires the modification of the model and the introduction of additional elements. In the case of elastic manipulators it seems to be a not desirable procedure.

Firstly this chapter presents a brief description of the two above mentioned Lagrangian based methods. It will be shown, how the equations of the inverse and direct dynamics can be obtained, subsequently, the main features of these methods are discussed. These formulations will be extended in comparison with previous research to consider the elastic degrees of freedom. The presentation of elasticities as discrete degrees of freedom does not introduce any limitations of the method and is a conventional method for the analysis of elastic robots (Beres & Sasiadek, 1995, Robinett et al., 2002). In addition to that, a new method for the derivation of the Jacobian matrix of the parallel manipulators will be presented (Stachera & Schumacher, 2007). This method allows the Jacobian matrix of the parallel manipulator to be derived systematically from the Jacobian matrices of the individual serial kinematic chains. Based on these procedures, the method - *Simultaneous Calculation of the Direct Dynamics* (SCDD) for elastic parallel manipulators will be presented. The idea of the "reduced system", which was already used to calculate the inverse dynamics, will be considered. The kinematic constraints of the closed loops are introduced here with the help of the forces and torques of the tree structure. Therefore the equations remain simple and their complexity should not rise. This feature is very important for simulations, for the application of an observer for complex systems or in a feedback control. The new method will then be compared with the existing one and the results will be discussed thereafter.

2. Lagrangian equations of the first type

The Lagrangian equations of the first type are formulated in a set of redundant coordinates (Kang & Mills, 2002, Miller & Clavel, 1992, Tsai, 1999). We assume that the manipulator possesses in all *n* joints, *e* and *p* of them are respectively discrete elastic DOF and passive joints as redundant coordinates. All of which joints have one degree of freedom. Coordinates of the end-effector or moving platform can be also used as redundant coordinates. The coordinates of the actuated joints n_a and the elastic DOF $n_e=e$ form a set of non-redundant coordinates. We assume the controllability of the manipulator structure in absence of elasticity. The coordinates of the structure are:

$$\mathbf{q}_t = \mathbf{q}_t(\mathbf{q}_a, \mathbf{q}_p, \mathbf{q}_e), \tag{1}$$

where $\mathbf{q_a} \in R^{(na \times 1)}$, $\mathbf{q_p} \in R^{(np \times 1)}$, $\mathbf{q_e} \in R^{(ne \times 1)}$ and the dimension of $\mathbf{q_t} \in R^{(nt \times 1)}$, where $n_t = n_a + n_p + n_e$. The $\mathbf{q}_t$ coordinates comprise the redundant degrees of freedom of the rigid movement of the manipulator augmented by degrees of freedom of the elastic deformation of the robot's structure. The redundant coordinates of the passive joints $\mathbf{q}_p$ depend on the remaining coordinates:

$$\mathbf{q_p} = \mathbf{q_p}(\mathbf{q_a}, \mathbf{q_e}). \tag{2}$$

Using (2) we can further write (1) as $\mathbf{q_t} = \mathbf{q_t}(\mathbf{q_a}, \mathbf{q_e})$. In order to solve the dynamics equation, due to redundant coordinates, the formulation of dynamics requires a set of additional constraint equations. These can be determined by examining the structure of the system, with respect to the closed kinematic loop constraints of the parallel manipulator. The constraints equations and their derivatives supplement the original equations of the machine dynamics, so that the number of equations is equal to that of the unknowns. Therefore the Lagrangian equations of the first type are formulated as follows

$$\frac{d}{dt}\frac{\partial L_t}{\partial \dot{\mathbf{q}}_t} - \frac{\partial L_t}{\partial \mathbf{q_t}} + \frac{\partial Q_t}{\partial \dot{\mathbf{q}}_t} = \boldsymbol{\tau}_t + \sum_{i=1}^{np} \lambda_i \frac{\partial h_i}{\partial \mathbf{q_t}}, \tag{3}$$

where L_t is the Lagrange function consists of the kinetic and potential energy of the system, Q_t means the function of the dissipative energy, h_i denotes the i^{th} constraint function, n_p is the number of constraints and at the same time number of redundant coordinates, $\boldsymbol{\tau}_t$ are the generalised torques and forces and λ_i are the Lagrange multipliers. In order to simplify the solution of these equations, they will be divided into two sets (Tsai, 1999).

2.1 Inverse dynamics

The first set of n_p equations refers to the redundant coordinates and is associated with the kinematic constraints of the closed loops. Here, the unknowns are the Lagrange multipliers $\lambda_i \in R^{(np \times 1)}$. Hence, these equations take the form

$$\sum_{i=1}^{np} \lambda_i \frac{\partial h_i}{\partial \mathbf{q_t}} = \frac{d}{dt}\frac{\partial L_t}{\partial \dot{\mathbf{q}}_t} - \frac{\partial L_t}{\partial \mathbf{q_t}} + \frac{\partial Q_t}{\partial \dot{\mathbf{q}}_t} - \hat{\boldsymbol{\tau}}_t, \tag{4}$$

where $\hat{\boldsymbol{\tau}}_t$ represents generalised torques and forces. They represent the external potential and non-potential forces, acting on the manipulator, which are already known. Here, the torques of the actuators are not taken into account. From these n_p equations of the redundant coordinates the n_p Lagrange multipliers are calculated. The second set is related to the (n_a+n_e) non-redundant coordinates. The only unknowns in these equations are the forces and torques of the actuators, which can be computed from

$$\boldsymbol{\tau}_t = \frac{d}{dt}\frac{\partial L_t}{\partial \dot{\mathbf{q}}_t} - \frac{\partial L_t}{\partial \mathbf{q_t}} + \frac{\partial Q_t}{\partial \dot{\mathbf{q}}_t} - \sum_{i=1}^{np} \lambda_i \frac{\partial h_i}{\partial \mathbf{q_t}}. \tag{5}$$

With these two equation sets (4) and (5), the torques and forces of the actuators for a given trajectory are computed, and thus produce the desired movement of the elastic parallel manipulator.

