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# Power System Reliability: Mathematical Models and Applications

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Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/intechopen.71926>

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## Abstract

This chapter deals with power systems reliability including technical, economical, and decisional aspects. Knowing that almost 90% of failures occur in the distribution systems, great interest was dedicated to this part of the system, and the first work was oriented to reliability indices defined as objectives to attempt and as performance measures in the electricity market. Some works deal with the managers' behavior, and the customers reactions are modeled using economic criteria in uncertain future and inspired from game theory. When studying components, degradation models were introduced and combined with the effects of shocks to study the reliability changing during system operation. In some works, the correlation between maintenance policies and reliability aspects was highlighted. In a recent work, considering the importance of new technologies integration and renewable energy insertion to power systems, it was revealed that reliability aspects and energy sustainability are two fundamental issues of progress in a given society.

**Keywords:** power systems reliability, distribution functions, degradation modeling, maintenance, decision-making

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## 1. Introduction

In general way, power system reliability addresses the issues of service interruption and power supply loss. In several cases, it is defined as an objective to attempt in terms of indices directly related to the customer. Typical reliability index values for US utilities are SAIFI, SAIDI, and CAIDI. Over time, they become standard values for evaluating the reliability of electrical systems and used in several publications. Medjoudj et al. [1], in their recent publication, defined other indices as reliability subcriteria in their decision-making

attributes, giving the best model of a smart energy grid. These indices are discussed in Section 2 with an application to a real case study. After this classical definition, some works have integrated data analysis and processing taking into account the calculation of distribution parameters such as those used for Weibull and Weibull-Markov processes. These works were vulgarized with applications in power system reliability by Van Casteren et al. [2] and Medjoudj et al. [3]. The most techniques used in power system reliability optimization and processing is the reliability centered preventive maintenance (RCM). Several publications have highlighted that in most cases of multicomponent systems, the maintenance actions arrive very early without any effects on the system or very late, that is, the need of curative maintenance with its negative consequences. This issue is also treated in the case of a differentiated service of reliability in the case of customers with different requirements of the reliability level. These two concepts are discussed in Section 3. Recent publications have highlighted the interest of combining reliability attributes for maintenance actions in the case of degrading systems and components [4]; however, a novel work developed recently at the LaMOS unit research dealing with multiple degradation processes is applied to power switchgear and is discussed in Section 4. The discussions and the conclusion highlighting the place of reliability in a power energy smart grid are given in Section 5.

## 2. Power system reliability indices

In the beginning, the methods used were classical to evaluate reliability indices of distribution systems such as failure frequency, mean failure times, mean time between failure and energy not supplied. These indices help decision makers to define technical and management measures to perform systems. After that was introduced the notion of loss of load probability (LOLP), which has a lot of applications in load modeling and electrical parameters dimensioning. It is significant for any power enterprise to analyze customer satisfaction. A variety of indices have been developed to measure reliability and its cost in power systems area such as loss of load probability (LOLP), loss of load expectation (LOLE), expected frequency of load curtailment (EFLC), expected duration of load curtailment (EDLC), expected duration of a curtailment (EDC), and expected energy not supplied (EENS) [1].

### 2.1. Loss of load probability

LOLP is an expected value, sometimes calculated on the basis of the peak hourly load of each day and sometimes on each hour's load (24 in a day). Moreover, in the beginning, LOLP is used to characterize the adequacy of generation to serve the load on the bulk power system; it does not directly model the reliability of the transmission and distribution system where the majority of outages actually occur [5]. Nourelfeth and Ait Kadi [6] have recently made that the LOLP is usually used to estimate the reliability index. Considering  $S$  and  $D$  as the supply and the load demand respectively, they compute the reliability of a multistate system (MSS) as:

$$R = \Pr(S \geq D) \text{ Or } R = 1 - LOLP \quad (1)$$

Using the well-known formulation of the LOLP given in several publications and discussed in the following section, they generalized the MSS reliability index  $R$

$$R = \frac{1}{\sum_{j=1}^M T_j} \sum_{j=1}^M \Pr(S \geq D_j) T_j \quad (2)$$

where the operation period  $T$  is divided into  $M$  intervals, and each interval has duration  $T_j$  and a required demand level  $D_j$ . In the same context, as advancement in reliability applications, Taboada et al. [7] have generalized the use of LOLP as a reliability index. Using a series-parallel system, they calculate the availability of each part of the power including transmission and distribution system using LOLP model.

### 2.1.1. Loss of load probability formulation

The generation system reliability calculations are based mainly on two analytical methods, which differ by the load model used. The first method is the calculation of the LOLP; there the load is given by the load duration curve. The second method is the frequency and duration approach, by which, besides the probability, the frequency and duration of load levels higher than the generation capacity can be determined. The loss of load probability method associates each value of MW outage with a certain cumulative probability, thus producing a capacity outage table. The expected load loss of the system is obtained from the capacity outage table, and a daily load peak variation curve is derived from the daily load curves. The LOLP, number of days on which capacity is insufficient, is obtained by adding the probability that the amount of capacity on forced outage, on day  $i$  is greater than or equal to the reserve on day  $i$ , for all days of the period being studied.

For a system state where the remaining generating capacity is  $C_j$ , the percentage of time  $t_j$  during which the load demand exceeds  $C_j$  can be determined from the load curve  $L$ . The overall probability that the load demand will not be met is called the loss of load probability and is given by the Eq. [4]:

$$LOLP = \sum_j P[C = C_j] P[L > C_j] = \sum_j \frac{p_j t_j}{100} \quad (3)$$

where  $p_j$  is the probability associated to the number of the failed generating units at time  $t_j$ , and it is formulated through the following development.

In addition to complete failures, generating units may experience partial failure, when they continue to operate but at reduced capacity levels. They are also taken out of service from time to time for preventive maintenance. Using a simple two-state model for the operation of a unit, its failure probability is given depending on its failure outage rate (FOR), which can be

assumed as the unit steady-state unavailability denoted  $\bar{A}$ . If at time  $t_j$ ,  $r$  units have failed from a total of  $n$  identical and independent installed units in the generating system, the probability  $p_j$  is given by:

$$p_j = \binom{n}{r} \bar{A}^r (1 - \bar{A})^{n-r} \quad (4)$$

A case of unequal size of the units can appear.

