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Chapter 9

Review of Applying European Option Pricing Models

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Abstract

An option is a derivative financial instrument that establishes a contract between two parities concerning the buying or selling of an asset at a reference price. The price of an option derives from the difference between the reference price and the value of the underlying asset plus a premium based on the time remaining until the option. The paper illustrated in both the binomial and the Black-Scholes models, which value options by creating replicating portfolios composed of the underlying asset and riskless leading or borrowing.

Keywords: European options, the binomial option pricing model, the Black-Scholes model, JEL classification: G1, G12, G15, C5

1. Introduction

The goals of the paper present review of option pricing models illustrated in both the binomial and the Black-Scholes models. In the paper, it generally divides option pricing part and part of binomial model and the Black-Scholes model. The first section presents option pricing theory and models. The second section describes the binomial option pricing model. The third section is about the Black-Scholes model. The last section is the conclusion.

2. Option pricing theory

An option provides the holder with the right to buy or sell a specified quantity of an underlying asset at the exercise price on or before the expiration date of the option. Since it is a right and not an obligation, the holder can choose not to exercise the right and allow the option to

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expire. According to that summarizes the options, variables increasing with the effect of call/put option prices:

- Underlying asset's price: increase of call price and decrease of put price.
- Exercise pricing: decrease of call price and increase of put price.
- Variance of underlying asset: both increases of call and put prices.
- Time to expiration: both increases of call and put prices.
- Interest rates: increase of call price and decrease of put price.
- Dividends paid: decrease of call price and increase of put price.

Option pricing theory has made vast strides since 1972, when Black and Scholes published paper "the pricing of options and corporate liabilities" in the Journal of Political Economy. Black and Scholes used a "replicating portfolio": a portfolio constituted the underlying asset and the risk-free asset, and they had the same cash flows as the option being valued of final formulation. However, the mathematical derivation is complicated, although binomial model is simpler for options valuation with same logic.

3. The binomial option pricing model

The binomial option pricing model depends on a simple formulation for the asset price process in any time period can move to one of two possible prices. Suppose an investor focuses estimates on how stock prices change between sub periods, rather than on the dollar levels. That is, beginning with stock price, for the next sub period forecasts:

- first is 1+% change for an up movement(μ);
- second is 1+% change for a down movement(d).

Additional, to the point of accumulation, the number of requisite forecasts. Assume that the same values for up movement and down movement apply to price change in all subsequent sub period. Under these assumptions, the investor need only forecast up movement, down movement, and N-the total number of sub periods.

The binomial option pricing model consists of the forecasted stock price and option value trees. The upper panel presents after μ and d during the first two sub periods, the initial stock price of S will have changed to $(\mu d)S = (d\mu)S$, and it means the forecast does not depend on whether the stock price begins its journey by increasing or decreasing. As before, once μ,d, and N are determined, the expiration date payoffs to the option (i.e.,$c_{\mu\mu\mu}$, $c_{\mu\mu d}$, $c_{\mu dd}$, *and* c_{ddd}) are established.

Hence, the formula for an option in sub period t can be inserted into the right-hand side of the formula for sub period t−1. Carrying this logic all the way back to date 0, the binomial option valuation model becomes

$$c_0 = \left\{ \sum_{j=0}^{N} \frac{N!}{(N-j)!j!} p^j (1-p)^{N-j} max\left[0, (\mu^j d^{N-j})S - X\right] \right\} \div r^N, \tag{1}$$

where $N! = [(N)(N-1)(N-2)\ldots(2)(1)]$ to interpret (1), the ratio $[N! \div (N-j)!\,j!]$ states how many distinct paths lead to a particular terminal outcome, $p^j(1-p)^{N-j}$ states the outcome probability, and $\max[0, (\mu^j d^{N-j})S - X]$ states the payoff. Assume m be the smallest integer number of up movement, the option will be in the money at expiration (i.e., $(\mu^m d^{N-m})S > X$), this formula can be reduced by the following Eq. (2).