2.2 Direct dynamics

For the given torques, the direct dynamics can be computed in a similar way. The n_p redundant coordinates and their derivatives are calculated from the closed kinematic loop constraints h_i and their derivatives. These redundant coordinates result from the non-redundant coordinates of the active joints and elastic DOF. The constraint forces of the structure are then computed (4). Finally, now that the input torques of the parallel manipulator and the constraint forces are known, (5) must be solved for the unknown accelerations of the non-redundant coordinates. Further, these equations can be solved by numerical integration, and the n_p Lagrange multipliers from (4) can also be computed on this way.

2.3 Features of the method of the Lagrangian equations of the first type

The coordinates of the active joints and elastic DOF form a subset of the selected generalised redundant coordinates. The remaining coordinates can be selected freely. These can be the coordinates of the platform, the end-effector or of the passive joints (Kang & Mills, 2002, Miller & Clavel, 1992, Tsai, 1999). Here the Lagrange multipliers might also have the meaning of generalised torques and forces, which determine the constraints of the closed loops for the serial kinematic chains. The disadvantage of this method is that, for the modelling of the manipulator, various simplifications must be made. In order to consider the Lagrange multipliers, the methods for the modelling of the dynamics that are used for the serial kinematic chains can require a modification. However, due to the equations' structures, a clear physical interpretation of the terms is not always possible, and therefore the employment of this method remains slightly complicated.

3. Lagrange-D'Alembert formulation (L-D'A)

3.1 Inverse dynamics

The Lagrange-D'Alembert formulation represents an elegant and effective consideration of the problem of manipulator's dynamics (Nakamura, 1991, Nakamura & Ghodoussi, 1989, Park et al., 1999, Yiu et al., 2001). Here, no additional multipliers are calculated. A set of independent and dependent generalized coordinates which satisfy the constraints of the mechanical system is chosen. The coordinates of the elastic DOF belong to the group of independent coordinates and are associated with the corresponding internal forces, resulting from the stress induced in the material. The procedure corresponds to the methods, which are known from the serial manipulators and consists of the following three steps:

1. *Transformation of the System:* Each closed kinematic loop of the parallel manipulator is separated at a passive joint, end-effector or link. The result is a tree structure as a reduced system (Nakamura & Ghodoussi, 1989). Consequently, only serial kinematics chains can be found in this system. Furthermore it is assumed that all remaining passive joints are equipped with virtual actuators.
2. *Computation of the Torques:* The torques and forces of the real and virtual actuators are computed for each kinematic chain. These torques and forces cause a movement in every chain, and these movements correspond to the movement of the original closed-link structure.
3. *Transformation of the Torques:* The torques and forces of the original parallel manipulator's actuators are calculated from the forces and torques of the tree structure by considering the additional closed kinematic loop constraints.

We assume that the manipulator consists of l closed kinematic loops. It possesses in all n joints, e and p of them are respectively discrete elastic DOF and passive joints. All of which joints have one degree of freedom. The coordinates of the active joints and elastic DOF form a set of non-redundant coordinates. We assume the controllability of the manipulator in absence of elasticity. According to the first step we divide this system into a tree structure. The number of active joints remains the same as in the original structure $n_a = (n-p-e)$. The number of passive joints amounts to $n_p = (p-l)$ and the number of the elastic DOF amounts to $n_e=e$. The coordinates of the tree structure are:

$$\mathbf{q}_t = \mathbf{q}_t\left(\mathbf{q}_a, \mathbf{q}_p, \mathbf{q}_e\right), \tag{6}$$

where $\mathbf{q}_e \in R^{(na\times1)}$, $\mathbf{q}_p \in R^{(np\times1)}$, $\mathbf{q}_e \in R^{(ne\times1)}$ and the dimension of $\mathbf{q}_t \in R^{(nt\times1)}$, where $n_t = n_a + n_p + n_e$. The redundant coordinates of the passive joints $\mathbf{q}_p$ depend on the coordinates of the active joints $\mathbf{q}_a$ and the elastic DOF $\mathbf{q}_e$:

$$\mathbf{q}_p = \mathbf{q}_p\left(\mathbf{q}_{ae}\right), \tag{7}$$

where $\mathbf{q}_{ae} \in R^{(nae\times1)}$ and $n_{ae} = n_a + n_e$. Using (7) we can further write (6) as $\mathbf{q}_t = \mathbf{q}_t(\mathbf{q}_{ae})$. Generally, the relation represented in (7) does not exist analytically, but the quantity of redundant coordinates can always be determined by the consideration of the geometrical dependencies in the manipulator structure (Merlet, 2000, Stachera, 2005). Therefore, in order to determine the relationship between the velocities and accelerations of the active and passive joints, a more suitable solution must be derived (Yiu et al., 2001). For this purpose we introduce the closed kinematic loop constraints of the parallel manipulator:

$$h(\mathbf{q}_t) = h\left(\mathbf{q}_{ae}, \mathbf{q}_p\right) = 0\,. \tag{8}$$

By differentiation of (8) we obtain the constraints in the Pfaffian form:

$$\frac{\partial h}{\partial \mathbf{q}_{ae}^T}\dot{\mathbf{q}}_{ae} + \frac{\partial h}{\partial \mathbf{q}_p^T}\dot{\mathbf{q}}_p = 0\,. \tag{9}$$