### 2.1.2. Purposes

#### 2.1.2.1. First purpose

It is well known that availability is a measure of success used primarily for repairable systems. For nonrepairable systems, availability  $A(t)$  equals reliability  $R(t)$ . In repairable systems,  $A(t)$  will be equal to or greater than  $R(t)$ . In the optimistic case, the availability is greater than the reliability. Following the Levitin and Lisnianski development for a multistate generating system (MSGs), the availability expectation is the function of demand  $D$  and may be defined as [8]:

$$E_A(D_j, T_j) = \frac{1}{\sum_{j=1}^M T_j} \sum_{j=1}^M A(D_j) T_j \quad (5)$$

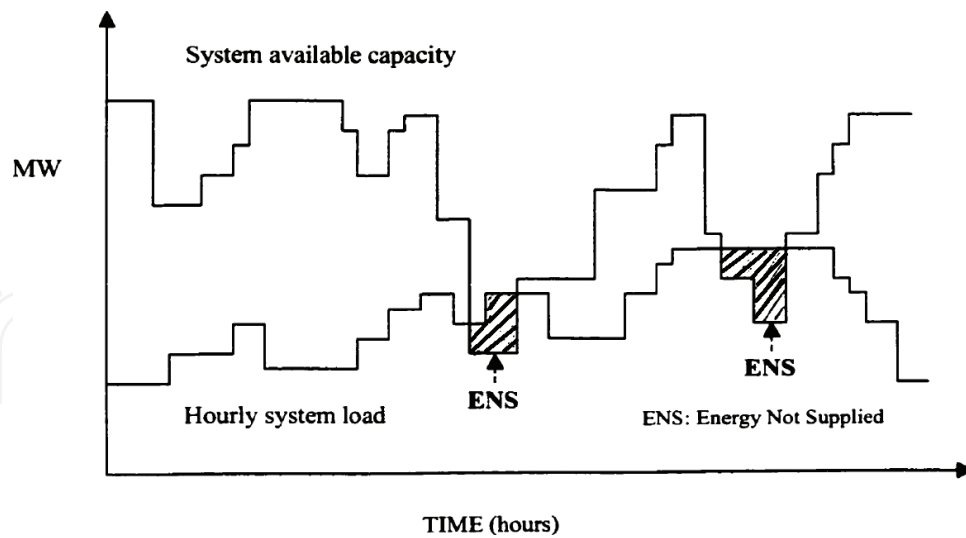
The index  $(1 - E_A)$  is often used and treated as loss of load probability and can be written as:

$$LOLP = (1 - E_A) \text{ Or } E_A = 1 - LOLP \quad (6)$$

#### 2.1.2.2. Second purpose

This purpose highlights the correlation between the system reliability, the energy availability, and the loss of load probability. To understand this correlation, we consider a multistate generating repairable system MSGS connected to a load  $L$ , and on a given period of time, we draw two curves representing the evolutions of the system available capacity (SAC) and the hourly system load (HSL), respectively, as shown in **Figure 1**. Depending on the states of generating units (up or down) that involve partial or total failure of a simple unit or of several units, the appearance of dips in the same curve reflects units' breakdowns, and the resumption to the initial level of capacity indicates that repairs were made. One of the most reliability indices that concerns more the utility than the customer is the energy not supplied (ENS) given by the dashed lines under the curve. Their corresponding time intervals denote durations, where the consumption exceeded the production, and therefore, we have loss of load. The decreasing level of system reliability is highlighted by degrading state, corresponding to each decreasing in the SAC curve behavior.

The generating system failures can occur in two ways: either through unit failures or through load increases. There is a loss of load when the demand is greater than the supply. However, there is a loss of supply when a failure occurs in the upstream of the load point. It is important



**Figure 1.** Superimposition of the system available capacity and the load model.

to retain this difference. In this context, two relevant questions are to be asked and the answers were given in [9, 10] with a case study application, such as: What happens when load increases? What is the consequence of generating system failure?

## 2.2. Frequency and duration indices

Almost every electricity utility computes reliability indices on an annual basis. The most important reliability indices involving decision-making criteria are given as follows [1]:

The Expected Frequency of Load Curtailment in (fault/yr):

$$EFLC = \sum_{k=1}^n \lambda_k \quad (7)$$

The Expected Duration of Load Curtailment in (hrs/yr):

$$EDLC = \sum_{k=1}^n \lambda_k T_k \quad (8)$$

The Expected Energy Not Supplied in (kWh/yr):

$$EENS = L.(EDLC) \quad (9)$$

where  $\lambda_k$ ,  $T_k$  are failure rate and failure duration of an item  $k$  and  $L$  is the load curtailed at a considered load point, respectively. Application is done for a part of the distribution system of Algiers city (Algeria). Considering the electrical characteristics (network topology, section length, power value at load points and the fault search method) and reliability parameters mentioned earlier, the overall system reliability indices are computed.



### 2.3. Reliability indices improvement

To improve the reliability level, technical and organizational measures are considered during system planning and operation. The actions currently carried out are as follows: intensifying the operations of maintenance; networks reorganization, looping and meshing systems, and automation of networks. In [11], some options are added such as load transfer between feeders, undergrounding circuits, and replacement of aging equipment. From a practical standpoint, this application allows to highlight the goodness of each measure to the system performances by a simple comparison of reliability indices. The results of reliability indices improvement are published in [1].

## 3. Interruptions modeling and reliability service differentiation

In several studies dealing with electrical distribution system reliability, the objective often sought by the energy distributor is the balance between the required reliability level and its cost. In the following, we develop two important points of view in the reliability of the electrical systems: that relating to modeling and that of the differentiation of electricity prices according to the level of reliability required with a minimal guaranteed reliability level for customers without any prior requirement [12].

