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Network Optimization as a Controllable Dynamic Process

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1. Introduction

One of the basic challenges in the design of large systems is how to reduce time spent to attain the optimal point of the objective function of the design process. The design process itself includes optimization of the structure of the future system, but since this stage is related to an artificial intelligence problem still unresolved, in the general case it is performed "by hand", and thus is absent in the CAD systems. In other words, the traditional approach to computer-aided design consists of two main parts: a model of the system set up in the form of a network described by some algebraic or integro-differential equations, and the parametric optimization procedure – to seek the optimum of the objective function corresponding to the sought characteristics of the system under design.
There are some powerful methods that reduce the necessary time for the circuit analysis. Because a matrix of the large-scale circuit is a very sparse, the special sparse matrix techniques are used successfully for this purpose (Osterby & Zlatev, 1983). Other approach to reduce the amount of computational required for the linear and nonlinear equations is based on the decomposition techniques. The partitioning of a circuit matrix into bordered-block diagonal form can be done by branches tearing as in (Wu, 1976), or by nodes tearing as in (Sangiovanni-Vincentelli et al., 1977) and jointly with direct solution algorithms gives the solution of the problem. The extension of the direct solution methods can be obtained by hierarchical decomposition and macromodel representation (Rabat et al., 1985). Other approach for achieving decomposition at the nonlinear level consists on a special iteration techniques and has been realized for example in (Ruehli et al., 1982; George, 1984) for the iterated timing analysis and circuit simulation. Optimization technique that is used for the circuit optimization and design exert a very strong influence on the total necessary computer time too. The numerical methods are developed both for the unconstrained and for the constrained optimization (Fletcher, 1980; Gill et al., 1981). The practical aspects of use of these methods are developed for VLSI circuit design, yield, timing and area optimization (Brayton et al., 1981; Ruehli, 1987; Massara, 1991). It is possible to suppose that the circuit analysis methods and the optimization procedures will be improved later on. Meanwhile, it is possible to reformulate the total design problem and generalize it to obtain a set of different design strategies. It is clear that a finite but a large number of different strategies

include more possibilities for the selection of one or several design strategies that are time-optimal or quasi-time-optimal ones. This is especially right if we have infinite number of the different design strategies.

The time required for optimization grows rapidly as the system complexity increases. The known measures of reduction of the time for system analysis (in the traditional approach) turned out to be insufficiently advanced.

By convention, the generally accepted ideas of network design will be called the traditional strategy of design, meaning that the method of analysis is based on Kirchhoff's laws. A new formulation of the network optimization problem without strict adherence to Kirchhoff's laws was suggested in (Kashirskiy, 1976; Kashirskiy & Trokhimenko, 1979). This process was called the generalized optimization and used the idea of ignoring Kirchhoff's laws for the whole network or some part of it. In this case, apart from minimization of the previously defined objective function, we also had to minimize the residual of the equation system describing the network model. In the extreme case, when the residual function included all equations of the network mathematical model, this idea was practically implemented in two CAD systems (Rizzoli et al., 1990; Ochotta et al., 1996). The authors of these works asserted that overall time of design was reduced considerably. This latter idea may be termed the modified traditional design strategy. As distinct from the traditional approach proper, including network model analysis at every step of the optimization procedure, the modified traditional strategy of deign may be defined as a strategy which does not include at all the model analysis in the process of optimization.

Another formulation of the network design problem based on the idea proposed in (Zemliak, 2001) can be introduced by generalization and formulation of this idea to obtain a set of different design strategies. Here we may pass to the problem of selecting, among this set, a strategy optimal in some sense – for instance, from the running-time viewpoint. Then the optimal strategy of design may be defined as a strategy permitting us to reach the optimal point of the objective function in minimal time. The main issue in this definition is what conditions have to be fulfilled to construct the algorithm providing for the optimal time. The answer to this question will make it possible to reduce substantially the computer time necessary for the design.

