

We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900

Open access books available

186,000

International authors and editors

200M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com



Kinetic Theory of Creep and Long-Term Strength of Metals

Alexander Lokoshchenko and Leonid Fomin

Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/intechopen.70768>

Abstract

This chapter deals with the simulation of the creep process and the effect of long-term strength of metals, notably, in both uniaxial and complex stress states. A description of a creep experiment and the simplest creep models are presented, that is, the theory of steady creep, the theory of ageing, the theory of flow and the theory of hardening. In creep process simulation, a kinetic theory based on the introduction of structural parameters characterising the state of the metal at a given time is widely used. Among such parameters, metal damage in the creep process, work of stresses on creep deformations (energy version) and concentration of an aggressive medium in the metal were studied. The coupled problem of creep and long tensile strength is also considered taking into account the mutual influence of damage accumulation and one-dimensional diffusion of the aggressive medium. The times to fracture are determined both in the presence of an aggressive medium and in the absence of one. A significant contribution of Soviet (Russian), European, American and Japanese scientists to the development of continuum damage mechanics is highlighted.

Keywords: kinetic theory, continuum damage mechanics, creep, long-term strength, metals, scalar and vector damage parameters, fracture criterion, aggressive medium, diffusion

1. Introduction

In the late 1950s of the twentieth century, two outstanding Soviet scientists Kachanov and Rabotnov concluded that the terms of the deformed solid mechanics (stress and strain tensors and displacement vector) used at that time were insufficient to describe the process of creep and delayed fracture of materials and structural elements under creep conditions. They proposed a new approach to study the long-term strength called kinetic. It is based on the use of Kachanov [1] and Rabotnov's [2] damage parameter and the subsequently developed kinetic

theory of creep and long-term strength by Rabotnov [3, 4]. The core of this approach is the introduction of a scalar parameter of damage $\omega(t)$ characterising the structural state of the material at an arbitrary time t . The initial state of the material (at $t=0$) corresponds to $\omega=0$; when the destruction occurs, the damage $\omega(t^*)$ takes the value of 1. Later, important results in this area were obtained by A.A. Ilyushin, S.A. Shesterikov, O.V. Sosnin and other Russian scientists.

Following the studies by Kachanov and Rabotnov, the mechanics of continuum fracture started to develop in different countries, mainly for the creep processes of metals. In the last 50 years, the continuum damage mechanics (CDM) has been extensively developing. Representatives of the English schools of mechanics F.A. Leckier and D.R. Hayhurst provided a significant contribution to the development of the theory of damage accumulation. Certain success was also achieved in the studies of Polish scientists M. Chrzanowski and W. Tramaczynskii. In France, the CDM fundamentals were formulated using thermodynamic considerations (J. Lemaitre). In the beginning of the 1980s of the twentieth century, this section of mechanics was rapidly developing in the USA as a result of the work of many scientists. Since then, this area has been the centre of attention worldwide both with respect to the development of the fundamentals (some theoretical problems have still remained unsolved) and applications.

2. Creep under uniaxial tension

In the mechanics of solids, it is common to differentiate the materials under investigation by their reaction to load. When under an arbitrary loading process, the material immediately returns to its original state after the load is removed; this means that the material exhibits elastic properties. If after unloading there appear residual deformations that depend only on the load values and the order of their application, but do not depend on the loading rates and durations, such medium is called elastoplastic. When these deformations essentially depend on the duration of loading, such media have creep properties or, more generally, rheological properties.

Below, the simplest form of high-temperature mechanical testing—the uniaxial tensile test—is considered. It is assumed that the entire process of loading or deformation occurs under isothermal conditions.

The tensile loading of a cylindrical specimen made of a homogeneous material with force P will be examined. Supposedly, the specimen L and its cross-sectional area d satisfy the condition $L \gg d$. In this case, it can be further assumed that at a certain distance from the ends the section of the specimen with length l is subjected to uniform tensile loading. It is also assumed that the strains are low, the variation of the cross-sectional area is insignificant and the specimen is deformed uniformly, without the formation of local constrictions. The ratio of force P to the cross-sectional area of the specimen is referred to as stress σ , and the relative variation of length l is referred to as strain $\varepsilon = \Delta l/l$, where Δl is the increment of length l .

This chapter considers a case in which a stress $\sigma(t)$ programme is given in the experiment and the dependency $\varepsilon(t)$ is registered. It should be noted that at this stage, only those programmes that do not lead to the appearance of strain heterogeneities (as a result of the shear lines, neck formation, etc.) are examined.

The chapter contains various terms that are quite familiar to the reader. Therefore, it appears excessive to define such characteristics as deformation, strain tensor, deformation rate, stress and stress tensor.

Previously, it was mentioned that the creep of metals is manifested in the development of the deformation process with time, usually at elevated temperatures. Thus, even in the case of the uniaxial stress state, it is necessary to consider the four macroquantities—temperature, stress, time and strain. Creep characteristics determined in the experiments at constant temperatures can also often be used when evaluating the efficiency of structures at varying temperatures. To determine the dependences describing the creep process, it is usually necessary to use the data obtained in the standard uniaxial tension tests. Creep is mostly common with metals and alloys at absolute temperatures T higher than $(0.4-0.5)T^*$ (T^* is the melting point on the absolute scale, i.e., in Kelvin (K)).

In the creep test, a cylindrical specimen with thermocouples attached to it is secured in the clamps of the loading machine and placed in the furnace. The temperature of the specimen is controlled using the thermocouples, and the results are sent to a tracking system. This system ensures heating of the specimen to the required level, and the temperature is then maintained constant with a specific accuracy. After complete heating of the furnace area, a tensile force is applied to the specimen. This force changes with time under a given law (in most cases, this force is constant or a fractional-constant time function). A strain measurement device is used to record the variation of the length of the specimen with time during continuous recording of the deformation diagram. The elongation of the specimen as a result of the creep of the material is accompanied by a decrease in the cross-sectional area and, consequently, the tensile stress increases continuously at a constant load.

In the tests of the materials characterised by high creep strains (of 4–5% order or more), there are used systems where the load is self-compensated so that the stress in the specimen remains constant. When testing a number of creep-resistant alloys, it appears that the creep strains remain relatively small (approximately 1–2%) up to the moment of fracture. In these conditions, the tests can be carried out at a constant load, and it can be assumed that the stresses remain unchanged during the experiment. The creep experiments show that even for the specimens taken from the same blank part (plate or bar) the creep strain values are greatly scattered for the same values of time (by up to 20–30% or more). The scatter is explained by the specific features of the individual specimens.