### 3.1. Interruption modeling using the Weibull-Markov process

In the last decade, a novel vision of interruption modeling in power systems was developed and consists of the Weibull-Markov process. The purpose is to model the failure and operating data according to Weibull distribution proprieties, while retaining those assigned to the Markov model where the system occupies discrete states. This process was initially developed by Van Castaren [2] and was applied successfully by Pivatolo [13] and Medjoudj et al. [3]. Applications were made to highlight maintenance policies gathered on three types of actions: namely nondestructive action which does not improve reliability level but slows the system degradation. This action, denoted (1a) as a minor maintenance, is characterized by an improvement factor  $m_1$ . A second action is considered and denoted (2b) and can touch some of the components of a system up to their replacement. To this action is associated an improvement factor  $m_2$ , and the maintenance is a major one. The third and final proposed maintenance action is on the renewal of equipment, and it is assumed to be perfect, and after its implementation, the system is assumed as good as new, and it is denoted (2p). From a practical standpoint, this action is highlighted by taking  $m_1$  and  $m_2$  equal to the unity. This notion is introduced by Tsai et al. [14] for a mechatronic system and applied for power systems by Medjoudj et al. [3]. In this part of the section, we introduce the concept of the differentiated reliability with an application to the case of an electrical MV/LV substation. Starting from the expression of reliability function expressed by a desired threshold, the need of performing preventive maintenance action at time is decided regarding the behavior of this function at the coming stage of maintenance. Then, the choice of the type of action to perform is dictated by the value of the maximum benefit brought by this action. Threshold reliability is allocated to the opposite risk of system failure occurrence.

### 3.1.1. Reliability data analysis

Considering the formulations of mean up time and mean down time of an item is given respectively by:

$$MUT = t_m - t_b \int_0^{t_m} h(t) dt \quad (10)$$

and

$$MDT = t_a - t_b \int_0^{t_m} h(t) dt, \quad (11)$$

where,  $t_m$ ,  $t_a$  and  $t_b$  are respectively, the preventive maintenance (PM) interval, the PM and corrective maintenance (CM) times on replacement; the operational availability is defined as:

$$A = \frac{t_m - t_b \int_0^{t_m} h(t) dt}{t_m + t_a} \quad (12)$$

Subsequently, the PM interval for maximizing the availability can be derived by differentiating Eq. 9 to time  $t_m$ , such as  $\frac{dA}{dt} = 0$ , and the differential result is:

$$(t_m + t_a)h(t_m) - \int_0^{t_m} h(t) dt = \frac{t_a}{t_b} \quad (13)$$

For data treatment and statistical processing, forced and planned outages are collected over 17 years of system operation continuously at the national company of electricity and gas center (SONELGAZ) of Bejaia city, Algeria. For an MV/LV transformer which is a critical item of an electrical substation, the estimated parameters and the adequate probability distribution functions are listed in **Table 1**.

The obtained results show that, based on the Kolmogorov Smirnov (KS) test [15],  $d_{ks}$  is lower than  $d_{(n,0.05)}$ , the Weibull distribution is not rejected; however, with the exponential law,  $d_{ks}$  is greater than  $d_{(n,0.05)}$ , the hypothesis is not accepted. In **Table 2** are gathered reliability indices, where both the feeder failure frequency ( $F_i$ ) and the transformer failure rate ( $h$ ) are added. The nonacceptation of the exponential distribution is comforted by the review results of reference [16], where the authors state that the exponential law, usually used to describe failures, is not always 100% suitable for electricity distribution systems.

In this study, it is assumed that the substation failures are due either to the transformer or to the internal cable connector failures. A three states diagram (working, failure and maintenance) is dressed for the life cycle modeling of the substation as shown in **Figure 2**.

Let  $X_{12}$ ,  $X_{13}$ ,  $X_1$ ,  $X_2$ ,  $X_3$  be the random variables representing the duration of the operation until failure, the duration of the operation until maintenance, the duration of the operation (state  $S_1$ ), the duration of the interruption (state  $S_2$ ) and the duration of the maintenance (state  $S_3$ ), respectively. The estimated parameters of the random variables following the Weibull distributions are listed in **Table 3**.



Component or Subsystem	$n$	Distribution	Parameters	$d_{ks}$	$d_{(n,0.05)}$	Decision
MT/LV Transformer	17	Weibull	$\beta = 2.459579$ ; $\eta = 5.909754 \times 10^6$	0.2264	0.308	Not rejected
(Tr)		Exponential	$\lambda = 0.007785467$	0.3136		Rejected

Table 1. Distribution functions parameters estimation.

Component or Subsystem	MUT (hours)	MDT (hours)	MTBF (hours)	$A$	$F_i$ (1/year)	$h$ (1/year)
MT/LV Transformer	87358.33	9.97	87368.32	0.9998	0.10024	0.00778

Table 2. Reliability indices of the power transformer.

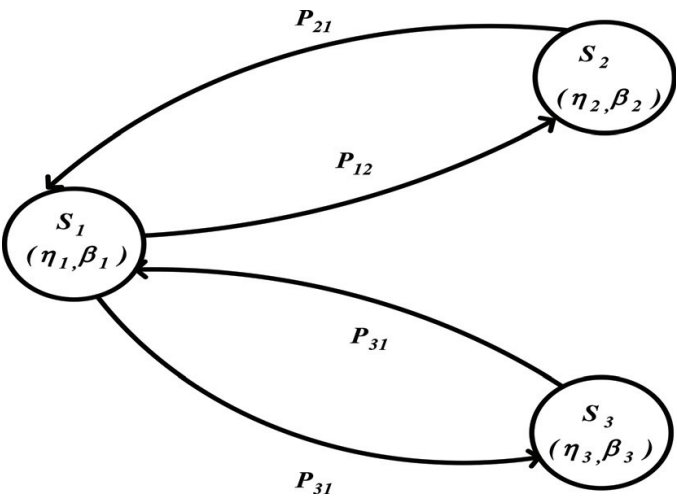


Figure 2. Three states diagram.

Variable	$N$	Distribution	Parameters	$d_{ks}$	$d_{(n,0.05)}$	Decision
$X_{12}$	27	Weibull	$\beta = 1.0644$ ; $\eta = 3.2827 \times 10^6$	0.1356	0.25438	Not rejected
$X_{13}$	27	Weibull	$\beta = 1.6689$ ; $\eta = 7.2589 \times 10^5$	0.2173	0.25438	Not rejected
$X_2$	27	Weibull	$\beta = 0.6764$ ; $\eta = 196.00159$	0.1219	0.25438	Not rejected
$X_3$	27	Weibull	$\beta = 1.03894$ ; $\eta = 123.6005$	0.1219	0.25438	Not rejected

Table 3. Parameters estimation of the random variables following Weibull distributions.

