Chapter 14

# Vibration Characteristics of Fluid-Filled Functionally Graded Cylindrical Material with Ring Supports

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**Abstract**

Vibration analysis of fluid-filled functionally graded material (FGM) cylindrical shells (CSs) is investigated with ring supports. The shell problem is formulated by deriving strain and kinetic energies of a vibrating cylindrical shell (CS). The method of variations of Hamiltonian principle is utilized to change the shell integral problem into the differential equation (DE) expression. Three differential equations (DE) in three unknown for displacement functions form a system of partial differential equations (PDEs). The shells are restricted along the thickness direction by ring supports. The polynomial functions describe the influence of the ring supports and have the degree equal to the number of ring supports. Fluid loaded terms (FLT) are affixed with the shell motion equations. The acoustic wave equation states the fluid pressure designated by the Bessel functions of first kind. Axial modal deformation functions are specified by characteristic beam functions which meet end conditions imposed on two ends of the shell. The Galerkin method is employed to get the shell frequency equation. Natural frequency of FGM cylindrical shell is investigated by placing the ring support at different position with fluid for a number of physical parameters. For validity and accuracy, results are obtained and compared with the data in open literature. A good agreement is achieved between two sets of numerical results.

**Keywords:** functionally graded material (FGM), ring supports, cylindrical material, Galerkin technique (GT), Hamiltonian principle

## 1. Introduction

All over the word, applications of fluid-filled cylindrical shells have grown in engineering and science. Amendments in shell physical quantities are inducted to enhance strength and stability of cylindrical shells (CSs) [1, 2]. Additional burden due to fluid factor on a physical system may cause damage to it. In the recent times, this feature has appealed to scientists doing research on dynamical properties of materials to explore more about specific strength, stiffness and super corrosion resistance [3–7]. It has been acquired by highly developed complex materials. Study of vibratory response of cylindrical shells (CSs) containing fluids is very beneficial to study dynamic behavior for their applications. This presents a direct contact between a solid composition and a liquid material [8]. In a lot of fields of engineering and technology (mechanical, civil, aeronautics), its useful implications can be seen. Thin-walled cylindrical shells (CSs) have extensive applications in engineering and industry. They are found in chimney design, pipe flow, nuclear reactors and submarines.

For theoretical point of view, study of cylindrical shell vibrations is done to investigate analytical results and their closeness with experimental ones. Here the Galerkin procedure is employed to solve the shell governing equations. For the present cylindrical shells, functionally graded materials are utilized for their structure construction. In the radial thickness direction, material distribution is handled by the exponential volume fraction law. Due to this law, special types of integrals are evolved and are approximated numerically or analytically to evaluate material stiffness modulus. These integrals involve the material parameters of thickness variable by assuming the Poisson ratios of functionally graded constituent materials. For simplifying the integrals, these are presumed to be nearly equal to each other. This assumption simplifies the material stiffness integrals. Shell dynamical equations are framed by applying the Hamilton's variational principle to the Lagrangian functional that is obtained from the shell strain and kinetic energy expressions. These equations govern the shell vibration behavior. For this problem, a suitable and effective method is employed to achieve the shell frequency equation in the eigenvalue problem expression. Normally energy variational approaches are used to solve cylindrical shells problem. They consist of the Raleigh - Ritz method and the Galerkin method. The axial deformation functions are estimated by characteristic beam functions. They are achieved from the solutions of beam differential equation.

Pioneering research work on vibrations of cylindrical shells (CSs) has performed by Arnold and Warburton [8]. The consequence of end conditions on vibration characteristics of a circular cylindrical shell (CS) was considered by Fosberg in 1964 by using shell equation.

Najafizadeh and Isvandzibaei [5] analyzed vibration characteristics of functionally graded cylindrical shells with ring supports. They based their analysis higher order shear deformation theory of shells. It was perceived that the influence of ring supports and fluid terms was very significant on shell frequencies. Vibration characteristics of cylindrical shells containing fluid were studied experimentally and theoretically by Chung et al. [9]. Goncalves and Batista [10] presented a theoretical vibration study of cylindrical shells (CSs) partially filled and submerged in a fluid. Simply supported end conditions were imposed on both edges. Goncalves et al. [11] investigated the transient stability of empty and fluid-filled cylindrical shells and used to study the non-linear dynamic behavior of shallow cylindrical shells under axial loading. Gasser [12] studied the frequency spectra of bi-layered cylindrical shells by taking different materials in both layers such as isotropic as well as functionally graded material (FGM) and by taking two different FGM at the inner and outer layers of the CSs respectively. Sharma and Johns [13] explored the vibrations of CSs with clamped-free and clamped-ring-stiffeners

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conditions by applying the Raleigh-Ritz method and estimated the axial displacement deformation with the help of beam functions. Xi et al. [14] have studied vibrations of cross-ply plastic-coated circular fluid-filled CSs by applying a semi-analytical method based on Reissner-Mindline theory. Zhang et al. [4] studied vibrations of CSs and applied the wave propagation approach (WPA) to solve shell dynamical equations. This method depended on the eigenvalue of characteristic beam functions. Axial wave number was designated to a boundary condition (BC) by a simple formula. They compared the results determined by this method to ones found by a FEM to check the efficiency, robustness and accuracy of the procedure. It was seen by making these comparisons that their approach is more victorious and exact for shell vibration difficulties. It was concluded that the proposed approach could be useful for a problem with compound end states and also for fluid-filled cylindrical shells.

Zhang et al. [15] examined vibrations of CSs containing fluid by applying WPA. After that a similarity evaluation was conducted between uncoupled frequencies with the numerical outcome obtained in the literature. They also put side by side the coupled frequencies estimated by the WPA with those attained by FEM. Xiang et al. [16] accessed exact solution for the vibration characteristics of CS placed at intermediate position and used the domain decomposition technique for the sake of ordering in the segment of the shells. Zhao et al. [17] investigated the effects of vibration with ring stiffeners and stringer for the laminated cross-ply rotating CS and two methods: variational method and averaging approach are used for these effects. They determined that averaging method produced the inexact values and was sensitive whereas the fast and better results were deduced with variational method. Xiang et al. [18] accessed the vibration characteristics of CS placed at intermediate position with axially dense ring supports and used Flügge shell theory and the Timoshenko thin shell theory to analyze the buckling shells as composite materials. Due to FGM their composition vary constantly and smoothly through thickness.

Vibration characteristic of FGM shell with ring supports has investigated by Isvandzibaei and Awasare [19] and they used third order deformation shear theory and Hamilton's principle for free-free end. Lee and Chang [20] gave a numerical study of coupled problems of fluid conveying dual walled carbon annotates and examined the effects of characteristic ratio and Van der Waals forces on basic frequencies. Silva et al. [21] investigated the nonlinear vibrant behavior and instabilities of partially fluid-filled CS constrained to axial load and resulting in a distinct low-dimensional model for the analysis of the vibrations to observe the shell vibration. Shah et al. [22] gave a vibration analysis of a functionally graded CS containing a fluid. The shells were rested on elastic foundations. They analyzed effects of Winkler and Pasternak moduli on shell vibration characteristics. Xiong et al. [23] investigated the free vibration analysis of fluid-filled elliptical cylindrical shells and explained the sensitivity of frequency parameters to the elliptical parameter with length of CS. The cylinder is filled with a compressible non-viscous fluid and may be subjected to arbitrary time-harmonic on-surface mechanical drives is investigated by Hasheminejad and Alaei-Varnosfadrani [24]. The free vibration of fluid-filled CS covered partially in elastic foundation is investigated by Kim [25] and the elastic foundation of partial axial and angular dimensions is represented by the Pasternak model. The variation of the frequency parameters with respect to the layer thickness, the length-to-radius ratio, the length-to-thickness ratio, and circumferential node number are analyzed by Izyan et al. [26]. Soutis et al. [27] investigated influence of ring supports on free vibration of FGM which is placed on the middle layer and Study is carried out for placing ring support in different position of FGM's, to find the natural frequencies by Rayleigh–Ritz approach.

In the present paper, vibration frequency characteristics of fluid-filled CSs are investigated. The shells are constrained in the radial direction by ring supports. The present problem is formulated in integral form and is converted into a system of three partial differential equations (PDEs) with the unknown displacement functions. Modal forms for the three unknown functions are assumed such that the special and temporal variables are separated. Energy variation approach is used to solve PDEs so that an eigenvalue problem is cropped up. Axial modal dependence is roughly estimated by trigonometric functions for a simply supported CSs. For other end conditions, characteristic beam functions are taken. The radial constraints are presumed by the polynomial functions having degree equal to number of ring supports. Fluid pressure is stated by the acoustic wave equation and Bessel's functions of first kind. Axial modal displacement deformations are measured by beam characteristic functions which ensure to meet boundary conditions. The Galerkin technique is implemented to form the shell frequency equation which is solved by using MATLAB coding. The radial deflection is restrained by ring supports. This factor is expressed by the polynomial functions which carry the degree equal to the number of ring supports.