Figure 1 schematically shows the curves characterising the strain dependence $\varepsilon(t)$ on time t at the different stresses σ .

The conventional $\varepsilon(t)$ curve corresponding to the average stress level ($\sigma = \sigma_2$) has three distinctive sections as follows: Section 1 with the constantly decreasing creep rate (unsteady creep), Section 2 with a constant (minimum) creep rate (steady-state creep) and Section 3 with accelerating creep preceding fracture. At relatively low stresses ($\sigma = \sigma_1$), the $\varepsilon(t)$ curve can have only the nonsteady section. The curves leading to relatively high stresses ($\sigma = \sigma_3$ and $\sigma = \sigma_4$) may not have the first section, and when $\sigma = \sigma_4$, only the third section is present. All these special features are satisfactorily explained by the presence of at least two structural deformation mechanisms (hardening and softening), which are determined by changes of the dislocation

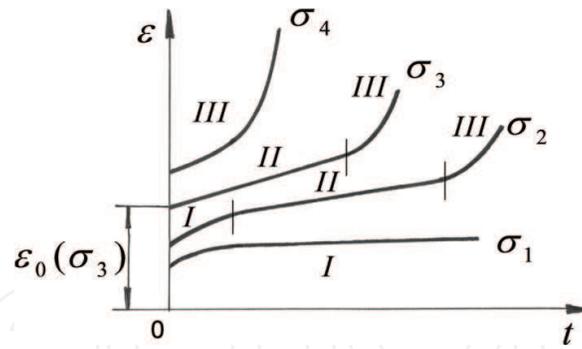


Figure 1. Dependence of strain on time at different stresses.

structure, the vacancy processes, phase transitions, grain size changes in the deformation process and other reasons. The preferential effect of one mechanism in comparison with the others leads to a change of the stages on the creep curve.

When constructing these curves, it is assumed that the loading time of the specimen to a given stress is very short compared to the test time. Therefore, the curves $\varepsilon(t)$ start at the strain corresponding to the 'instantaneous' loading.

The creep theory seeks to determine a relationship between stress σ , time t , creep strain p and temperature T ; this relationship, which is universal, should be capable of determining the creep curve $p(t) = \varepsilon(t) - \varepsilon_0(\sigma)$ at the arbitrary laws of stress $\sigma(t)$ and temperature $T(t)$ variations with time.

Different problems of the creep theory have been investigated in a number of monographs ([3–7], and others).

Without loss of generality, here and further on, it is possible to consider isothermal processes occurring at a constant temperature. The transition to other temperatures in creep and long-term strength simulation should be carried out using known temperature-time analogies specified, for example, in [6, 7].

Here, the case is examined where $\sigma(t) = \text{const}$ and the specimen is at the steady-state creep stage $\dot{p}(\sigma, t) = \text{const}$ most of the time. In this case, to describe the behaviour of the material, it is natural to use the relationship of the non-linear viscous flow called the theory of the steady-state creep:

$$\dot{p} = f(\sigma) \quad (1)$$

(the dot above the symbol indicates the differentiation with respect to time t).

The steady-state creep rate \dot{p} is of special importance, because in many technical applications it accounts for the main part of the accumulated creep strain. In most studies, the function $f(\sigma)$ is a power function of the mechanical stress σ :

$$\dot{p} = A\sigma^n, \quad (2)$$

where A and n are the constants of the material.

To describe the first nonsteady section of the creep curve alongside the second one, it is possible to use different theories with the simplest one being the ageing theory:

$$p = F(t, \sigma). \quad (3)$$

The first analytical description of the ageing theory for metals was proposed by E.N. Andrade in 1920:

$$p = At^{1/3} + Bt, \quad (4)$$

where the coefficients A and B depend on stress σ .

An important feature of the creep theory constructed using the relationships explicitly containing the time (this takes place when introducing the hypothesis of ageing) is that they are valid only at constant or relatively slowly changing stresses. For a sudden stress variation, these theories lead to a stepwise variation of the creep strains, too, which is naturally impossible. However, since the ageing hypothesis leads to smaller mathematical difficulties in comparison with the other theories, it is used in calculations with considerable success taking into account the scope of its applicability.

The first attempt to overcome the shortcomings of the above-described ageing theory was considered to be the C.C. Davenport's (1938) hypothesis, according to which relation (3) should be replaced by a relation of the following form:

$$\dot{p} = f(t, \sigma), \quad (5)$$

or taking the elastic properties into account:

$$\dot{\epsilon} = \frac{\sigma}{E} + f(t, \sigma). \quad (6)$$

This theory is called the theory of ageing in the form of a flow or, briefly, the flow theory.

The simplest consistent assumption used to describe an unstable creep area at a constant temperature is that the creep rate $\dot{p}(t)$ for an arbitrary value of t is determined by the stress σ and the current value of the creep strain:

$$\dot{p} = f(\sigma, p). \quad (7)$$

From Eq. (7), which is the basis of the hardening theory, it follows that the creep rate does not depend explicitly on time t . This fact indicates a significant advantage of the hardening theory over other theories.

The most promising theory in the mechanics of solid media to describe the creep processes of structural metals is the concept of the mechanical equation of state, proposed by Rabotnov [3, 4].

According to this concept, the creep rate \dot{p} of a structurally stable material at every moment of time t depends on the magnitude of applied stress, temperature and the structural state of the material at this moment t . The structural state of the material is characterised by the set of values q_1, q_2, \dots, q_N , which are called the structural parameters. The kinetic creep theory consists of the mechanical equation of state

$$\dot{p} = \dot{p}(\sigma, T, q_1, q_2, \dots, q_N) \quad (8)$$

and a system of kinetic equations to determine the parameters q_i . The structural parameters q_i ($i=1, 2, \dots, N$) used in Eqs. (8) and (3) vary during deformation in accordance with the kinetic equations:

$$dq_i = a_i dp + b_i d\sigma + c_i dt + g_i dT, \quad (9)$$

and the coefficients a_i, b_i, c_i, g_i are the functions of p, σ, t, T and also of q_1, q_2, \dots, q_N . Relationships (8) and (9) widen the range of theories available to describe greatly varying experimental results. Extensive studies of the creep of metals using the mechanical equation of state in form (8) supplemented by kinetic equations (9) were carried out in a large number of researches by Rabotnov and his colleagues [8].

A creep process with a stepwise increase of the stress $\sigma(t)$ is investigated below. For an analytical description of the creep curve after changing the mechanical stress, the system of equations (8) and (9) is used. Consider the energy version of the kinetic theory.