3.1.2. Reliability under preventive maintenance

The improvement of maintenance to reliability is developed using two factors, and the selection of the action to do for the components on every  $PM$  stage is decided by maximizing

system benefit in maintenance. Depending on the percent of the survival parts of system when it is maintained, the reliability function is:

$$R_j(t) = R_{0,j} \cdot R_{V,j}(t) \quad (14)$$

where  $R_{0,j}$  is the initial reliability of the  $j^{th}$  stage and  $R_{V,j}(t)$  is the reliability degradation of surviving parts on this stage. Considering periodical PM which interval is  $t_m$ , the reliability of surviving parts is defined as:

$$R_{V,j}(t) = R \left( \frac{1}{m_1} (t - (j-1)t_m) \right) \quad (15)$$

With:  $(j-1)t_m \leq t \leq jt_m$  and  $m_1$ , ( $0 < m_1 \leq 1$ ) is the improvement factor of action (1a).

To model the reliability of systems following PM, the effects of various actions on  $R_{0,j}$  and  $R_{V,j}$  must be evaluated.

$$R_{0,j} = R_{f,j-1} = R_{0,j-1} \cdot R(t_m) \quad (16)$$

where  $R_{0,j-1}$ ,  $R_{f,j-1}$  indicate the initial and final reliability values of the system on the  $(j-1)^{th}$  stage.

Action (1b) can improve the surviving parts of the system and also recover the failed parts. Generally, the impact of this action on the failed parts can be measured by an improvement factor  $m_2$ , which is also set between 0 and 1 representing the restored level except the surviving parts. According to the definition, the initial reliability on the action (1b) can be expressed as:

$$R_{0,j} = R_{f,j-1} + m_2(R_0 - R_{f,j-1}) \quad (17)$$

where  $R_0$  denotes the initial reliability of the new system.

The system reliability is expressed as:

$$R_j(t) = R_{0,j} \exp \left[ (t - (j-1)t_m) / (m_1 \eta) \right]^\beta \quad (18)$$

where  $R_{0,j}$  is the initial reliability of the  $j^{th}$  stage and  $m_1$  is the improvement factor of action (1a). The benefit of component maintenance on the  $j^{th}$  stage is defined as [14]:

$$B_{i,k} = \frac{\int_{t_j}^{\infty} R_{i,j+1}(t) dt - \int_{t_j}^{\infty} R_{i,j}(t) dt}{C_{i,k}} \quad (19)$$

where  $i, k$  denote, respectively, the  $i^{th}$  subsystem or component and the maintenance action considered and  $C_{i,k}$ , the action cost. The advantageous one will correspond to the maximum of the benefit, that is,  $B_i^* = \text{Max}(B_{i,k})$ . Once the action of maintenance is defined and retained, the availability of the system at any stage is processed as:

$$A_{s,j} = \frac{T - t_{b,m} \sum_{i=1}^n \int_{t_{j-1}}^{t_j} h_{i,j}(t) dt}{T + \sum_i^n t_{i,k,a}} \quad (20)$$

where  $n$  is the number of components or subsystems and  $t_{i,k,a}$  is the time of the  $PM$  actions (1a), (1b) and (2p) and  $T$ , the cycle time. In the following are described the different types of  $PM$  actions in the case of the power transformer.

- Action (1a): cleaning, lubricating, tightening and oil-level verification,
- Action (1b): oil and internal cable connectors' (I.C.C) replacement,
- Action (2p): transformer replacement.

Parameters are needed to compute the benefits such as the distribution function parameters  $(\beta, \eta)$   $PM$  and  $CM$  times ( $t_a$  and  $t_b$ ), maintenance actions costs ( $C_{1a}$ ,  $C_{1b}$ , and  $C_{2p}$ ) listed in **Table 4**, and the threshold value of reliability ( $R_{th} = 0.8$ ).

The obtained results are: ( $t_m$  (I.C.C) = 715 days,  $t_m$  (Tr) = 2415 days); however, the maintenance interval for the system is  $T_m = \min\{715, 2415\} = 715$  days. The maintenance action to retain is based on the maximum benefit value, and the results at different maintenance stages are listed in **Table 5** using the following notations:

\*: no maintenance is needed,

0: nothing to do (after an inspection),

1: action (1a) is carried out,

2: action (1b) is carried out,

3: action (2p) is carried out,

$R((j+1)T_m)$ : The instantaneous reliability at  $(j+1) \times T_m$ .

The results listed in **Table 5** can be interpreted as follows: at every maintenance stage, verify for each component if its reliability for the coming stage is greater or equal to the  $R_{th}$ .

- If the condition is realized, the decision is doing nothing. For the example of the transformer, at  $j = 1$ , the reliability is  $R((j+1) \times T_m) = R(2 \times T_m) = 0.9272 > R_{th} = 0.80$ ; however, no maintenance is needed for the stage  $j = 1$ .
- If no, compute the benefit for each action proposed and choose the maximum value. For example, for the threshold value  $R_{th} = 0.95$ , at the first stage,  $j = 1$ , the reliability at the coming

$\beta$	$\eta$	$t_a(days)$	$t_b(days)$	$m_1$	$m_2$	$C_{1a}$ \$	$C_{1b}$ \$	$C_{2p}$ \$
2.45	4103.99	3.5	28	0.80	0.90	600	1500	8600