## 2. Formation of shell problem

### 2.1. Functionally graded shells

In practice a CS is constructed from a FGM which consists of two constituent materials. Two constituent materials having material parameters: $E_1$, $E_2$, $\nu_1$, $\nu_2$, and $\rho_1$, $\rho_2$. Then the effective material quantities: $E_{fgm}$, $v_{fgm}$ and $\rho_{fgm}$ are given as:

$$E_{fgm} = [E_1 - E_2]\left[\frac{z}{h} + \frac{1}{2}\right]^p + E_2, \nu_{fgm} = [\nu_1 - \nu_2]\left[\frac{z}{h} + \frac{1}{2}\right]^p + \nu_2, \rho_{fgm} = [\rho_1 - \rho_2]\left[\frac{z}{h} + \frac{1}{2}\right]^p + \rho_2 \quad (1)$$

The value of $z$ lies as $0 < z < \infty$ in the radial direction. The volume fraction $V_r$ for a functionally graded constituent material can be defined by the following function:

$$V_r = \left[\frac{z}{h} + \frac{1}{2}\right]^p \quad (2)$$

where $p$ is the power law exponent which indicates the material variation profile through the shell thickness and keeps its real between zero and infinity.

### 2.2. Theoretical investigation

Consider **Figure 1** in which a geometrical sketch of a CS is given. $L$, $R$, and $h$ are termed as shell geometrical parameters. Other shell basic quantities are material parameters and are designate by $E$, $v$ and $\rho$.

where the forces and moments are designated by $N$ and $M$ have the directions along the longitudinal, tangential and shear directions correspondingly.

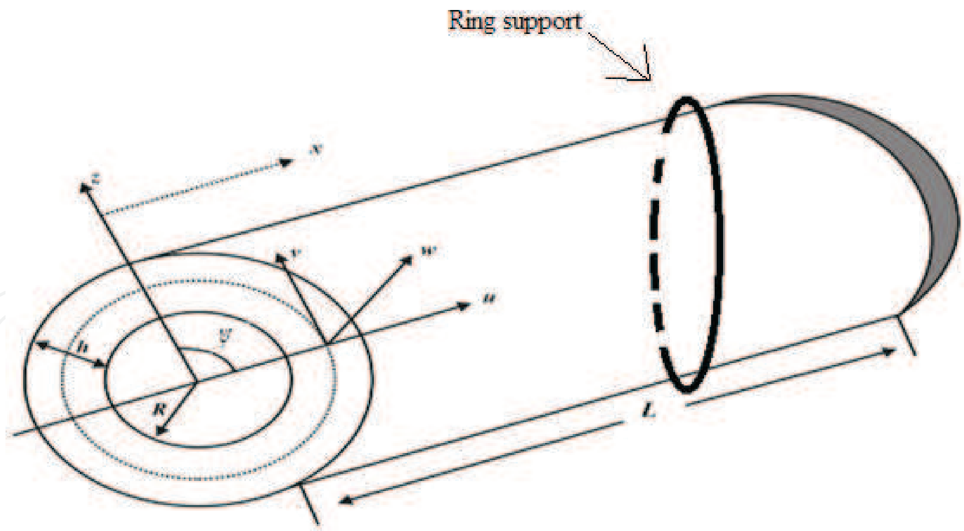


**Figure 1.** Geometrical sketch of a FG cylindrical shell with ring support.

$$(N_x, N_\Psi, N_{x\Psi}) = \int_{\frac{-h}{2}}^{\frac{h}{2}} (\sigma_x, \sigma_\Psi, \sigma_{x\Psi}) dz, \quad (M_x, M_\Psi, M_{x\Psi}) = \int_{\frac{-h}{2}}^{\frac{h}{2}} (\sigma_x, \sigma_\Psi, \sigma_{x\Psi}) z dz \tag{3}$$

where $\sigma_x$, $\sigma_\Psi$ are the linear stresses along $x$ and $\Psi$-directions respectively and $\sigma_{x\Psi}$ represents the shear stress along $x\Psi$-direction. For a cylindrical shell, the stresses defined in Eq. (3) are defined by the two dimensional Hook's law.

$$\begin{bmatrix} \sigma_x \\ \sigma_\Psi \\ \sigma_{x\Psi} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} e_x \\ e_\Psi \\ e_{x\Psi} \end{bmatrix} \tag{4}$$

where the strains along $x$ and $\Psi$ directions are labeled by $e_x$ and $e_\Psi$ respectively and the shear strain is denoted by $e_{x\Psi}$ in the $x\Psi$ - direction. The first thin shell theory was developed by Love [7] which is based on Kirchhoff's perception for plates. Various thin shell theories have been deduced from this theory by modifying the geometrical and physical parameters. The members of the strain vector $[e]$ in Eq. (4) have been defined as linear functions of thickness coordinate $z$ which taken from Love's [32] theory are stated as:

$$e_x = e_1 + z\kappa_1, e_\Psi = e_2 + z\kappa_2, e_{x\Psi} = \gamma + 2z\tau \tag{5}$$

here $e_1$, $e_2$ and $\gamma$ denote strains with regard to the shell middle reference surface. $\kappa_1$, $\kappa_2$ and $\tau$ stand for the surface curvatures. The expressions for strain and curvature displacement relationship are written as:

$$\{e_1, e_2, \gamma\} = \left\{ \frac{\partial u}{\partial x}, \frac{1}{R}\left(\frac{\partial v}{\partial \Psi} + w\right), \left(\frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \Psi}\right) \right\}$$

$$\{\kappa_1, \kappa_2, \tau\} = \left\{ -\frac{\partial^2 w}{\partial x^2}, -\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \Psi^2} - \frac{\partial v}{\partial \Psi}\right), -\frac{1}{R}\left(\frac{\partial^2 w}{\partial x \partial \Psi} - \frac{\partial v}{\partial x}\right) \right\} \tag{6}$$

By substituting Eqs. (5) and (6) into Eq. (4) and then substituting the resulting equation into Eq. (3). The force and moment results can be written as:

$$\begin{bmatrix} N_x \\ N_\Psi \\ N_{x\Psi} \\ M_x \\ M_\Psi \\ M_{x\Psi} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \gamma \\ \kappa_1 \\ \kappa_2 \\ 2\tau \end{bmatrix} \tag{7}$$

where $A_{ij}$, $B_{ij}$ and $D_{ij}$ ($i$, $j$ = 1, 2 and 6) are the extentional, coupling and bending stiffness defined respectively, as:

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}\left(1, z, z^2\right) dz, \tag{8}$$

where coupling stiffness, $B_{ij}$'s vanish for a CS structured from isotropic materials where they exist for heterogeneous and an isotropic materials such as laminated FGM. For an isotropic material, $Q_{ij}$ ($i$, $j$ = 1, 2 and 6) are expressed as

$$Q_{11} = Q_{22} = \frac{E}{1 - v^2}, Q_{12} = \frac{vE}{1 - v^2}, Q_{66} = \frac{E}{2(1 + v)} \tag{9}$$

where $E$ and $\nu$ are the Young's modulus and Poisson's ratio for the shell's material. In this study, the cylindrical shell is considered thin and valid with thickness-to- radius ratio is less than 0.05. For vibrating thin cylindrical shell, the strain energy expressed as

$$\begin{aligned} S = \frac{R}{2} \int_0^L \int_0^{2\pi} & \left[A_{11} {e_1}^2 + A_{22} {e_2}^2 + 2A_{12} e_1 e_2 + A_{66} \gamma^2 + 2B_{11} e_1 k_1 + 2B_{12} e_1 k_2 + 2B_{12} e_2 k_1 \right. \\ & \left. + 2B_{22} e_2 k_2 + 4B_{66} \gamma \tau + D_{11} {k_1}^2 + D_{22} {k_2}^2 + 2D_{12} k_1 k_2 + 4D_{66} \tau^2 \right] d\Psi dx \end{aligned} \tag{10}$$

Substituting the expression for the surface strains and the curvatures from the relationships (6)

$$\begin{aligned} S = \frac{R}{2} \int_0^L \int_0^{2\pi} & \left[ A_{11} \left(\frac{\partial u}{\partial x}\right) + \frac{A_{22}}{R^2} \left(\frac{\partial v}{\partial \Psi} + w\right)^2 + \frac{2A_{12}}{R} \left(\frac{\partial u}{\partial x}\right) \left(\frac{\partial v}{\partial \Psi} + w\right) + A_{66} \left(\frac{\partial v}{\partial x} + \frac{1}{R} \frac{\partial u}{\partial \Psi}\right)^2 \right. \\ & -2B_{11} \left(\frac{\partial u}{\partial x}\right) \left(\frac{\partial^2 w}{\partial x^2}\right) - \frac{2B_{12}}{R} \left(\frac{\partial u}{\partial x}\right) \left(\frac{\partial^2 w}{\partial x \partial \Psi} - \frac{\partial v}{\partial x}\right) - \frac{2B_{12}}{R} \left(\frac{\partial v}{\partial \Psi} + w\right) \left(\frac{\partial^2 w}{\partial x^2}\right) \end{aligned}$$