Our goal is now to find the transformation between the tree structure and the original parallel manipulator. According to the D'Alembert principle the performed virtual work for both systems, the reduced and the original one, has to be equal:

$$\delta\mathbf{q}_c^T\boldsymbol{\tau}_c = \delta\mathbf{q}_t^T\boldsymbol{\tau}_t\,, \tag{10}$$

where $\boldsymbol{\tau}_t \in R^{(nt\times1)}$ represents all forces and torques of the real and virtual drives of the tree system and $\boldsymbol{\tau}_c \in R^{(nae\times1)}$ the drive torques of the original parallel manipulator. Hence, the Lagrange equations for the reduced system can be formulated:

$$\left(\frac{d}{dt}\frac{\partial L_t}{\partial \dot{\mathbf{q}}_t} - \frac{\partial L_t}{\partial \mathbf{q}_t} + \frac{\partial Q_t}{\partial \dot{\mathbf{q}}_t} - \boldsymbol{\tau}_t\right)^T \delta\mathbf{q}_t = 0\,, \tag{11}$$

where L_t is the Lagrange function of the tree structure and Q_t is the function of the dissipative energy. This Lagrange function consists of the kinetic and potential energy of the system $L_t = T_t - V_t$. We assume that the robot is normally actuated and away from actuator singularity. The matrix from (9) - $\frac{\partial h}{\partial \mathbf{q}_p^T}$ is square and invertible. The configuration space of the manipulator can be smoothly parameterised by the coordinates of the active joints and the elastic DOF $\mathbf{q}_{ae}$:

$$\dot{\mathbf{q}}_p = \left(\frac{\partial h}{\partial \mathbf{q}_p^T}\right)^{-1}\left(\frac{\partial h}{\partial \mathbf{q}_{ae}^T}\right)\dot{\mathbf{q}}_{ae} = \left(\frac{\partial \mathbf{q}_p}{\partial \mathbf{q}_{ae}^T}\right)\dot{\mathbf{q}}_{ae} \,. \tag{12}$$

Therefore the equations of the tree structure can be expressed in the non-redundant coordinates $\mathbf{q}_{ae}$. Considering (6), (11) and (12):

$$\begin{gathered}\left(\frac{d}{dt}\frac{\partial L_t}{\partial \dot{\mathbf{q}}_{ae}} - \frac{\partial L_t}{\partial \mathbf{q}_{ae}} + \frac{\partial Q_t}{\partial \dot{\mathbf{q}}_{ae}} - \boldsymbol{\tau}_{ae}\right)^T \delta\mathbf{q}_{ae} \\ + \left(\frac{d}{dt}\frac{\partial L_t}{\partial \dot{\mathbf{q}}_p} - \frac{\partial L_t}{\partial \mathbf{q}_p} + \frac{\partial Q_t}{\partial \dot{\mathbf{q}}_p} - \boldsymbol{\tau}_p\right)^T \delta\mathbf{q}_p = 0\end{gathered} \,, \tag{13}$$

and it is:

$$\begin{gathered}\left(\frac{d}{dt}\frac{\partial L_t}{\partial \dot{\mathbf{q}}_{ae}} - \frac{\partial L_t}{\partial \mathbf{q}_{ae}} + \frac{\partial Q_t}{\partial \dot{\mathbf{q}}_{ae}}\right) + \left(\frac{\partial \mathbf{q}_p}{\partial \mathbf{q}_{ae}^T}\right)^T \cdot \\ \left(\frac{d}{dt}\frac{\partial L_t}{\partial \dot{\mathbf{q}}_p} - \frac{\partial L_t}{\partial \mathbf{q}_p} + \frac{\partial Q_t}{\partial \dot{\mathbf{q}}_p}\right) = \boldsymbol{\tau}_{ae} + \left(\frac{\partial \mathbf{q}_p}{\partial \mathbf{q}_{ae}^T}\right)^T \boldsymbol{\tau}_p\end{gathered} \,. \tag{14}$$

The equations of motion of the entire parallel manipulator are similar to (11) and take the following form:

$$\left(\frac{d}{dt}\frac{\partial L_c}{\partial \dot{\mathbf{q}}_{ae}} - \frac{\partial L_c}{\partial \mathbf{q}_{ae}} + \frac{\partial Q_c}{\partial \dot{\mathbf{q}}_{ae}}\right) = \boldsymbol{\tau}_c \,. \tag{15}$$

Regarding (10), (14) and (15) we can finally write:

$$\begin{gathered}\frac{d}{dt}\frac{\partial L_c}{\partial \dot{\mathbf{q}}_{ae}} - \frac{\partial L_c}{\partial \mathbf{q}_{ae}} + \frac{\partial Q_c}{\partial \dot{\mathbf{q}}_{ae}} = \\ \left(\frac{d}{dt}\frac{\partial L_t}{\partial \dot{\mathbf{q}}_{ae}} - \frac{\partial L_t}{\partial \mathbf{q}_{ae}} + \frac{\partial Q_t}{\partial \dot{\mathbf{q}}_{ae}}\right) \\ + \left(\frac{\partial \mathbf{q}_p}{\partial \mathbf{q}_{ae}^T}\right)^T \cdot \left(\frac{d}{dt}\frac{\partial L_t}{\partial \dot{\mathbf{q}}_p} - \frac{\partial L_t}{\partial \mathbf{q}_p} + \frac{\partial Q_t}{\partial \dot{\mathbf{q}}_p}\right)\end{gathered} \,, \tag{16}$$

$$\boldsymbol{\tau}_c = \boldsymbol{\tau}_{ae} + \left(\frac{\partial \mathbf{q}_p}{\partial \mathbf{q}_{ae}^T}\right)^T \boldsymbol{\tau}_p \,. \tag{17}$$

From these derivations, the transformation matrix between the tree structure and the original closed-link structure can be formulated:

$$\mathbf{W} = \frac{\partial \mathbf{q}_t}{\partial \mathbf{q}_{ae}^T} = \begin{bmatrix} \mathbf{I} \\ \dfrac{\partial \mathbf{q}_p}{\partial \mathbf{q}_{ae}^T} \end{bmatrix}. \tag{18}$$

Proofs of these derivations can be found in works (Nakamura, 1991, Nakamura & Ghodoussi, 1989).

