Chapter 10

Adaptive Gain Robust Control Strategies for Uncertain Dynamical Systems

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Additional information is available at the end of the chapter

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Abstract

In this chapter, adaptive gain robust control strategies for uncertain dynamical systems are presented. Firstly, synthesis of centralized adaptive gain robust controllers for a class of uncertain linear systems is shown. The design problem of the centralized controller is reduced to the constrained convex optimization problem, and allowable perturbation regions of unknown parameters are discussed. Next, the result for the centralized robust controller is extended to uncertain large-scale interconnected systems, that is, an LMI-based design approach for decentralized adaptive gain robust controllers is suggested.

Keywords: adaptive gain robust control, adjustable time-varying parameter, allowable perturbation regions of unknown parameters, LMIs

1. Introduction

It is well known that control systems can be found in abundance in all sectors of industry such as robotics, power systems, transportation systems space technologies, and many others, and thus control theory has been well studied. In order to design control systems, designers have to derive mathematical models for dynamical systems, and there are mainly two types of representations for mathematical models, that is, transfer functions and state equations. In other words, control theory is divided into "classical control" and "modern control" (e.g., see [12]).

Classical control means an analytical theory based on transfer function representations and frequency responses, and for classical control theory, we can find a large number of useful and typical results such as Routh-Hurwitz stability criterion [20] based on characteristic equations in the nineteenth century, Nyquist criterion [28] in the 1930s, and so on. Moreover, by using classical control ideas, some design methods of controllers such as proportional, derivative, and integral (PID) controllers and phase lead-lag compensators have also been presented [21]. In classical control, controlled systems are mainly linear and time-invariant and have a single input and a single output only. Furthermore, it is well known that design approaches based on classical control theory need experiences and trial and error. On the other hand, in the 1960s, state variables and state equations (i.e., state-space representations) have been introduced by Kalman as system representations, and he has proposed an optimal regulator theory [14–16] and an optimal filtering one [17]. Namely, controlled systems are represented by state equations, and controller design problems are reduced to optimization problems based on the concept of state variables. Such controller design approach based on the state-space representation has been established as "modern control theory." Modern control is a theory of time domain, and whereas the transfer function and the frequency response are of limited applicability to nonlinear systems, state equations and state variables are equally appropriate to linear multi-input and multi-output systems or nonlinear one. Therefore, many existing results based on the state-space representation for controller design problems have been suggested (e.g., [7, 43]).

Now, as mentioned above, in order to design control systems, the derivation of a mathematical model for controlled system based on state-space representation is needed. If the mathematical model describes the controlled system with sufficient accuracy, a satisfactory control performance is achievable by using various controller design methods. However, there inevitably exists some gaps between the controlled system and its mathematical model, and the gaps are referred to as "uncertainties." The uncertainties in the mathematical model may cause deterioration of control performance or instability of the control system. From this viewpoint, robust control for dynamical systems with uncertainties has been well studied, and a large number of existing results for robust stability analysis and robust stabilization have been obtained [34, 36, 47, 48]. One can see that quadratic stabilization based on Lyapunov stability criterion and $\mathcal{H}^\infty$ control is a typical robust controller (e.g., [1, 6]). Furthermore, some researchers investigated quadratic stabilizing control with an achievable performance level in Ref. to such as a quadratic cost function [23, 28, 35, 37], robust $\mathcal{H}^2$ control [18, 39], and robust $\mathcal{H}^\infty$-type disturbance attenuation [46]. However, these approaches result in worst-case design, and, therefore, these controllers with a fixed feedback gain which is designed by considering the worst-case variations of uncertainties/unknown parameters become cautious when the perturbation region of uncertainties has been estimated larger than the proper region. In contrast with the conventional robust control with fixed gains, several design methods of some robust controllers with time-varying adjustable parameters have also been proposed (e.g., [3, 24, 36]). In the work of Maki and Hagino [25], by introducing time-varying adjustable parameters, adaptation mechanisms for improving transient behavior have been suggested. Moreover, robust controllers with adaptive compensation inputs have also been shown [29–31]. In particular, for linear systems with matched uncertainties, Oya and Hagino [29] have introduced an adaptive compensation input which is determined so as to reduce the effect of unknown parameters. Furthermore, a design method of a variable gain robust controller based on LQ optimal control for a class of uncertain linear system has also been shown [32]. These robust controllers have fixed gains and variable ones tuned by updating laws and are more flexible and adaptive compared with the conventional robust controllers with fixed gains only, and one can easily see that these robust controllers with adjustable parameters differ from gain-scheduling control techniques [22, 41, 42]. Additionally, these robust controllers with time-varying adjustable parameters may also be referred to as "variable gain robust controller" or "adaptive gain robust controller."

In recent years, a great number of control systems are brought about by present technologies and environmental and societal processes which are highly complex and large in dimension, and such systems are referred to as "large-scale complex systems" or "large-scale interconnected systems." Namely, large-scale and complex systems are progressing due to the rapid development of industry, and large-scale interconnected systems can be seen in diverse fields such as economic systems, electrical systems, and so on. For such large-scale interconnected systems, it is difficult to apply centralized control strategies because of calculation amount, physical communication constraints, and so on. Namely, a notable characteristic of the most large-scale interconnected systems is that centrality fails to hold due to either the lack of centralized computing capability of or centralized information. Moreover, large-scale interconnected systems are controlled by more than one controller or decision-maker involving decentralized computation. In the decentralized control strategy, large-scale interconnected systems are divided into several subsystems, and various types of decentralized control problems have been widely studied [13, 38, 44]. The major problem of large-scale interconnected systems is how to deal with the interactions among subsystems. A large number of results in decentralized control systems can be seen in the work of Šijjak [38]. Moreover, a framework for decentralized fault-tolerant control has also been studied [44]. Additionally, decentralized robust control strategies for uncertain large-scale interconnected systems have also attracted the attention of many researchers (e.g., [3–5, 11]). Moreover, in the work of Mao and Lin [24], for large-scale interconnected systems with unmodeled interaction, the aggregative derivation is tracked by using a model following the technique with online improvement, and a sufficient condition for which the overall system when controlled by the completely decentralized control is asymptotically stable has been established. Furthermore, decentralized guaranteed cost controllers for uncertain large-scale interconnected systems have also been suggested [26, 27].

In this chapter, for a class of uncertain linear systems, we show LMI-based design strategies for adaptive gain robust controllers for a class of uncertain dynamical systems. The adaptive gain robust controllers consist of fixed gains and adaptive gains which are tuned by time-varying adjustable parameters. The proposed adaptive gain robust controller can achieve asymptotical stability but also improving transient behavior of the resulting closed-loop system. Moreover, by adjusting design parameters, the excessive control input is avoided [32]. In this chapter, firstly, a design method of the centralized adaptive gain robust stabilizing controllers for a class of uncertain linear systems has been shown, and the maximum allowable perturbation region of uncertainties is discussed. Namely, the proposed adaptive gain robust controllers can achieve robustness for the derived perturbation regions for unknown parameters. Additionally, the result for the centralized adaptive gain robust stabilizing controllers is extended to the design problem of decentralized robust control systems.

The contents of this chapter are as follows, where the item numbers in the list accord with the section numbers:

2. Synthesis of centralized adaptive gain robust controllers.

3. Synthesis of decentralized adaptive gain robust controllers.

4. Conclusions and future works.

The basic symbols are listed below.

$\mathbb{R}$	The set of the real number
$\mathbb{R}^{\mathrm{n}}$	The set of n-dimensional vectors
$\mathbb{R}^{n\times m}$	The set of $n \times m$-dimensional matrices
$\mathbb{C}$	The set of complex numbers

Other than the above, we use the following notation and terms: For a matrix $\mathcal{A}$, the transpose of matrix A and the inverse of one are denoted by $\mathcal{A}^T$ and $\mathcal{A}^{-1}$, respectively. The notations $H_e\{\mathcal{A}\}$ and $\mathrm{diag}(\mathcal{A}_1, \cdots, \mathcal{A}_\mathcal{N})$ represent $\mathcal{A} + \mathcal{A}^T$ and a block diagonal matrix composed of matrices $\mathcal{A}_i$ for $i = 1, \cdots, \mathcal{N}$. The n-dimensional identity matrix and $n \times m$-dimensional zero matrix are described by I_n and $0_{n\times m,}$ and for real symmetric matrices $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{A} > \mathcal{B}$ $(\text{resp. } \mathcal{A} \geq \mathcal{B})$ means that $\mathcal{A} - \mathcal{B}$ is a positive (resp. nonnegative) definite matrix. For a vector $\alpha \in \mathbb{R}^{\mathrm{n}}$, $||\alpha||$ denotes standard Euclidian norm, and for a matrix $\mathcal{A}$, $||\mathcal{A}||$ represents its induced norm. The real part of a complex number s (i.e., $s \in \mathbb{C}$) is denoted by $\mathbb{R}_{\mathrm{e}}\{s\}$, and the symbols "$\stackrel{\Delta}{=}$" and "$\star$" mean equality by definition and symmetric blocks in matrix inequalities, respectively.

Furthermore, the following useful lemmas are used in this chapter.

Lemma 1.1. *For arbitrary vectors λ and ξ and the matrices $\mathcal{G}$ and $\mathcal{H}$ which have appropriate dimensions, the following relation holds:*

$$2\lambda^T \mathcal{G}\Delta(t)\mathcal{H}\xi \leq 2\left\|\mathcal{G}^T\lambda\right\| \| \mathcal{H}\xi \|,$$

where $\Delta(t) \in \mathbb{R}^{p\times q}$ is a time-varying unknown matrix satisfying $\| \Delta(t) \| \leq 1$.

Proof. The above relation can be easily obtained by Schwartz's inequality (see [9]).

Lemma 1.2. *(Schur complement) For a given constant real symmetric matrix Ξ, the following arguments are equivalent:*

(i) $\Xi = \begin{pmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{12}^T & \Xi_{22} \end{pmatrix} > 0.$

(ii) $\Xi_{11} > 0$ *and* $\Xi_{22} - \Xi_{12}^T \Xi_{11}^{-1} \Xi_{12} > 0.$

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(iii) $\Xi_{22} > 0$ *and* $\Xi_{11} - \Xi_{12}\Xi_{22}^{-1}\Xi_{12}^{T} > 0$.

Proof. See Boyd et al. [2].

2. Synthesis of centralized adaptive gain robust controllers

A centralized adaptive gain robust state feedback control scheme for a class of uncertain linear systems is proposed in this section. The adaptive gain robust controller under consideration is composed of a state feedback with a fixed gain matrix and a time-varying adjustable parameter. In this section, we show an LMI-based design method of the adaptive gain robust state feedback controller, and the allowable perturbation region of unknown parameters is discussed.

2.1. Problem statement

Consider the uncertain linear system described by the following state-space representation:

$$\frac{d}{dt}\mathrm{x}(t) = (A + \Delta(t))\mathrm{x}(t) + Bu(t), \tag{1}$$

where $\mathrm{x}(t) \in \mathbb{R}^{\mathrm{n}}$ and $u(t) \in \mathbb{R}^{\mathrm{m}}$ are the vectors of the state (assumed to be available for feedback) and the control input, respectively. In Eq. (1) the constant matrices A and B mean the nominal values of the system, and (A, B) is stabilizable pair. Moreover, the matrix $\Delta(t) \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ represents unknown time-varying parameters which satisfy $\Delta^{T}(t)\Delta(t) \le \delta^{\star} I_n$, and the elements of $\Delta(t) \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ are Lebesgue measurable [1, 34]. Namely, the unknown time-varying matrix $\Delta(t) \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ is bounded, and the parameter $\delta^{\star}$ denotes the upper bound of the perturbation region for the unknown parameter $\Delta(t) \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}}$. Additionally, we suppose that the nominal system which can be obtained by ignoring the unknown parameter $\Delta(t)$ in Eq. (1) is given by

$$\frac{d}{dt}\overline{\mathrm{x}}(t) = A\overline{\mathrm{x}}(t) + B\overline{u}(t). \tag{2}$$

In Eq. (2), $\overline{\mathrm{x}}(t) \in \mathbb{R}^{\mathrm{n}}$ and $\overline{u}(t) \in \mathbb{R}^{\mathrm{m}}$ are the vectors of the state and the control input for the nominal system, respectively.