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$$-\frac{2B_{22}}{R^3}\left(\frac{\partial v}{\partial\Psi}+w\right)\left(\frac{\partial^2 w}{\partial\Psi^2}-\frac{\partial v}{\partial\Psi}\right)-\frac{8B_{66}}{R}\left(\frac{\partial v}{\partial x}+\frac{1}{R}\frac{\partial u}{\partial\Psi}\right)\left(\frac{\partial^2 w}{\partial x\partial\Psi}-\frac{\partial v}{\partial x}\right)$$

$$+D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\frac{D_{22}}{R^4}\left(\frac{\partial^2 w}{\partial\Psi^2}-\frac{\partial v}{\partial\Psi}\right)^2+\frac{2D_{12}}{R^2}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial\Psi^2}-\frac{\partial v}{\partial\Psi}\right)$$

$$+\frac{4D_{66}}{R^2}\left(\frac{\partial^2 w}{\partial x\partial\Psi}-\frac{\partial v}{\partial x}\right)^2 d\Psi dx \tag{11}$$

Also for a cylindrical shell, its kinetic energy expression represented by $T$, is given as:

$$T=\frac{R}{2}\int_0^L\int_0^{2\pi}\rho_T\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]d\Psi dx \tag{12}$$

where $\rho_T$ is expressed as:

$$\rho_T=\int_{\frac{-h}{2}}^{\frac{h}{2}}\rho dz \tag{13}$$

Now the shell problem is framed by the Lagrangian energy functional which is the difference between the shell kinetic and strain energies and is given as:

$$\prod = T-S \tag{14}$$

where $\prod$ denotes the Lagrangian functional. Substituting the expressions for strains and kinetic energies of the shell from Eqs. (11) and (12) respectively into Eq. (14). Calculus of variations process is applied to the integral terms to derive the Euler-Lagrange equations. Hamilton's variational principle is a process in which the variations in the variables are assumed to be zero. Implementation this principle to the Lagrangian functional which is an integral expression. The subsequent dynamical equations are obtained in the following system of PDEs:

$$A_{11}\frac{\partial^2 u}{\partial x^2}+\frac{A_{66}}{R^2}\frac{\partial^2 u}{\partial\Psi^2}+\left(\frac{A_{12}+A_{66}}{R}+\frac{B_{12}+2B_{66}}{R^2}\right)\frac{\partial^2 v}{\partial x\partial\Psi}+\frac{A_{12}}{R}\frac{\partial w}{\partial x}-B_{11}\frac{\partial^3 w}{\partial x^3}-\frac{B_{12}+2B_{66}}{R^2}\frac{\partial^3 w}{\partial x\partial\Psi^2}=\rho_T\frac{\partial^2 u}{\partial t^2}$$

$$\left(\frac{A_{12}+A_{66}}{R}+\frac{B_{12}+B_{66}}{R^2}\right)\frac{\partial^2 u}{\partial x\partial\Psi}+\left(A_{66}+\frac{3B_{66}}{R}+\frac{3D_{66}}{R^2}\right)\frac{\partial^2 v}{\partial x^2}+\left(\frac{A_{22}}{R^2}+\frac{2B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)\frac{\partial^2 v}{\partial\Psi^2}$$
$$-\left(\frac{B_{12}+2B_{66}}{R}+\frac{D_{12}+2D_{66}}{R^2}\right)\frac{\partial^3 w}{\partial x^2\partial\Psi}+\left(\frac{A_{22}}{R^2}+\frac{B_{22}}{R^3}\right)\frac{\partial w}{\partial\Psi}-\left(\frac{B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)\frac{\partial^3 w}{\partial^3\Psi}=\rho_T\frac{\partial^2 v}{\partial t^2}$$

$$B_{11}\frac{\partial^3 u}{\partial x^3}-\frac{A_{12}}{R}\frac{\partial u}{\partial x}+\frac{B_{12}+2B_{66}}{R^2}\frac{\partial^3 u}{\partial x\partial\Psi^2}+\left(\frac{B_{12}+2B_{66}}{R}+\frac{D_{12}+4D_{66}}{R^2}\right)\frac{\partial^3 v}{\partial x^2\partial\Psi}+\left(\frac{B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)\frac{\partial^3 v}{\partial\Psi^3}$$
$$-\left(\frac{A_{22}}{R^2}+\frac{B_{22}}{R^3}\right)\frac{\partial v}{\partial\Psi}-D_{11}\frac{\partial^4 w}{\partial x^4}-\frac{2(D_{12}+2D_{66})}{R^2}\frac{\partial^4 w}{\partial x^2\partial\Psi^2}-\frac{D_{22}}{R^4}\frac{\partial^4 w}{\partial\Psi^4}+\frac{2B_{12}}{R}\frac{\partial^2 w}{\partial x^2}$$
$$+\frac{2B_{22}}{R^3}\frac{\partial^2 w}{\partial\Psi^2}-\frac{A_{22}}{R^2}w=\rho_T\frac{\partial^2 w}{\partial t^2} \tag{15}$$

## 3. Application of Galerkin technique

Two energy variational techniques namely viz., the Rayleigh-Ritz method and the Galerkin technique (GT) are exploited to solve shell motion equations because these methods yield results fast with enough accuracy. The present CS is analyzed by applying the Galerkin technique (GT) for their vibrations. This approach is very expedient, simple and convenient to use to find vibration frequencies and has been widely engaged by numerous mathematicians [25, 28, 29]. Modal displacement forms are designated by $x$, $\Psi$ and $t$. The following modal deformation displacement functions for $u$, $v$ and $w$ are adopted as:

$$u(x,\Psi,t)=p_m\frac{d\varphi}{dx}\sin n\Psi\cos\omega t$$

$$v(x,\Psi,t)=q_m\varphi(x)\cos n\Psi\cos\omega t$$

$$w(x,\Psi,t)=r_m\varphi(x)\sum_{i=1}^{k}(x-a_i)^{z_i}\sin n\Psi\cos\omega t \tag{16}$$

Here the parameters $p_m$, $q_m$ and $r_m$ present vibration amplitudes in the $x$, $\Psi$ and $z$ directions respectively. The position of i$^{th}$ ring support with the circular direction of the shell is denoted by $a_i$ and $z_i$ has value, 1 when a ring support exists and is 0, when no rings hold up. For this purpose the modal displacement forms for $u$, $v$ and $w$ given in the relation (12) respectively and their corresponding partial derivatives are substituted into Eq. (15) by taking, $z_i=1$ for a single ring support, the resulting equations are integrated with respect to $x$ from 0 to $L$, the following equations are got:

$$\left(A_{11}I_1-n^2\frac{A_{66}}{R^2}I_2\right)p_m-n\left(\frac{A_{12}+A_{66}}{R}+\frac{B_{12}+2B_{66}}{R^2}\right)I_2q_m+\begin{bmatrix}\frac{A_{12}}{R}(I_6+I_7)-B_{11}(I_8+3I_9)\\+n^2\frac{B_{12}+2B_{66}}{R^2}(I_6+I_7)\end{bmatrix}r_m=-\omega^2\rho_T I_2 p_m$$

$$n\left(\frac{A_{12}+A_{66}}{R}+\frac{B_{12}+2B_{66}}{R^2}\right)I_3p_m+\left[\left(A_{66}+\frac{3B_{66}}{R}+\frac{4D_{66}}{R^2}\right)I_3-n^2\left(\frac{A_{22}}{R^2}+\frac{2B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)I_4\right]q_m$$
$$+\left[n\left(\frac{A_{22}}{R^2}+\frac{B_{22}}{R^3}\right)I_{10}+n^3\left(\frac{B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)I_{10}-n\left(\frac{B_{12}+2B_{66}}{R}+\frac{D_{12}+4D_{66}}{R^2}\right)(I_{11}+2I_{12})\right]r_m=-\omega^2\rho_T I_4 p_m$$

$$\left(-\frac{A_{12}}{R}I_{18}+B_{11}I_{19}-n^2\frac{B_{12}+2B_{66}}{R^2}I_{18}\right)p_m+\left[n\left(\frac{A_{22}}{R^2}+\frac{B_{22}}{R^3}\right)I_{10}+n^3\left(\frac{B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)I_{10}\right.$$
$$\left.-n\left(\frac{B_{12}+2B_{66}}{R}+\frac{D_{12}+4D_{66}}{R^2}\right)I_{18}\right]q_m+\left[-\frac{A_{22}}{R^2}I_3+\frac{2B_{12}}{R}(I_{14}+2I_{15})-2n^2\frac{B_{22}}{R^3}I_{13}\right.$$
$$\left.-D_{11}(I_{16}+4I_{17})+2n^2\frac{D_{12}+2D_{66}}{R^2}(I_{14}+2I_{15})-\frac{D_{22}}{R^4}I_{13}\right]r_m=-\omega^2\rho_T I_{13}r_m \tag{17}$$

where the integral terms are listed in Appendix-I. Terms in Eq. (17) are arranged to form the homogeneous algebraic linear equations (HALEs) in $p_m$, $q_m$ and $r_m$. This leads to the formation of the shell frequency equation the eigenvalue shape as:

$$\begin{aligned} d_{11}p_m+d_{12}q_m+d_{13}r_m&=-\omega^2\rho_T p_m I_2\\ d_{21}p_m+d_{22}q_m+d_{23}r_m&=-\omega^2\rho_T q_m I_4\\ d_{31}p_m+d_{32}q_m+d_{33}r_m&=-\omega^2\rho_T r_m I_{13}\end{aligned} \tag{18}$$