2. Problem formulation

By the traditional design strategy we mean the problem of design of an analogue network with a given topology based on the process of unconditional minimization of an objective function $C(X)$ in a space R^K, where *K* is the number of independent variables. Simultaneously, we are seeking the solution to a system of *M* dependent on some components of the vector *X*. It is assumed that the physical model can be described by a system of nonlinear algebraic equation:

$$g_j(X)=0, \quad j = 1,2,\ldots, M \tag{1}$$

The vector $X \in R^N$ is broken into two parts: $X = (X', X'')$, where the vector $X' \in R^K$ is the vector of independent variables, the vector $X'' \in R^M$, is the vector of dependent variables and $N = K + M$. This partition into independent and dependent variables is a matter of convention, because any parameter may be considered independent or dependent. Due to such definition, some parameters of the design process, for example, frequency, temperature, etc., are beyond our consideration. We can easily include them in the general design procedure, but here we presume them to be constant and include them the coefficients of system (1).

In the general case, the process of minimization of the objective function $C(X)$ in the space R^K of independent variables for the two-step procedure can be described by the following vector equation:

$$X'^{s+1} = X'^{s} + t_s \cdot H^s \tag{2}$$

where *s* is the iterations number, t_s is an iteration parameter, $t_s \in R^1$, and *H* is a function establishing the direction of lowering the objective function $C(X)$. The constraints for independent variables can be bypassed easily, which is shown in the examples given in the second part of this work.

A particular feature of the design process, at least for electronic network applications, is that we are not obliged to fulfill conditions (1) at every step of the optimization procedure. It is sufficient to satisfy these conditions at the final point of the design process. In this event the vector function H depends on the objective function $C(X)$ and on some additional penalty function $\varphi(X)$, whose structure includes all the equations of system (1) and can be defined, for instance, as:

$$\varphi\left(X^s\right) = \frac{1}{\varepsilon} \sum_{j=1}^{M} g_j^2\left(X^s\right) \tag{3}$$

In this case we define the design process as an unconditional optimization problem:

$$X^{s+1} = X^s + t_s \cdot H^s \tag{4}$$

in the space R^N without any additional system of constraints, but for a new objective function $F(X)$, which can be defined, for instance, as an additive function:

$$F(X) = C(X) + \varphi(X) \tag{5}$$

Then at the point of minimum of the objective function $F(X)$ we also have the minimum of the objective function $C(X)$, and system (1) is satisfied at the final point of the optimization process. This method can be called the modified traditional method of design: it reproduces a different strategy of design and a different trajectory in the space R^N.

On the other hand, we can generalize the idea of using of an additional penalty function, if the penalty function is formed only from a part of system (1) while the remaining part is regarded as a system of constraints. In this event the penalty function includes, for example, only Z first terms of $\varphi\left(X^{s}\right)=\frac{1}{\varepsilon}\sum_{i=1}^{Z} g_i^2\left(X^{s}\right)$, where $Z\in\left[0, M\right]$ and other M - Z equations form, instead of (1), a modified system make up one modification of the system (1):

$$g_j(X)=0,\quad j=Z+1, Z+2,\ldots, M \tag{6}$$

Obviously, every new value of the parameter Z generates a new design strategy and a new trajectory. This notion can be easily extended to a situation when the penalty function $\varphi\left(X\right)$ includes Z arbitrary equations of system (1). The overall number of different design strategies in this case equals 2^{M}. All these strategies exist within the same optimization procedure. The optimization procedure is realized in the space R^{K+Z}. The number of dependent parameters M grows together with complexity of the system while the number of different design strategies grows by exponential law. These strategies are characterized by different numbers of operations and different overall running time. Accordingly, we may formulate the problem of searching for the design strategy optimal in time, i.e., having a minimum running time of the processor.