First of all, we design the state feedback control for the nominal system of Eq. (2) so as to generate the desirable transient behavior in time response for the uncertain linear system of Eq. (1). Namely, the nominal control input is given as

$$\overline{u}(t) = K\overline{\mathrm{x}}(t), \tag{3}$$

and thus the following nominal closed-loop system is obtained:

$$\frac{d}{dt}\overline{\mathrm{x}}(t) = A_K\overline{\mathrm{x}}(t), \tag{4}$$

where A_K is a matrix given by $A_K \stackrel{\Delta}{=} A + BK$. Note that the standard LQ control theory for the nominal system of Eq. (2) for designing the fixed feedback gain $K \in \mathbb{R}^{m \times n}$ is adopted in the existing result [32]. In this section, for the nominal system of Eq. (2), we derive a state feedback controller with pole placement constraints [8]. Note that for simplicity the sector constraints are introduced only in this chapter, and of course, one can adopt some other design constraints or another controller design approach for designing the fixed gain matrix $K \in \mathbb{R}^{m \times n}$. Therefore, we consider the matrix inequality condition:

$$(A_K + \alpha I_n)^T \mathcal{P} + \mathcal{P}(A_K + \alpha I_n) + \mathcal{Q} < 0, \tag{5}$$

where $\mathcal{P} \in \mathbb{R}^{n \times n}$ and $\mathcal{Q} \in \mathbb{R}^{n \times n}$ are a symmetric positive definite matrix and a symmetric semi-positive definite matrix, respectively, and the matrix $\mathcal{Q} \in \mathbb{R}^{n \times n}$ is selected by designers. If the symmetric positive definite matrix $\mathcal{P} \in \mathbb{R}^{n \times n}$ satisfying the matrix inequality of Eq. (5) exists, then poles for the nominal closed-loop system of Eq. (4) are located into the subspace $\mathcal{S}_\alpha = \{s | \mathbb{R}_e\{s\} \leq -\alpha\}$ in the complex plane. Namely, the nominal closed-loop system of Eq. (4) is asymptotically stable, and the quadratic function $\mathcal{V}(\overline{x}, t) \stackrel{\Delta}{=} \overline{x}^T(t)\mathcal{P}\overline{x}(t)$ becomes a Lyapunov function for the nominal closed-loop system of Eq. (4), because the time derivative of the quadratic function $\mathcal{V}(\overline{x}, t)$ can be expressed as

$$\begin{aligned} \frac{d}{dt}\mathcal{V}(\overline{x}, t) &< -\overline{x}^T(t)(\mathcal{Q} + 2\alpha\mathcal{P})\overline{x}(t) \\ &< 0, \quad \forall\, \overline{x}(t) \neq 0. \end{aligned} \tag{6}$$

Now, we introduce complementary matrices $\mathcal{Y} \in \mathbb{R}^{n \times n}$ and $\mathcal{W} \in \mathbb{R}^{m \times m}$ which satisfy the relations $\mathcal{Y} \stackrel{\Delta}{=} \mathcal{P}^{-1}$, $K = -\mathcal{W}B^T\mathcal{P}$, and $\mathcal{W} = \mathcal{W}^T > 0$, respectively. Then, some algebraic manipulations gives

$$\mathcal{Y}A^T + A\mathcal{Y} - B\mathcal{W}^T B^T - B\mathcal{W}B^T + 2\alpha\mathcal{Y} + \mathcal{Y}\mathcal{Q}\mathcal{Y} < 0. \tag{7}$$

Additionally, applying **Lemma 1.2** (Schur complement) to Eq. (7), one can easily see that the matrix inequality condition of Eq. (7) is equivalent to

$$\begin{pmatrix} \mathcal{Y}A^T + A\mathcal{Y} - B\mathcal{W}^T B^T - B\mathcal{W}B^T + 2\alpha\mathcal{Y} & \mathcal{Y} \\ \star & -\mathcal{Q}^{-1} \end{pmatrix} < 0. \tag{8}$$

Thus, the control gain matrix $K \in \mathbb{R}^{m \times n}$ is determined as $K = -\mathcal{W}B^T\mathcal{P} = -\mathcal{W}B^T\mathcal{Y}^{-1}$.

Now, for the uncertain linear system of Eq. (1), we define the following control input [37]:

$$u(t) \stackrel{\Delta}{=} (1 + \theta(x, t))Kx(t), \tag{9}$$

where $\theta(x, t)$: $\mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ is an adjustable time-varying parameter [32] which plays the important role for correcting the effect of uncertainties, that is, the control input $u(t) \in \mathbb{R}^m$

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consists of a fixed gain matrix $K \in \mathbb{R}^{m \times n}$ and $\theta(x, t) \in \mathbb{R}$. Note that, the robust control input of the form of Eq. (9) is called "adaptive gain robust control" in this chapter. Thus, from Eqs. (1) and (9), the uncertain closed-loop system can be written as

$$\frac{d}{dt}x(t) = A_K x(t) + \Delta(t)x(t) + \theta(x, t)BKx(t). \tag{10}$$

From the above, the control objective in this section is to design the adaptive gain robust control which achieves satisfactory transient behavior. Namely, the control problem is to derive the adjustable time-varying parameter $\theta(x, t) \in \mathbb{R}$ such that the closed-loop system of Eq. (10) can achieve the desired transient response. In addition, we evaluate the allowable perturbation region of the unknown parameter $\Delta(t) \in \mathbb{R}^{n \times n}$.

2.2. Synthesis of centralized adaptive gain robust state feedback controllers

In this subsection, we deal with design problems for the adjustable time-varying parameter $\theta(x, t) \in \mathbb{R}$ so that the satisfactory transient response for the uncertain linear system of Eq. (1) can be achieved. For the proposed adaptive gain robust control, the following theorem gives an LMI-based design synthesis.

Theorem 1: *Consider the uncertain linear system of Eq. (1) and the adaptive gain robust control of Eq. (9) with the adjustable time-varying parameter $\theta(x, t) \in \mathbb{R}$.*

For a given design parameter $\vartheta > 0$ and the known upper bound $\delta^\star$ for the unknown parameter $\Delta(t) \in \mathbb{R}^{n \times n}$, if the scalar parameter $\gamma > 0$ exists satisfying

$$\begin{pmatrix} A_K^T \mathcal{P} + \mathcal{P} A_K + \gamma \mathcal{P}^2 & I_n \\ \star & -\dfrac{\gamma}{\delta^\star} I_n \end{pmatrix} < 0, \tag{11}$$

the adjustable time-varying parameter $\theta(x, t) \in \mathbb{R}$ is determined as

$$\theta(x, t) = \begin{cases} \dfrac{\sqrt{\delta^\star} \| \mathcal{P}x(t) \| \| x(t) \|}{\| \mathcal{W}^{1/2} B^T \mathcal{P} x(t) \|^2} & if \quad x^T(t)\mathcal{P}B\mathcal{W}B^T\mathcal{P}x(t) \geq \vartheta x^T(t)x(t), \\ \\ \dfrac{\sqrt{\delta^\star} \| \mathcal{P}x(t) \| \| x(t) \|}{\vartheta x^T(t)x(t)} & if \quad x^T(t)\mathcal{P}B\mathcal{W}B^T\mathcal{P}x(t) < \vartheta x^T(t)x(t). \end{cases} \tag{12}$$

Then, the uncertain closed-loop system of Eq. (10) is asymptotically stable.

Proof. In order to prove Theorem 1, by using symmetric positive definite matrix $\mathcal{P} \in \mathbb{R}^{n \times n}$ which satisfies the standard Riccati equation of Eq. (4), we introduce the quadratic function

$$\mathcal{V}(x, t) \stackrel{\Delta}{=} x^T(t)\mathcal{P}x(t), \tag{13}$$

as a Lyapunov function candidate. Let $\mathrm{x}(t)$ be the solution of the uncertain closed-loop system of Eq. (10) for $t \geq t_0$, and then the time derivative of the quadratic function $V(\mathrm{x}, t)$ along the trajectory of the uncertain closed-loop system of Eq. (10) can be written as

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(\mathrm{x}, t) &= \mathrm{x}^T(t)\left(A_K^T\mathcal{P} + \mathcal{P}A_K\right)\mathrm{x}(t) \\ &+2\mathrm{x}^T(t)\mathcal{P}\Delta(t)\mathrm{x}(t) + 2\theta(\mathrm{x}, t)\mathrm{x}^T(t)\mathcal{P}BK\mathrm{x}(t).\end{aligned} \tag{14}$$

Firstly, the case of $\mathrm{x}^T(t)\mathcal{P}B\mathcal{W}B^T\mathcal{P}\mathrm{x}(t) \geq \vartheta\mathrm{x}^T(t)\mathrm{x}(t)$ is considered. In this case, one can see from the relation $\| \Delta(t) \| \leq \sqrt{\delta^\star}$, Eq. (14), and **Lemma 1.1** that the following inequality holds:

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(\mathrm{x}, t) &\leq \mathrm{x}^T(t)\left(A_K^T\mathcal{P} + \mathcal{P}A_K\right)\mathrm{x}(t) + 2\sqrt{\delta^\star}\| \mathcal{P}\mathrm{x}(t) \|\| \mathrm{x}(t) \| \\ &+2\theta(\mathrm{x}, t)\mathrm{x}^T(t)\mathcal{P}BK\mathrm{x}(t).\end{aligned} \tag{15}$$

Moreover, since the relation $K = -\mathcal{W}B^T\mathcal{P}$ holds, the inequality of Eq. (15) can be rewritten as

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(\mathrm{x}, t) &\leq \mathrm{x}^T(t)\left(A_K^T\mathcal{P} + \mathcal{P}A_K\right)\mathrm{x}(t) + 2\sqrt{\delta^\star}\| \mathcal{P}\mathrm{x}(t) \|\| \mathrm{x}(t) \| \\ &-2\theta(\mathrm{x}, t)\mathrm{x}^T(t)\mathcal{P}B\mathcal{W}B^T\mathcal{P}\mathrm{x}(t).\end{aligned} \tag{16}$$

Substituting the adjustable time-varying parameter $\theta(x, t)$ of Eq. (12) into Eq. (16) gives

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(\mathrm{x}, t) &\leq \mathrm{x}^T(t)\left(A_K^T\mathcal{P} + \mathcal{P}A_K\right)\mathrm{x}(t) + 2\sqrt{\delta^\star}\| \mathcal{P}\mathrm{x}(t) \|\| \mathrm{x}(t) \| \\ &-2\mathrm{x}^T(t)\mathcal{P}\left(\frac{\sqrt{\delta^\star}\| \mathcal{P}\mathrm{x}(t) \|\| \mathrm{x}(t) \|}{\| \mathcal{W}^{1/2}B^T\mathcal{P}\mathrm{x}(t) \|^2}\right)B\mathcal{W}B^T\mathcal{P}\mathrm{x}(t) \\ &\leq \mathrm{x}^T(t)\left(A_K^T\mathcal{P} + \mathcal{P}A_K\right)\mathrm{x}(t).\end{aligned} \tag{17}$$

If the solution of the LMI of Eq. (11) exists, then the inequality

$$A_K^T\mathcal{P} + \mathcal{P}A_K < 0 \tag{18}$$

is satisfied. Thus, one can see that the following relation holds:

$$\frac{d}{dt}\mathcal{V}(\mathrm{x}, t) < 0, \quad \forall \mathrm{x}(t) \neq 0. \tag{19}$$

Next, we consider the case of $\mathrm{x}^T(t)\mathcal{P}B\mathcal{W}B^T\mathcal{P}\mathrm{x}(t) < \vartheta\mathrm{x}^T(t)\mathrm{x}(t)$. By using the well-known inequality for any vectors α and β with appropriate dimensions and a positive scalar ζ

$$2\alpha^T\beta \leq \zeta\alpha^T\alpha + \frac{1}{\zeta}\beta^T\beta, \tag{20}$$

we see from Eq. (14) that some algebraic manipulations give

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(\mathrm{x},t) &\leq \mathrm{x}^T(t)\left(A_K^T\mathcal{P}+\mathcal{P}A_K\right)\mathrm{x}(t)+\gamma\mathrm{x}^T(t)\mathcal{P}^2\mathrm{x}(t)+\frac{1}{\gamma}\mathrm{x}^T(t)\Delta^T(t)\Delta(t)\mathrm{x}(t)\\ &\quad +2\theta(\mathrm{x},t)\mathrm{x}^T(t)\mathcal{P}BK\mathrm{x}(t)\\ &\leq \mathrm{x}^T(t)\left(A_K^T\mathcal{P}+\mathcal{P}A_K+\gamma\mathcal{P}^2+\frac{\delta^\star}{\gamma}I_n\right)\mathrm{x}(t)+2\theta(\mathrm{x},t)\mathrm{x}^T(t)\mathcal{P}BK\mathrm{x}(t)\end{aligned} \tag{21}$$

where γ is a positive constant.