$$c_0 = \left\{ \sum_{j=m}^{N} \frac{N!}{(N-j)!j!} p^j (1-p)^{N-j} \left[(\mu^j d^{N-j})S - X\right] \right\} \div r^N. \tag{2}$$

3.1. Illustration 1 valuing an option using the binomial model

SPDR S&P 500 ETF Trust (SPY) is an exchange-traded fund. The trust corresponds to the price and yield performance of the S&P 500 Index. The S&P 500 Index is composed of 500 selected stocks and spans over 24 separate industry groups. The Fund's investment sectors include information technology, financials, energy, health care, consumer staples, industrials, consumer discretionary, materials, utilities, and telecommunication services.

Suppose that the riskless rate r is 10% p.a., the time to maturity T is 0.5 year, the initial price of the underlying asset is 134.76 m.u., the volatility σ = 18.06% p.a., the exercise price X is 70.26 m.u. It is also supposed that the time interval T is split into n = 1000 subintervals of equal length.

Task is to determine the price of the European call option on the basis of the multi period binomial model. Illustrate the probability distribution for both, the underlying asset price and the intrinsic value at maturity T graphically.

Following **Table 1** presents input data of determined parameters.

First step (**Table 2**) to calculate the probabilities for a state j (using the sample of number 30)π_j and stock price for $S_{T,j}$, intrinsic value of the option for $IV_{T,j}$, and product of π_j, $IV_{T,j}$.

Finally, the option price calculated by discounting the mean value of the intrinsic value, the result of option price is 68 m.u. Following **Figure 1** shows illustration depicts that the probability distribution for both, the stock price and the intrinsic value of the option at maturity time.

Risk-less rate, r	Number of steps, n	Time to maturity, T	Initial underlying asset price, S0	Exercise price, X	Volatility, σ	Up-ratio, μ	Down-ratio, d	Probability, p	Discount factor, df
10%	1000	0.5	134.76	70.26	18.06%	1.004046504	0.995969804	0.505181219	0.951230613

Table 1. Determined parameters.

Up movements	Probability	Stock price	Intrinsic value	Productπj, $IV_{T,j}$
0	0.0000	66.9559	0.0000	0.0000
1	0.0000	70.1521	0.0000	0.0000
2	0.0000	73.5008	3.2408	0.0000
3	0.0000	77.0093	6.7493	0.0000
4	0.0000	80.6854	10.4254	0.0001
5	0.0000	84.5369	14.2769	0.0005
6	0.0002	88.5723	18.3123	0.0033
7	0.0007	92.8003	22.5403	0.0155
8	0.0022	97.2301	26.9701	0.0602
9	0.0061	101.8714	31.6114	0.1944
10	0.0146	106.7342	36.4742	0.5310
11	0.0298	111.8292	41.5692	1.2405
12	0.0533	117.1674	46.9074	2.4986
13	0.0832	122.7603	52.5003	4.3654
14	0.1138	128.6203	58.3603	6.6432
15	0.1369	134.7600	64.5000	8.8292
16	0.1447	141.1928	70.9328	10.2625
17	0.1343	147.9326	77.6726	10.4334
18	0.1094	154.9942	84.7342	9.2674
19	0.0779	162.3928	92.1328	7.1749
20	0.0483	170.1447	99.8847	4.8232
21	0.0259	178.2665	108.0065	2.7999
22	0.0120	186.7761	116.5161	1.3931
23	0.0047	195.6918	125.4318	0.5881
24	0.0015	205.0332	134.7732	0.2078
25	0.0004	214.8205	144.5605	0.0603
26	0.0001	225.0749	154.8149	0.0140
27	0.0000	235.8189	165.5589	0.0025
28	0.0000	247.0757	176.8157	0.0003
29	0.0000	258.8699	188.6099	0.0000
30	0.0000	271.2270	200.9670	0.0000
sum	1.00			71.4093

Table 2. Calculation of probabilities, stock price, intrinsic value, and product.