Now, the equations of the manipulator's dynamics will be written in matrix form. The equations of motion of the tree structure are described by the following expression:

$$\begin{aligned} &\mathbf{M}_t(\mathbf{q}_t)\ddot{\mathbf{q}}_t + \mathbf{C}_t(\dot{\mathbf{q}}_t, \mathbf{q}_t)\dot{\mathbf{q}}_t + \boldsymbol{\eta}_t(\mathbf{q}_t) \\ &+ \mathbf{K}_t\mathbf{q}_t + \mathbf{D}_t\dot{\mathbf{q}}_t = \boldsymbol{\tau}_t \end{aligned}, \tag{19}$$

where the $\mathbf{M}_t(\mathbf{q}_t), \mathbf{C}_t(\dot{\mathbf{q}}_t, \mathbf{q}_t) \in \mathrm{R}^{(nt \times nt)}$ are the inertia matrix and the Coriolis matrix of the tree structure respectively. These matrices satisfy the following structural properties:

1. $\mathbf{M}_t(\mathbf{q}_t)$ is symmetric and positive definite matrix,
2. $\dot{\mathbf{M}}_t(\mathbf{q}_t) - 2\mathbf{C}_t(\dot{\mathbf{q}}_t, \mathbf{q}_t)$ is a skrew-symmetric matrix.

$\boldsymbol{\eta}(\mathbf{q}_t) \in \mathrm{R}^{(nt \times 1)}$ is the vector of the gravity force reflected in the joints' space. $\mathbf{K}_t \in \mathrm{R}^{(nt \times nt)}$ and $\mathbf{D}_t \in \mathrm{R}^{(nt \times nt)}$ represent the diagonal matrices of the lumped elasticities and lumped dampings in the joints' space. By using the matrix $\mathbf{W}$ from (18) the equations of the dynamics of the tree structure (19) can be transformed into the equations of the closed-link mechanism. Then, they are expressed only in dependence on the coordinates of the active joints $\mathbf{q}_a$ and the elastic DOF $\mathbf{q}_e$:

$$\begin{aligned} &\mathbf{M}_c(\mathbf{q}_t)\ddot{\mathbf{q}}_{ae} + \mathbf{C}_c(\dot{\mathbf{q}}_t, \mathbf{q}_t)\dot{\mathbf{q}}_{ae} + \boldsymbol{\eta}_c(\mathbf{q}_t) \\ &+ \mathbf{K}_c\mathbf{q}_{ae} + \mathbf{D}_c\dot{\mathbf{q}}_{ae} = \boldsymbol{\tau}_c \end{aligned}, \tag{20}$$

where:

$$\mathbf{M}_c = \mathbf{W}^T\mathbf{M}_t\mathbf{W} \quad \in \mathrm{R}^{(nae \times nae)}, \tag{21}$$

$$\mathbf{C}_c = \mathbf{W}^T\mathbf{M}_t\dot{\mathbf{W}} + \mathbf{W}^T\mathbf{C}_t\mathbf{W} \quad \in \mathrm{R}^{(nae \times nae)}, \tag{22}$$

$$\boldsymbol{\eta}_c = \mathbf{W}^T\boldsymbol{\eta}_t \quad \in \mathrm{R}^{(nae \times 1)}, \tag{23}$$

$$\mathbf{K}_c = \mathbf{W}^T\mathbf{K}_t\mathbf{W} \quad \in \mathrm{R}^{(nae \times nae)}, \tag{24}$$

$$\mathbf{D}_c = \mathbf{W}^T\mathbf{D}_t\mathbf{W} \quad \in \mathrm{R}^{(nae \times nae)}. \tag{25}$$

From these considerations, two methods for the computation of the inverse dynamics of the parallel manipulator result. In the first method, the real and virtual forces and torques of the tree structure (11) are computed. These torques are then transformed with (17) or (18) into the drive torques of the closed-link structure. In the second method, the equations of the dynamics of the tree structure (19) are transformed into the compact equations of the closed-link mechanism (20) and parameterised (21)-(25) by the non-redundant coordinates $\mathbf{q}_{ae}$. With these the drive torques can then be calculated.

3.2 Direct dynamics

In this method the equations of the direct dynamics are obtained from the compact equations (20) of the manipulator's inverse dynamics:

$$\begin{aligned}\ddot{\mathbf{q}}_{ae} &= \mathbf{M}_c(\mathbf{q}_t)^{-1}\left(\boldsymbol{\tau}_c - \mathbf{C}_c(\dot{\mathbf{q}}_t, \mathbf{q}_t)\dot{\mathbf{q}}_{ae}\right.\\ &\left.- \boldsymbol{\eta}_c(\mathbf{q}_t) - \mathbf{K}_c\mathbf{q}_{ae} - \mathbf{D}_c\dot{\mathbf{q}}_{ae}\right)\end{aligned} . \tag{26}$$

According to this, the complex equations of the direct dynamics, parameterized by the coordinates of the active joints and elastic DOF $\mathbf{q}_{ae}$ are obtained. The redundant coordinates of the passive joints, which are necessary for the computation of the matrices, result from the closed kinematic loop constraints of the parallel manipulator (7) as well as their first (12) and second derivatives.

3.3 Features of the Lagrange-D'Alembert formulation

In this method, for the modelling of the elastic parallel manipulators' dynamics one can use, without modifications, the well known methods and techniques from serial robotics (Khalil & Gautier, 2000, Piedboeuf, 2001, Robinett et al., 2002). The computations of the inverse dynamics can be carried out in parallel, and therefore can be faster, which is an advantage for the real time calculation of the manipulator's model. The method of the direct dynamics produces the compact equations of the elastic parallel manipulator. The disadvantage is that the computations cannot be executed in parallel. If more parameters are to be considered for the modelling, e.g. Finite-Element-Method (Beres & Sasiadek, 1995, Wang & Mills, 2004), the compact matrices of the system can reach such dimensions, that their calculation is simply not possible in real time.