**Table 4.** The useful parameters for benefit evaluation.

Stage	Action proposed	$R((j+1)T_m)$			Benefit \$			Action Retained		
		$R_{th} = 0.8$	$R_{th} = 0.9$	$R_{th} = 0.95$	$R_{th} = 0.8$	$R_{th} = 0.8$	$R_{th} = 0.95$	$R_{th} = 0.8$	$R_{th} = 0.8$	$R_{th} = 0.95$
1	(1a)	0.9272	0.9272	0.9272	■	■	0.4378	0	0	2
	(1b)				■	■	0.8285			
	(2p)				■	■	0.1601			
2	(1a)	0.8155	0.8155	0.9234	■	2.0944	0.6371	0	1	2
	(1b)				■	1.7618	0.9393			
	(2p)				■	0.3254	0.1827			
3	(1a)	0.6618	0.8286	0.9230	3.0284	1.2526	0.6601	1	2	2
	(1b)				2.5859	1.8010	0.9394			
	(2p)				0.4710	0.3326	0.1844			
4	(1a)	0.7287	0.9220	0.9222	1.7524	■	0.6830	2	0	2
	(1b)				2.1457	■	0.9393			
	(2p)				0.3945	■	0.1861			
5	(1a)	0.9200	0.8056	0.9222	■	1.5305	0.7058	0	1	2
	(1b)				■	1.0851	0.9393			
	(2p)				■	0.3429	0.1877			
6	(1a)	0.8038	0.8032	0.9219	3.1444	1.5599	0.7514	1	1	2
	(1b)				2.6712	0.3430	0.9392			
	(2p)				0.4881	0.3188	0.1910			
7	(1a)	0.6455	0.8721	0.9215	1.7271	1.3359	0.7741	2	1	2
	(1b)				2.1735	0.6047	0.9392			
	(2p)				0.4016	0.3687	0.1927			
8	(1a)	0.7181	0.8376	0.9211	1.7271	1.3359	0.7741	2	1	2
	(1b)				2.1735	0.6047	0.9392			
	(2p)				0.4016	0.3687	0.1927			
9	(1a)	0.9190	0.7916	0.9207	■	1.3303	0.7967	0	1	2
	(1b)				■	0.9271	0.9392			
	(2p)				■	0.4300	0.1944			
10	(1a)	0.8024	0.7385	0.9203	■	1.3084	0.8193	0	1	2
	(1b)				■	1.2177	0.9391			
	(2p)				■	0.4867	0.1960			

**Table 5.** Maintenance plan depending on reliability thresholds.

stage of maintenance  $R((j+1) \times T_m) = R(2 \times T_m) = 0.9272 < R_{crit} = 0.95$  and actions ((1a), (1b)) benefits are (0.4378, 0.8285), respectively. The action (1b) is retained looking at the maximum value of the benefit.

The risk management is highlighted by thresholds of reliability. Depending on the reliability level reached, or fixed a priori, maintenance operations can be decided. The objectives are the determination of maintenance frequencies on an item and consequently their costs. It will be remarked that a high level of reliability is required (i.e., the risk of failure is minimized), the maintenance frequency increases, and subsequently, the cost increases.

#### 4. Competing failure processes of oil circuit breaker

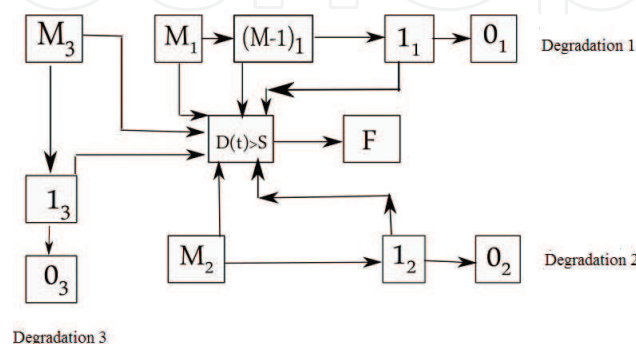
The components constituting a high voltage oil circuit breaker (HVOCB) are subject to various degradations, namely the aging of the insulating oil in the arc's extinguishing chamber, the contacts wear out and the sharp breakdown of bus bars supports. In this section, we have modeled the behavior of this item as subject to a competing three degradation processes by using the Markov state diagram as given in **Figure 3**. The states were defined using thresholds of degradation parameters. To the degradation processes was associated a shock process highlighting the effects of short circuit arrivals on the HVOCB when defaults occur at the downstream feeder. The novelty in this work is outlined by the use of three-dimensional matrix to show the possible states, where the HVOCB can sojourn.

##### 4.1. Case of three degradation processes modeling

We consider that the processes of degradation are modeled using continuous probability functions, and the operating condition of the system is characterized by a number of states which space is noted by  $\Omega_\mu$ .

Following the Li and Pham theory [17], we consider the three state spaces  $\Omega_1$ ,  $\Omega_2$ , and  $\Omega_3$  corresponding to the degradation processes  $Y_1(t)$ ,  $Y_2(t)$ , and  $Y_3(t)$ , respectively. After obtaining the state spaces  $\Omega_1$ ,  $\Omega_2$ , and  $\Omega_3$ , we develop a methodology to establish a relationship between the states of the system  $\Omega_\mu$ , the set of degradation states and catastrophic state due to shocks arrivals  $\{\Omega_1, \Omega_2, \Omega_3, F\}$ .

The study deals with the three processes of degradation  $Y_1(t)$ ,  $Y_2(t)$ , and  $Y_3(t)$  combined with the shock process denoted  $D(t)$  as given in **Figure 3**. The sets of states are represented by  $\Omega_1 = M_1, \dots, 1_1, 0_1$ , which corresponds to the degradation 1 with  $(M_1 + 1)$  states,  $\Omega_2 = M_2, \dots,$



**Figure 3.** Diagram of transition states of a system subject to four failure processes.

$1_2, 0_2$ , corresponds to the degradation 2 with  $(M_2 + 1)$ , and  $\Omega_3 = M_3, \dots, 1_3, 0_3$  corresponds to the degradation 3 with  $(M_3 + 1)$  states. **Figure 3** shows the transition between states of a system submitted to four failure processes.