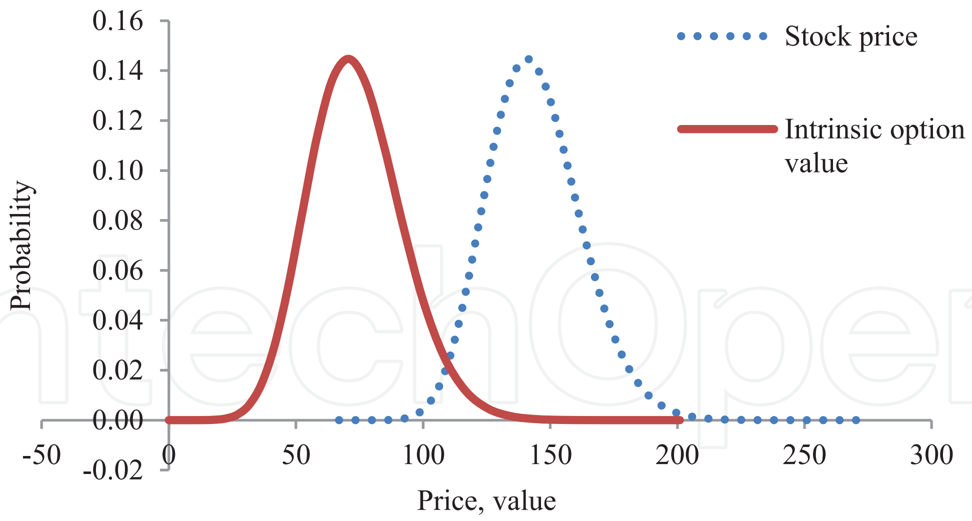

Figure 1. Probability distribution.

4. The Black-Scholes model

The Black-Scholes model assumes that a statistical process known as geometric Brownian motion can describe stock price movements. This statistical process summarized by a volatility factor σ, which is analogous to the investor's stock price forecasts in the previous models. Formally, assumed the Black and Scholes' stock price process is

$$\frac{\Delta S}{S} = \mu[\Delta T] + \sigma \in [\Delta T]^{1/2}. \tag{3}$$

Hence, the equation presents stock's return ($\Delta S/S$) that relates to expected component ($\mu[\Delta T]$) and a "noise" component ($\sigma \in [\Delta T]^{1/2}$) in any future period T. μ is the mean return and $\in$ states the standard normally distributed random error.

Assuming Black and Scholes used the riskless hedge to get the following formula for no dividend-paying stock call option valuation:

$$c_0 = \mathrm{SN}(d_1) - X(e^{-(\mathrm{RFR})T})N(d_2), \tag{4}$$

where $e^{-(RFR)T}$ is the continuously compounded variables discount function.

The variable $N(d)$ represents the cumulative probability, the value from the standard normal distribution $\leq d$. As the standard normal distribution is symmetric around zero, a value of $d = 0$ would lead to $N(d) = 0.5000$:

- positive values of d would then have cumulative probabilities > 50%,
- negative values of d leading to cumulative probabilities < 50%.

The option's value is a function of five variables, there are current security price (S), exercise price (X), time to expiration (T), risk-free rate (RFR), and security price volatility (σ). Hence, the Black-Scholes model holds that $c = f(S, X, T, RFR, \sigma)$. S and RFR are observable market prices, and X and T are defined by the contract itself. Thus, the only variable an investor must provide is the volatility factor.

4.1. Illustration 2 valuing an option using the Black-Scholes model

Suppose that known all parameters that are needed to apply the Black-Scholes model, r, S0, dt, X, and σ. All input data are shown in **Table 3**.

Task is to determine the price of the European call option on the Black-Scholes model.

First step to calculate the prices of options. Following **Table 4** presents the procedure of prices of options.

Following **Table 5** and **Figure 2** describe the result of options prices.