Let us consider the last term of the right-hand side of Eq. (21). We see from Eq. (12) and the relation $K=-\mathcal{W}B^T\mathcal{P}$ that the last term of the right-hand side of Eq. (21) is nonpositive. Thus, if the scalar parameter γ exists satisfying

$$A_K^T\mathcal{P}+\mathcal{P}A_K+\gamma\mathcal{P}^2+\frac{\delta^\star}{\gamma}I_n<0, \tag{22}$$

then the following relation for the quadratic function $\mathcal{V}(\mathrm{x},t)$ holds:

$$\frac{d}{dt}\mathcal{V}(\mathrm{x},t)<0, \quad \forall\mathrm{x}(t)\neq 0. \tag{23}$$

Furthermore, applying **Lemma 1.2** (Schur complement) to Eq. (22), we find that the matrix inequality condition of Eq. (22) can be transformed into the LMI of Eq. (11). Namely, the quadratic function $\mathcal{V}(\mathrm{x},t)$ of Eq. (13) becomes a Lyapunov function of the uncertain closed-loop system of Eq. (10) with the adjustable time-varying parameter of Eq. (12), that is, asymptotical stability of the uncertain closed-loop system of Eq. (10) is ensured. It follows that the result of this theorem is true.

From the above, we show an LMI-based design strategy for the proposed adaptive gain robust control. Namely, the design problem of the proposed adaptive gain robust controller can be reduced to the feasibility of the LMI of Eq. (11). Note that the LMI of Eq. (11) defines a convex solution set of γ, and therefore one can easily see that various efficient convex optimization algorithms can be used to test whether the LMI is solvable and to generate particular solution. Furthermore, the LMI of Eq. (11) can also be exploited to design the proposed adaptive gain robust controller with some additional requirements. Thus, in this paper, we consider the allowable region of the unknown parameter $\Delta(t)\in\mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ and introduce the additional constraints $\gamma=\delta^\star$ and

$$\gamma-\frac{1}{\varepsilon}>0, \tag{24}$$

where ε is a positive constant. From the relation of Eq. (24), we find that the minimization of the parameter ε means the maximization of the upper bound $\delta^\star$. Then, by using Lemma 2 (Schur complement), we find that the LMI of Eq. (11) is equivalent to

$$A_K^T\mathcal{P}+\mathcal{P}A_K+\gamma\mathcal{P}^2+I_n<0, \tag{25}$$

and the constraint of Eq. (24) can be transformed into

$$\begin{pmatrix} \gamma & 1.0 \\ \star & \varepsilon \end{pmatrix} > 0. \tag{26}$$

From the above, we consider the following constrained optimization problem:

$$\underset{\gamma>0,\,\varepsilon}{\text{Minimize}}\,[\varepsilon] \ \text{ subject to } \ (25) \ \text{ and } \ (26). \tag{27}$$

If the optimal solution of the constrained optimization problem of Eq. (27) exists, in which are denoted by $\gamma^\star$ and $\varepsilon^\star$, the proposed adaptive gain robust controller can be done, and the allowable upper bound of the unknown parameter $\Delta(t) \in \mathbb{R}^{n\times n}$ is given by

$$\delta^\star = \gamma^\star. \tag{28}$$

Consequently, the following theorem for the proposed adaptive gain robust control with guaranteed allowable region of unknown parameter $\Delta(t) \in \mathbb{R}^{n\times n}$ is developed.

Theorem 2: *Consider the uncertain linear system of Eq. (1) and the adaptive gain robust control of Eq. (8) with the adjustable time-varying parameter $\theta(\mathrm{x}, t) \in \mathbb{R}$.*

If the optimal solution $\gamma^\star$ of the constrained optimization problem of Eq. (27) exists, then the adjustable time-varying parameter $\theta(\mathrm{x}, t) \in \mathbb{R}$ is designed as Eq. (12), and asymptotical stability of the uncertain closed-loop system of Eq. (10) is ensured. Moreover, the upper bound $\delta^\star$ for the unknown parameter $\Delta(t) \in \mathbb{R}^{n\times n}$ is given by Eq. (28).

Remark 1: *In this section, the uncertain linear dynamical system of Eq. (1) is considered, and the centralized adaptive gain robust controller has been proposed. Although the uncertain linear system of Eq. (1) has uncertainties in the state matrix only, the proposed adaptive gain robust controller can also be applied to the case that the uncertainties are included in both the system matrix and the input one. Namely, by introducing additional actuator dynamics and constituting an augmented system, unknown parameters in the input matrix are embedded in the system matrix of the augmented system [45]. As a result, the proposed controller design procedure can be applied to such case.*

Remark 2: *In* Theorem 1*, the design problem of the proposed adaptive gain robust controller can be reduced to the feasibility of the LMI of Eq. (11). Namely, in order to design the proposed robust control system, designers have to solve the LMI of Eq. (11). If the LMI of Eq. (11) is feasible for $\exists\delta^\star > 0$, then one can easily see that the LMI of Eq. (11) is always satisfied for the positive scalar $\forall\delta^- < \delta^\star$. Moreover, if a positive scalar γ exists satisfying the LMI of Eq. (11) for $\exists\delta^+ > \delta^\star$, then the proposed adaptive gain robust controller can also be designed, and note that the adaptive gain robust controller for $\delta^\star > 0$ coincides exactly with the one for $\delta^+ > \delta^\star > 0$. Furthermore, one can see from* Theorem 2 *that the resultant adaptive gain robust controller derived by solving the constrained convex optimization problem of Eq. (27) is same, because the solution of LMI of Eq. (8) or one of the constrained convex optimization problem of Eq. (27) cannot be reflected the resultant controller. Note that in the general controller design strategies for the conventional fixed gain robust control, the solution of the some constraints can be applied to the resultant robust controller. This is a fascinating fact for the proposed controller design strategy.*

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Remark 3: *The proposed adaptive gain robust controller with the adjustable time-varying parameter has some advantages as follows: the proposed controller design approach is very simple, and by selecting the design parameter, the proposed adaptive gain robust control system can achieve good transient performance which is close to the nominal one or avoid the excessive control input (see [32]). Besides, the structure of the proposed control system is also simple compared with the existing results for robust controllers with adjustable parameters* (e.g., *[29, 30]). However, the online adjustment strategy for the design parameter ϑ has not been established, and this problem is one of our future research subjects.*

Remark 4: *In this section, firstly the nominal control input is designed by adopting pole placement constraints, and the fixed gain $K \in \mathbb{R}^{m \times n}$ can be derived by using the solution of the LMI of Eq. (8). Note that the quadratic function $\mathcal{V}(x, t)$ is a Lyapunov function for both the uncertain linear system of Eq. (1) and the nominal system of Eq. (2), that is, the Lyapunov function for the uncertain linear system of Eq. (1) and one for the nominal system of Eq. (2) have same level set. Therefore, by selecting the design parameter $\vartheta > 0$, the proposed adaptive gain robust control system can achieve good transient performance which is close to the nominal one or avoid the excessive control input.*

On the other hand, if the design problem for a state feedback control $u(t) = K_s x(t)$ is considered, the quadratic function $\mathcal{V}(x, t)$ is replaced as $\mathcal{V}_s(x, t) = x^T(t)\mathcal{P}_s x(t)$ where $\mathcal{P}_s \in \mathbb{R}^{n \times n}$ is a Lyapunov matrix. Moreover, $\mathcal{P}_s \in \mathbb{R}^{n \times n}$ becomes a variable for resultant LMI conditions, and the standard techniques for the quadratic stabilization can also be used.

2.3. Illustrative examples

In order to demonstrate the efficiency of the proposed control strategy, we have run a simple example.

Consider the following linear system with unknown parameter $\Delta(t) \in \mathbb{R}^{2 \times 2}$:

$$\frac{d}{dt}x(t) = \begin{pmatrix} 1.0 & 4.0 \\ 0.0 & -1.0 \end{pmatrix} x(t) + \Delta(t)x(t) + \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix} u(t). \tag{29}$$

Firstly, we design the nominal control input $\overline{u}(t) = K\overline{x}(t)$. By selecting the design parameters α and Q in Eq. (5) such as $\alpha = 3.0$ and $Q = 1.0 \times I_2$ and solving the LMI of Eq. (8), we obtain the following solution:

$$\begin{aligned} \mathcal{Y} &= \begin{pmatrix} 1.0855 & -1.5356 \\ \star & 4.5318 \end{pmatrix}, \\ \mathcal{W} &= 2.1708 \times 10^1. \end{aligned} \tag{30}$$

Thus, the following fixed gain matrix can be computed:

$$K = \begin{pmatrix} -1.3017 \times 10^1 & -9.2008 \end{pmatrix}. \tag{31}$$

Next, we solve the constrained optimization problem of Eq. (27), then the solutions

$$\begin{aligned}\gamma &= 3.1612,\\ \varepsilon &= 3.1633 \times 10^{-1},\end{aligned} \tag{32}$$

can be derived, and therefore the allowable upper bound of unknown parameter is given as

$$\delta^{\star} = 3.1612. \tag{33}$$

In this example, we consider the following two cases for the unknown parameter $\Delta(t) \in \mathbb{R}^{2\times 2}$:

- Case 1) $\Delta(t) = \delta^{\star} \times \begin{pmatrix} 0.0 & 0.0 \\ -7.2289 & 6.8530 \end{pmatrix} \times 10^{-1}$.
- Case 2) $\Delta(t) = \delta^{\star} \times \begin{pmatrix} \sin(5.0 \times \pi \times t) & -\cos(5.0 \times \pi \times t) \\ \star & -\sin(5.0 \times \pi \times t) \end{pmatrix}$.

Note that the unknown parameter of Case 1 satisfies the matching condition [45]. In addition, for the design parameter ϑ, the numerical simulation for two cases such as $\vartheta = 1.0 \times 10^2$ and $\vartheta = 5.0 \times 10^{-1}$ is run. Moreover, the initial values of the uncertain system of Eq. (29) and the nominal system are selected as $x(0) = \overline{x}(0) = (1.0\ -2.0)^T$. The results of the simulation of this example are shown in **Figures 1–4** and **Table 1**. In these figures, "Case 1)" and "Case 2)" represent the time histories of the state variables $x_1(t)$ and $x_2(t)$ and the control input $u(t)$ and Lyapunov function $\mathcal{V}(x.t)$ for the proposed adaptive gain robust control, and "nominal" means the desired time response and the desired control input and Lyapunov function $\mathcal{V}(\overline{x}, t)$ for the nominal system. In **Table 1**, $\mathcal{J}_e$ means

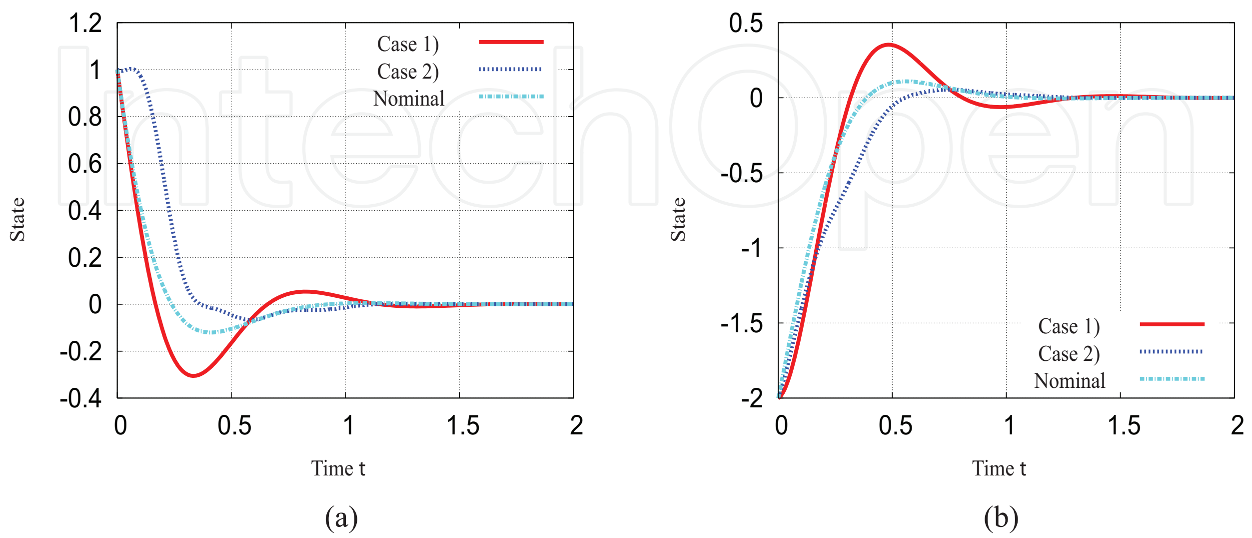

Figure 1. Time histories of the states for $\vartheta = 1.0 \times 10^2$. (a) The time histories of $x_1(t)$, (b) The time histories of $x_2(t)$.