4. New method for derivation of the Jacobian matrix of the parallel manipulator

In the conventional methods the Jacobian of the parallel manipulator will be derived from the velocity vector-loop method (Tsai, 1999) or from the analysis of the parallel manipulator's statics (Kock, 2001, Merlet, 2000). The Lagrange-D'Alembert Method (Nakamura, 1991, Nakamura & Ghodoussi, 1989, Park et al., 1999, Yiu et al., 2001) makes it possible to systematically convert between the single models of serial kinematic chains to the model of the compact parallel manipulator. In this method, however, it was not shown how the Jacobian matrices of the serial kinematic chains of the tree structure $\mathbf{J}_i$ can be transformed into the Jacobian of the parallel manipulator $\mathbf{G}$. An algorithm was developed because of that, in order to perform this transformation. For this purpose, the $\mathbf{W}$ matrix, representing the parameterisation of the configuration space from (18) and the static matrix $\mathbf{S}$ of the parallel manipulator are used. The static matrix $\mathbf{S}$ describes the relationship between the forces in the arms of the parallel manipulators $\mathbf{f}_B \in \mathrm{R}^{(n_a \times 1)}$ and the force on its end-effector $\mathbf{f}_{XYZ}$. The external force $\mathbf{f}_{XYZ}$ acting on the end-effector is distributed into the corresponding branches, shown in the Fig. 1.
These forces in the branches can be calculated through the following relationship:

$$\mathbf{f}_B = \begin{bmatrix}\mathbf{s}_1 & \dots & \mathbf{s}_{na}\end{bmatrix}^{-1}\mathbf{f}_{XYZ} = \mathbf{S}^{-1}\mathbf{f}_{XYZ} . \tag{27}$$

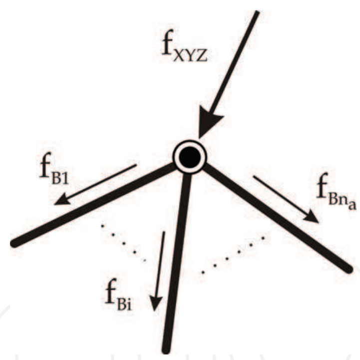

Fig. 1. Force distribution on the manipulator

If the matrix $\mathbf{S}$ is not square then $\mathbf{S}^{-1}$ is pseudo-inverse $\mathbf{S}^{+}$. The elements of the $\mathbf{S}$-Matrix $\mathbf{s}_i(\mathbf{q}_{ai}, \mathbf{q}_{pi}, \mathbf{q}_{ei})$ comprise the vector that relates $\mathbf{f}_{Bi}$, the force of the i^{th} chain, and $\mathbf{f}_{XYZi}$, the Cartesian force, which is a result of this force. It can be described in the following formula:

$$\mathbf{f}_{XYZi} = \mathbf{s}_i \mathbf{f}_{Bi} . \tag{28}$$

In the matrix form with the matrix $\mathbf{U}$ takes this relation the form:

$$\mathbf{f}_{BXYZ} = \begin{bmatrix} \mathbf{s}_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \mathbf{s}_{n_a} \end{bmatrix} \mathbf{f}_B = \mathbf{U}\mathbf{f}_B . \tag{29}$$

Now we introduce the Jacobian matrix $\mathbf{J}_t$ of the tree structure:

$$\mathbf{J}_t = \begin{bmatrix} \mathbf{J}_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \mathbf{J}_{na} \end{bmatrix}, \tag{30}$$

where $\mathbf{J}_i(\mathbf{q}_{ai}, \mathbf{q}_{pi}, \mathbf{q}_{ei})$ are the n_a - Jacobian matrices of the serial kinematic chains. In order to eliminate the dependencies of the coordinates of the passive joint $\mathbf{q}_p$ for the calculation of the $\mathbf{G}$ Jacobian matrix of parallel manipulator, the matrix $\mathbf{J}_t$ will be parameterised with the matrix $\mathbf{W}$. After this parameterisation the new matrix no longer represents the mapping between the joint and Cartesian velocity and force space of the parallel manipulator. In order to obtain this mapping, the matrices $\mathbf{S}$ and $\mathbf{U}$ have to be introduced. The matrix $\mathbf{S}^{-1}$ represents the transformation between the forces from the Cartesian space into the branch forces. The matrix $\mathbf{U}$ constitutes the relation between the forces in the branches and these Cartesian components. With regards to this transformations, the Jacobian matrix of the parallel manipulator can be derived from the following relation:

$$\mathbf{G}^{+T} = \mathbf{W}^T \mathbf{J}_t^T \mathbf{U} \mathbf{S}^{-1} . \tag{31}$$

This pseudo-inverse Jacobian matrix $\mathbf{G}^{+T}$ represents the mapping between the Cartesian force $\mathbf{f}_{XYZ}$ on the end-effector of parallel manipulator and the forces/torques $\boldsymbol{\tau}_{ae} \in R^{(nae \times 1)}$ in the manipulator's structure in the joint space:

$$\boldsymbol{\tau}_{ae} = \mathbf{G}^{+T} \mathbf{f}_{XYZ} . \tag{32}$$

The presented method has the great advantage that the derivation of the serial Jacobian matrices is much easier than the derivation of the compact Jacobian matrix.

5. A new method for the calculation of the direct dynamics of elastic parallel manipulators

In this section, the formation of a system as a tree structure for the simultaneous calculation of the direct dynamics of the elastic parallel manipulator – SCDD is suggested. It is the same idea as the one used for the inverse dynamics of the Lagrange-D'Alembert formulation. However, in this system the closed kinematic loop constraints of the elastic parallel structure are represented by forces and torques, just like in the case of the Lagrangian equations of the first type. These forces and torques are distributed in the tree structure such that they cause motion and internal forces which match the motion and mechanical stress in the structure of the original parallel manipulator.