So the above equations are written in the eigenvalue problem notation as:

$$\begin{pmatrix} d_{11} & d_{12} & d_{13}\\ d_{21} & d_{22} & d_{23}\\ d_{31} & d_{32} & d_{33}\end{pmatrix}\begin{pmatrix} p_m\\ q_m\\ r_m\end{pmatrix}=-\omega^2\rho_T\begin{pmatrix} I_2 & 0 & 0\\ 0 & I_4 & 0\\ 0 & 0 & I_{13}\end{pmatrix}\begin{pmatrix} p_m\\ q_m\\ r_m\end{pmatrix} \tag{19}$$

### 3.1. Annexation of fluid terms

The acoustic pressure generated by a fluid is described by the wave equation in cylindrical coordinate system $(x, \Psi, r)$ and is presented as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial q}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 q}{\partial\Psi^2}+\frac{\partial^2 q}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 q}{\partial t^2} \tag{20}$$

where $t$, $q$, $c$ represents respectively the time, acoustic pressure and speed of sound in the fluid. The acoustic pressure produced in the fluid meets the equation of motion Eq. (20) and is articulated by the following modal function expression:

$$q=q_m\sin n\theta J_n(k_r r)\Psi(x)\cos(\omega t) \tag{21}$$

Here $J_n(k_r r)$ represents the Bessel's function of first kind with order $n$. It is the same number as the circumferential wave number. $\omega$ denotes the natural frequency for the CS, $k_r$ stands for the radial wave number and axial wave number $k_m$ is designated by for a number of boundary conditions that has been indicates in Ref. [10]. There exists a relation between $k_m$ and $k_r$ is written as:

$$(k_r R)^2=\Omega^2\left(\frac{c_L}{c_f}\right)^2-(k_m R)^2 \tag{22}$$

where $c_L$ and $c_f$ denote the speeds of the sound in the empty and fluid-filled cylindrical shells respectively. $\Omega$ is the non-dimensional frequency parameter. An assumption is made that the fluid and the shell radial displacement must be the same at the borderline of the interior wall of the shell and it is made sure that the fluid is kept with the interaction between the fluid and the shell wall. A coupling condition relation of a fluid with the shell wall exists and is defined by the following expression:

$$\left\{\frac{1}{i\omega\rho_f}\right\}\left(\frac{\partial q}{\partial r}\right) = \frac{\partial w}{\partial t} \tag{23}$$

By applying this condition at $r = R$, the fluid loading term ($FLT$) owing to the existence of the fluid pressure is given by

$$FLT = \left(\frac{\omega^2 \rho_f J_n(k_r R)}{k_r J_n'(k_r R)}\right) r_m \tag{24}$$

$J_n'(K_r r)$ denotes the differentiation of the Bessel's function with respect to the argument $(k_r R)$. The fluid loaded term ($FLT$), which represents the fluid pressure, is annexed with the frequency Eq. (18) for an empty CS. Ultimately the shell frequency equation for fluid-filled functionally graded CS is articulated in the following forms:

$$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix} \begin{pmatrix} p_m \\ q_m \\ r_m \end{pmatrix} = -\omega^2 \rho_T \begin{pmatrix} I_2 & 0 & 0 \\ 0 & I_4 & 0 \\ 0 & 0 & I_{13} + \dfrac{\rho_f}{k_r}\dfrac{J_n(k_r R)}{J_n'(k_r R)} \end{pmatrix} \begin{pmatrix} p_m \\ q_m \\ r_m \end{pmatrix} \tag{25}$$

The expressions for the terms $d_{ij}^{'}s$, $I_2$, $I_4$ and $I_{13}$ are given in Appendix-I. This is an eigenvalue problem involving the shell frequency. Presently, the MATLAB is generally used to calculate the physical problems in engineering and science. In our case, MATLAB computer software has been used to compute the shell frequencies through eigenvalues and eigenvectors. A single command 'eig' furnishes shell frequencies and mode shapes by calculating eigenvalues and eigenvectors respectively. It is significant that MATLAB can compute integrals that fairly easy to apply the Galerkin method with polynomial basis functions. The exact solution of integral equation is known and presents a sample of MATLAB code to illustrate the success of the method. In this way the Galerkin method is implemented to form the shell frequency equation that is solved by employing MATLAB software.

### 3.2. Effective material

Materials of cylindrical shells have a paramount role in analyzing shell vibrations. They impress their stability. In practice isotropic, laminated and functionally graded materials are used to manufacture them. Here functionally graded materials are benefitted to form the shells. These materials are advanced and useful in a highly environs. Their material properties are temperature - dependents. One of their material properties $C$ is stated by the following relation:

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$$C = C_0\left(C_{-1}T^{-1} + C_1T + C_2T^2 + C_3T^3\right) \tag{26}$$

where $C_{-1}$, $C_0$, $C_1$, $C_2$ and $C_3$ designate the temperature dependent constants. $T$ is calculated in the Kelvin scale. These constants differ from material to material. Formula (26) is due to Toulokian [38]. A functionally graded cylindrical shell comprising of two constituent materials can be classified into two categories. This depends upon the arrangement of the two materials forming the shell. It is known that the stainless steel and nickel are used for structuring such types of shells. Stainless steel and nickel are used in its external and internal surfaces respectively for Category- I *(C-I)* CS structure, while for Category-II *(C-II)* cylindrical shell, stainless steel and nickel are taken for constituting its internal and external surfaces respectively. At temperature 300 K, the material properties for stainless steel and nickel of functionally graded cylindrical shell are: $E$, $v$, $\rho$ for nickel are $2.05098 \times 10^{11}\,\mathrm{N/m^2}$, 0.31, 8900 $\mathrm{kg/m^3}$ and stainless steel are $2.07788 \times 10^{11}\,\mathrm{N/m^2}$, 0.317756 and $81,666\,\mathrm{N/m^3}$. These values have been taken from Ref. [3].

## 4. Numerical results

In this section, to check the validity and accuracy, for the determination of the natural frequencies with present methodology, empty and fluid-filled cylindrical shells with ring supports are analyzed and the results are compared with experimental and other numerical values found in literature. The stainless steel and nickel are used for structuring such types of shells. Stainless steel and nickel are used in its external and internal surfaces respectively for Category- I *(C-I)* CS structure, while for Category-II *(C-II)* cylindrical shell, stainless steel and nickel are taken for constituting its internal and external surfaces respectively. At temperature 300 K, the material properties for stainless steel and nickel of functionally graded cylindrical shell are: $E$, $v$ , $\rho$ for nickel are $2.05098 \times 10^{11}\,\mathrm{N/m^2}$, 0.31, 8900 $\mathrm{kg/m^3}$ and stainless steel are $2.07788 \times 10^{11}\,\mathrm{N/m^2}$, 0.317756 and $81,666\,\mathrm{N/m^3}$.

A few comparisons of analytical frequencies for isotropic cylindrical shells are exhibited to validate the present Galerkin technique. As a first example, the lowest dimensionless frequency $\Omega = \omega R\sqrt{(1-\nu^2)\rho/E}$ of a simply supported empty cylinder are compared with the solution derived using generalized differential quadrature method (DQM) by Loy et al. [2] as shown in **Table 1**. They are varied with circumferential wave mode, $n$ for axial wave number $m = 1$. There is an excellent agreement is seen between two sets of frequency parameters results as the percentage difference is negligible.

As another example, the natural frequency of a simply supported empty cylinder is compared with Loy et al. [3], Ansari et al. [30] and Warburton [35] as shown in **Figure 2(a)**. They are varied with axial wave number, $m$ for circumferential wave mode, $n$ = 2, 3. For the same shell, the present results for a fluid-filled shell are compared with those numerical results obtained by Gonçalves and Batista [10], Gonçalves et al. [11] and obtained experimentally by Gasser [12] in **Figure 2(b)**. They are varied with circumferential wave mode, $n$ for axial wave number, $m$ = 1. There is an excellent agreement is seen between two sets of frequency parameters results.

| $n$ | Loy et al. [2] | Present | Difference % |
|---|---|---|---|
| 1 | 0.016101 | 0.016102 | 0.006 |
| 2 | 0.009382 | 0.009383 | 0.010 |
| 3 | 0.022105 | 0.022106 | 0.004 |
| 4 | 0.042095 | 0.042097 | 0.004 |
| 5 | 0.068008 | 0.068009 | 0.001 |
| 6 | 0.099730 | 0.099732 | 0.002 |
| 7 | 0.137239 | 0.137241 | 0.001 |
| 8 | 0.180527 | 0.180528 | 0.000 |
| 9 | 0.229594 | 0.229596 | 0.000 |
| 10 | 0.284435 | 0.284436 | 0.000 |

**Table 1.** Comparison of frequency parameter $\Omega = \omega R\sqrt{(1-v^2)\rho/E}$ for a simply supported CS ($m = 1$, $L/R = 20$, $h/R = 0.01$, $\nu = 0.3$).