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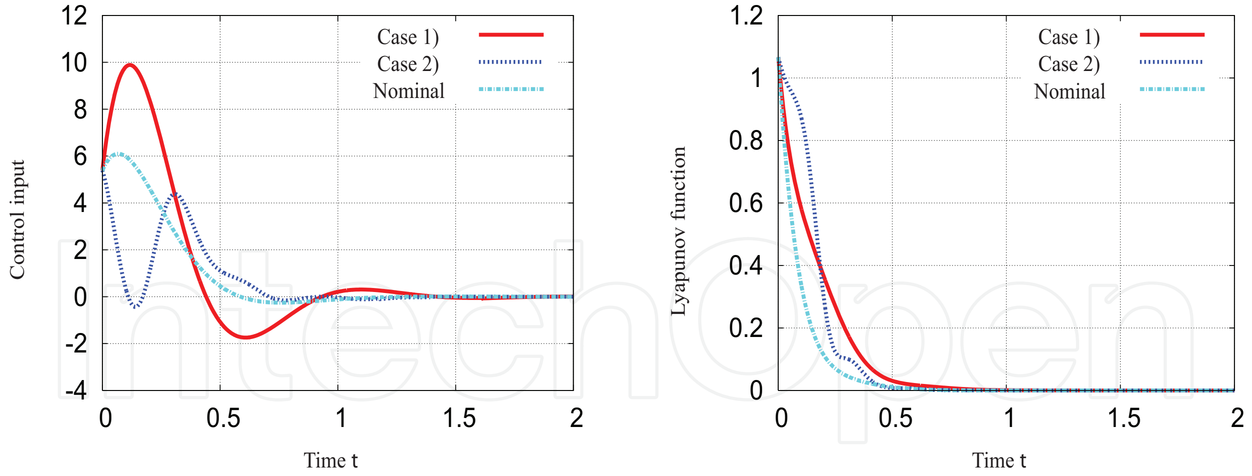
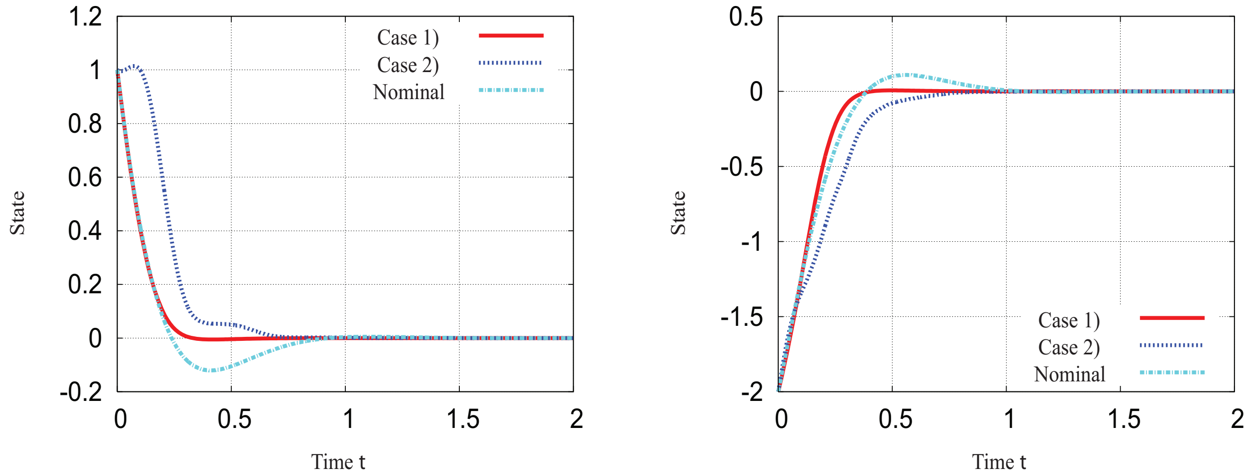


(a) (b)

Figure 2. Time histories of the control inputs and the Lyapunov function for $\vartheta = 1.0 \times 10^2$. (a) The time histories of $u(t)$ and $\overline{u}(t)$, (b) The time histories of $\mathcal{V}(x, t)$ and $\mathcal{V}(\overline{x}, t)$.



(a) (b)

Figure 3. Time histories of the states for $\vartheta = 5.0 \times 10^{-1}$. (a) The time histories of $x_1(t)$, (b) The time histories of $x_2(t)$.

$$\mathcal{J}_e \triangleq \int_0^\infty e^T(t)e(t)dt, \tag{34}$$

where $e(t)$ is an error vector between the time response and the desired one generated by the nominal system, that is, $e(t) \triangleq x(t) - \overline{x}(t)$. Namely, $\mathcal{J}_e$ of Eq. (34) is a performance index so as to evaluate the transient performance.

From **Figures 1–4** the proposed adaptive gain robust state feedback controller stabilizes the uncertain linear system of Eq. (29) in spite of uncertainties. Furthermore, we also find that the proposed adaptive gain robust controller achieves the good transient performance close to the nominal system.

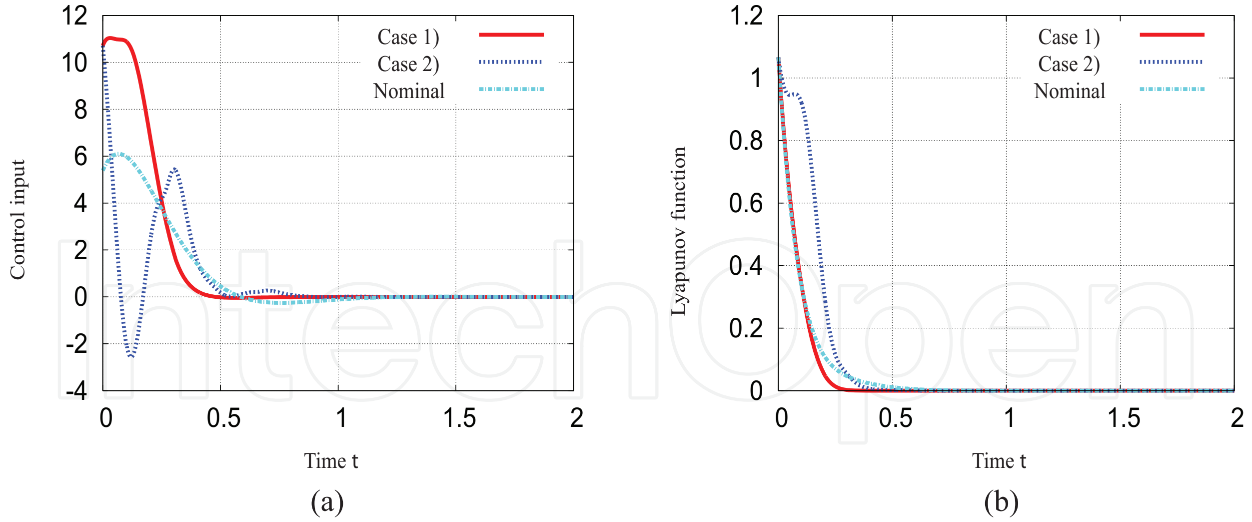


Figure 4. Time histories of the control inputs and the Lyapunov function for $\vartheta = 5.0 \times 10^{-1}$. (a) The time histories of $u(t)$ and $\overline{u}(t)$, (b) The time histories of $\mathcal{V}(x, t)$ and $\mathcal{V}(\overline{x}, t)$.

	$\boldsymbol{\vartheta = 1.0 \times 10^2}$	$\boldsymbol{\vartheta = 5.0 \times 10^{-1}}$
Case 1)	4.2584×10^{-2}	1.0160×10^{-2}
Case 2)	9.7403×10^{-2}	1.0038×10^{-1}

Table 1. The performance index $\mathcal{J}_e$.

For Case 1 in this example, one can see from **Table 1** that the adaptive gain robust controller for $\vartheta = 5.0 \times 10^{-1}$ is more desirable comparing with one for $\vartheta = 1.0 \times 10^2$, that is, the error between the time response and the desired one generated by the nominal system ("nominal" in figures) is small. But for the result of Case 2), we find that the robust controller with the parameter $\vartheta = 1.0 \times 10^2$ achieves more desirable performance. Additionally, one can see from **Figures 2(a)** and **4(a)** that by selecting the design parameter ϑ the proposed adaptive gain robust controller can adjust the magnitude of the control input. In this example, the magnitude of the control input for $\vartheta = 1.0 \times 10^2$ is suppressed comparing with one for $\vartheta = 5.0 \times 10^{-1}$. However, the online adjustment way of the design parameter ϑ for the purpose of improving transient behavior and avoiding excessive control input cannot to developed, and thus it is an important problem of our research subjects.

Therefore, the effectiveness of the proposed adaptive gain robust controller is shown.

2.4. Summary

In this section, an LMI-based design scheme of the centralized adaptive gain robust state feedback controller for a class of uncertain linear systems has been proposed, and by simple numerical simulations, the effectiveness of the proposed robust control strategy has been

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presented. Since the proposed adaptive gain robust controller can easily be obtained by solving the constrained convex optimization problem, the proposed design approach is simple. Moreover, by selecting the design parameter, the proposed adaptive gain robust controller can achieve good transient performance and/or avoid excessive control input. Note that there are trade-offs between achieving good transient performance and avoiding excessive control input.

The future research subject is the extension of proposed robust control scheme to such a broad class of systems as linear systems with state delays, uncertain systems with some constraints, and so on. Additionally, we will discuss the online adjustment for the design parameter ϑ and the design problem for output feedback control systems.

3. Synthesis of decentralized adaptive gain robust controllers

In this section, on the basis of the result derived in Section 2, an LMI-based design method of decentralized adaptive gain robust state feedback controllers for a class of uncertain large-scale interconnected systems is suggested. The design problem of the decentralized adaptive gain robust controller under consideration can also be reduced to the feasibility of LMIs, and the allowable perturbation region of uncertainties is also discussed.

3.1. Problem statement

Consider the uncertain large-scale interconnected system composed of $\mathcal{N}$ subsystems described as

$$\frac{d}{dt}\mathrm{x}_i(t) = A_{ii}(t)\mathrm{x}_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} A_{ij}(t)\mathrm{x}_j(t) + B_i u_i(t), \tag{35}$$

where $\mathrm{x}_i(t) \in \mathbb{R}^{\mathrm{n}_i}$ and $u_i(t) \in \mathbb{R}^{\mathrm{m}_i}$ $(i = 1, \cdots, \mathcal{N})$ are the vectors of the state and the control input for the ith subsystem, respectively, and $\mathrm{x}(t) = \left(\mathrm{x}_1^T(t), \cdots, \mathrm{x}_{\mathcal{N}}^T(t)\right)^T$ is the state of the overall system. The matrices $A_{ii}(t) \in \mathbb{R}^{\mathrm{n}_i \times \mathrm{n}_i}$ and $A_{ij}(t) \in \mathbb{R}^{\mathrm{n}_i \times \mathrm{n}_j}$ in Eq. (35) are given by

$$\begin{aligned} A_{ii}(t) &= A_{ii} + \Delta_{ii}(t), \\ A_{ij}(t) &= A_{ij} + \Delta_{ij}(t). \end{aligned} \tag{36}$$

In Eqs. (35) and (36), the matrices $A_{ii} \in \mathbb{R}^{\mathrm{n}_i \times \mathrm{n}_i}$, $A_{ij} \in \mathbb{R}^{\mathrm{n}_i \times \mathrm{n}_j}$, and $B_i \in \mathbb{R}^{\mathrm{n}_i \times \mathrm{m}_i}$ denote the nominal values of the system, and matrices $\Delta_{ii}(t) \in \mathbb{R}^{\mathrm{n}_i \times \mathrm{n}_i}$ and $\Delta_{ij}(t) \in \mathbb{R}^{\mathrm{n}_i \times \mathrm{n}_j}$ show unknown parameters which satisfy $\Delta_{ii}^T(t)\Delta_{ii}(t) \le \rho_{ii}^{\star} I_{n_i}$ and $\Delta_{ij}^T(t)\Delta_{ij}(t) \le \rho_{ij}^{\star} I_{n_j}$, respectively. Note that the elements of these unknown parameters are Lebesgue measurable [1, 34]. For Eq. (35), the nominal subsystem, ignoring the unknown parameters, is given by

$$\frac{d}{dt}\overline{x}_i(t) = A_{ii}\overline{x}_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} A_{ij}\overline{x}_j(t) + B_i\overline{u}_i(t), \tag{37}$$

where $\overline{x}_i(t) \in \mathbb{R}^{n_i}$ and $\overline{u}_i(t) \in \mathbb{R}^{m_i}$ are the vectors of the state and the control input for the ith nominal subsystem, respectively. Furthermore, the control input for the nominal subsystem of Eq. (37) is determined as

$$\overline{u}_i(t) = -K_i\overline{x}_i(t), \tag{38}$$

where $K_i \in \mathbb{R}^{m_i \times n_i}$ is a fixed gain matrix. From Eqs. (37) and (38), the following nominal closed-loop subsystem is obtained:

$$\frac{d}{dt}\overline{x}_i(t) = A_{K_i}\overline{x}_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} A_{ij}\overline{x}_j(t), \tag{39}$$

where $A_{K_i} \stackrel{\Delta}{=} A_{ii} - B_iK_i$.