Let us estimate the number of operations for several design strategies. The traditional design strategy includes two systems of equations. To be specific, assume that the optimization procedure is based on a gradient method and can be defined by a system of ordinary differential equations for independent variables in the form

$$\frac{dx_i}{dt}=-b\cdot\frac{\delta}{\delta x_i}C(X),\quad i=1,2,\ldots,K \tag{7}$$

where b is the iterative parameter. The operator $\frac{\delta}{\delta x_i}$ means that

$$\frac{\delta}{\delta x_i}\varphi\left(X\right)=\frac{\partial\varphi\left(X\right)}{\partial x_i}+\sum_{p=K+1}^{K+M}\frac{\partial\varphi\left(X\right)}{\partial x_p}\frac{\partial x_p}{\partial x_i}.$$

Nevertheless, the use of the gradient method does not mean loss of generality of the results obtained. It is necessary only that we represent the optimization process as a system of ordinary differential equations for independent variables. The mathematical model of an electronic system in this case is a system of constraints and is described by equation (1). The number of operations for solution of system (1) by Newton's method will be $S\cdot\left[M^{3}+M^{2}(1+P)+MP\right]$, where P is the average number of operations for calculation of $g_j(X)$, and S is the number of iterations in Newton's method for resolving system (1). The number of operations in a single step of integration of system (7) by Newton's method is $K + C \cdot (1+K) + (1+K) \cdot S \cdot \left[M^3 + M^2(1+P) + MP \right]$, where C is the number of operations for calculation of the objective function. The overall number of operations for resolving the problem (1) and (7) by Newton's method, i.e.,

$$N_1 = L_1 \left\{ K + (1+K) \left\{ C + S \cdot \left[M^3 + M^2(1+P) + MP \right] \right\} \right\} \tag{8}$$

where L_1 is the overall number of steps in the optimization algorithm.

The modified traditional strategy of design is fully defined by the equation system of the optimization procedure without any additional limitations. In this case the number of independent variables equals $K+M$. The fundamental system has the form

$$\frac{dx_i}{dt} = -b \cdot \frac{\delta}{\delta x_i} F(X), \; i = 1,2,\ldots,K+M \tag{9}$$

where $F(X)$ is the generalized objective function: $F(X) = C(X) + \frac{1}{\varepsilon} \sum_{j=1}^{M} g_j^2(X) \cdot$

The overall number of operations for resolving (9) is

$$N_2 = L_2 \left\{ K + M + (1+K+M) \cdot \left[C + (P+1) M \right] \right\} \tag{10}$$

A more general strategy of design can be defined as a strategy having a variable number of independent parameters equal to $K+Z$. Here we use two systems of equations, (6) and (11):

$$\frac{dx_i}{dt} = -b \cdot \frac{\delta}{\delta x_i} F(X), \; i = 1,2,\ldots,K+Z \tag{11}$$

where $F(X) = C(X) + \frac{1}{\varepsilon} \sum_{j=1}^{Z} g_j^2(X) \cdot$

Then the overall number of operations N_3 for resolution of systems (6) and (11) can be evaluated as:

$$N_3 = L_3 \{ K + Z + (1+K+Z) \{ C + (P+1) Z + S \cdot [(M-Z)^3 + (M-Z)^2 (1+P) + (M-Z) P] \} \} \tag{12}$$

This formula turns into (8) if *Z=0*, and into (10) when *Z=M*. Analysis of the number of operations N_3 as a function of the parameter *Z* permits us to find the conditions for defining the strategy characterized by a minimum running time. For linear system (1), in the Newton's method the number of iterations *S*=1, and the traditional approach is optimal, but for a nonlinear system this is not the case. Also, we assume that the iterations number L_3 and the number of operations *C* for objective function calculation depend on the number of independent parameters as $L_3 = L_0 \cdot (K+Z)^n$, $C = C_0 \cdot (K+Z)^m$. This assumption may be considered trivial, but the main difficulty consists in indeterminacy of the powers *n* and *m*. On the other hand, the number of iterations *S* in Newton's method does not depend, to a first approximation, on the order of system (12), and represents some constant S_0. In practice, to obtain the accuracy $\delta = 10^{-10} - 10^{-12}$, this constant value is within 4 - 5. The average number of operations *P* for calculation of the function $g_j(X)$ is invariant to Z in the case of analysis of an electronic system. This is true, because the conductance matrix of an electronic network is sparse. We assume that this number of operations is constant and equal to P_0. Then expression (12) for calculation of the function $N_3(Z)$ can be reduced to the form

$$\begin{aligned} N_3(Z) = L_0 \cdot (K+Z)^n \cdot \{K+Z+(1+K+Z)[C_0 \cdot (K+Z)^m + Z(1+P_0)] \\ + S_0 \cdot ((M-Z)^3 + (M-Z)^2(1+P_0) + (M-Z)P_0)]\} \end{aligned} \tag{13}$$