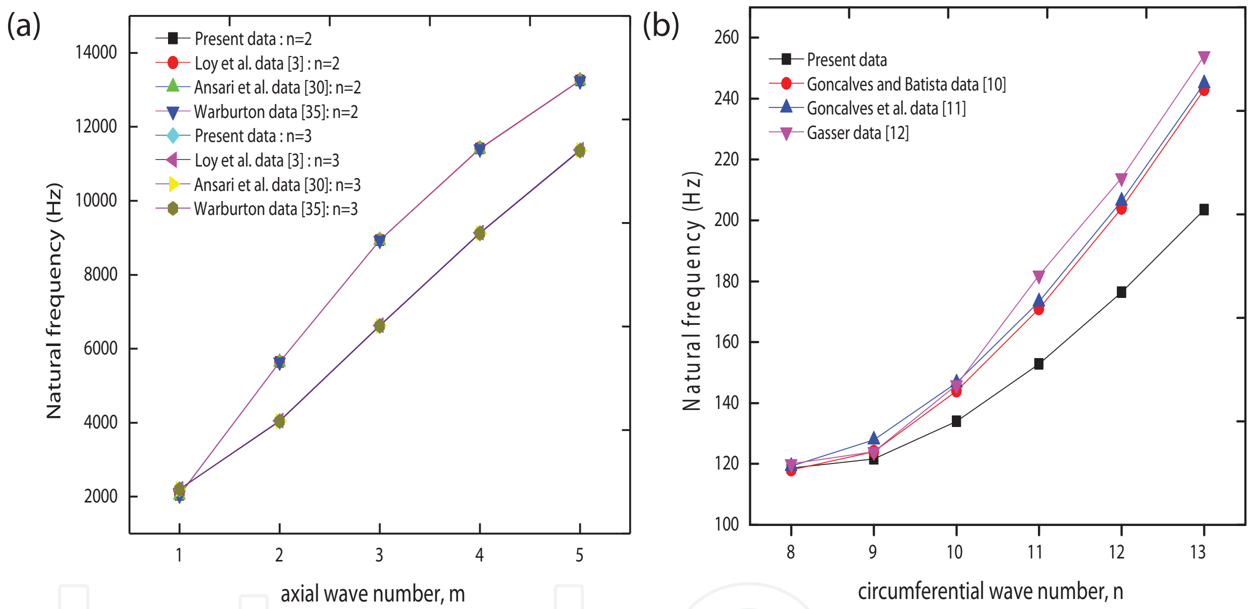


**Figure 2.** **(a)** Comparison of natural frequencies (Hz) for a simply supported CS ($L$ = 8 in, $h$ = 0.1 in, $R$ = 2 in, $E = 30 \times 10^6$ lbf in$^{-2}$, $\nu$ = 0.3, $\rho = 7.35 \times 10^{-4}$ lbf s$^2$ in$^{-4}$). **(b)** Comparison of natural frequencies (Hz) for a fluid-filled CS ($m$ = 1, $L$ = 0.41 m, $h$ = 0.001 m, $R$ = 0.3015 m, $E = 2.1 \times 10^{11}$ N/m$^2$, $v$ = 0.3, $\rho$ = 7850 kg/m$^3$).

When $n$ = 2, 3, $m$ = 1, the present values are 2046.4, 2199.0 and the values of Loy et al. [3] are 2050.7, 2204.0 which is good agreement between each other. **Figure 2(b)** exhibits a good coincidence between respective counters of the frequencies. The values at $n$ = 8, 9 are less significant and on enhancing the value of $n$, the present numerical values are smaller than those of Gonçalves and Batista [10] and Gonçalves et al. [11] and Gasser [12]. This difference is the result of two separate analytical techniques.

### 4.1. Frequency analysis of fluid-filled cylindrical shell

**Figure 3(a)** reveals variations of natural frequencies (Hz) for a CS including fluid against $L/R$ for circumferential wave modes $n$ = 1, 2, 3, 4, 5 and the longitudinal wave mode, $m$ = 1.

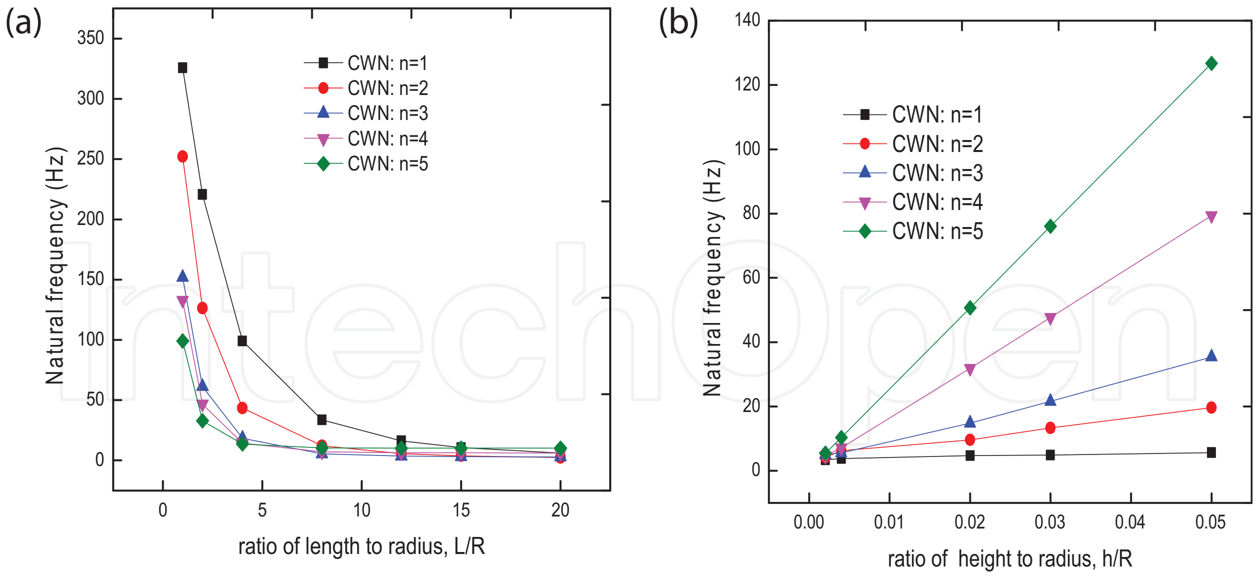


**Figure 3.** Variations of natural frequency **(a)** for fluid-filled CS against $L/R$. ($m = 1$, $h = 0.004$ m, $R = 1$ m, $E = 2.1 \times 10^{11}$ N/m$^2$, $v = 0.3$, $\rho = 7850$ kg/m$^3$, $\rho_f = 1000$ kg/m$^3$) **(b)** for fluid-filled CS with $h/R$ ($m = 1$, $L = 8$ m, $R = 1$ m, $E = 2.1 \times 10^{11}$ N/m$^2$, $v = 0.3$, $\rho = 7850$ kg/m$^3$, $\rho_f = 1000$ kg/m$^3$).

The value of length-to-radius ratio is composed as $1 \le L/R \le 20$, the values of $L/R = 1$, at CWN: $n$ = 1, 2, 3, 4, 5 are 325.625, 252.019, 151.816, 132.818, 99.128 and $L/R = 20$ are 6.2054, 2.3505, 2.8798, 6.3196, 10.0948. For the wave mode, $n$ = 1, 2, 3, 4, 5 as $L/R$ is made to grow i.e., the shell becomes longer and longer, the frequency reduces for each tangential wave mode.

Now the values of length-to-radius ratio $1 \le L/R \le 20$, at CWN: $n$ = 1, the frequencies are 325.675, 220.745, 99.005, 33.693, 16.348, 10.770, 6.2054 and at CWN: $n$ = 5 are 99.128, 33.017, 13.756, 10.3603, 10.1506, 10.1148, 10.0948 respectively. These results show that, on increasing the values of $L/R$ the frequencies reduce to each CWN mode [36]. **Figure 3(b)** demonstrates variations of natural frequencies (Hz) for a CS enclosing fluid versus the $h/R$ for the tangential wave modes, $n$ = 1, 2, 3, 4, 5 and the longitudinal wave mode, $m$ = 1. The value of height-to-radius ratio is composed as $0.002 \le h/R \le 0.05$, the values of $h/R = 0.002$, at CWN: $n$ = 1, 2, 3, 4, 5 are 3.36929, 4.1911, 5.0052, 4.5812, 5.4979 and $h/R = 0.05$, are 5.6977, 19.6873, 35.4274, 79.3621, 126.739 respectively. As $h/R$ is made to increase i.e., the shell gets thicker and thicker, the frequency increases. Now the values of height-to-radius ratio $0.002 \le h/R \le 0.05$, at CWN: $n$ = 1, the frequencies are 3.36929, 3.7929, 4.6937, 4.9046, 5.6977 and at CWN: $n$ = 5 are 5.4979, 10.3603, 50.7402, 76.0671, 126.7139. As $h/R$ is made to increase i.e., these results shows that the frequency increases very slowly for $n$ = 1 but for higher values of $n$, there is seen an appreciable increments in frequency values with each tangential wave mode. Previous study reveals that when the value of $h/R$ increases then frequencies also increases [37].