Now, by using symmetric positive definite matrices $\mathcal{P}_i \in \mathbb{R}^{n_i \times n_i}$, we consider the quadratic function

$$\mathcal{V}(\overline{x}, t) \stackrel{\Delta}{=} \sum_{i=1}^{\mathcal{N}} \mathcal{V}_i(\overline{x}_i, t), \tag{40}$$

$$\mathcal{V}_i(\overline{x}_i, t) \stackrel{\Delta}{=} \overline{x}_i^T(t)\mathcal{P}_i\overline{x}_i(t), \tag{41}$$

as a Lyapunov function candidate. For the quadratic function $\mathcal{V}_i(\overline{x}_i, t)$ of Eq. (41), its time derivative along the trajectory of the nominal closed-loop subsystem of Eq. (39) is given by

$$\frac{d}{dt}\mathcal{V}_i(\overline{x}_i, t) = \overline{x}_i^T(t)\left(A_{K_i}^T\mathcal{P}_i + \mathcal{P}_iA_{K_i}\right)\overline{x}_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} 2\overline{x}_i^T(t)\mathcal{P}_iA_{ij}\overline{x}_j(t). \tag{42}$$

For the second term on the right side of Eq. (42), by using the well-known relation of Eq. (20), we can obtain the following relation:

$$\frac{d}{dt}\mathcal{V}_i(\overline{x}_i, t) \leq \overline{x}_i^T(t)\left(A_{K_i}^T\mathcal{P}_i + \mathcal{P}_iA_{K_i}\right)\overline{x}_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \mu_{ij}\overline{x}_i^T(t)\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}\overline{x}_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{\mathcal{N}} \frac{1}{\mu_{ij}}\overline{x}_j^T(t)\overline{x}_j(t). \tag{43}$$

From Eqs. (40) and (43), we have

$$\frac{d}{dt}\mathcal{V}(\overline{x},t) \leq \sum_{i=1}^{\mathcal{N}} \overline{x}_i^T(t)\left(A_{K_i}^T\mathcal{P}_i + \mathcal{P}_iA_{K_i}\right)\overline{x}_i(t) + \sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \mu_i\overline{x}_i^T(t)\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_i\overline{x}_i(t)$$
$$+ \sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\mu_{ij}}\overline{x}_j^T(t)\overline{x}_j(t). \tag{44}$$

The inequality of Eq. (44) can also be rewritten as

$$\frac{d}{dt}\mathcal{V}(\overline{x},t) \leq \sum_{i=1}^{\mathcal{N}} \overline{x}_i^T(t)\left(A_{K_i}^T\mathcal{P}_i + \mathcal{P}_iA_{K_i} + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \mu_i\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_i + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\mu_{ji}}I_n\right)\overline{x}_i(t). \tag{45}$$

Therefore, if the matrix inequality

$$A_{K_i}^T\mathcal{P}_i + \mathcal{P}_iA_{K_i} + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \mu_i\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_i + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\mu_{ji}}I_n < 0 \tag{46}$$

holds, then the following relation for the time derivative of $\mathcal{V}(\overline{x},t)$ is satisfied:

$$\frac{d}{dt}\mathcal{V}(\overline{x},t) < 0, \quad \forall \overline{x}(t) \neq 0. \tag{47}$$

Now, as with Section 2, we derive a decentralized controller with pole placement constraints for the nominal subsystem of Eq. (37). Namely, from Eq. (46), the matrix inequality

$$\left(A_{K_i} + \alpha_i I_n\right)^T\mathcal{P}_i + \mathcal{P}_i\left(A_{K_i} + \alpha_i I_n\right) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \mu_i\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_i + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\mu_{ji}}I_n + \mathcal{Q}_i < 0, \tag{48}$$

is considered. In Eq. (48), $\alpha_i \in \mathbb{R}$ is a positive scalar and is selected by designers.

We introduce symmetric positive definite matrices $\mathcal{Y}_i \triangleq \mathcal{P}_i^{-1}$ and $\mathcal{W}_i \in \mathbb{R}^{m_i \times m_i}$ and define the fixed gain K_i as $K_i \triangleq \mathcal{W}_iB_i^T\mathcal{P}_i$. Then for the matrix inequality of Eq. (48), by pre- and post-multiplying both sides of the matrix inequality of Eq. (48) by $\mathcal{Y}_i$, it can be obtained that

$$A_{ii}\mathcal{Y}_i - B_i\mathcal{W}_iB_i^T + \mathcal{Y}_iA_{ii}^T - B_i\mathcal{W}_i^TB_i^T + 2\alpha_i\mathcal{Y}_i + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \mu_{ij}A_{ij}A_{ij}^T + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\mu_{ji}}\mathcal{Y}_i\mathcal{Y}_i + \mathcal{Y}_i\mathcal{Q}_i\mathcal{Y}_i < 0. \tag{49}$$

Thus, by applying **Lemma 1.2** (Schur complement) to Eq. (49), we find that the matrix inequality of Eq. (49) is equivalent to the following LMI:

$$\begin{pmatrix} \Lambda_i\left(\mathcal{Y}_i, \mathcal{W}_i, \mu_{ij}\right) & \Theta_i(\mathcal{Y}_i) \\ \star & -\Gamma_i\left(\mu_{ij}\right) \end{pmatrix} < 0. \tag{50}$$

In Eq. (50), matrices $\Lambda_i\left(\mathcal{Y}_i, \mathcal{W}_i, \mu_{ij}\right) \in \mathbb{R}^{n_i \times n_i}$, $\Theta_i(\mathcal{Y}_i) \in \mathbb{R}^{n_i \times \mathcal{N} n_i}$, and $\Gamma_i\left(\mu_{ij}\right) \in \mathbb{R}^{\mathcal{N} n_i \times \mathcal{N} n_i}$ are given by

$$\begin{aligned} \Lambda_i\left(\mathcal{Y}_i, \mathcal{W}_i, \mu_{ij}\right) &\triangleq A_{ii}\mathcal{Y}_i - B_i\mathcal{W}_i B_i^T + \mathcal{Y}_i A_{ii}^T - B_i \mathcal{W}_i^T B_i^T + 2\alpha_i \mathcal{Y}_i + \sum_{\substack{j=1 \\ j \neq i}}^{\mathcal{N}} \mu_{ij} A_{ij} A_{ij}^T, \\ \Theta_i(\mathcal{Y}_i) &\triangleq \left(\overbrace{\mathcal{Y}_i \; \mathcal{Y}_i \; \dots \; \mathcal{Y}_i}^{\mathcal{N}} \right), \\ \Gamma_i\left(\mu_{ij}\right) &\triangleq \operatorname{diag}\left(\mathcal{Q}_i^{-1}, \mu_{1i} I_n, \mu_{2i} I_n, \cdots, \mu_{i-1i} I_n, \mu_{i+1i} I_n, \cdots, \mu_{\mathcal{N}i} I_n\right). \end{aligned} \tag{51}$$

Therefore, if matrices $\mathcal{Y}_i \in \mathbb{R}^{n_i \times n_i}$ and $\mathcal{W}_i \in \mathbb{R}^{m_i \times m_i}$ and positive scalars μ_{ij} exist, the nominal closed-loop subsystem is asymptotically stable, and the fixed gain matrix K_i is determined as $K_i = \mathcal{W}_i B_i^T \mathcal{Y}_i^{-1}$.

Now, by using the fixed gain matrix $K_i \in \mathbb{R}^{m_i \times n_i}$ which is designed for the nominal subsystem, we define the control input

$$u_i(t) \triangleq -(1 + \theta_i(t)) K_i x_i(t), \tag{52}$$

where $\theta_i(t) \in \mathbb{R}^1$ is an adjustable time-varying parameter. From Eqs. (35) and (52), the uncertain closed-loop subsystem can be obtained as

$$\frac{d}{dt} x_i(t) = A_{K_i} x_i(t) + \Delta_{ii}(t) x_i(t) + \sum_{i=1}^{\mathcal{N}} \left(A_{ij} + \Delta_{ij}(t)\right) x_j(t) - \theta_i(t) B_i K_i x_i(t). \tag{53}$$

From the above discussion, the designed objective in this section is to determine the decentralized robust control of Eq. (52) such that the resultant overall system achieves robust stability. That is to design the adjustable time-varying parameter $\theta_i(t) \in \mathbb{R}^1$ such that asymptotical stability of the overall system composed of $\mathcal{N}$ subsystems of Eq. (53) is guaranteed.

3.2. Decentralized variable gain controllers

The following theorem shows sufficient conditions for the existence of the proposed decentralized adaptive gain robust control system.

Theorem 3: *Consider the uncertain large-scale interconnected system of Eq. (35) and the control input of Eq. (52).*

For a given positive constant ϑ_i, if positive constants ξ_{ii}, σ_{ij}, and ε_{ij} exist which satisfy the LMIs

$$\begin{pmatrix} \Pi_i\left(\xi_{ii}, \varepsilon_{ij}, \sigma_{ij}\right) & \Xi_i \\ \star & -\Omega_i\left(\xi_{ii}, \varepsilon_{ij}, \sigma_{ij}\right) \end{pmatrix} < 0, \tag{54}$$

the time-varying adjustable parameters $\theta_i(t) \in \mathbb{R}$ *are determined as*

$$\theta_i(t) \triangleq \begin{cases} \dfrac{\sqrt{\rho_{ii}^\star}\|\mathcal{P}_i x_i(t)\|\|x_i(t)\|}{x_i^T(t)\mathcal{P}_i B_i \mathcal{W}_i B_i^T \mathcal{P}_i x_i(t)} & \text{if} \quad x_i^T(t)\mathcal{P}_i B_i \mathcal{W}_i B_i^T \mathcal{P}_i x_i(t) \geq \vartheta_i x_i^T(t) x_i(t), \\ \dfrac{\sqrt{\rho_{ii}^\star}\|\mathcal{P}_i x_i(t)\|\|x_i(t)\|}{\vartheta_i x_i^T(t) x_i(t)} & \text{if} \quad x_i^T(t)\mathcal{P}_i B_i \mathcal{W}_i B_i^T \mathcal{P}_i x_i(t) < \vartheta_i x_i^T(t) x_i(t), \end{cases} \tag{55}$$

where matrices $\Pi_i\left(\xi_i, \varepsilon_{ij}, \sigma_{ij}\right) \in \mathbb{R}^{n_i \times n_i}$, $\Xi_i \in \mathbb{R}^{n_i \times (2\mathcal{N}-1)n_i}$, *and* $\Omega_i\left(\xi_{ii}, \varepsilon_{ij}, \sigma_{ij}\right) \in \mathbb{R}^{(2\mathcal{N}-1)n_i \times (2\mathcal{N}-1)n_i}$ *are given by*

$$\Pi_i\left(\xi_{ii}, \varepsilon_{ij}, \sigma_{ij}\right) \triangleq \left(A_{K_i}^T \mathcal{P}_i + \mathcal{P}_i A_{K_i}\right) + \xi_{ii}\mathcal{P}_i\mathcal{P}_i + \sum_{\substack{j=1 \\ j \neq i}}^{\mathcal{N}} \varepsilon_{ij}\mathcal{P}_i A_{ij} A_{ij}^T \mathcal{P}_i + \sum_{\substack{j=1 \\ j \neq i}}^{\mathcal{N}} \sigma_{ij}\mathcal{P}_i\mathcal{P}_i,$$

$$\Xi_i \triangleq \left(\overbrace{I_n \ I_n \ \cdots \ I_n}^{2\mathcal{N}-1}\right),$$

$$\begin{aligned} \Omega_i\left(\xi_{ii}, \varepsilon_{ij}, \sigma_{ij}\right) \triangleq \ \mathrm{diag}\big(&\xi_{ii}\rho_{ii}^\star I_n, \varepsilon_{1i} I_n, \varepsilon_{2i} I_n, \cdots, \varepsilon_{i-1i} I_n, \varepsilon_{i+1i} I_n, \cdots, \varepsilon_{\mathcal{N}i} I_n, \sigma_{1i}\rho_{1i}^\star I_n, \sigma_{2i}\rho_{2i}^\star I_n, \\ &\cdots, \sigma_{i-1i}\rho_{i-1i}^\star I_n, \sigma_{i+1i}\rho_{i+1i}^\star I_n, \cdots, \sigma_{\mathcal{N}i}\rho_{\mathcal{N}i}^\star I_n\big). \end{aligned} \tag{56}$$