5.1 Simultaneous Calculation of the Direct Dynamics (SCDD)

The equations of the tree structure have the known form shown in (19). These equations will now be factored into the equations of motion of the individual serial kinematic chains:

$$\begin{aligned}&\mathbf{M}_{ti}(\mathbf{q}_{ti})\ddot{\mathbf{q}}_{ti} + \mathbf{C}_{ti}(\dot{\mathbf{q}}_{ti},\mathbf{q}_{ti})\dot{\mathbf{q}}_{ti} + \boldsymbol{\eta}_{ti}(\mathbf{q}_{ti})\\ &+ \mathbf{J}_{ti}^{T}\mathbf{f}_{XYZi} + \mathbf{K}_{ti}\mathbf{q}_{ti} + \mathbf{D}_{ti}\dot{\mathbf{q}}_{ti} = \boldsymbol{\tau}_{ti}\end{aligned}, \tag{33}$$

for i=1 … n_a, where i designates one kinematic chain with the associated variables $\mathbf{q}_{ti}$ = [$\mathbf{q}_{ai}$ $\mathbf{q}_{pi}$ $\mathbf{q}_{ei}$] and torques τ_{ti} = [τ_{ai} τ_{pi} τ_{ei}] of the active τ_{ai} and passive τ_{pi} joints and additionally structure torques τ_{ei}. $\mathbf{f}_{XYZi}$ represents an external force acting on the end of the ith-branches of the tree structure. The equations of the direct dynamics of each such chain can then be formulated:

$$\begin{aligned}\ddot{\mathbf{q}}_{ti} &= \mathbf{M}_{ti}(\mathbf{q}_{ti})^{-1}(\boldsymbol{\tau}_{ti} - \mathbf{C}_{ti}(\dot{\mathbf{q}}_{ti},\mathbf{q}_{ti})\dot{\mathbf{q}}_{ti}\\ &- \boldsymbol{\eta}_{ti}(\mathbf{q}_{ti}) - \mathbf{K}_{ti}\mathbf{q}_{ti} - \mathbf{D}_{ti}\dot{\mathbf{q}}_{ti} - \mathbf{J}_{ti}^{T}\mathbf{f}_{XYZi})\end{aligned}, \tag{34}$$

for i=1 … n_a. Thus, the direct dynamics of each serial kinematic chain can be calculated. The input for each of these equations are the external forces acting on the end of the particular serial kinematic chain $\mathbf{f}_{XYZi}$ and the input torques τ_{ai} and τ_{pi}. The torques of the elastic DOF τ_{ei} result from the material properties like stiffness and damping. Additionally, they can be also produced by attached adaptronic actuators. They are independent. The input of the tree-structure (19) and of the compact parallel manipulator (20) is the torque vector τ_c. The virtual work of both systems is equal (10). The torques of the tree-structure are interdependent and result from the input torque vector. They represent the constraint torques/forces of the structure and the drive torques that induce the movement of the manipulator. The relation between these torques and the input torque vector is established in (17). However, before these torques can be calculated, one must first calculate the position (7), velocity (12) and after the differentiation of velocity, the acceleration of the redundant passive joints as a function of the active joints and the elastic DOF (Beyer, 1928, Stachera, 2005). This is done with the use of the closed kinematic loop constraints (8). Then from the equations of the reduced system (33) the partial matrices and vectors are taken, which are associated with the virtual torques of the passive joints:

$$\begin{aligned}\boldsymbol{\tau}_{pj} &= \mathbf{M}_{pj}(\mathbf{q}_{ti})\ddot{\mathbf{q}}_{ti} + \mathbf{C}_{pj}(\dot{\mathbf{q}}_{ti}, \mathbf{q}_{ti})\dot{\mathbf{q}}_{ti} \\ &+ \boldsymbol{\eta}_{pj}(\mathbf{q}_{ti}) + \mathbf{J}_{pj}^T \mathbf{f}_{XYZi} + \mathbf{D}_{pj}\dot{\mathbf{q}}_{ti}\end{aligned}, \tag{35}$$

for j=1 … n_p, i=1…n_a, where the j[th] passive joint belongs to the i[th] kinematic chains. Finally, the torques τ_a that arise from the computed virtual torques $\tau_p = [\tau_1 \ldots \tau_{np}]$ and from the drive torques τ_c of the original parallel structure can be calculated. For that, the Jacobian matrix (12) is used, which was already derived for the inverse dynamics (17) and (18). This matrix exists already in a symbolic form, which reduces the amount of work:

$$\boldsymbol{\tau}_a = \boldsymbol{\tau}_c - \left(\frac{\partial \mathbf{q}_p}{\partial \mathbf{q}_a^T}\right)^T \boldsymbol{\tau}_p . \tag{36}$$

Only the virtual torques (35) of the passive joints from all of the torques and forces in the robot's structure are used for the calculation of the torques τ_a of active joints. The influence of the torques and forces τ_e of the elastic DOF on the manipulator's movement is reflected in the coordinates of the elastic DOF and they were already used for the calculation of the virtual torques τ_p. These torques of the passive τ_p and active τ_a joints cause movement in the tree structure, that correspond to the movement of the original parallel manipulator, according to the D'Alembert principle of virtual work. In the compact equations of direct dynamics, the compact torques affect the active joints (20). These torques are accounted for by the torque and force distributions in the closed-link mechanism. For this reason, the compact torques should be applied to the active joints of the reduced tree structure in order to ensure the same operation conditions. Namely:

$$\begin{aligned}\boldsymbol{\tau}_a &= \boldsymbol{\tau}_c, \\ \boldsymbol{\tau}_p &= 0.\end{aligned} \tag{37}$$