At $N=1$, $a_1 = a_1(\sigma)$, $b_1=0$, $c_1=0$, $g_1=0$, in a partial case $a_1(\sigma) = \sigma$ and

$$dq = \sigma dp \quad (10)$$

Here, the parameter q is the work of stresses acting at the creep strains. Application of this version in the theory to describe the creep process based on the energy approach has been widely used in the studies by Sosnin and his colleagues [9].

In [10], the results of experimental verification of this version of the theory for D16AT aluminium alloy at the temperature of 150°C are presented. The circles in **Figure 2** show the experimental creep curves at $\sigma_1 = 150$ MPa and $\sigma_2 = 250$ MPa.

When simulating these experimental data, the simplest form of the hardening theory is initially examined assuming that it is expressed as follows:

$$\dot{p} = p^{-\alpha} f(\sigma), \quad \alpha > 0. \quad (11)$$

It is further assumed that $f(\sigma)$ has the form of a power function. Therefore,

$$\dot{p} p^\alpha = B \sigma^n. \quad (12)$$

Integration of differential equation (12) separately over the intervals $0 < t < t_1$ and $t > t_1$ results in the following expressions:

$$p(t) = [B(\alpha + 1)\sigma_1^n]^{1/(\alpha+1)} t^{1/(\alpha+1)} \quad \text{if } 0 \leq t < t_1, \quad (13)$$

$$p(t) = [B(\alpha + 1)\sigma_2^n(t - t_1) + p_1^{\alpha+1}]^{1/(\alpha+1)} \quad \text{if } t > t_1, \quad (14)$$

where $p_1 = p(t_1) = [B(\alpha + 1)\sigma_1^n]^{1/(\alpha+1)} t_1^{1/(\alpha+1)}$.

Now, the version of equations (8) and (9) is selected in such a way that at a constant stress ($\sigma_1 = \sigma_2$) they coincide with (12):

$$\dot{p}q^\alpha = B\sigma^{n+\alpha}. \quad (15)$$

In the first creep section under the effect of the stress $\sigma = \sigma_1$, there is obtained $q(t) = \sigma_1 p(t)$. Therefore, differential equations (12) and (15) coincide and, consequently, the creep curve, corresponding to the defining Eq. (15) at $0 < t < t_1$ coincides with the curve (13). After additional step loading,

$$q(t) = \sigma_2 p(t) - p_1(\sigma_2 - \sigma_1). \quad (16)$$

Substituting $q(t)$ (16) into differential equation (15) and integrating this equation at $t > t_1$ taking into account the initial condition $p(t_1) = p_1$, it is obtained that

$$p(t) = \frac{\sigma_2 - \sigma_1}{\sigma_2} + \left[B(\alpha + 1)\sigma_2^n(t - t_1) + \left(\frac{\sigma_1 p_1}{\sigma_2} \right)^{\alpha+1} \right]^{1/(\alpha+1)}. \quad (17)$$

The dashed line in **Figure 2** shows the creep curve calculated using Eq. (14), and the solid line represents the curve calculated using Eq. (17). Evidently, the hardening theory set up by Eq. (15) taking into account kinetic Eq. (10) describes the experimental data more efficiently than the simplest hardening theory (12).

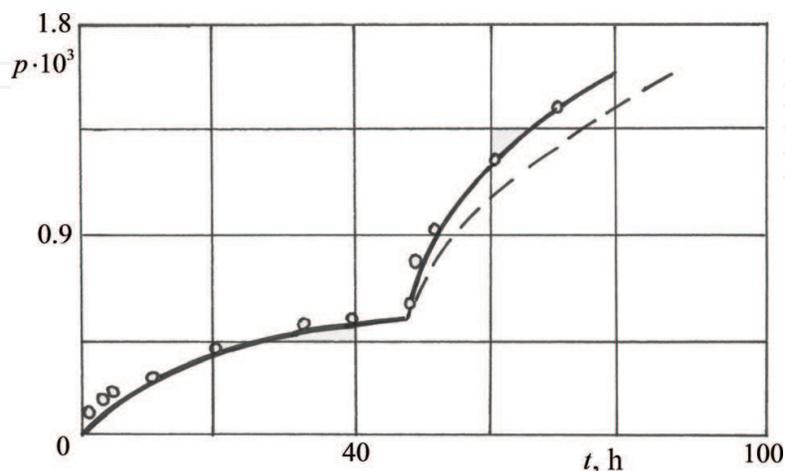


Figure 2. Creep of D16AT alloy [10].

Considering the system of Eqs. (8) and (9) with two kinetic parameters—in addition to the generally accepted hardening measure $q_1 = p$, we introduce the second parameter q_2 determined by the following relationship [11]:

$$dq_2 = \begin{cases} pd\sigma & npu & d\sigma > 0, \\ 0 & npu & d\sigma \leq 0. \end{cases} \quad (18)$$

If at the initial moment of time ($t=0, p=0$), there is applied stress σ_1 that subsequently remains constant, and then $q_2=0$ if $0 < t < t_1$.

At a stepwise increase of stress at the time t_1 , the parameter q_2 , according to Eq. (18), receives the increment $\Delta q_2 = p_1(\sigma_2 - \sigma_1) > 0$. The creep law in [11] is represented by the equation

$$\dot{p}p^\alpha = k \exp\left(\frac{\sigma}{A} + \frac{q_2}{B}\right). \quad (19)$$

Eq. (19) implies that additional introduction of the second kinetic parameter q_2 according to (18) leads to a higher creep rate increase after the instantaneous loading compared with the standard hardening theory (12). In case of a step load, it follows from kinetic equation (18) that the parameter q_2 does not change.

Figure 3 shows the experimental points [11] obtained on samples of D16T alloy, tested at the stress of $\sigma_1 = 80$ MPa for the time $t_1 = 24$ h and at $\sigma_2 = 160$ MPa and at $t > t_1$ (temperature of 200°C). The dashed line corresponds to standard hardening theory (12), the solid line—to the theory with two kinetic parameters described by Eq. (19).