The equivalence relations between degradation states  $\Omega_1 = M_1, \dots, 1_1, 0_1$ ;  $\Omega_2 = M_2, \dots, 1_2, 0_2$  and  $\Omega_3 = M_3, \dots, 1_3, 0_3$ , and their corresponding intervals are given as follows:

Degradation Process 1:

$$0 < Y_1(t) \leq W_M \implies \text{State } M_1.$$

$$W_M < Y_1(t) \leq W_{M-1} \implies \text{State } (M-1)_1.$$

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$$W_2 < Y_1(t) \leq W_1 \implies \text{State } 1_1.$$

$$G_1 = W_1 < Y_1(t) \implies \text{State } 0_1.$$

Degradation Process 2:

$$0 < Y_2(t) \leq A_M \implies \text{State } M_2.$$

$$A_M < Y_2(t) \leq A_{M-1} \implies \text{State } (M-1)_2.$$

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$$A_2 < Y_2(t) \leq A_1 \implies \text{State } 1_2.$$

$$G_2 = A_1 < Y_2(t) \implies \text{State } 0_2.$$

Degradation Process 3:

$$0 < Y_3(t) \leq Z_M \implies \text{State } M_3.$$

$$Z_M < Y_3(t) \leq Z_{M-1} \implies \text{State } (M-1)_3.$$

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$$Z_2 < Y_3(t) \leq Z_1 \implies \text{State } 1_3.$$

$$G_3 = Z_1 < Y_3(t) \implies \text{State } 0_3.$$

The states' space of the system is defined by  $\Omega_\mu = \{M, \dots, 1, 0, F\}$  with  $(M+2)$  states. In this part, we develop a function that generates the relation between the states' space of the system



$\Omega_\mu$  and degradation states' space  $\{\Omega_1, \Omega_2, \Omega_3, F\}$ . For example, at a given time  $t$ , it is assumed that the degradation process 1 is at the state  $i_1 \in \Omega_1$  and the degradation process 2 is at state  $j_2 \in \Omega_2$  and the process of degradation 3 is in the state  $k_3 \in \Omega_3$ . It is assumed that at the present time, the system is not at fault condition (catastrophic state  $F$ ). Thus, state  $F$  can be ignored for the moment, we must therefore seek a function relationship between  $\Omega$  and  $\{\Omega_1, \Omega_2, \Omega_3\}$ . Instead of  $\Omega_\mu$  and  $\{\Omega_1, \Omega_2, \Omega_3, F\}$ . The operation can be described by the mathematical function formulated as follows:

$$f: R = \Omega_1 \times \Omega_2 \times \Omega_3 \rightarrow \Omega = \{M, \dots, 1, 0\} \quad (21)$$

where  $R = \Omega_1 \times \Omega_2 \times \Omega_3 = \{(i_1, j_2, k_3) \mid i_1 \in \Omega_1, j_2 \in \Omega_2, k_3 \in \Omega_3\}$

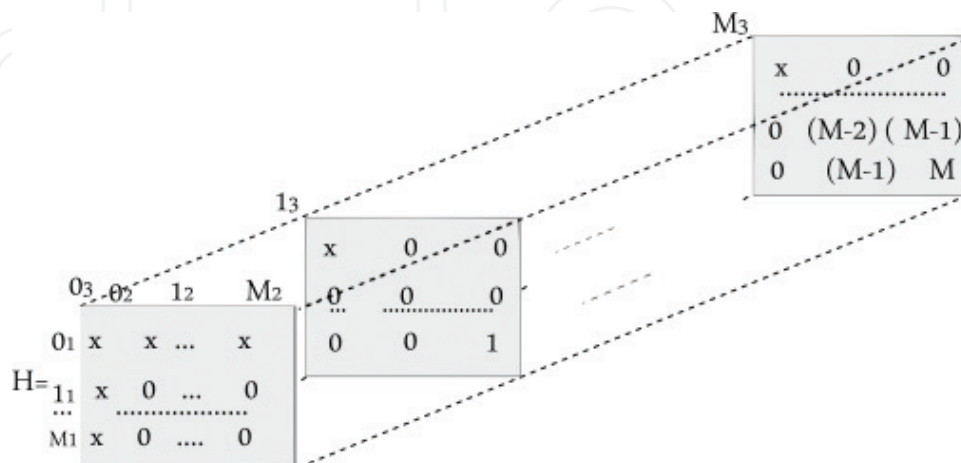
The function  $f$  is defined by:  $\Omega_1 \times \Omega_2 \times \Omega_3 \Rightarrow f \Rightarrow H$ .

The matrix  $H$  represented in **Figure 4** gives information about the resulting states space and component of  $(M+1)$  elements corresponding to each space leaving by the function  $f$ . The line at the top of the matrix  $H$  represents the states of the degradation process 1, the right column of the matrix represents the states of the degradation process 2, and the top page of the matrix  $H$  represents the degradation process 3. The elements of the matrix  $H$  represent the states  $f(i_1, j_2, k_3) = L$ .

We note that in the matrix  $H$ , some elements are zeros and it can be assumed that when degradation 1 is in a certain advanced state  $I_1 (0_1 < I_1 < M_1)$ , the degradation 2 is also in a certain weak state  $I_2 (0_2 < I_2 < M_2)$ , and the degradation 3 is also in a certain weak state  $I_3 (0_3 < I_3 < M_3)$ , it is considered as a failure condition. We also notice that  $f(M_1; M_2; M_3) = M$ , and initially, the system is in a perfect state.

We define time until failure by:  $T = \inf\{t : Y_1(t) > G_1, Y_2(t) > G_2, Y_3(t) > G_3 \text{ where } D > S\}$ .

It is important to know that the life of the system depends on a single process among the three degradations and of that of the shock. However, the system failure is caused by the process



**Figure 4.** Three-dimensional matrix of system' states, where  $I_1 \in \Omega_1$ ,  $J_2 \in \Omega_2$ ,  $K_3 \in \Omega_3$ , and  $L \in \Omega$ .

that occurs first, exceeding its critical value corresponding to the level which can bring the system back to failure.

#### 4.2. System case study modeling and application

Initially, the system is considered in good states of operation ( $M_1, M_2$ , and  $M_3$ ). It can pass first, to the degradation states  $((M-1)_1, (M-1)_2, (M-1)_3)$  or to the state of catastrophic failure (state  $F$ ), due to random shock. When the system reaches the first state of degradation, it can either remain in this state or go to the second degradation state  $((M-2)_1, (M-2)_2, (M-2)_3)$ , or it passes to state  $F$ . The same process is repeated at each degradation stage with the exception of the states  $(0_1, 0_2, 0_3)$

Assumptions:

1. The system occupies  $(M+2)$  states, where 0 and  $F$  are the states of failure, state  $i$  is a degradation state,  $1 < i < M$ ;
2. No repair or maintenance is carried out on the system;
3.  $Y_i(t)$ ;  $i = 1, 2, 3$ ; is a not decreasing and not negative function. With respect to time  $t$ , it corresponds to an irreversible accumulation of damage;
4.  $Y_i(t)$ ;  $i = 1, 2, 3$ ; and  $D(t)$  is statistically independent implying that the state of one process will have no effect on the other state;
5. At time  $t = 0$ , the system is at state  $M$ ;
6. The system may fail due to;
  - Degradation process if:  $Y_i(t) > G_i$  for  $i = 1, 2, 3$ ;
  - Random shock process (the system passes to the condition of the catastrophic failure state  $F$ ), if:  $D(t) = \sum_{t=1}^{N(t)} X_t > S$ ; with  $G_i$  and  $S$  are critical level of degradation and shocks, respectively.