Options	Riskless rate	S0	dt	σ	X
Option 1	0.1	134.76	0.5	18.060%	70.26
Option 2	0.1	134.76	0.5	21.060%	80.26
Option 3	0.1	134.76	0.5	24.060%	90.26
Option 4	0.1	134.76	0.5	27.060%	100.26
Option 5	0.1	134.76	0.5	30.060%	110.26
Option 6	0.1	134.76	0.5	33.060%	120.26
Option 7	0.1	134.76	0.5	36.060%	130.26
Option 8	0.1	134.76	0.5	39.060%	140.26
Option 9	0.1	134.76	0.5	42.060%	150.26
Option 10	0.1	134.76	0.5	45.060%	160.26
Option 11	0.1	134.76	0.5	48.060%	170.26
Option 12	0.1	134.76	0.5	51.060%	180.26
Option 13	0.1	134.76	0.5	54.060%	190.26
Option 14	0.1	134.76	0.5	57.060%	200.26
Option 15	0.1	134.76	0.5	60.060%	210.26
Option 16	0.1	134.76	0.5	63.060%	220.26
Option 17	0.1	134.76	0.5	66.060%	230.26
Option 18	0.1	134.76	0.5	69.060%	240.26
Option 19	0.1	134.76	0.5	72.060%	250.26
Option 20	0.1	134.76	0.5	75.060%	260.26

Table 3. Input data.

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d1	d2	N(d1)	N(−d1)	N(d2)	N(−d2)
2.7777	2.6500	0.9973	0.0027	0.9960	0.0040
1.9451	1.7962	0.9741	0.0259	0.9638	0.0362
1.3674	1.1973	0.9143	0.0857	0.8844	0.1156
0.9513	0.7599	0.8293	0.1707	0.7763	0.2237
0.6428	0.4302	0.7398	0.2602	0.6665	0.3335
0.4089	0.1751	0.6587	0.3413	0.5695	0.4305
0.2284	−0.0266	0.5903	0.4097	0.4894	0.5106
0.0871	−0.1890	0.5347	0.4653	0.4250	0.5750
−0.0246	−0.3220	0.4902	0.5098	0.3737	0.6263
−0.1138	−0.4325	0.4547	0.5453	0.3327	0.6673
−0.1855	−0.5253	0.4264	0.5736	0.2997	0.7003
−0.2434	−0.6044	0.4039	0.5961	0.2728	0.7272
−0.2902	−0.6724	0.3858	0.6142	0.2507	0.7493
−0.3281	−0.7315	0.3714	0.6286	0.2322	0.7678
−0.3587	−0.7834	0.3599	0.6401	0.2167	0.7833
−0.3834	−0.8293	0.3507	0.6493	0.2035	0.7965
−0.4031	−0.8702	0.3434	0.6566	0.1921	0.8079
−0.4188	−0.9071	0.3377	0.6623	0.1822	0.8178
−0.4310	−0.9405	0.3332	0.6668	0.1735	0.8265
−0.4403	−0.9710	0.3299	0.6701	0.1658	0.8342

Table 4. Procedure of prices of options (a).

4.2. Summary: the binomial model vs. the Black-Scholes model

The number of steps affects the option price and the price determined by the binomial model converges to the analytical solution of the Black-Scholes model. Then we will get the options prices to compare with Black-Scholes model and binomial model in different number of steps. It is easy to see that the result of binomial model is around by the continuous time of Black-Scholes model. Following **Figure 3** presents verification of options prices between two models, the results of number of steps will be select by sample.

When the process is continuous, the binomial model for pricing options coverages on the Black-Scholes model. The advantage of the Black-Scholes approach:

- riskless hedge method leads to a relatively simple,
- closed-form equation capable of valuing options accurately under extensive situation.