In conformity with the fundamental definition of optimal design strategy, we can find this strategy from analysis of this formula. We have to find the optimum point Z_{opt}, in which the function $N_3(Z)$ has a minimum value. If the case of Z_{opt} *=0*, the traditional strategy is optimal. If Z_{opt} *=M*, the modified traditional strategy is optimum. If Z_{opt} is confined within (*0, M*) interval, then some intermediate strategy is optimal. The derivative of the function $N_3(Z)$ defined by equation (13) is defined by the formula:

$$\begin{aligned} N_3'(Z) = L_0 n(K+Z)^{n-1}\{K+Z+(1+K+Z)[C_0(K+M)^m + Z(1+P_0) + S_0((M-Z)^3 + (M-Z)^2(1+P_0) \\ + (M-Z)P_0)]\} + L_0(K+Z)^n\{1+C_0(K+M)^m + (1+K+2Z)(1+P_0) + S_0[(M-Z)^3 \\ + (M-Z)^2(1+P_0) + (M-Z)P_0 - (1+K+Z)(3(M-Z)^2 + 2(M-Z)(1+P_0) + P_0)]\} \end{aligned}$$

To ensure that the optimal point lies within the [*0, M*] interval, it is necessary and sufficient to fulfill the following two conditions for the derivative at the interval boundaries: $N_3'(0) < 0$ and $N_3'(M) > 0$. It is expedient to introduce an additional parameter $q = \dfrac{M}{K}$. Then the value of derivative $N_3'(0)$, under the conditions *m*=1 and $M, K \to \infty$, can be calculated by the formula $N_3'(0) = L_0 K^{n+1} M^2 S_0 \left[(1+n)q - 3 \right]$. We have to impose a special condition for the parameter n to meet the inequality $N_3'(0) < 0$. This condition is set by the formula $n < \frac{3}{q} - 1$. In the majority of systems, $q \le 1$. In this case, for the parameter n the condition is set in the form $n < 2 + \varepsilon$. On the other hand, the derivative $N_3'(Z)$ in the point *Z=M*, under the condition $M, K \to \infty$ has the form:

$$N_3'(M) = L_0 (K+M)^{n+1} \left[C_0(1+n) + \frac{(1+K+2M+nM)(1+P_0)}{K+M} - S_0 P_0 \right].$$

Provided that *n=2,* the inequality $N_3'(M) > 0$ provides the condition $3C_0 + \frac{1+4q}{1+q}\left(1+P_0\right) - S_0 P_0 > 0$. When $q \to 1$ and $C_0 \approx P_0$, this formula turns into $P_0\left(5.5 - S_0\right) + 2.5 > 0$. If *n=1,* then the condition $P_0\left(4 - S_0\right) + 2 > 0$ is valid. The condition $N_3'(M) > 0$ can be fulfilled if the number of iterations S_0 equals 4 or 5. Consequently, in this case the optimal point Z_{opt} lies within [*0, M*] interval.

3. Problem formulation by control theory approach

The most general approach to the problem of construction of the optimal design algorithm can be worked out based on the optimal control theory. We can define the design strategy with the aid of equations (4) and (6) with the variable parameter Z during the whole process of optimization. It means that we may change the number of independent variables and the number of terms in the penalty function formula at every point of optimization procedure. Also, it is worth introducing into our consideration a vector of control functions $U = \left(u_1, u_2, ..., u_m\right)$, where $u_j \in \Omega$ and $\Omega = \{0;1\}$. In other words, every control function u_j can take the value 0 or 1. These functions have the meaning of control functions of the design process and generalize this process. Particularly, the meaning of the control function u_j is as follows: the equation with the ordinal number j belongs to system (6), while the term $g_j^2(X)$ is eliminated from the right-hand part of formula (3) if $u_j = 0$, and vice versa - the *j*-th equation is excluded from system (6) and the respective term appears in the right-hand part of formula (3) if $u_j = 1$. Then the system model equations and the type of the penalty function can be rewritten in the form:

$$\left(1 - u_j\right) g_j(X) = 0, \quad j = 1, 2, \ldots, M \tag{14}$$

$$\varphi(X)=\frac{1}{\varepsilon}\sum_{j=1}^{M}u_j\cdot g_j^2(X) \tag{15}$$

All the control functions u_j are functions of a current point of the design process. In this case the directed motion vector $H=f(X,U)$ is the function of the vectors *X* and *U*. The number of various design strategies generated within a single optimization procedure is virtually unlimited. Among all these strategies, there are one or several strategies, which are optimal and accomplish all the goals of design in a minimum possible time. Hence, the problem of search for the optimal strategy is now formulated as a typical problem of minimization of some functional in the optimal control theory. The functional value represents the actual running time of the processor. The main difficulty of such definition consists in unknown optimal dependencies of the control functions u_j. However, if we have an optimal vector of control functions, the optimal design strategy will be realized with the aid of this vector.

The idea of formulation of optimal design of a system from the viewpoint of time as a problem of minimization of a functional invoked from the optimal control theory, does not depend on some specific realization of the optimization algorithm, and can be embedded into an arbitrary optimization procedure. All this has been shown in (Zemliak, 2001), with approbation of three different algorithms, which are typical representatives of three major groups of optimization methods: the gradient method, the Newton's method and the Davidon-Fletcher-Powell method (DFP).

Now the process of network optimization is formulated as a controllable dynamic process. We have possibility to control the design process by means of the control vector *U* variation. In the above formulation, every possible design strategy, defined by the vector *U*, has its own trajectory in the space of variables. Obviously, the comparison of different trajectories by the time of moving over them, or by some other parameters, is consistent only in situation when these trajectories have identical initial and final points. On the other hand, the objective function $C(X)$ has a number of local minima, since design problems are nonlinear in principle, even if the design concerns a physically linear system. In this case, for consistent comparison of different strategies and their trajectories, it is desirable to impose additional conditions of single-valuedness for attainment of one and the same point in the space parameters. At the same time, the problem of ambiguity is not a peculiar feature of the new formulation of the design methodology. We face this problem every time when starting the design process from different initial points. In future, both in theoretical reasoning and in practical examples, we shall assume that the problem of single-valuedness of the final point is overcome by imposing some additional conditions on the variables. It should be specially stressed that this problem is essential only in comparison of different strategies and their trajectories, while in actual design we do not need any additional conditions except for feasibility requirements.

The process of system design, formulated in the terms of the control theory, can be described either in discrete or in continuous form. The continuous form is traditional for the optimal control theory. To represent the problem in the continuous form, we assume that numerical equation (2) corresponding to the optimization process can be replaced by a differential equation:

$$\frac{dX}{dt} = f(X,U) \tag{16}$$

where the right-hand part $f(X,U)$ is the vector of directed motion of H and depends on the generalized objective function $F(X,U)$.

What this means is that the design process is formulated as the problem of integration of system (16) with additional conditions (14). The structure of the function $f(X,U)$ can be defined as follows:

$$f(F(X,U)) = -F'(X,U) \tag{17}$$

- for the gradient method,

$$f(F(X,U)) = -\{F''(X,U)\}^{-1} \cdot F'(X,U) \qquad (17')$$

- for the Newton method, where $F''(X,U)$ is a matrix of second derivatives,

$$f(F(X,U)) = -B(X,U) \cdot F'(X,U) \qquad (17'')$$

- for the DFP method, where $B(X,U)$ is a symmetric, positive definite matrix of the DFP algorithm.