### 4.2. Frequency analysis of fluid-filled cylindrical shells with ring support

In this section, frequency analysis for an isotropic CS enclosing fluid is performed by attaching some ring supports. These rings are located at some distance from one end of a cylindrical shell as shown in **Figure 4**.

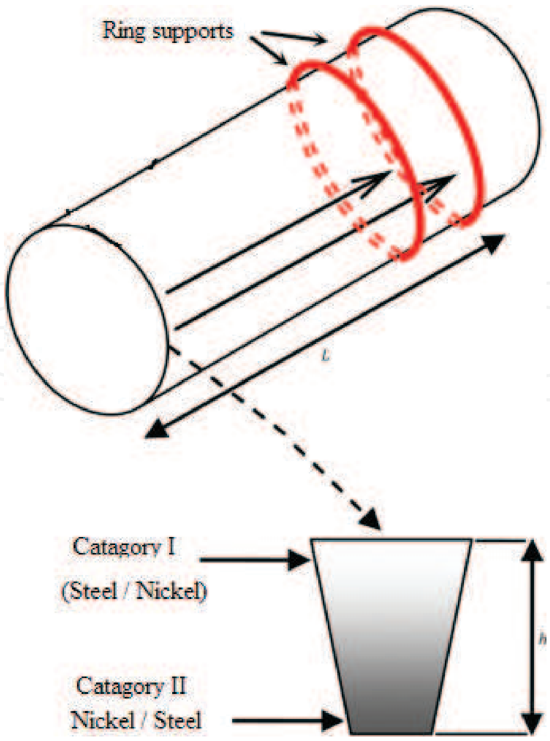


**Figure 4.** FGM shell with two ring supports (Ansari et al. [30]).

In **Figure 5(a)** variations of natural frequencies (Hz) for a CS containing fluid with ring supports versus $L/R$ are demonstrated for three positions of the ring supports at $a$ = 0.3$L$, 0.5$L$, $L$. The value of $L/R$ is taken as $1 \leq L/R \leq 20$, when $L/R$ at ring support $a$ = 0.3$L$, 0.5$L$, $L$ without fluid, the frequencies are 790.31, 863.378, 745.085 and $L/R$ = 20 the values are 420.639, 516.775, 208.819 and now with fluid at $L/R$ = 1 the values are 352.975, 362.764, 314.385 and $L/R$ = 20, the values are 187.549, 217.132, 86.109 respectively. There is a substantial decrease in frequency values as the shell length gets higher [36]. Influence of fluid on vibration frequency is seen significantly visible that reduced them approximately to (40~50%). In **Figure 5(b)** natural frequencies (Hz) for a CS containing fluid without and with ring supports are exhibited with $h/R$. The value of $h/R$ is taken as: $0.002 \leq h/R \leq 0.05$, when $h/R$ = 0.002 at ring support $a$ = 0.3$L$, 0.5$L$, $L$ without fluid are 413.2180, 548.4688, 215.4291 and $h/R$ = 0.05 the values are 413.3080, 548.4888, 215.5019 respectively. Now with fluid at $h/R$ = 0.002 the values are 273.6543, 365.6459, 144.5833 and $h/R$ = 0.05, the values are 273.7139, 365.6592, 144.6321 respectively. The ring supports are located at $a$ = 0.3$L$, 0.5$L$, $L$. It is observed that frequencies enhance minutely for the thicker CSs [37]. Again the frequency has been considerably reduced when the fluid is added. From **Figure 5(a)** and **(b)**, it is observed that in both the cases, without fluid and with fluid for $L/R$ and $h/R$, the value of ring support $a$ = 0.3$L$ which is sandwich between $a$ = 0.5$L$, $L$. It is seen that the influences of ring supports and fluid terms are converse to each other. The ring supports increase the frequencies whereas the fluid loaded terms lower them [32–34].

### 4.3. Frequency analysis of empty and fluid-filled cylindrical shells with ring supports

It is noted that the natural frequencies of the shells have varied by the location of ring support in the shell and this change also varies WOF and WF as shown in **Figure 6(a)–(d)**. It is evident

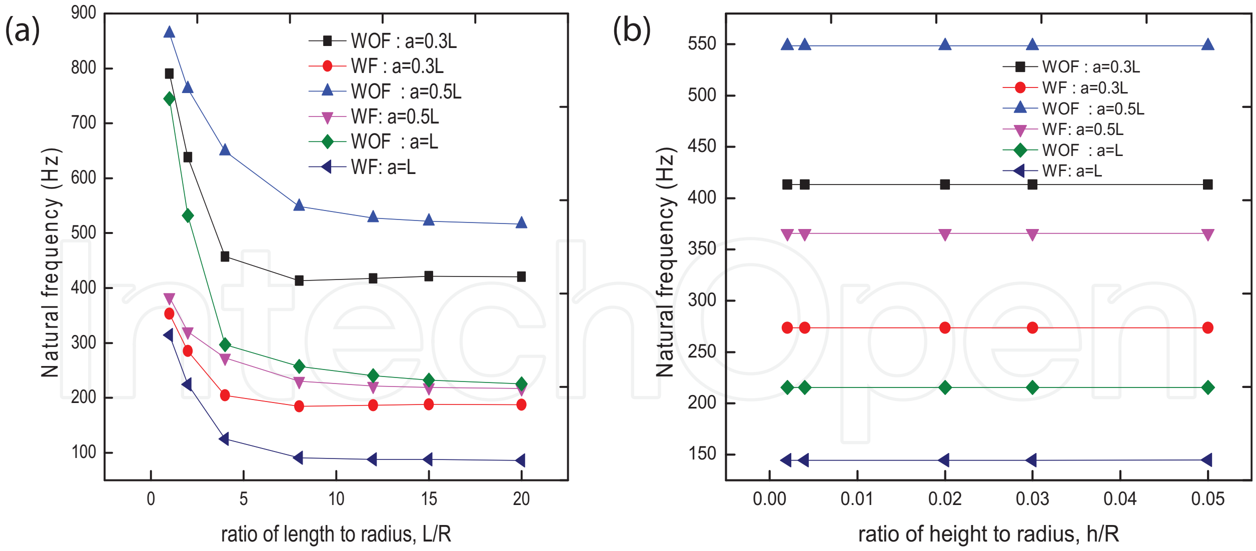


**Figure 5.** Variations of natural frequencies (Hz) for a WOF and WF cylindrical shell with ring support against (a) $L/R$ (b) $h/R$ ($n = 1$, $m = 1$, $L = 8$ m, $h = 0.004$ m, $R = 1$ m, $E = 2.1 \times 10^{11}$ N/m$^2$, $v = 0.3$, $\rho = 7850$ kg/m$^3$, $\rho_f = 1000$ kg/m$^3$).

from these figures that when the values of $a$ $(0 < a < 0.5)$ increases then the natural frequencies also increases and $a = 0.5L$, it reaches its peak value but for $0.5L < a < 0.1L$, on increasing the value of $a$, it begins to decrease and rust itself as bell shape symmetric curve. It is clear seen that when the ring support is positioned at the center of the fluid-filled cylindrical shell, the natural frequency attains its extreme value; it is observed that the natural frequency decreases as the ring supports shifts from center toward right/left side of the fluid-filled cylindrical shell. Thus, when different exponent is adopted, the frequency curve is also symmetrical about the center of fluid-filled cylindrical shell for simply supported symmetric end condition imposed on the both shell ends.

In **Figure 6(a)**, variations of natural frequencies (Hz) with locations of the ring supports are listed with ring supports for WOF and WF. The ring support is composed as $0 \le a \le L$ and for the values at $a = 0$ for WOF and WF are 215.429, 140.665 and at $a = L$ the values are 215.429, 146.095 respectively. At $0.4L < a < 0.6L$ means $a = 0.5L$ for WOF and WF have a peak value 548.468, 548.358 but for $a = 0.4L$ the values are 518.609, 496.419 and $a = 0.6L$ are 518.609, 504.140 very closed to each other. In each case the shell frequencies goes up with the position of the ring supports to the highest values at the mid of the shell and then start to lower down to gain their initial values. It is noticed that the fluid addition has made frequency to decrease. From the previous data, the shell frequencies are affected highly as the fluid quantities [11, 31–35] and ring supports are appended [2, 5, 27, 36]. A beam type vibration of CSs crops up when the addition of ring supports are made.