Then, the overall close-loop system composed of $\mathcal{N}$ *closed-loop subsystems is asymptotically stable.*

Proof. In order to prove Theorem 3, the following Lyapunov function candidate is introduced by using symmetric positive definite matrices $\mathcal{P}_i \in \mathbb{R}^{n_i \times n_i}$ which satisfy the LMIs of (50):

$$\mathcal{V}(x, t) \triangleq \sum_{i=1}^{\mathcal{N}} \mathcal{V}_i(x_i, t), \tag{57}$$

where $\mathcal{V}_i(x_i, t)$ is a quadratic function given by

$$\mathcal{V}_i(x_i, t) \triangleq \ x_i^T(t)\mathcal{P}_i x_i(t). \tag{58}$$

We can obtain the following relation for the time derivative of the quadratic function $\mathcal{V}_i(x_i, t)$ of Eq. (58):

$$\begin{aligned} \frac{d}{dt}\mathcal{V}_i(x_i, t) = x_i^T(t)\left(A_{K_i}^T \mathcal{P}_i + \mathcal{P}_i A_{K_i}\right)x_i(t) + 2x_i(t)\mathcal{P}_i\Delta_{ii}(t)x_i(t) \\ + 2x_i^T(t)\mathcal{P}_i \sum_{\substack{j=1 \\ j \neq i}}^{\mathcal{N}} \left(A_{ij} + \Delta_{ij}(t)\right)x_j(t) - 2\theta_i(t)x_i^T(t)\mathcal{P}_i B_i K_i x_i(t). \end{aligned} \tag{59}$$

Firstly, we consider the case of $\mathrm{x}_i^T(t)\mathcal{P}_iB_iW_iB_i^T\mathcal{P}_i\mathrm{x}_i(t)\geq\vartheta_i\mathrm{x}_i^T(t)\mathrm{x}_i(t)$. In this case, one can see from the relations $\Delta_{ii}^T(t)\Delta_{ii}(t)\leq\rho_{ii}^\star I_{n_i}$ and $\Delta_{ij}^T(t)\Delta_{ij}(t)\leq\rho_{ij}^\star I_{n_j}$, the well-known inequality of Eq. (20), and **Lemma 1.1** that the following relation for the quadratic function $\mathcal{V}_i(x_i,t)$ of Eq. (58) can be obtained:

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_i(\mathrm{x}_i,t)\leq{}&\mathrm{x}_i^T(t)\left(A_{K_i}^T\mathcal{P}_i+\mathcal{P}_iA_{K_i}\right)\mathrm{x}_i(t)+2\sqrt{\rho_{ii}^\star}\|\mathcal{P}_i\mathrm{x}_i(t)\|\|\mathrm{x}_i(t)\|\\&+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\varepsilon_{ij}\mathrm{x}_i^T(t)\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_i\mathrm{x}_i(t)+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{1}{\varepsilon_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t)+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\sigma_{ij}\mathrm{x}_i^T(t)\mathcal{P}_i\mathcal{P}_i\mathrm{x}_i(t)\\&+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{\rho_{ij}^\star}{\sigma_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t)-2\theta_i(t)\mathrm{x}_i^T(t)\mathcal{P}_iB_iK_i\mathrm{x}_i(t).\end{aligned}\tag{60}$$

Substituting the adjustable time-varying parameter $\theta_i(t)$ of Eq. (55) into Eq. (60) gives

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_i(\mathrm{x}_i,t)\leq{}&\mathrm{x}_i^T(t)\left(A_{K_i}^T\mathcal{P}_i+\mathcal{P}_iA_{K_i}\right)\mathrm{x}_i(t)+2\sqrt{\rho_{ii}^\star}\|\mathcal{P}_i\mathrm{x}_i(t)\|\|\mathrm{x}_i(t)\|\\&+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\varepsilon_{ij}\mathrm{x}_i^T(t)\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_i\mathrm{x}_i(t)+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{1}{\varepsilon_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t)+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\sigma_{ij}\mathrm{x}_i^T(t)\mathcal{P}_i\mathcal{P}_i\mathrm{x}_i(t)\\&+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{\rho_{ij}^\star}{\sigma_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t)-2\left(\frac{\sqrt{\rho_{ii}^\star}\|\mathcal{P}_i\mathrm{x}_i(t)\|\|\mathrm{x}_i(t)\|}{\mathrm{x}_i^T(t)\mathcal{P}_iB_i\mathcal{W}_iB_i^T\mathcal{P}_i\mathrm{x}_i(t)}\right)\mathrm{x}_i^T(t)\mathcal{P}_iB_iK_i\mathrm{x}_i(t)\\={}&\mathrm{x}_i^T(t)\left(A_{K_i}^T\mathcal{P}_i+\mathcal{P}_iA_{K_i}\right)\mathrm{x}_i(t)+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\varepsilon_{ij}\mathrm{x}_i^T(t)\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_i\mathrm{x}_i(t)+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{1}{\varepsilon_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t)\\&+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\sigma_{ij}\mathrm{x}_i^T(t)\mathcal{P}_i\mathcal{P}_i\mathrm{x}_i(t)+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{\rho_{ij}^\star}{\sigma_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t),\end{aligned}\tag{61}$$

and, thus, we have the following inequality for the function $\mathcal{V}(\mathrm{x},t)$ of Eq. (57):

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(\mathrm{x},t)\leq{}&\sum_{i=1}^{\mathcal{N}}\mathrm{x}_i^T(t)\left(A_{K_i}^T\mathcal{P}_i+\mathcal{P}_iA_{K_i}\right)\mathrm{x}_i(t)+\sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\varepsilon_{ij}\mathrm{x}_i(t)\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_i\mathrm{x}_i(t)\\&+\sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{1}{\varepsilon_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t)+\sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\sigma_{ij}\mathrm{x}_i^T(t)\mathcal{P}_i\mathcal{P}_i\mathrm{x}_i(t)+\sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{\rho_{ij}^\star}{\sigma_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t).\end{aligned}\tag{62}$$

Furthermore, the inequality of Eq. (62) can be rewritten as

$$\frac{d}{dt}\mathcal{V}(\mathrm{x},t) \leq \sum_{i=1}^{\mathcal{N}} \mathrm{x}_i^T(t)\left(A_{K_i}^T\mathcal{P}_i + \mathcal{P}_i A_{K_i} + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \varepsilon_{ij}\mathcal{P}_i A_{ij} A_{ij}^T \mathcal{P}_i + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\varepsilon_{ji}} I_{\mathrm{n}} + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \sigma_{ij}\mathcal{P}_i\mathcal{P}_i + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{\rho_{ji}^{\star}}{\sigma_{ji}} I_{\mathrm{n}}\right)\mathrm{x}_i(t). \quad (63)$$

Therefore, if the matrix inequality

$$A_{K_i}^T\mathcal{P}_i + \mathcal{P}_i A_{K_i} + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \varepsilon_{ij}\mathcal{P}_i A_{ij} A_{ij}^T \mathcal{P}_i + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\varepsilon_{ji}} I_{\mathrm{n}} + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \sigma_{ij}\mathcal{P}_i\mathcal{P}_i + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{\rho_{ij}^{\star}}{\sigma_{ji}} I_{\mathrm{n}} < 0 \quad (64)$$

holds, then the following relation for the time derivative of $\mathcal{V}(\mathrm{x},t)$ is satisfied:

$$\frac{d}{dt}\mathcal{V}(\mathrm{x},t) < 0, \quad \forall \mathrm{x}(t) \neq 0. \quad (65)$$

Next, we consider the case of $\mathrm{x}_i^T(t)\mathcal{P}_i B_i \mathcal{W}_i B_i^T \mathcal{P}_i \mathrm{x}_i(t) < \vartheta_i \mathrm{x}_i^T(t)\mathrm{x}_i(t)$. In this case, by using the relations $\Delta_{ii}^T(t)\Delta_{ii}(t) \leq \rho_{ii}^{\star} I_{n_i}$ and $\Delta_{ij}^T(t)\Delta_{ij}(t) \leq \rho_{ij}^{\star} I_{n_j}$, and Eq. (20) and substituting the adjustable time-varying parameter $\theta_i(t)$ of Eq. (55) into Eq. (59), we have

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_i(\mathrm{x}_i,t) \leq{}& \mathrm{x}_i^T(t)\left(A_{K_i}^T\mathcal{P}_i + \mathcal{P}_i A_{K_i}\right)\mathrm{x}_i(t) + \xi_{ii}\mathrm{x}_i^T(t)\mathcal{P}_i\mathcal{P}_i\mathrm{x}_i(t) + \frac{\rho_{ii}^{\star}}{\xi_{ii}}\mathrm{x}_i^T(t)\mathrm{x}_i(t) \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \varepsilon_{ij}\mathrm{x}_i(t)\mathcal{P}_i A_{ij}A_{ij}^T\mathcal{P}_i\mathrm{x}_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\varepsilon_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \sigma_{ij}\mathrm{x}_i^T(t)\mathcal{P}_i\mathcal{P}_i\mathrm{x}_i(t) \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{\rho_{ij}^{\star}}{\sigma_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t) - 2\left(\frac{\sqrt{\rho_{ii}^{\star}}\|\mathcal{P}_i\mathrm{x}_i(t)\|\|\mathrm{x}_i(t)\|}{\vartheta_i\mathrm{x}_i^T(t)\mathrm{x}_i(t)}\right)\mathrm{x}_i^T(t)\mathcal{P}_i B_i K_i \mathrm{x}_i(t).\end{aligned} \quad (66)$$

The last term on the right side of Eq. (66) is less than 0 because the matrix $K_i \in \mathbb{R}^{\mathrm{m}_i\times\mathrm{n}_i}$ is defined as $K_i = \mathcal{W}_i B_i^T \mathcal{P}_i$ and $\theta_i(t)$ is a positive scalar function. Therefore, we find that the following relation for the quadratic function $\mathcal{V}_i(\mathrm{x}_i,t)$ is satisfied:

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_i(\mathrm{x}_i,t) \leq{}& \mathrm{x}_i^T(t)\left(A_{K_i}^T\mathcal{P}_i + \mathcal{P}_i A_{K_i}\right)\mathrm{x}_i(t) + \xi_{ii}\mathrm{x}_i^T(t)\mathcal{P}_i\mathcal{P}_i\mathrm{x}_i(t) + \frac{\rho_{ii}^{\star}}{\xi_{ii}}\mathrm{x}_i^T(t)\mathrm{x}_i(t) \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \varepsilon_{ij}\mathrm{x}_i(t)\mathcal{P}_i A_{ij}A_{ij}^T\mathcal{P}_i\mathrm{x}_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{1}{\varepsilon_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t) + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \sigma_{ij}\mathrm{x}_i^T(t)\mathcal{P}_i\mathcal{P}_i\mathrm{x}_i(t) \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \frac{\rho_{ij}^{\star}}{\sigma_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t).\end{aligned} \quad (67)$$

Therefore, we see from Eqs. (57) and (67) that the following inequality:

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(\mathrm{x},t)\leq\ &\sum_{i=1}^{\mathcal{N}}\mathrm{x}_i^T(t)\left(A_{K_i}^T\mathcal{P}_i+\mathcal{P}_iA_{K_i}\right)\mathrm{x}_i(t)+\sum_{i=1}^{\mathcal{N}}\xi_{ii}\mathrm{x}_i^T(t)\mathcal{P}_i\mathcal{P}_i\mathrm{x}_i(t)+\sum_{i=1}^{\mathcal{N}}\frac{\rho_{ii}^\star}{\xi_{ii}}\mathrm{x}_i^T(t)\mathrm{x}_i(t)\\&+\sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\varepsilon_{ij}\mathrm{x}_i(t)P_iA_{ij}A_{ij}^TP_i\mathrm{x}_i(t)+\sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{1}{\varepsilon_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t)\\&+\sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\sigma_{ij}\mathrm{x}_i^T(t)\mathcal{P}_i\mathcal{P}_i\mathrm{x}_i(t)+\sum_{i=1}^{\mathcal{N}}\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{\rho_{ij}^\star}{\sigma_{ij}}\mathrm{x}_j^T(t)\mathrm{x}_j(t)\end{aligned}\tag{68}$$

can be derived. Moreover, one can easily see that the inequality of Eq. (68) can be rewritten as

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(\mathrm{x},t)\leq\ \sum_{i=1}^{\mathcal{N}}\mathrm{x}_i^T(t)\Bigg(&A_{K_i}^T\mathcal{P}_i+\mathcal{P}_iA_{K_i}+\xi_{ii}\mathcal{P}_i\mathcal{P}_i+\frac{\rho_{ii}^\star}{\xi_{ii}}I_{\mathrm{n}}+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\varepsilon_{ij}\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_i\\&+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{1}{\varepsilon_{ji}}I_{\mathrm{n}}+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\sigma_{ij}\mathcal{P}_i\mathcal{P}_i+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{\rho_{ji}^\star}{\sigma_{ji}}I_{\mathrm{n}}\Bigg)x_i(t).\end{aligned}\tag{69}$$

Therefore, if the matrix inequality

$$\begin{aligned}&A_{K_i}^T\mathcal{P}_i+\mathcal{P}_iA_{K_i}+\xi_{ii}\mathcal{P}_i\mathcal{P}_i+\frac{\rho_{ii}^\star}{\xi_{ii}}I_{\mathrm{n}}+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\varepsilon_{ij}\mathcal{P}_iA_{ij}A_{ij}^T\mathcal{P}_i+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{1}{\varepsilon_{ji}}I_{\mathrm{n}}+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\sigma_{ij}\mathcal{P}_i\mathcal{P}_i\\&+\sum_{\substack{j=1\\j\neq i}}^{\mathcal{N}}\frac{\rho_{ji}^\star}{\sigma_{ji}}I_{\mathrm{n}}<0\end{aligned}\tag{70}$$

holds, then the relation of Eq. (65) for the time derivative of the function $\mathcal{V}(\mathrm{x},t)$ of Eq. (57) is satisfied. Due to the 3rd and 4th terms on the left side of Eq. (70) which are positive definite, if the inequality of Eq. (70) is satisfied, then the inequality of Eq. (64) is also constantly satisfied.

For the matrix inequality of Eq. (70), by applying **Lemma 1.2** (Schur complement), one can find that the matrix inequalities of Eq. (70) are equivalent to the LMIs of Eq. (54). Therefore, by solving the LMIs of Eq. (54), the adjustable time-varying parameter is given by Eq. (55), and proposed control input of Eq. (52) stabilizes the overall system of Eq. (35). Thus, the proof of Theorem 3 is completed.

Next, as mentioned in Section 2, we discuss the allowable region of the unknown parameters $\Delta_{ii}(t)\in\mathbb{R}^{\mathrm{n}_i\times\mathrm{n}_i}$ and $\Delta_{ij}(t)\in\mathbb{R}^{\mathrm{n}_i\times\mathrm{n}_j}$. Thus, the following additional constraints are introduced:

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$$\begin{aligned}\rho_{ii}^{\star} &= \xi_{ii},\\ \rho_{ij}^{\star} &= \sigma_{ij}.\end{aligned} \tag{71}$$

From the relations of Eq. (71), one can find that the maximization of ξ_{ii} and σ_{ij} is equivalent to the maximization of $\rho_{ii}^{\star}$ and $\rho_{ij}^{\star}$. Then, the LMIs of Eq. (54) can be rewritten as

$$\begin{pmatrix} \Pi_{i'}\left(\xi_{ii}, \varepsilon_{ij}, \sigma_{ij}\right) & \Xi_{i'} \\ \star & -\Omega_{i'}\left(\varepsilon_{ij}\right) \end{pmatrix} < 0, \tag{72}$$

$$\begin{aligned}\Pi_{i'}\left(\xi_{ii}, \varepsilon_{ij}, \sigma_{ij}\right) &\overset{\Delta}{=} A_{K_i}^T\mathcal{P}_i + \mathcal{P}_i A_{K_i} + \xi_{ii}\mathcal{P}_i\mathcal{P}_i + \mathcal{N} I_n + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \varepsilon_{ij}\mathcal{P}_i A_{ij} A_{ij}^T \mathcal{P}_i + \sum_{\substack{j=1\\ j\neq i}}^{\mathcal{N}} \sigma_{ij}\mathcal{P}_i\mathcal{P}_i,\\ \Xi_{i'} &\overset{\Delta}{=} \Big(\overbrace{I_{\mathrm{n}} \quad I_{\mathrm{n}} \quad \cdots \quad I_{\mathrm{n}}}^{\mathcal{N}-1}\Big),\\ \Omega_{i'}\left(\varepsilon_{ij}\right) &\overset{\Delta}{=} \mathrm{diag}(\varepsilon_{1i} I_{\mathrm{n}}, \varepsilon_{2i} I_{\mathrm{n}}, \cdots, \varepsilon_{i-1i} I_{\mathrm{n}}, \varepsilon_{i+1i} I_{\mathrm{n}}, \cdots, \varepsilon_{\mathcal{N}i} I_{\mathrm{n}}).\end{aligned} \tag{73}$$

Furthermore, we introduce a positive scalar λ and a complementary matrix $\Gamma \in \mathbb{R}^{\mathcal{N}^2 \times \mathcal{N}^2}$ defined as

$$\Gamma \overset{\Delta}{=} \mathrm{diag}(\xi_{11}, \xi_{22}, \cdots, \xi_{\mathcal{N}\mathcal{N}}, \sigma_{12}, \sigma_{13}, \cdots, \sigma_{1\mathcal{N}}, \sigma_{21}, \sigma_{23}, \cdots, \sigma_{\mathcal{N}\mathcal{N}-1}), \tag{74}$$

and consider the following additional condition:

$$\Gamma - \frac{1}{\lambda} I_{\mathcal{N}^2} > 0. \tag{75}$$

Namely, we can replace the maximization problem of ξ_{ii} and σ_{ij} with the minimization problem of λ. From Eq. (75) and **Lemma 1.2** (Schur complement), one can easily see that the constraint of Eq. (75) can be transformed into

$$\begin{pmatrix} \Gamma & I_{\mathcal{N}^2} \\ \star & \lambda I_{\mathcal{N}^2} \end{pmatrix} > 0. \tag{76}$$

Thus, in order to design the proposed decentralized adaptive gain robust controller, the constrained convex optimization problem

$$\underset{\xi_{ii}>0,\quad \varepsilon_{ij}>0,\quad \sigma_{ij}>0}{\mathrm{Minimize}} \quad [\lambda] \text{ subject to } (72) \text{ and } (76) \tag{77}$$

should be solved.

As a result, the following theorem can be obtained:

Theorem 4: *Consider the uncertain large-scale interconnected system of Eq. (35) and the control input of Eq. (52).*

If positive constants ξ_{ii}, ε_{ij}, σ_{ij}, and λ exist which satisfy the constrained convex optimization problem of Eq. (77), the adjustable time-varying parameter $\theta_i(t)$ is designed as Eq. (55). Then, the overall *uncertain closed-loop system of Eq. (53) is asymptotically stable. Furthermore, by using the optimal solution* $\xi_{ii}^{\star}$ *and* $\sigma_{ij}^{\star}$ *for Eq. (77), the upper bound of unknown parameters* $\Delta_{ii}(t) \in \mathbb{R}^{\mathrm{n}_i \times \mathrm{n}_i}$ *and* $\Delta_{ij}(t) \in \mathbb{R}^{\mathrm{n}_i \times \mathrm{n}_j}$ *is given by*

$$\begin{aligned} \rho_{ii}^{\star} &= \xi_{ii}^{\star}, \\ \rho_{ij}^{\star} &= \sigma_{ij}^{\star}. \end{aligned} \tag{78}$$

3.3. Illustrative examples

To demonstrate the efficiency of the proposed decentralized robust controller, an illustrative example is provided. In this example, we consider the uncertain large-scale interconnected system consisting of three two-dimensional subsystems, that is, $\mathcal{N} = 3$. The system parameters are given as follows:

$$\begin{aligned}
&A_{11} = \begin{pmatrix} -1.0 & 1.0 \\ 0.0 & 1.0 \end{pmatrix}, A_{22} = \begin{pmatrix} 0.0 & 1.0 \\ -1.0 & -1.0 \end{pmatrix}, A_{33} = \begin{pmatrix} 1.0 & 0.0 \\ 1.0 & -3.0 \end{pmatrix}, \\
&B_1 = \begin{pmatrix} 0.0 \\ 1.0 \end{pmatrix}, B_2 = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix}, B_3 = \begin{pmatrix} 1.0 \\ 0.0 \end{pmatrix}, \\
&A_{12} = \begin{pmatrix} 0.5 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}, A_{13} = \begin{pmatrix} 0.0 & 0.5 \\ 0.0 & 0.0 \end{pmatrix}, A_{21} = \begin{pmatrix} 0.0 & 0.0 \\ 0.0 & 0.5 \end{pmatrix}, \\
&A_{23} = \begin{pmatrix} 0.0 & 0.5 \\ 1.0 & 0.0 \end{pmatrix}, A_{31} = \begin{pmatrix} 0.5 & 0.0 \\ 0.0 & 0.0 \end{pmatrix}, A_{32} = \begin{pmatrix} 0.0 & 0.5 \\ 0.0 & 0.5 \end{pmatrix}.
\end{aligned} \tag{79}$$

Firstly, by selecting the design parameters $\alpha_i \in \mathbb{R}^1$ and $\mathcal{Q}_i \in \mathbb{R}^{2\times 2}$ $(i = 1, 2, 3)$ as $\alpha_1 = \alpha_2 = \alpha_3 = 1.0$ and $\mathcal{Q}_1 = \mathcal{Q}_2 = \mathcal{Q}_3 = 2.0 \times I_2$ and solving LMIs of Eq. (50), we have the symmetric positive definite matrices $\mathcal{Y}_i \in \mathbb{R}^{2\times 2}$ and $\mathcal{W}_i \in \mathbb{R}^{1\times 1}$, and positive scalars μ_{ij} can be obtained:

$$\begin{aligned}
&\mathcal{Y}_1 = \begin{pmatrix} 1.8972 & -2.1976 \\ \star & 8.1021 \end{pmatrix} \times 10^{-1}, \qquad \mathcal{W}_1 = 3.9298, \\
&\mathcal{Y}_2 = \begin{pmatrix} 3.4941 & 4.7825 \\ \star & 8.8702 \end{pmatrix} \times 10^{-1}, \qquad \mathcal{W}_2 = 2.2200, \\
&\mathcal{Y}_3 = \begin{pmatrix} 4.0414 \times 10^{-1} & 3.2732 \times 10^{-2} \\ \star & 3.2709 \times 10^{-1} \end{pmatrix}, \quad \mathcal{W}_3 = 3.2166,
\end{aligned} \tag{80}$$

$$\begin{aligned}
&\mu_{12} = 7.0526 \times 10^{-1}, \ \ \mu_{13} = 4.5522 \times 10^{-1}, \ \ \mu_{21} = 1.3986, \\
&\mu_{23} = 3.2285 \times 10^{-1}, \ \ \mu_{31} = 3.4477, \ \ \mu_{32} = 2.0763.
\end{aligned}$$

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Thus, the symmetric positive definite matrices $\mathcal{P}_i = \mathcal{Y}_i^{-1}$ and the fixed gain matrices $K_i = \mathcal{W}_i B_i^T \mathcal{Y}_i^{-1}$ can be calculated as

$$\begin{aligned}
\mathcal{P}_1 &= \begin{pmatrix} 7.6854 & 2.0845 \\ \star & 1.7996 \end{pmatrix}, & K_1 &= \begin{pmatrix} 8.1918 & 7.0723 \end{pmatrix}, \\
\mathcal{P}_2 &= \begin{pmatrix} 1.0923 \times 10^1 & -5.8891 \\ \star & 4.3025 \end{pmatrix}, & K_2 &= \begin{pmatrix} 1.1174 \times 10^1 & -3.5221 \end{pmatrix}, \\
\mathcal{P}_3 &= \begin{pmatrix} 2.4946 & -2.4964 \times 10^{-1} \\ \star & 3.0823 \end{pmatrix}, & K_3 &= \begin{pmatrix} 8.0240 & -8.0297 \times 10^{-1} \end{pmatrix}.
\end{aligned} \tag{81}$$