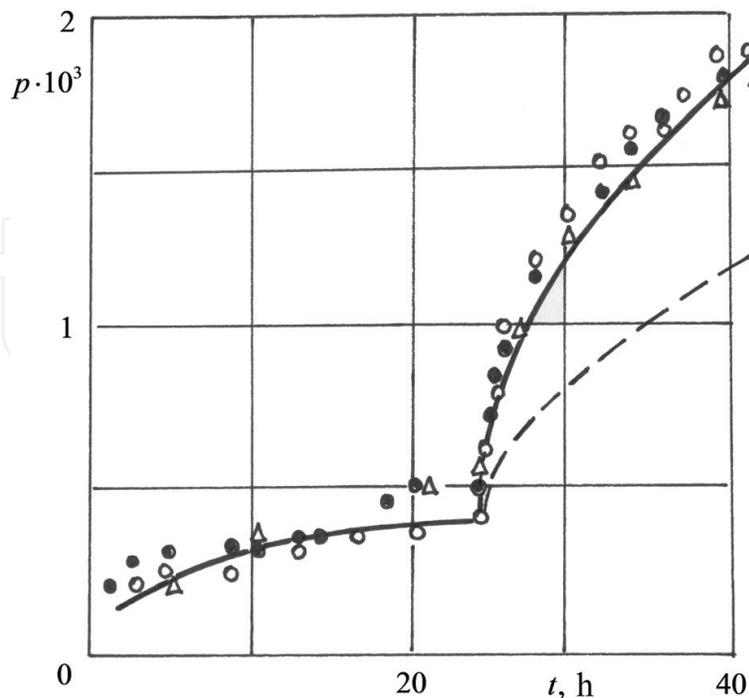


Figure 3. Creep of D16T alloy [11].

These considerations show that the concept of equation of the mechanical state (8) with the system of the kinetic equation (9) proposed by Rabotnov to determine the structural parameters is highly promising for describing different special features of the material behaviours in the creep conditions.

3. Long-term strength under uniaxial tension

In the majority of cases when the levels of temperatures and stresses in the experiments are quite high, the deformation process over time ends with the fracture of the specimen. This moment is characterised by certain time t^* determined by the given values of stress σ and temperature T . If a sufficiently large series of experiments is carried out, then for a number of temperature values T , it is possible to construct a series of curves $t^*(\sigma)$ which are referred to as the long-term strength curves. It should be noted that the actual experimental data for the majority of metals and alloys are greatly scattered.

Typical long-term strength curves are shown schematically in **Figure 4**. In the first case, the points in the logarithmic coordinates are distributed over a single straight line. In the second case, the diagram consists of two straight sections. In this case, section AB of the diagram corresponds to ductile fracture, and section BC corresponds to brittle fracture. The diagram does not always consist of two straight lines characterised by a distinctive intersection point. Sometimes, a curvilinear transition region occurs between the straight lines AB and BC indicated by the dashed line—the region of mixed fracture.

A general approach to the fracture problem implies that the value ω (the extent of cracking) is regarded as a structural parameter. Therefore, the creep process is described by the creep equation

$$\dot{p} = f(\sigma, \omega), \quad p(t = 0) = 0, \quad (20)$$

and the fracture process is described by the kinetic equation of gradual fracture

$$\dot{\omega} = \varphi(\sigma, \omega), \quad \omega(t = 0) = 0. \quad (21)$$

If the fracture is accompanied by a small elongation only, the stress σ at a constant tensile loading force may be regarded as constant ($\sigma(t) = \sigma = \text{const}$). In this case, Eq. (21) is integrated

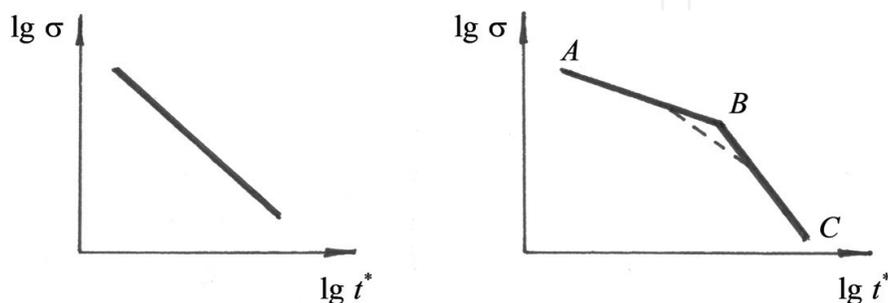


Figure 4. Typical long-term strength curves.

independently of (20) and provides the equation $\omega = \omega(\sigma, t)$. Therefore, the moment of fracture t^* is determined as the value $t = t^*$ at which $\omega = 1$. Further, substitution of $\omega = \omega(t)$ into Eq. (20) leads to the creep curve equation $p(t)$. The following simplest Eqs. (20) and (21) can be used

$$\dot{p} = A\sigma^n(1 - \omega)^{-k}, \quad (22)$$

$$\dot{\omega} = B\sigma^m(1 - \omega)^{-s}. \quad (23)$$

As previously, here σ is the mean macrostress determined by the tensile force divided by the area of the sample. The deformation process is completed at the moment of fracture $t = t^*$ corresponding to the value of the parameter $\omega(t^*) = 1$. Integration of the system of Eqs. (22) and (23) at $s + 1 - k > 0$ provides the equation of the creep curve

$$p = p^* \left[1 - \left(1 - \frac{t}{t^*} \right)^{\frac{s+1-k}{s+1}} \right], \quad (24)$$

where the time to fracture t^* and the respective limiting creep strain p^* depend on the stress σ and the material constants as follows:

$$t^* = [B(s + 1)\sigma^m]^{-1}, \quad p^* = \frac{A}{B(s + 1 - k)}\sigma^{(n-m)}. \quad (25)$$

Some tests of metals in the creep conditions before fracture showed a non-monotonic change of the limiting creep strain p^* corresponding to the moment of fracture t^* in the investigated range of the constant tensile stress σ . In [12], it was reported that when simulating the non-monotonic dependence $p^*(\sigma)$, various functional relationships should be used to take into account the effect of stress on the creep rate and damage accumulation rate.