Options	Call option
Option 1	67.8267
Option 2	57.6927
Option 3	47.2716
Option 4	37.7111
Option 5	29.7951
Option 6	23.6162
Option 7	18.9135
Option 8	15.3523
Option 9	12.6407
Option 10	10.5544
Option 11	8.9299
Option 12	7.6504
Option 13	6.6325
Option 14	5.8168
Option 15	5.1603
Option 16	4.6318
Option 17	4.2081
Option 18	3.8721
Option 19	3.6109
Option 20	3.4145

Table 5. Options prices.

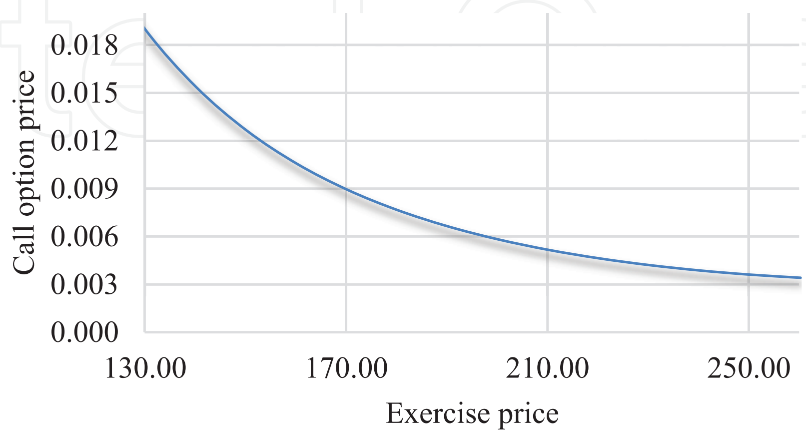

Figure 2. Dependency of a call option price on an exercise price.

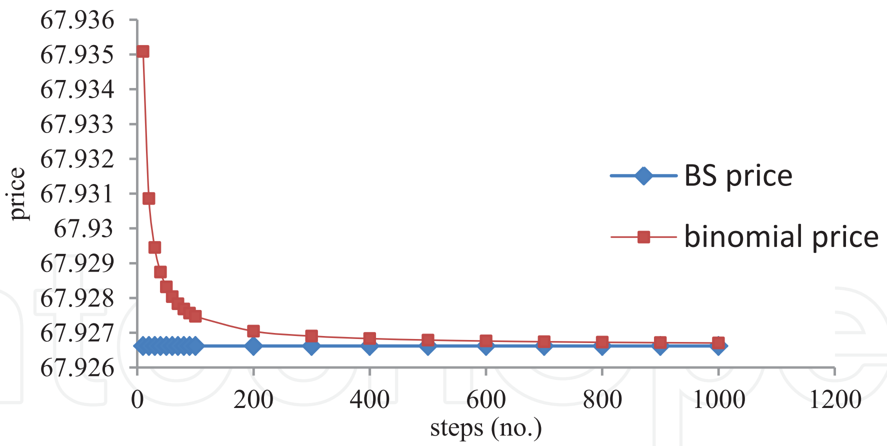

Figure 3. Verification of applying prices of European options using binomial model and Black-Scholes model.

5. Conclusion

This paper presents two classic option pricing model and illustrated the binomial model and the Black-Scholes model based on the same theoretical foundations and assumptions (such as the geometric Brownian motion theory of stock price behavior and risk-neutral valuation). The Black-Scholes option pricing model is the first successful option pricing model, published in 1973, and is based on stochastic calculus. It focuses on the pricing of European options, in which the underlying does not pay a dividend in the option period. The option is priced according to the value of the underlying, the volatility of the value of the underlying, the exercise price, the time to maturity, and the risk-free rate of interest. The model provided a general approach to option pricing and has given rise to a number of other option pricing models. The same underlying assumptions regarding stock prices underpin both the binomial and Black-Scholes models: that stock prices follow a stochastic process described by geometric Brownian motion. As a result, for European options, the binomial model converges on the Black-Scholes formula as the number of binomial calculation steps increases. In fact, the Black-Scholes model for European options is really a special case of the binomial model where the number of binomial steps is infinite.

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