The reliability function is defined as follows:

$$\begin{aligned}
 R_M(t) &= P(\text{state} \geq 1) \\
 &= \sum_{i=1}^M P_i(t) \\
 &= P\{Y_1 \leq G_1, Y_2 \leq G_2, Y_3 \leq G_3, D(t) \leq S\}
 \end{aligned} \tag{22}$$

The system will fail if any of the degradation rates exceeds the critical level  $G_i$ ;  $i = 1, 2, 3$  or the process of shock also exceeds the critical level  $S$ .

The system subject to three displacement processes is defined by:

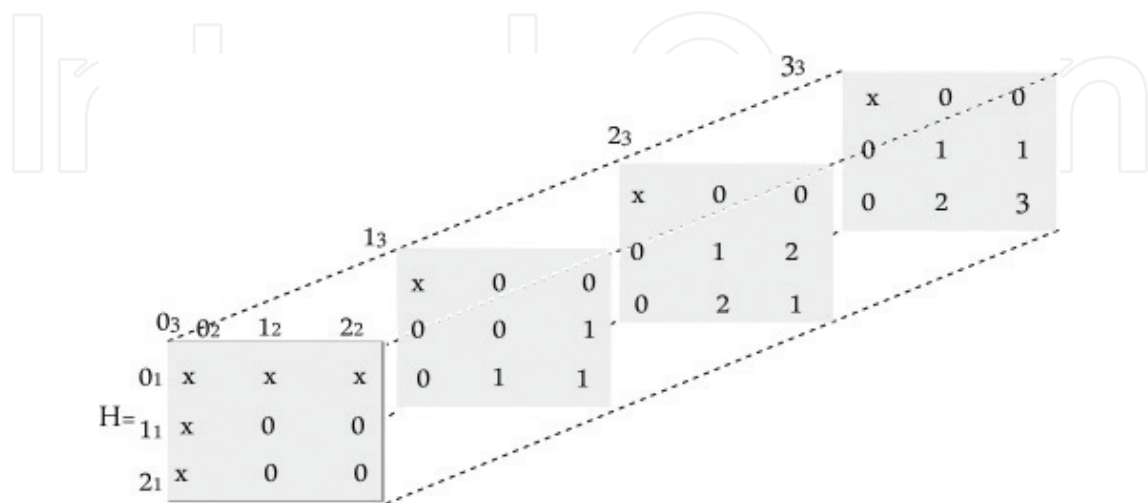
1. The process of increasing degradation representing the wear of the contacts of the circuit breaker is denoted by  $Y_1(t)$ ;

- 2. The process of increasing degradation representing the aging of oil insulating circuit of the circuit breaker is denoted by  $Y_2(t)$ ;
- 3. The degradation process of bus bars supports is denoted by  $Y_3(t)$ ;
- 4. A random process of cumulative shock damages is given by  $D(t) = \sum_{t=1}^{N(t)} X_t$ .