To describe the creep process at a constant stress up to the moment of fracture and to determine deformation p^* , consider the exponential dependence of the creep rate on the stress:

$$\dot{p} = C(\text{sh}(\sigma/c))(1 - \omega)^{-n} \quad (26)$$

and the power-law dependence of the change rate of the kinetic parameter ω on σ :

$$\dot{\omega} = D\sigma^k(1 - \omega)^{-k}, \quad k > 1. \quad (27)$$

Consider the relation $\dot{p}/\dot{\omega}$ and integrate it. As a result, in accordance with Eqs. (26) and (27), the following dependence of the limiting strain p^* on the level of mechanical stress σ is obtained:

$$p^* = \frac{C}{D} \cdot \frac{\text{sh}(\sigma/c)}{\sigma^k}. \quad (28)$$

For relatively small values σ , the dependence $p_i^*(\sigma)$ in (28) is decreasing; for sufficiently large values σ , this dependence is increasing. Consequently, at some intermediate value of the stress, the limiting strain is minimal. The condition

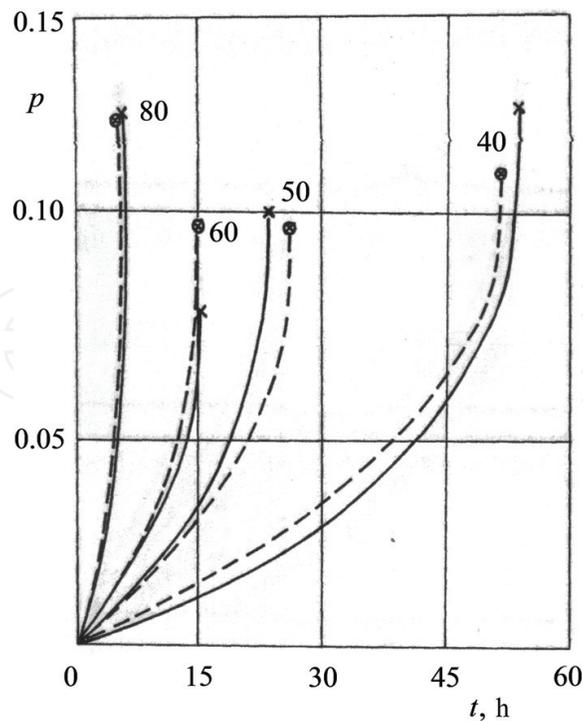


Figure 5. Creep curves of Cr18Ni10Ti steel with the non-monotonic dependence $p^*(\sigma)$.

$$(dp^*/d\sigma)|_{\sigma=\sigma_i} = 0 \quad (29)$$

with due account for Eq. (28) allows determining this value for kinetic equation (27):

$$\text{th}(\sigma/c) = \sigma/(kc). \quad (30)$$

Figure 5 shows the creep curves for stainless steel Cr18Ni10Ti at 850°C and at stresses $\sigma = 40 - 80$ MPa [12]. The experimental curves are shown by solid lines, and the theoretical curves are shown by dashed lines ($n=2.28$, $k=3.1$, $C = 2.02 \cdot 10^{-4} \text{ h}^{-1}$, $D = 5.1 \cdot 10^{-8} (\text{MPa})^{-3.1} \text{ h}^{-1}$, $c=17.8$ MPa).

When describing a series of creep curves before fracture with an internal minimum of the dependence $p^*(\sigma)$, the types of dependencies \dot{p} and $\dot{\omega}$ on σ should be interchanged.

4. Creep under complex stress state

In the previous paragraphs, attention was given to the main assumptions regarding the metal creep phenomenon, and the basic models describing the creep of bars under uniaxial tensile loading were described. In the determination of the mechanical behaviour of the structural elements in the creep conditions, however, it is necessary, as a rule, to consider the multiaxial complex stress state. Experimental studies of creep in these conditions are associated with considerable technical difficulties, and, therefore, the currently available experimental data are not extensive and do not allow a reliable justification of any creep theory in the complex stress state conditions. To determine the characteristics of the material, tests are usually carried

out on thin-walled, tubular specimens—the stress state in the specimens is usually generated by a combination of tensile loading with torsion or tensile loading with internal pressure. To generate the quasi-homogeneous stress state in the tubular specimens, the latter must be thin walled. In rare instances, experimental studies of creep in the complex stress state are carried out on a rectangular plate in biaxial loading conditions.

As noted in the beginning of the chapter, for the readers familiar with the basics of the theory of elasticity and plasticity, the concepts of stress tensors, strains and strain rates are also known.

Creep strains in the complex stress state p_{ij} , just as in the case of uniaxial tension, mean the differences between total deformations ε_{ij} and instantaneous deformations arising under quasi-static loading.

The first results of researching the creep of metals under multiaxial stress state conditions were published in the 1930s of the twentieth century. The basic principles of constructing the creep theory are considered in various monographs and textbooks.

Creep theories for the complex stress state usually consider the following three hypotheses:

1. The volume deformation is elastic (the material is always considered as incompressible).
2. The hypothesis of the proportionality of the stress deviators and creep strain rates (the flow-type theory) or the deviators of the stresses and strains (deformation theory).
3. A functional relationship between the second invariants of the tensors of the stresses and creep strain rates (or the stress and strain tensors) is assumed to have such a form that the relationships of one of the well-known creep theories are fulfilled in the partial case of uniaxial tensile loading. The presence of such a relationship implies that the dependence of the intensity of creep strain rates \dot{p}_u on stress intensity σ_u (or intensity of creep strains p_u on σ_u) is the same at different types of the stress state (i.e., the 'single curve' hypothesis is satisfied). Sometimes, the dependence between the stress and strain intensities can be replaced by a similar dependence between the maximum tangential stress and the maximum shear stress.

Different approaches to the construction of metal creep models under the conditions of the multiaxial complex stress state have been discussed in a number of monographs and journal articles (e.g., [3–7]).

As an example of such models, consider the steady-state creep model.

The hypothesis of proportionality of stress deviators and creep strain rates for an incompressible body can be presented as follows:

$$\dot{p} = 0, \quad \dot{p}_{ij} = \frac{3}{2} \cdot \frac{f(\sigma_u)}{\sigma_u} s_{ij}, \quad \delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j \end{cases} \quad \sigma = (1/3) \sum_{k=1}^3 \sigma_{kk}, \quad \dot{p}_u = f(\sigma_u). \quad (31)$$

The relationship of the tensors σ_{ij} and \dot{p}_{ij} in the Cartesian coordinates can have the form:

$$\begin{cases} \dot{p}_{xx} = \frac{3}{2} \cdot \frac{f(\sigma_u)}{\sigma_u} (\sigma_{xx} - \sigma), \dots, \\ \dot{p}_{xy} = \frac{3}{2} \cdot \frac{f(\sigma_u)}{\sigma_u} \sigma_{xy}, \dots \end{cases} \quad (32)$$

The simplest version of the theory of steady-state creep under uniaxial tensile loading is the power dependence $\dot{p}_1 = B\sigma_1^n$; in a general case, the identical relationship of the second invariant of the corresponding tensors has the form

$$\dot{p}_u = f(\sigma_u) = B\sigma_u^n. \quad (33)$$

Creep equation (31) is the equation of the non-linear viscous flow. They were derived by R. Bailey in 1935 and J. Marin in 1942.