In order to fulfill this condition, the new forces of the closed-loop constraints acting on the end of each i[th] –branch, must be calculated, and together with the drive torques supplied to the partial equations of direct dynamics (34). The difference between the acting torques of the compact manipulator and the acting torques of the tree structure amounts to:

$$\Delta\boldsymbol{\tau}_a = \boldsymbol{\tau}_c - \boldsymbol{\tau}_a = \left(\frac{\partial \mathbf{q}_p}{\partial \mathbf{q}_a^T}\right)^T \boldsymbol{\tau}_p . \tag{38}$$

The new constraints forces can be calculated:

$$\hat{\mathbf{f}}_{XYZi} = \mathbf{J}_{ti}^{-T}\left[\Delta\boldsymbol{\tau}_{ai} \quad -\boldsymbol{\tau}_{pi}\right], \tag{39}$$

for i=1 … n_a. Distribution of this force on the manipulator's structure imply the condition (37). Now the external forces acting on the end-effector of the manipulator have to be distributed between all the separate serial kinematic chains. The relation of static (27) and (29) will be used:

$$\mathbf{f}_{BXYZ} = \mathbf{U}\mathbf{S}^{-1}\mathbf{f}_{XYZ} , \tag{40}$$

where $\mathbf{f}_{BXYZ}=[\mathbf{f}_{XYZ1} \ldots \mathbf{f}_{XYZi} \ldots \mathbf{f}_{XYZna}]^T$. These forces (40) and the forces resulting from the constraints (39) form the common force acting on each serial kinematic chain. The final formulation for the forces takes the form:

$$\mathbf{f}_{XYZi} = \mathbf{f}_{XYZi} + \hat{\mathbf{f}}_{XYZi}\,, \tag{41}$$

for i=1 … n_a. The movement of the tree structure and the movement of the original parallel manipulator as well as the force and torques distribution in the structure are equal.
This algorithm can be summarized in the followings steps:

1. *Transformation of the system* in a reduced system and calculation of the direct dynamics for each serial kinematic chain separately – simultaneous (34). In order to compute these equations (in a calculation loop), the torques and forces resulting from the constraints and from the actuation have to be calculated first.
2. *Calculation of the trajectory* of the passive joints based on the non-redundant DOF (coordinates of the actuated joints and elastic DOF) and the constraints of the closed kinematic loops of the parallel structure.
3. *Calculation of the virtual torques* of the passive joints using the partial equations of the inverse dynamics of serial kinematic chains (35) and the difference between the torques of the actuated joints of the reduced system and the original manipulator (38).
4. *Calculation of the forces of constraints* for each serial kinematic chain from the virtual torques of the passive joints and the torque differences (39).
5. *Fusion of the forces of constraints* with the external forces acting on the end of each kinematic chain (41). Setting of the torques and forces of the reduced system (34) to those of the original parallel manipulator (37).

5.2 Features of the new method

In the Method - *Simultaneous Calculation of the Direct Dynamics*, SCDD – the system is segmented into a tree structure, as in the case of the inverse dynamics of Lagrange-D'Alembert formulation. This is done in order to accelerate the inversion of the inertia matrix. The most frequently used method, the LU-Gaussian elimination, has the complexity $0(n^3)$. For the symmetrical manipulator's structure with only the rotational joints the complexity can be written as $O((n_a+n_a n_{ek})^3)$, where n_{ek} means the number of the elastic DOF in particular kinematic chain. In comparison, the complexity for the new distributed calculation performed for the same type of robots amounts to $O(n_a(1+n_{pk}+n_{ek})^3)$, where n_{pk} represents the number of the passive joints in one kinematic chain. For complex systems the relation $n_{pk} \ll n_{ek}$ is valid. The avoidance of the multiplication between n_a and n_{ek} under the power of three reduces the computational effort. Therefore, the computation speed of the direct dynamics in joint space of large scale systems can be significantly accelerated by using several small matrices instead of one complex matrix. Additionally, the computations of the direct dynamics with this decomposition can be performed in parallel. In the Table 1 the complexity of the matrix inversion, number of the necessary arithmetical operations, for three different robots from the *Collaborative Research Center 562* is shown (Hesselbach et al., 2005). These calculations were done, with the assumption that in each kinematic chain one elastic DOF n_{ek} exists.
These results show considerable reduction of the calculation complexity by using the proposed algorithm, even with only one additional elastic DOF in each kinematic chain. Therefore each kinematic chain can be modelled with more parameters, what is a common procedure for elastic manipulators.

	n_a	n_{pk}	n_{ek}	SCDD	L-D'A	Reduction
FIVE-BAR	2	1	1	54	64	16 %
HEXA	6	2	1	384	1728	78 %
TRIGLIDE	3	2	1	162	729	78 %

Table 1. Complexity of the matrix inversion

Also the calculation of the direct dynamics of rigid body parallel manipulators can benefit from this new method. The reduced form of the dynamics' equations can decrease the number of arithmetic operations needed for the calculation of the model. This problem was investigated on the base of rigid parallel manipulator FIVE-BAR (Stachera, 2006b). The model derived with this new method was compared with a model gained with the standard Lagrange-D'Alembert Formulation. Since it is a comparison study the exact form of the manipulator's model is here not important. The symbolic equations were derived and simplified with the use of Mathematica®. All the operations and transformations that are necessary for the computations of the direct dynamics have been considered.

Operation Type	Number of the operations		Reduction
	SCDD	L-D'A	
+	192	670	71 %
-	80	302	74 %
*	432	2482	83 %
/	38	36	-6 %

Table 2. Complexity of the arithmetic operations

The results presented in the Table 2 show a considerable reduction of the computational effort for each kind of operation excepting division (increasing about 6 %). A digital processor needs many machine steps for the multiplication, therefore the reduction of this operation's number is essential for the general reduction of the computational power for a model computation. In this case a reduction of 84 % was achieved. This confirms the applicability of this procedure for the effective reduction of computing power even for a rigid parallel manipulators.