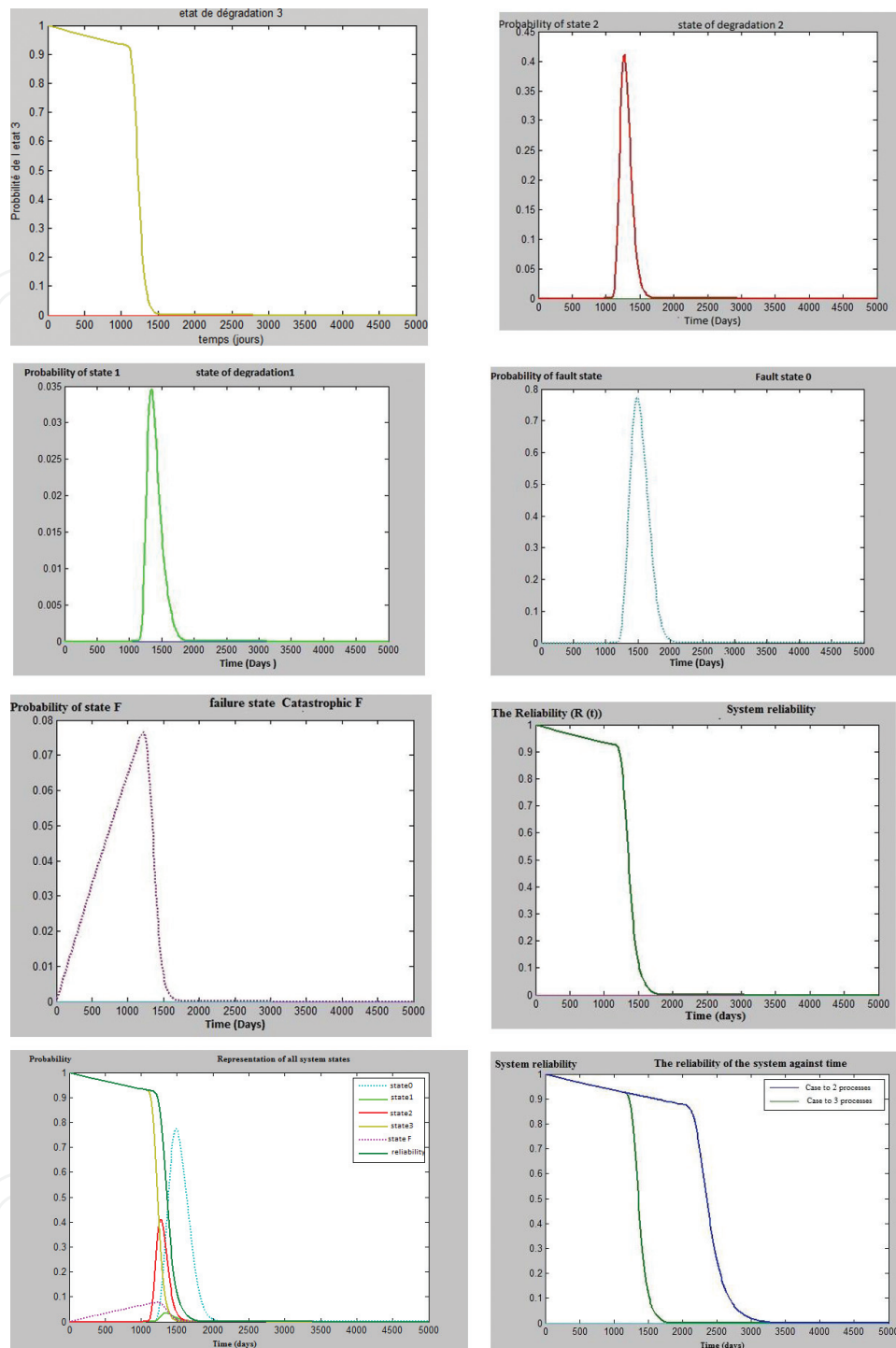
We obtain a system with four competing degradation processes. For the wear of the contacts:  $Y_1(t)$ ,  $M_1 = 2$ ; for the aging of the oils  $Y_2(t)$ :  $M_2 = 2$ ; and for the degradation of the supports:  $Y_3(t)$ ,  $M_3 = 3$ ; the system fails if the process of degradation  $Y_i(t)$  exceeds a level  $G_i$ ;  $i = 1, 2, 3$  or the process of damages cumulates shocks, and  $D(t)$  exceeds the level  $S$ . It is assumed that the state spaces associated to  $Y_1$ ,  $Y_2$  and  $Y_3$  are  $\Omega_1 = \{2_1, 1_1, 0_1\}$ ,  $\Omega_2 = \{2_2, 1_2, 0_2\}$ ,  $\Omega_3 = \{3_3, 2_3, 1_3, 0_3\}$ , respectively. Consequently, the space of the system is defined as follows:  $\Omega = \{3; 2; 1; 0; F\}$ . Thus, the function  $f$  is defined as being the Cartesian product of three sets following  $f : R = \Omega_1 \times \Omega_2 \times \Omega_3 \rightarrow \Omega = \{3, 2, 1\}$ , and the result is illustrated by the three-dimensional matrix given in **Figure 5**.

The implementation of the abovementioned models under the MATLAB software which has given the results of the probabilities of the sojourns in different states and the system reliability changing is shown in **Figure 6**. The system (oil circuit breaker, bus bars) is in good condition for 3 years (1100 days) with a probability greater than 0.9. After this period, the latter decreases exponentially to reach the zero value after 4 years (1490 days) without any maintenance actions.

The evolution of the probability of the system in degradation state 2 is complementary to that of the probability of the degradation state 3. Indeed, during 3 years of operation, the probability of being in state 2 is zero. Then, it increases exponentially to reach its maximum value of 0.41 up to 4 years (1280 days). As this state is transient, its probability function decreases to 0 after 5 years. For the system reliability, we note that during the first 8 years, it is expected to decrease by 20% due to the random shock process that governs the system during this period.



**Figure 5.** The three-dimensional matrix of the studied system.



**Figure 6.** Probabilities of the states and the system reliability changing.

## 5. Conclusion and discussions

Recently researchers in electrical systems have proposed differentiated electricity service based on reliability and have shown some inconveniences to apply it into a real case. In the same

location are connected both consumers with high reliability requirements, with an agreement to pay more and others who are not concerned. Because the technical measure proposed is to add a reliable feeder, the differentiation is not quite possible. Our proposition consists of organizational measure and is oriented to maintenance actions on MV/LV substations, which are directly connected to the end users of the network. The differentiated service is directly related to the reliability of the substation where the improvement is a function of maintenance actions and the frequency of interventions. For statistical considerations and for interruption (forced or scheduled outages) modeling, we have applied the Weibull-Markov approach rather than the Markov method, which is usually used for the case of electrical systems. It has been proven that it is possible to maintain in another way than the classical one based on systematic preventive maintenance. In this chapter, it is shown that the maintenance is decided on the reliability level and benefit bases. Another critical component of electrical substation is studied using competing failure processes and consists a circuit breaker. The reliability aspects are formulated in the bases of oil aging, contacts wear and bus bars support degradation. Investigations conducted by Pham in a theoretical framework have been applied successfully to complement system such as electrical system. The models applied on simple numerical examples have been validated by application to a real case engineering. During system operation, the results analysis of the network current state allows to the decision maker to reach better information and target the equipment that reduces the performances of the system and practicing suitable maintenance actions. Recent studies in energy sustainability and smart energy grid have revealed that reliability is the main criterion taken into account by decision makers in electricity market behavior and a performance index for electric utilities classifications.

## Acknowledgements and tributes

In 1988, a group of professors in mathematics and engineering (A. Aissani, D. Aissani, K. D. Haim, A. Boubakeur, and A. Ouabdeslam) had organized the national conference named MFSI at the University of Bejaia, in Algeria, where during 2 days, the notion of reliability was vulgarized in the field of engineering. From that manifestation was born the group of work in power system reliability FSE2. Over than 1500 various works were conducted in engineer, master and doctorate theses dealing with all the aspects of power systems reliability and with a large cooperation with other universities and various manufactures and services. A great number of applications were done around the power systems including production, transportation and distribution parts. In recent years, a lot of novelties were developed compared to what is done over the word, such as the Weibull-Markov modeling in data analysis, nonparametric distributions in switching components behavior, Box and Jenkins models in blackouts forecasting and reliability aspects in smart grids development and multicriteria optimization. The results were valorized in a great number of international conference proceedings and in valuable international journals. This chapter dealing with power system reliability constitutes an interesting opportunity to express our acknowledgments and tributes to these pioneers of reliability in Algeria for what they have given for research.



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