5. Long-term strength under complex stress state

In the kinetic theory, damage accumulation is investigated as a process of gradual fracture of the material. In many studies of Russian and foreign scientists when examining the multiaxial complex stress state, special attention is given to the damage parameters that are not only of the scalar but also of vector and tensor nature. Modern versions of the kinetic theory allow describing the deformation and long-term fracture of metals in nonproportional loading taking into account the anisotropy of metal properties, using the theory to solve technological problems, etc.

Consider the uniaxial ($\sigma_1 = \sigma_0 > 0, \sigma_2 = \sigma_3 = 0$) and equiaxed plane ($\sigma_1 = \sigma_2 = \sigma_0 > 0, \sigma_3 = 0$) stress states for the same stress level σ_0 (**Figure 6**). The available test results show that the time to

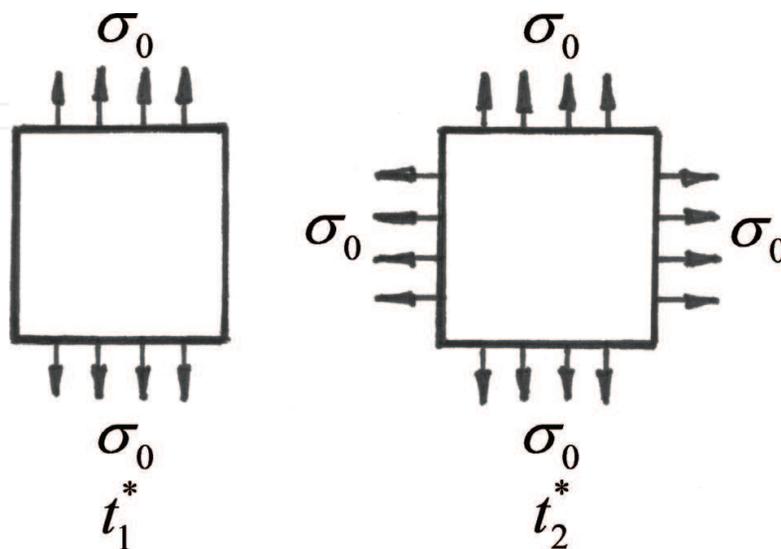


Figure 6. Uniaxial and equiaxed planar stress states.

fracture t_1^* in the uniaxial tensile loading is considerably greater than the time to fracture t_2^* in biaxial loading under these conditions ($s = t_1^*/t_2^* > 1$) [13, 14].

All the investigated experimental data indicate that the addition to the axial tensile loading stress of a transverse tensile stress of the same magnitude decreases the time to fracture several times.

Calculations of the long-term strength of structural members loaded under the conditions of the stationary multiaxial complex stress state are usually carried out using a criterial approach. In this approach, the only characteristic of the stress state taken into account is the so-called equivalent stress σ_e . This characteristic is represented by different combinations of the stress tensor components with a distinctive mechanical meaning such as the maximum tensile stress, the intensity of tangential stresses, the difference of the maximum and minimum main stresses and other expressions. Since these equivalent stresses coincide ($\sigma_e = \sigma_0$) for the investigated uniaxial and biaxial tensile loading, it is not possible to obtain different values of t_1^* and t_2^* using the criteria relationship $t^* = t^*(\sigma_e)$. Further, two versions of the systems of kinetic equations with the vector damage parameter to describe different values of the time to fracture t_1^* and t_2^* are examined.

First, the dependence of the time to fracture t^* on the type of stress state will be described taking into account the instantaneous damage for an isotropic material. This will be carried out using the generalisation of the vector approach taking into account the damage accumulated during loading. One of the possible models for describing different times t_1^* and t_2^* is the system or relationships:

$$d\omega_i = \frac{d\varphi(\sigma_i)}{d\sigma_i} \cdot d\sigma_i + f(\sigma_i) \cdot dt, \quad i = 1, 2. \quad (34)$$

where function $\varphi(\sigma_i)$ characterises the projection of ω_i on the x_i axis of the vector of damage accumulated during loading; $f(\sigma_i)$ is a constant rate of increase of the projection ω_i with time t . In uniaxial tensile loading, from relationships (34), it follows that

$$\omega_1(t) = \varphi(\sigma_0) + f(\sigma_0) \cdot t, \quad \omega_2 = 0, \quad t_1^* = [1 - \varphi(\sigma_0)]/f(\sigma_0), \quad (35)$$

and in the case of the equal biaxial tensile loading, relationships (34) provide

$$\omega_1(t) = \omega_2(t) = \varphi(\sigma_0) + f(\sigma_0) \cdot t, \quad t_2^* = [\sqrt{2}/2 - \varphi(\sigma_0)]/f(\sigma_0). \quad (36)$$

Relationships (35) and (36) show that the instantaneous value of damage $\varphi(\sigma_0)$ should be in the range $0 < \varphi(\sigma_0) < \sqrt{2}/2$, and the ratio

$$s = t_1^*/t_2^* = (1 - \varphi(\sigma_0))/(\sqrt{2}/2 - \varphi(\sigma_0)) \quad (37)$$

should exceed $\sqrt{2}$ at any values of σ_0 in this range. As an example of using Eq. (34), the results of tests [13] which at $\sigma_0 = 56.2$ MPa lead to the following values are considered: $t_1^* = 900$ h and

$t_2^* = 280$ h. At $\varphi(\sigma_0) = 0.57$ and $f(\sigma_0) = 4.78 \cdot 10^{-4} \text{ h}^{-1}$, the theoretical values of t_1^* and t_2^* calculated from the relationship (36) coincide with the respective experimental values.

The results show that the ratio s depends only on the level of damage $\varphi(\sigma_0)$ accumulated under quasi-static loading. Taking into account the instantaneous damage in the form (5.1) makes it possible to describe the experimental data only for $s \geq \sqrt{2}$. In this case, the result does not depend on the nature of damage accumulation during creep.

Now, the dependence of time to fracture t^* will be described taking into account the anisotropy of the material. When determining the long-term strength of thin-walled pipes, it is taken into account that in the process of such pipes manufacturing the material may acquire anisotropic strength properties (difference in material properties in different directions). To facilitate a quantitative analysis, there are introduced anisotropy coefficients α_1 and α_2 characterising the anisotropy of the instantaneous and long-term strength properties, respectively,

$$f(\sigma_{zz}/\alpha_1) = f(\sigma_{\theta\theta}), \quad \varphi(\sigma_{zz}/\alpha_2) = \varphi(\sigma_{\theta\theta}), \quad (38)$$

Further on, these coefficients will be assumed to be equal to $\alpha_1 = \alpha_2 = \alpha \geq 1$.