Next, by solving the constrained convex optimization problem of Eq. (77), the following solution can be obtained:

$$\begin{aligned}
&\xi_{11} = 3.4167 \times 10^{-2}, \quad \xi_{22} = 3.5524 \times 10^{-2}, \quad \xi_{33} = 1.5590 \times 10^{-1}, \\
&\varepsilon_{12} = 8.5122 \times 10^{-1}, \quad \varepsilon_{13} = 5.9622 \times 10^{-1}, \quad \varepsilon_{21} = 1.4174, \\
&\varepsilon_{23} = 3.1440 \times 10^{-1}, \quad \varepsilon_{31} = 9.9709, \quad \varepsilon_{32} = 1.9446, \\
&\sigma_{12} = 3.4167 \times 10^{-2}, \quad \sigma_{13} = 3.4167 \times 10^{-2}, \quad \sigma_{21} = 3.5524 \times 10^{-2}, \\
&\sigma_{23} = 3.5524 \times 10^{-2}, \quad \sigma_{31} = 1.5590 \times 10^{-1}, \quad \sigma_{32} = 1.5590 \times 10^{-1}, \\
&\lambda = 1.0001.
\end{aligned} \tag{82}$$

Therefore, the allowable upper bound of unknown parameters is given as

$$\begin{aligned}
&\rho_{11}^{\star} = 3.4167 \times 10^{-2}, \quad \rho_{22}^{\star} = 3.5524 \times 10^{-2}, \quad \rho_{33}^{\star} = 1.5590 \times 10^{-1}, \\
&\rho_{12}^{\star} = 3.4167 \times 10^{-2}, \quad \rho_{13}^{\star} = 3.4167 \times 10^{-2}, \quad \rho_{21}^{\star} = 3.5524 \times 10^{-2}, \\
&\rho_{23}^{\star} = 3.5524 \times 10^{-2}, \quad \rho_{31}^{\star} = 1.5590 \times 10^{-1}, \quad \rho_{32}^{\star} = 1.5590 \times 10^{-1}.
\end{aligned} \tag{83}$$

In this example, unknown parameters $\Delta_{ii}(t) \in \mathbb{R}^{2\times 2}$ and $\Delta_{ij}(t) \in \mathbb{R}^{2\times 2}$ are chosen as

$$\begin{aligned}
\Delta_{ii}(t) &= \rho_{ii}^{\star} \times \begin{pmatrix} \sin(5.0 \times \pi \times t) & -\cos(2.0 \times \pi \times t) \\ \star & \cos(5.0 \times \pi \times t) \end{pmatrix}, \\
\Delta_{ij}(t) &= \rho_{ij}^{\star} \times \begin{pmatrix} -\cos(\pi \times t) & \sin(3.0 \times \pi \times t) \\ \star & \sin(\pi \times t) \end{pmatrix}.
\end{aligned} \tag{84}$$

Moreover, the design parameters $\vartheta_i (i = 1, 2, 3)$, the initial value of the uncertain large-scale system with system parameters of Eq. (79), and one of the nominal systems are selected as $\vartheta_1 = \vartheta_2 = \vartheta_3 = 1.0 \times 10^{-1}$ and $\mathrm{x}(0) = \overline{\mathrm{x}}(0) = \begin{pmatrix} 1.5 & -1.0 & -1.0 & 5.0 \times 10^{-1} & 2.0 & -1.0 \end{pmatrix}^T$.

The result of this example is shown in **Figures 5** and **6**. In these figures, $\mathrm{x}_i^{(l)}(t)$, $u_i(t)$, $\overline{\mathrm{x}}_i^{(l)}(t)$, and $\overline{u}_i(t)$ denote the lth element $(l = 1, 2)$ of the state $\mathrm{x}_i(t)$ and the control input $u_i(t)$ for ith subsystem and one of the states $\overline{\mathrm{x}}_i(t)$ and the control input $\overline{u}_i(t)$ for ith nominal subsystem.

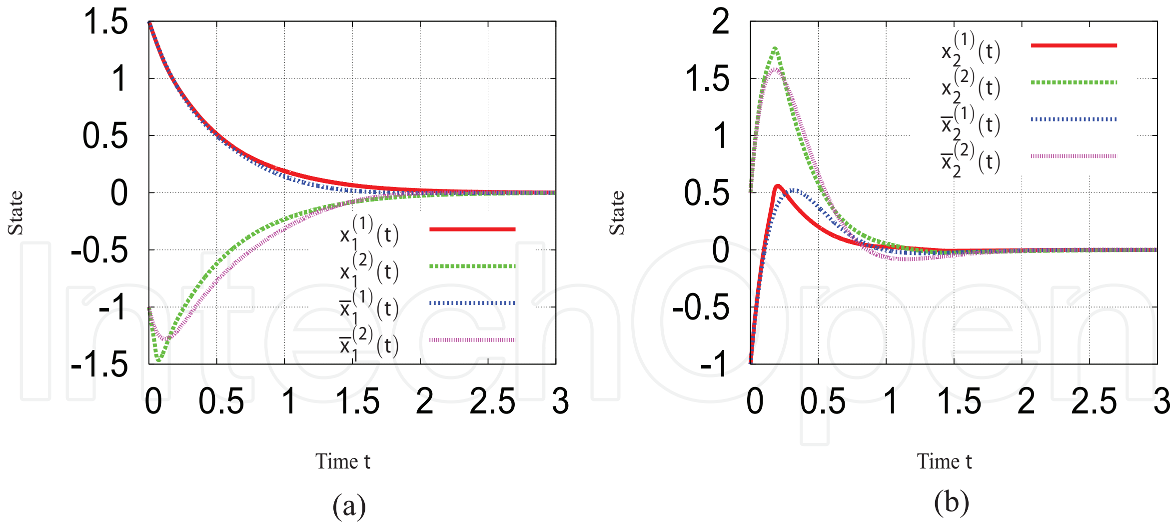

Figure 5. Time histories of $x_i(t)$ and $\overline{x}_i(t)$ ($i = 1, 2$). (a) The time histories of $x_1(t)$ and $\overline{x}_1(t)$, (b) Time histories of $x_2(t)$ and $\overline{x}_2(t)$.

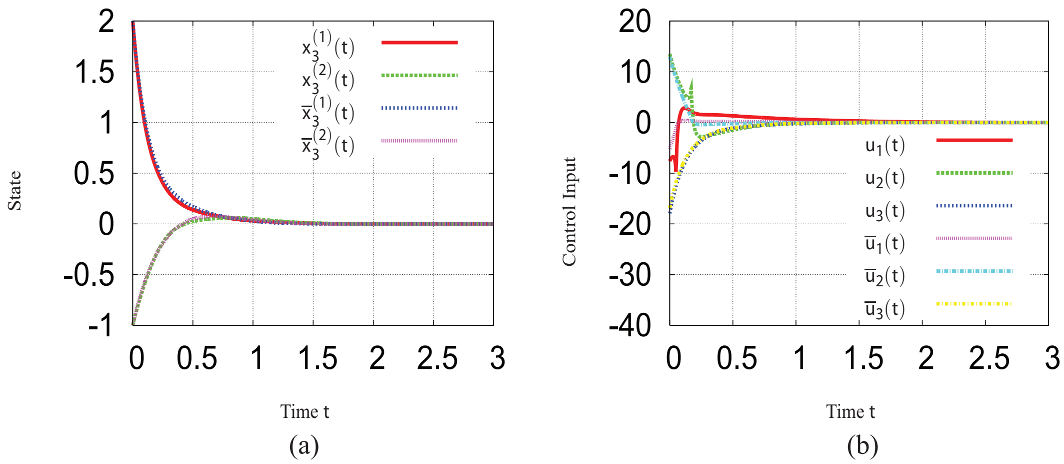

Figure 6. Time histories of $x_3(t)$, $\overline{x}_3(t)$, $u(t)$ and $\overline{u}(t)$. (a) Time histories of $x_3(t)$ and $\overline{x}_3(t)$, (b) Time histories of $u(t)$ and $\overline{u}(t)$.

From these figures, the proposed decentralized adaptive gain robust controller stabilizes the uncertain large-scale interconnected system with system parameters of Eq. (79). Furthermore, one can see that each subsystem achieves good transient behavior close to nominal subsystems by the proposed decentralized robust controller. Thus, the effectiveness of the proposed robust control strategy is shown.

3.4. Summary

In this section, on the basis of the result of Section 2, we have suggested the decentralized adaptive gain robust controller for the large-scale interconnected system with uncertainties.

Furthermore, the effectiveness of the proposed controller has been shown via an illustrative example. The proposed adaptive gain robust controller can be easily designed by solving a constrained convex optimization problem and adjust the magnitude of the control input for each subsystem. Therefore, we find that the proposed decentralized robust controller design method is very useful.

Future research subjects include analysis of conservatism for the proposed controller design approach and extension of the proposed adaptive gain robust control strategies to uncertain systems with time delay, decentralized output/observer-based control systems, and so on.

4. Conclusions and future works

In this chapter, firstly the centralized adaptive gain robust controller for a class of uncertain linear systems has been proposed, and through a simple numerical example, we have shown the effectiveness/usefulness for the proposed adaptive gain robust control strategy. Next, for a class of uncertain large-scale interconnected systems, we have presented an LMI-based design method of decentralized adaptive gain robust controllers. In the proposed controller robust synthesis, advantages are as follows: the proposed adaptive gain robust controller can achieve satisfactory transient behavior and/or avoid the excessive control input, that is, the proposed robust controller with adjustable time-varying parameters is more flexible and adaptive than the conventional robust controller with a fixed gain which is derived by the worst-case design for the unknown parameter variations. Moreover, in this chapter we have derived the allowable perturbation region of unknown parameters, and the proposed robust controller can be obtained by solving constrained convex optimization problems. Although the solution of the some matrix inequalities can be applied to the resultant robust controller in the general controller design strategies for the conventional fixed gain robust control, the solutions of the constrained convex optimization problem derived in this chapter cannot be reflected to the resultant robust controller. Note that the proposed controller design strategy includes this fascinating fact.

In Section 2 for a class of uncertain linear systems, we have dealt with a design problem of centralized adaptive gain robust state feedback controllers. Although the standard LQ regulator theory for the purpose of generating the desired response is adopted in the existing result [32], the nominal control input is designed by using pole placement constraints. By using the controller gain for the nominal system, the proposed robust control with adjustable time-varying parameter has been designed by solving LMIs. Additionally, based on the derived LMI-based conditions, the constrained convex optimization problem has been obtained for the purpose of the maximization of the allowable perturbation region of uncertainties included in the controlled system. Section 3 extends the result for the centralized adaptive gain robust state feedback controller given in Section 2 to decentralized adaptive gain robust state feedback controllers for a class of uncertain large-scale interconnected systems. In this section, an LMI-based controller synthesis of decentralized adaptive gain robust state feedback control has also been presented. Furthermore, in order to maximize the allowable region of uncertainties, the design problem of the decentralized adaptive gain robust controller for the uncertain large-scale interconnected system has been reduced to the constrained convex optimization problem.

In the future research, an extension of the proposed adaptive gain robust state feedback controller to output feedback control systems or observer-based control ones is considered. Moreover, the problem for the extension to such a broad class of systems as uncertain time-delay systems, uncertain discrete-time systems, and so on should be tackled. Furthermore, we will examine the conservativeness of the proposed adaptive gain robust control strategy and online adjustment way of the design parameter which plays important roles such as avoiding the excessive control input.

On the other hand, it is well known that the design of control systems is often complicated by the presence of physical constraints: temperatures, pressures, saturating actuators, within safety margins, and so on. If such constraints are violated, serious consequences may ensue. For example, physical components will suffer damage from violating some constraints, or saturations for state/input constraints may cause a loss of closed-loop stability. In particular, input saturation is a common feature of control systems, and the stabilization problems of linear systems with control input saturation have been studied (e.g., [33, 40]). Additionally, some researchers have investigated analysis of constrained systems and reference managing for linear systems subject to input and state constraints (e.g., [10, 19]). Therefore, the future research subjects include the constrained robust controller design reducing the effect of unknown parameters.

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