**Figure 6(b)** and **(c)** display the comparisons of variations of natural frequencies (Hz) with ring supports for WOF and WF for *C-I* and *C-II* respectively. The ring support is composed as $0 \le a \le L$ and for volume fraction law, the exponent $p = 0.5$, 1, 15 is adopted, the values at $a = 0$ for WOF and WF are 207.058, 135.199 ($p = 0.5$), 205.346, 134.082 ($p = 1$), 201.012, 131.252 ($p = 15$) and at $a = L$ the values are 207.058, 140.46 ($p = 0.5$), 205.346, 139.255 ($p = 1$), 201.012, 136.316 ($p = 15$) for *C-I* and for *C-II*: 204.048, 133.234 ($p = 0.5$), 205.729, 134.332 ($p = 1$), 210.311, 137.324 ($p = 15$) and at $a = L$ the values are 204.048, 138.375 ($p = 0.5$), 205.729, 139.515 ($p = 1$), 210.311,

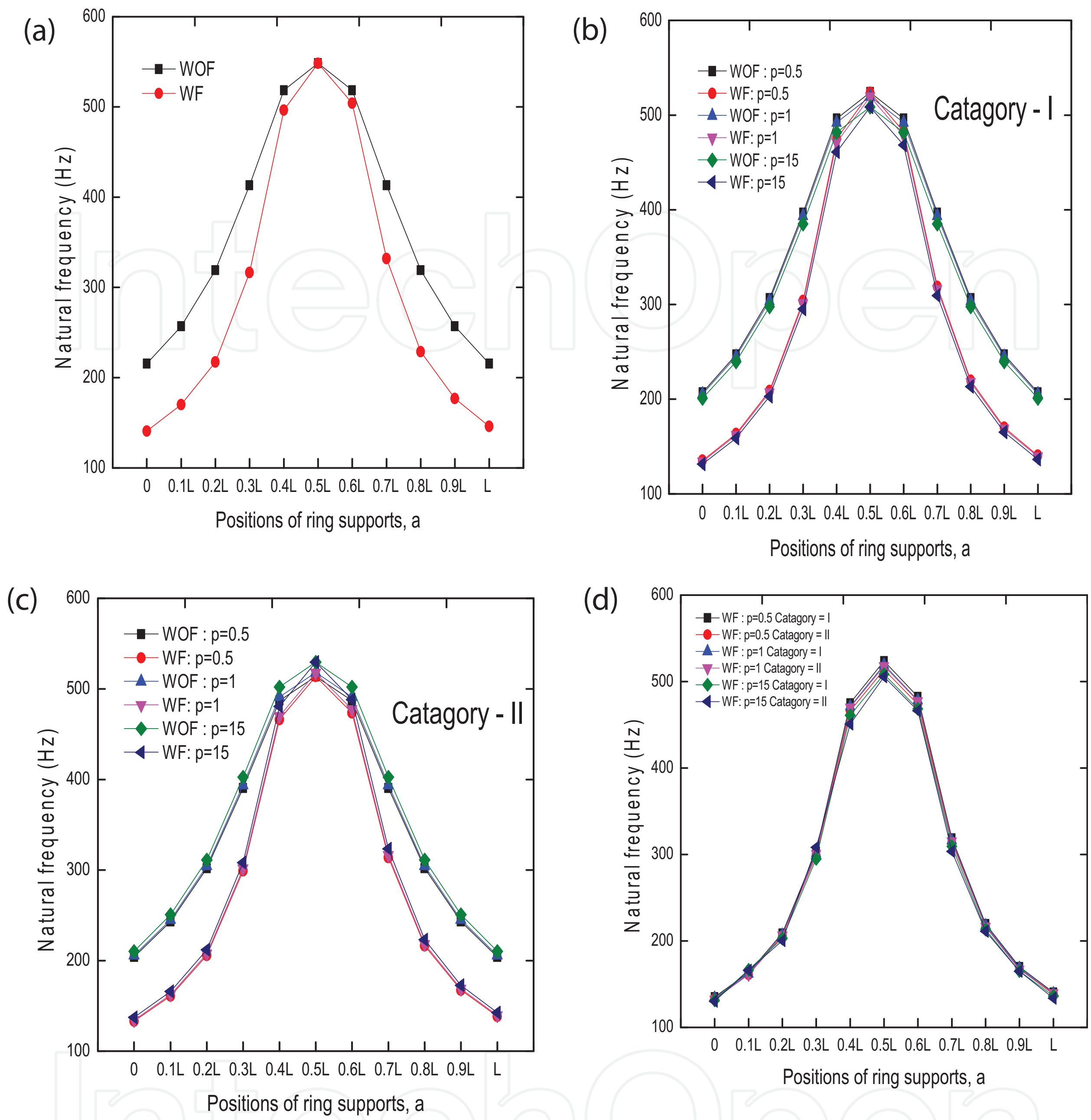


**Figure 6.** Variation of natural frequencies (Hz) (a) against the positions of the ring supports for WOF and WF cylindrical shell. ($n = 1$, $m = 1$, $L = 8$ m, $h = 0.004$ m, $R = 1$ m, $E = 2.1 \times 10^{11}$ N/m$^2$, $\upsilon = 0.3$, $\rho = 7850$ kg/m$^3$, $\rho_f = 1000$ kg/m$^3$) (b) versus the positions of ring supports for WOF and WF cylindrical shell with ring support: Category-I ($n = 1$, $m = 1$, $L = 8$ m, $h = 0.004$ m, $R = 1$ m) (c) Category-II (d) Category-I and II.

142.622 ($p = 15$) respectively. For *C-I*, at $0.4L < a < 0.6L$ means $a = 0.5L$ have a extreme value of frequency 523.924, 523.819 ($p = 0.5$), 519.591, 519.487 ($p = 1$), 508.626, 508.524) ($p = 15$), for both WOF and WF but for $a = 0.4L$ the values of frequencies are 496.188, 474.957 ($p = 0.5$), 492.085, 471.030 ($p = 1$), 481.699, 461.088 ($p = 15$) and $a = 0.6L$ are 496.188, 482.345 ($p = 0.5$), 492.085, 478.356 ($p = 1$), 481.699, 468.260 ($p = 15$) very closed to each other for both WOF and WF. For *C-II*: at $0.4L < a < 0.6L$ means $a = 0.5L$ have a maximum frequency value 513.823, 513.720 ($p = 0.5$), 518.059, 517.955 ($p = 1$), 529.595, 488.154 ($p = 15$) for both WOF and WF but for $a = 0.4L$ the values are 487.212, 466.366, 491.226, 470.208 ($p = 1$), 502.164, 480.678 ($p = 15$) and

$a = 0.6L$ are 487.212, 473.619, 491.226, 477.521 ($p$ = 1) 502.164, 488.154 ($p$ = 15) very closed to each other for both WOF and WF. In both cases, the frequencies first rise, attain their maximum at the mid position and then fall down to original values at the other shell end. There is observed a reduction in frequency values owing to fluid term induction [11, 32–34]. It is concluded that the maximum point and other relative value of *C-II* are bit smaller than that of corresponding value of *C-I*. This conclusion is also shown in **Figure 6(d)** where *C-I* and *C-II* are illustrated in one graph. In **Figure 6(d)** variations of frequencies for two categories with ring supports for fluid-filled for *C-I* and *C-II* are cataloged with the locations of the ring supports. The ring support is composed for fluid-filled as $0 \le a \le L$, for volume fraction law, the exponent $p$ = 0.5, 1, 15 is adopted, the values at $a = 0$ are 135.99, 133.234 ($p$ = 0.5), 134.082, 132.332 ($p$ = 1) 131.252, 130.324 ($p$ = 15) and at $a = L$ for category-I and for category-II, the values are 140.416, 138.375 ($p$ = 0.5), 139.255, 139.515 ($p$ = 1), 136.316, 133.622 ($p$ = 15). For *C-I*, *C-II* at $0.4L < a < 0.6L$ means $a = 0.5$ have a maximum value 513.819, 513.720 ($p$ = 0.5), 519.487, 517.955 ($p$ = 1), 508.524, 505.489 ($p$ = 15) but for $a = 0.4L$ the values are 474.957, 466.366 ($p$ = 0.5), 471.030, 470.208 ($p$ = 1), 461.088, 450.678 ($p$ = 15) and $a = 0.6$ are 482.342, 473.342 ($p$ = 0.5), 478.356, 477.521 ($p$ = 1), 468.260, 466.154 ($p$ = 15) respectively. Here for *C-I* cylindrical shells, frequencies are some bit higher for those of *C-II* ones [32–34]. The effects of fluid-filled cylindrical shell with ring supports are perceived to be very conspicuous in the variation of natural frequencies.