In Eq. (38), the components of the stress tensor in a cylindrical coordinate system are used. Anisotropy analysis of the long-term strength characteristics of metals is described in detail in [6, 7].

Consider the kinetic equation for the components of the damage vector ω_i in the following form:

$$d\omega_i = \omega_i^{-1} \cdot [d\varphi(\hat{\sigma}_i) + f_i(\hat{\sigma}_i)dt], \quad i = z, \theta \quad (39)$$

$\hat{\sigma}_i$ represents the transformed main stresses $\hat{\sigma}_{zz} = \sigma_{zz}/\alpha$, $\hat{\sigma}_{\theta\theta} = \sigma_{\theta\theta}$ ($\alpha > 1$). Simple transformations of Eq. (39) provide the relations for the square of the damage vector length under uniaxial and biaxial stretching, respectively:

$$\omega^2 = 2[\varphi(\sigma_0/\alpha) + f(\sigma_0/\alpha) \cdot t], \quad (40)$$

$$\omega^2 = 2[\varphi(\sigma_0/\alpha) + \varphi(\sigma_0) + f(\sigma_0/\alpha) \cdot t + f(\sigma_0) \cdot t].$$

Eq. (40) allows deriving the following expressions for t_1^* , t_2^* and s :

$$t_1^* = \frac{0.5 - \varphi(\sigma_0/\alpha)}{f(\sigma_0/\alpha)}, \quad t_2^* = \frac{0.5 - \varphi(\sigma_0/\alpha) - \varphi(\sigma_0)}{f(\sigma_0/\alpha) + f(\sigma_0)}, \quad (41)$$

$$s = \frac{t_1^*}{t_2^*} = \frac{[f(\sigma_0/\alpha) + f(\sigma_0)]}{f(\sigma_0/\alpha)} \cdot \frac{[0.5 - \varphi(\sigma_0/\alpha)]}{[0.5 - \varphi(\sigma_0/\alpha) - \varphi(\sigma_0)]}.$$

It is obvious from expression (41) that the value s is greater than 1 for any kinds of functions $f(x)$ and $\varphi(x)$, values $\alpha > 1$ and levels of the stress state σ_0 .

6. Creep and long-term strength in the presence of aggressive medium

Forecasting the life of materials and structural elements residing under conditions of prolonged high-temperature loading in the presence of an aggressive medium is an extremely topical task to ensure reliability throughout the working lifespan. In this section, the influence of an aggressive medium on the creep and the long-term strength of materials and structural elements are considered [6, 7, 15–18]. This influence is determined by the diffusion penetration of the medium elements into the material reducing the duration of operability.

Here, an analysis of the long-term tensile strength of a long, thin rod of a rectangular cross section with thickness H_0 and width b , with $H_0 < b$ residing in an aggressive medium, is presented [17]. Following the accepted geometric dimensions, a one-dimensional diffusion process in a rod along the axis x arranged along the thickness H_0 is considered. Together with the process of the aggressive medium diffusion in the rod, the accumulation of damage in the rod material during the creep process is taken into account. Examine the coupled problem of determining the long-term strength of a tension rod under the condition of mass transfer on its surface. This problem statement takes into consideration a mutual dependence of the concentration level of the medium $c(x, t)$ in the rod material and the amount of accumulated damage ω . To this end, the dependence of the diffusion coefficient D on the level of damage ω is considered. For simplicity, the dependence $D(\omega)$ is assumed to be linear:

$$D(\omega) = D_0(1 + k\omega), \quad D_0 = \text{const}, \quad k = \text{const}. \quad (42)$$

The following dimensionless variables,

$$\bar{t} = \frac{48D_0}{H_0^2}t, \quad \bar{x} = 2x/H_0, \quad \bar{c} = c/c_0, \quad \bar{A} = \frac{A\sigma_0^n H_0^2}{48D_0}, \quad (43)$$

are introduced, where t is the time, c is the concentration, σ_0 is the nominal stress and A and n are the material constants in the power law of creep.

Consider a simplified problem statement where $\omega(\bar{t})$ is understood as an integral mean damage in the rod cross section. The rod fracture criterion is considered as $\omega(\bar{t}^{**}) = 1$.

In this case, the system of equations in the adopted dimensionless variables consisting of the parabolic diffusion equation and the kinetic equation of damage accumulation has the following form:

$$\begin{cases} \frac{\partial \bar{c}}{\partial \bar{t}} = \frac{1}{12}(1 + k\omega) \frac{\partial^2 \bar{c}}{\partial \bar{x}^2}, \\ \frac{d\omega}{d\bar{t}} = \bar{A}(1 - \omega)^{-n} \cdot f(\bar{c}_m(\bar{t})), \end{cases} \quad (44)$$

The linear form of the function $f(\bar{c}_m(\bar{t})) = 1 + a\bar{c}_m(\bar{t})$ is chosen,

where $\bar{c}_m(\bar{t}) \equiv \int_0^1 c(x, t) dx$ is the dimensionless integral mean concentration of the medium in the rod material.

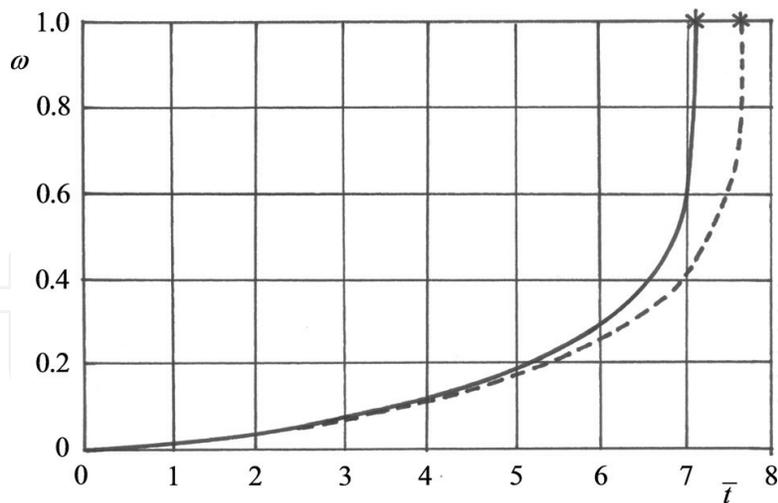


Figure 7. Dependences $\omega(\bar{t})$ for constant and variable diffusion coefficients.