5.3 Verification

The new SCDD method was compared with the L-D'A method in simulation. A model of elastic planar parallel manipulators FIVE-BAR was created. A lumped elasticity $c_L = c_R = 5.464 \cdot 10^6$ *N/m* was considered in the upper arms of the manipulator, shown in Fig. 2. M_L and M_R represent the motors. The other parameters of this model are not relevant, since it is a comparison study. A straight line trajectory between two points p^A and p^E was chosen. The models were then controlled by torques, which were created by a rigid body model without control. The black line represents the reference trajectory. The dark gray line is a result of the L-D'A model and the light gray line from the SCDD model. It can be seen, that the models both follow the trajectory with comparable accuracy.

For better comparison of these models, the same trajectories are expressed now with the help of the forces induced in the branches, F_L in the left branch and F_R in the right one, of the parallel manipulator, shown in Fig.3. A small difference between these forces can be noted. At the beginning of the simulation the differences are equal to zero, but with the time they change. Apart from the difference between these forces, a good agreement in the vibrations' behaviour of both systems, frequency, amplitude and phase, can be observed, which confirms the new proposed method.

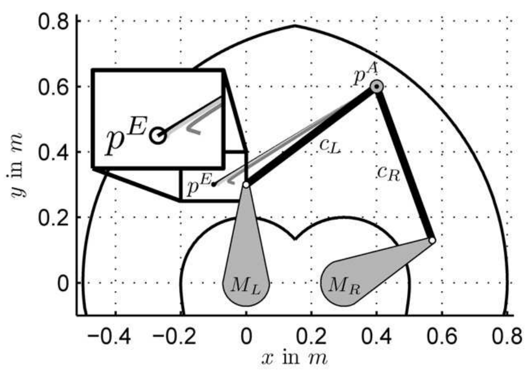

Fig. 2. Trajectory and workspace of elastic planar parallel manipulator FIVE-BAR

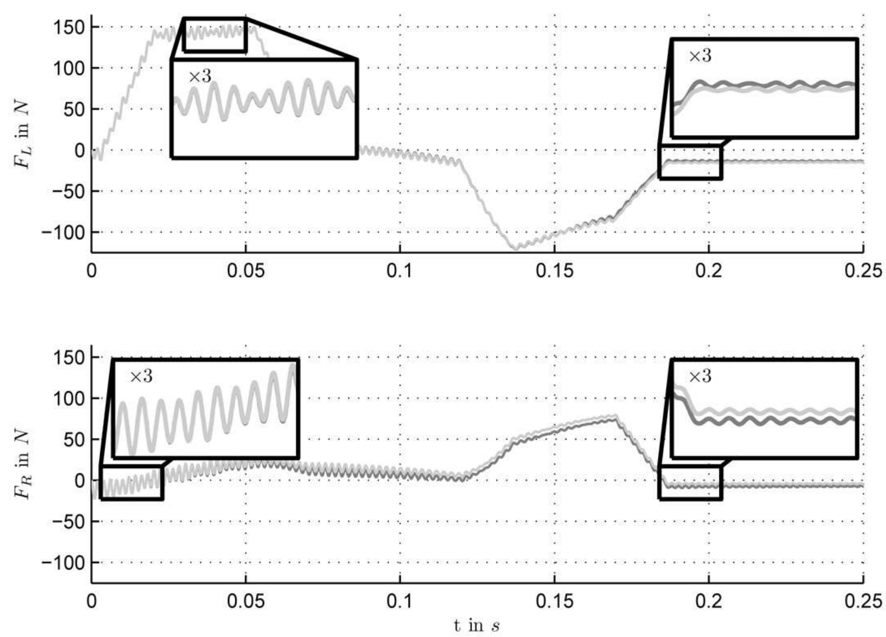

Fig. 3. Force in the active rods of the parallel manipulator - comparison

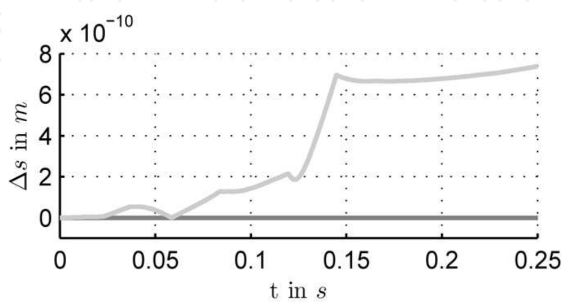

Fig. 4. Distance between two kinematic chains of FIVEBAR – numerical error of SCDD

The existing differences between the paths traveled by these two elastic models and the induced forces can be accounted for by the numerical precision. Fig. 4 shows the distance between the end points of both kinematic chains. This numerical precision causes the increase in time of the distance between two kinematic chains, that should have been equal to zero. The dark gray line $\Delta s = 1 \cdot 10^{-14}$ *m* shows the L-D'A and the light gray the SCDD model. The error is dependent on the sample interval of the simulation: the smaller the interval, the smaller the error. In the field of numerical methods algorithms are known that deal with the stabilization of the numerical calculation and increasing of the computation accuracy (Baumgarte, 1972), which will be the next step in the investigation of this new algorithm. Despite this numerical error, the analytical approach is confirmed by these presented results.

6. Conclusion

In this chapter, the Lagrange equation of the first type and Lagrange-D'Alembert Formulation were introduced around the consideration of elastic modes. To complete the standard method of Lagrange-D'Alembert, an algorithm for the derivation of the Jacobian matrix of the parallel manipulator based on the Jacobian matrices of the individual serial kinematic chains was presented. Originating from this knowledge, a new method was presented for the simultaneous calculation of the direct dynamics of the parallel and furthermore the elastic parallel manipulators. The new method shows a significant reduction of the complexity of the calculation, even for the rigid body manipulators. For the sophisticated systems this feature is a great advantage. The disadvantage of the presented method is the numerical stability over long periods of time, which will therefore be the topic of future researches.

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