## 5. Summary

Theoretical vibration analysis of cylindrical shells (CSs) has a significant importance in applied mathematics and mechanics in view of their practical uses. Here the shell problem has been associated with investigation vibrations of cylindrical shells with ring supports. They shells have been supposed to be constructed from functionally graded materials. They materials are advanced and smart for their physical properties. Moreover these shells have been assumed to contain fluid. Here the Galerkin technique is employed to obtain the shell frequency equation. Effect of ring supports on shell vibration is inducted by a polynomial function which has degree equal to the number of ring supports. From the study of results, it is observed that vibration frequencies of cylindrical shells decrease significantly when the fluid loaded terms are appended. However their variations are alike to that behavior which is noticed for the cylindrical shells not containing fluid and attachment of ring supports boost much the vibration frequencies. For functionally graded material cylindrical shells, variations of frequencies with the circumferential wave modes for length-to-radius ratio, height-to-radius ratio for fluid-filled cylindrical shell and also for fluid-filled cylindrical shell with ring supports has been analyzed. It is seen that the influences of ring supports and fluid terms are converse to each other. The ring supports increase the frequencies whereas the fluid loaded terms lower them. However, the increments and decrements in the shell frequency depend upon the order of functionally graded material constituents forming a cylindrical shell. The induction of fluid-filled with ring support on the cylindrical shell has a prominent effect on the natural frequency as compared to the shell frequency without fluid attached with ring support. Different position of ring supports with and without fluid with various exponent law for category-I, II or both are analyzed. It is concluded that in both cases, the frequencies first rise, attain their maximum values at the mid position and then fall down to original values at the other shell end.

Moreover, there is observed a reduction in frequency values owing to fluid term induction. It is also concluded that the maximum point and other relative value of category-II are bit smaller than that of corresponding value of category-I An extension of this analysis procedure can be performed to investigate vibrations of rotating fluid isotropic and functionally graded cylindrical shells with ring supports.

## List of symbols

| | |
|---|---|
| $A_{ij}$ | Extensional stiffness |
| $B_{ij}$ | Coupling stiffness |
| $D_{ij}$ | Bending stiffness |
| $E$ | Young's modulus |
| $h$ | Shell thickness |
| $L$ | Shell length |
| $v$ | Poisson's ratio |
| $\omega$ | Natural angular frequency |
| $d_{ij}(i, j = 1, 2, 3)$ | Shell parameters |
| q | Acoustic pressure |
| $c$ | Speed of sound in the fluid |
| $J_n(k_r r)$ | Bessel's function of first kind |
| $c_f$ | Speeds of the sound in fluid – filled cylindrical shell |
| $\Omega$ | Non-dimensional frequency parameter |
| $E_1$, $E_2$ | Young's moduli |
| $\nu_1$, $\nu_2$ | Poisson's ratios. |
| $\rho_1$, $\rho_2$ | Mass densities |
| $E_{fgm}$, $v_{fgm}$, $\rho_{fgm}$ | Effective material quantities |
| $T$ | Kelvin scale |
| $C_r$ | Material properties |
| $V_r$ | Volume fractions |
| $k$ | Number of ring supports |
| $R_0$, $R_1$ | Outer and inner radii of the shell |
| $h/R$ | Thickness to radius ratio |

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| | |
|---|---|
| $L/R$ | Length to radius ratio |
| $n$ | Circumferential wave number |
| $m$ | Half- axial wave number |
| $C$ | Material property |
| $C_0, C_{-1}, C_1, C_2, C_3$ | Coefficients of temperature |
| $p$ | Volume fraction law or Power law exponent |
| $Q_{ij}$ | Reduced stiffness |
| $R$ | Shell radius |
| $T(\mathrm{K})$ | Temperature function of Kelvin |
| $x$ | Axial coordinate |
| $\Psi$ | Circumferential coordinate |
| $z$ | Radial coordinate or thickness variable |
| $u(x, \Psi, t)$ | Deformation displacement functions in $x$ direction |
| $v(x, \Psi, t)$ | Deformation displacement functions in $\Psi$ direction |
| $w(x, \Psi, t)$ | Deformation displacement functions in $z$ direction |
| $V_{fi}$ | Volume fraction |
| $\rho$ | Mass density of shell material |
| $\rho_t$ | Mass density per unit length |
| $S_{11}, S_{22}, S_{12}$ | Surface curvatures |
| $k_r$ | Radial wave number |
| $k_m$ | Axial wave number |
| $c_L$ | Speeds of the sound in the empty cylindrical shell |
| $\rho_f$ | Indicates the density of the fluid density |
| $p_m$, $q_m$ and $r_m$ | Vibration amplitudes in the $x$, $\Psi$ and $z$ directions |

## Abbreviations

| | |
|---|---|
| CSs | Cylindrical shells |
| FGM | Functionally graded material |
| WF | With fluid |

| | |
|---|---|
| FLT | Fluid loading term |
| C-I | Category-I |
| WPA | Wave propagation approach |
| ODEs | Ordinary differential equations |
| PDEs | Partial differential equations |
| FFCS | Fluid-filled cylindrical shell |
| WOF | Without fluid |
| CWN | Circumferential wave number |
| C-II | Category-II |
| FEM | Finite element method |

## A. Appendix-I

$$d_{11} = \left(A_{11}I_1 - n^2\frac{A_{66}}{R^2}I_2\right)$$

$$d_{12} = -n\left(\frac{A_{12}+A_{66}}{R} + \frac{B_{12}+2B_{66}}{R^2}\right)I_2$$

$$d_{13} = \left[\frac{A_{12}}{R}(I_6+I_7) - B_{11}(I_8+3I_9) + n^2\frac{B_{12}+2B_{66}}{R^2}(I_6+I_7)\right]$$

$$d_{21} = n\left(\frac{A_{12}+A_{66}}{R} + \frac{B_{12}+2B_{66}}{R^2}\right)I_3$$

$$d_{22} = \left[\left(A_{66} + \frac{3B_{66}}{R} + \frac{4D_{66}}{R^2}\right)I_3 - n^2\left(\frac{A_{22}}{R^2} + \frac{2B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)I_4\right]$$

$$d_{23} = \left[n\left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3}\right)I_{10} + n^3\left(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)I_{10} - n\left(\frac{B_{12}+2B_{66}}{R} + \frac{D_{12}+4D_{66}}{R^2}\right)(I_{11}+2I_{12})\right]$$

$$d_{31} = \left(-\frac{A_{12}}{R}I_{18} + B_{11}I_{19} - n^2\frac{B_{12}+2B_{66}}{R^2}I_{18}\right)$$

$$d_{23} = \left[n\left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3}\right)I_{10} + n^3\left(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)I_{10} - n\left(\frac{B_{12}+2B_{66}}{R} + \frac{D_{12}+4D_{66}}{R^2}\right)I_{18}\right]$$

$$d_{33} = \left[-\frac{A_{22}}{R^2}I_3 + \frac{2B_{12}}{R}(I_{14}+2I_{15}) - 2n^2\frac{B_{22}}{R^3}I_{13} - D_{11}(I_{16}+4I_{17}) + 2n^2\frac{D_{12}+2D_{66}}{R^2}(I_{14}+2I_{15}) - \frac{D_{22}}{R^4}I_{13}\right]$$

where

| | | |
|---|---|---|
| $I_1=\int_0^L \frac{d^3\varphi}{dx^3}\frac{d\varphi}{dx}dx$ | $I_2=\int_0^L \frac{d\varphi}{dx}\frac{d\varphi}{dx}dx$ | $I_3=\int_0^L \frac{d^2\varphi}{dx^2}\varphi(x)dx$ |
| $I_4=\int_0^L \varphi(x)\varphi(x)dx$ | $I_5=\int_0^L \frac{d^4\varphi}{dx^4}\varphi(x)dx$ | $I_6=\int_0^L (x-a)\frac{d\varphi}{dx}\frac{d\varphi}{dx}dx$ |
| $I_7=\int_0^L \varphi(x)\frac{d\varphi}{dx}dx$ | $I_8=\int_0^L (x-a)\frac{d^3\varphi}{dx^3}\frac{d\varphi}{dx}dx$ | $I_9=\int_0^L \frac{d^2\varphi}{dx^2}\frac{d\varphi}{dx}dx$ |
| $I_{10}=\int_0^L (x-a)\varphi(x)\varphi(x)dx$ | $I_{11}=\int_0^L \frac{d^2\varphi}{dx^2}(x-a)\varphi(x)dx$ | $I_{12}=I_7$ |
| $I_{13}=\int_0^L \varphi^2(x)(x-a)^2dx$ | $I_{14}=\int_0^L \frac{d^2\varphi}{dx^2}\varphi(x)(x-a)^2dx$ | $I_{15}=\int_0^L \frac{d\varphi}{dx}\varphi(x)(x-a)dx$ |
| $I_{16}=\int_0^L \frac{d^4\varphi}{dx^4}\varphi(x)(x-a)^2dx$ | $I_{17}=\int_0^L \frac{d^3\varphi}{dx^3}\varphi(x)(x-a)dx$ | $I_{18}=\int_0^L \frac{d^2\varphi}{dx^2}\varphi(x)(x-a)dx$ |
| $I_{19}=\int_0^L \frac{d^4\varphi}{dx^4}\varphi(x)(x-a)dx$ | | |

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