The initial and boundary conditions are assumed to be as follows:

$$\omega(0) = 0, \quad \bar{c}(\bar{x}, 0) = 0, \quad \frac{\partial \bar{c}}{\partial \bar{x}}(1, \bar{t}) = \bar{\gamma}[\bar{c}(1, \bar{t}) - 1], \quad \frac{\partial \bar{c}}{\partial \bar{x}}(0, \bar{t}) = 0, \quad (45)$$

where $\bar{\gamma}$ is the dimensionless mass transfer coefficient.

The following values of the constants are used in calculation

$$n = 3, \quad \bar{\gamma} = 1, \quad k = 4, \quad \bar{A} = 0.01, \quad a = 9.5. \quad (46)$$

The dependence $\omega(\bar{t})$ for constants (46) is shown in **Figure 7** by a solid line, while the time to fracture $\bar{t}^{**} = 7.16$. In **Figure 7**, the dashed line additionally shows the dependence $\omega(t)$ corresponding to the solution of the system of Eq. (44) at $k=0$ ($\bar{t}^{**} = 7.65$). A comparison of the two curves $\omega(t)$ at $k=0$ and at $k=4$ confirms that in the coupled problem ($k>0$), the diffusion coefficient $\bar{D} = (1 + k\omega)$ increases with the increase of the damage so that the concentration level increases at a greater speed and the time to fracture decreases.

7. Conclusion

Analysis of various approaches to simulation of the metal creep phenomenon shows that the most promising is the kinetic theory concept. This concept allows describing the characteristic features of the metal deformation under creep conditions up to the moment of fracture under various temperature-force loading programmes.

Certain above-mentioned advantages of the kinetic theory of creep and long-term strength compared to other theories should be emphasised. As described in this chapter, various versions of the kinetic theory can describe deformation and long-term fracture of metals under step loading. After the introduction of the structural kinetic parameter of damage, the kinetic theory makes it possible to take into account the non-monotonic dependence of the creep strain limit magnitude on the stress. Also, the kinetic theory allows considering the anisotropy

of metal properties in simulating the long-term strength under complex stress state conditions. Moreover, this theory enables the researchers to take into account the influence of an aggressive medium on the creep and long-term strength of metals by introducing the kinetic parameter of aggressive medium concentration in the metal.

Apparently, it is not easy to highlight wide capabilities of the kinetic theory in one chapter. Therefore, this chapter naturally fails to show them all. However, the following promising research directions should be outlined:

1. Vibrocreep of metals under uniaxial and complex stress states
2. Dependence of the long-term strength under conditions of a biaxial stress state on the short-term loading programme
3. Simulation of a long-term fracture of a plate under a nonstationary complex stress state in the presence of an aggressive medium
4. Simulation of the blocking effect of the diffusion process.

Funding

The work was supported by the Russian Foundation for Fundamental Research (project No. 17-08-00210).

Author details

Alexander Lokoshchenko and Leonid Fomin*

*Address all correspondence to: fleonid1975@mail.ru

Research Institute of Mechanics, Lomonosov Moscow State University, Moscow, Russian Federation

References

- [1] Kachanov LM. About time of rupture in the creep condition. Academy of Sciences of the USSR. Department of Tech. Sciences. 1958;(8):26-31. [in Russian]
- [2] Rabotnov YN. On the mechanism of creep rupture. In: Questions of Strength of Materials and Structures. Moscow: Publishing House of the Academy of Sciences of the USSR; 1959, p. 5-7. [in Russian]
- [3] Rabotnov YN. Creep of Structural Elements. Moscow: Nauka; 1966, 752 p. [in Russian]
- [4] Rabotnov YN. Creep Problem in Structural Members. Amsterdam: North Holland; 1969

- [5] Kachanov LM. The Theory of Creep. Moscow: Fizmatgiz; 1960, 456 p. [in Russian]
- [6] Lokoshchenko AM. Creep and Creep Rupture of Metals. Moscow: Fizmatlit; 2016, 504 p. [in Russian]
- [7] Lokoshchenko AM. Cambridge. In: Creep and Creep Rupture of Metals, International Science Publishing. 2017 (in print)
- [8] Lokoshchenko AM. Use of a vector damage parameter in modeling of long-term strength of metals. *Mechanics of Solids*. 2016;**51**(3):315-320. DOI: 10.3103/S0025654416030080
- [9] Sosnin OV, Gorev BV, Nikitenko AF. In: Lavrentieva MA, editor. Energy Version of the Theory of Creep. Novosibirsk: Institute of Hydrodynamics; 1986 96 p
- [10] Vilesova NS, Namestnikov VS. About one hardening parameter. *Journal of Applied Mechanics and Technical Physics*. 1964;(3):43-47. [in Russian]
- [11] Namestnikov VS, Rabotnov Yu N. On the hypothesis of the equation of state for creep. *Journal of Applied Mechanics and Technical Physics*. 1961;(3):101-102. [in Russian]
- [12] Lokoshchenko AM, Shesterikov SA. Model of creep rupture with a non-monotonous dependence of deformation on stress. *Journal of Applied Mechanics and Technical Physics*. 1982;(1):160-163. [in Russian]
- [13] Hayhurst DR. Creep rupture under multi-axial states of stress. *Journal of the Mechanics and Physics of Solids*. 1972;**20**(6):381-390
- [14] Lokoshchenko AM, Myakotin EA, Shesterikov SA. Creep and long-term strength of steel Cr18Ni10Ti under conditions of a complex stress state. *Academy of Sciences of the USSR. Mechanics of Solids*. 1979;(4):87-94. [in Russian]
- [15] Lokoshchenko AM. The application of an approximate analysis of the diffusion process for a description of creep and creep rupture. *International Journal of Mechanical Science*. 2005;**47**:359-373. DOI: 10.1016/j.ijmecsci.2005.01.011
- [16] Fomin LV. Description of creep rupture strength of tensile rod with rectangular and circular cross-section at high temperature air media. *Journal of Samara State Technical University, Series Physical and Mathematical Sciences*. 2013;**3**(32):87-97
- [17] Lokoshchenko AM, Fomin LV. Creep and long-term strength of rods under tension and bending in the presence of an aggressive medium. *Izvestiya MSTU "MAMI". Natural Sciences. Scientific peer-reviewed journal. - M., Moscow State Technical University "MAMI", 2(24), 2015, v. 4. 76 – 85*
- [18] Fomin LV. Steady-state creep of a composite rod in tension in the presence of an aggressive environment. *Mechanics of Composite Materials*. 2017;**52**(6):741-750. DOI: 10.1007/s11029-017-9624-5