In this case the problem of construction of the optimal, in terms of running time, design algorithm is formulated as a typical problem of functional minimization of the control theory for differential equation system (16). The right-hand part of (16) depends on the particular method of optimization, for instance, (17), (17´), or (17´´), and has an objective function defined by formulas (5), (15) and constraints (14). An additional difficulty is that the right-hand parts of system (16) are piecewise continuous functions rather than strictly continuous. Such a problem for system (16) with piecewise continuous control functions can be resolved most effectively based on the known maximum principle (Pontryagin et al., 1962), but straightforward application of this principle for nonlinear problems of large dimensionality is highly problematical. This problem can be resolved based on provisions developed in the process of approximate solution of control theory problems (Fedorenko, 1969; Pytlak, 1999).
The fundamental system of equations of design process for the three algorithms of optimization is given below. System (16), which can be rewritten in component-wise form as

$$\frac{dx_i}{dt} = f_i(X,U), \quad i = 1,2,\ldots,N \tag{18}$$

in combination with (14) defines the design process.

In the case of gradient method, the right-hand part of (18), i.e.,

$$f_i\left(X,U\right)=-\frac{\delta}{\delta x_i}F\left(X,U\right), \qquad i=1,2,\ldots,K \tag{19}$$

$$f_i\left(X,U\right)=-u_{i-K}\frac{\delta}{\delta x_i}F\left(X,U\right)+\frac{\left(1-u_{i-K}\right)}{t_s}\left\{-x_i^s+\eta_i\left(X\right)\right\}, \quad i=K+1,K+2,\ldots,N \tag{19´}$$

where $F\left(X,U\right)=C\left(X\right)+\frac{1}{\varepsilon}\sum_{j=1}^{M}u_j g_j^2\left(X\right)$. The function $\eta_i\left(X\right)$, written in an implicit form, defines the current value of the variable x_i^{s+1} $\left(x_i^{s+1}=\eta_i\left(X\right)\right)$ obtained after resolving the system (14), and the control variables u_j are the functions of "current time".

In the case of Newton's method or DFP, equations (19) and (19´) undergo some modifications:

$$f_i\left(X,U\right)=-\sum_{k=1}^{N}b_{ik}\frac{\delta}{\delta x_k}F\left(X,U\right), \quad i=1,2,\ldots,K$$

$$f_i\left(X,U\right)=-u_{i-K}\sum_{k=1}^{N}b_{ik}\frac{\delta}{\delta x_k}F\left(X,U\right)+\frac{\left(1-u_{i-K}\right)}{t_s}\left\{-x_i^s+\eta_i\left(X\right)\right\}, \quad i=K+1,K+2,\ldots,N$$

where b_{ik} is an element of the inverse Hessian $\left\{F''\left(X,U\right)\right\}^{-1}$ for Newton's method, or an element of the matrix *B(X,U)* in DFP method.

In the latter case the matrix *B(X,U)* is defined by expressions $B_{s+1}=B_s+\frac{R^s\left(R^s\right)^T}{\left(R^s\right)^T Q^s}-\frac{\left(B_sQ^s\right)\left(B_sQ^s\right)^T}{\left(Q^s\right)^T B_sQ^s}$, where B_0 is the unitary matrix, $s=0,1,\ldots$, while $R^s=X^{s+1}-X^s$ and $Q^s=F'\left(X^{s+1},U^s\right)-F'\left(X^s,U^s\right)$.

4. Numerical results

Numerical results in conformity with the new approach to formulation of the design process are presented below. They point to the prospects arising in construction of the optimal (in terms of minimum running time) algorithm. The primary emphasis is placed on demonstration of new opportunities appearing due to application of the new methodology. The number of nodes in the networks taken for illustrations in this part varies from 3 to 5. We deal with the problem of dc analysis, where the objective function $C\left(X\right)$ is defined as the sum of squared differences between the preset and current values of nodal voltages for some

nodes, supplemented with additional inequalities for some elements of the network. The calculations presented below correspond to the different optimization methods: the gradient method, the Newton's method, and the Davidon-Fletcher-Powell method (DFP).
The basic system of equations (16) was integrated by the fourth-order Runge-Kutta method. The integration step was variable and optimal for every new strategy to minimize the processor running time. The processor operation time indicated in the calculations corresponds to a computer with the processor Pentium 4, 2.2 GHz.
Fig. 1 shows the equivalent circuit of the network to be designed. The circuit has four independent variables (*K*=4), conductances y_1, y_2, y_3, y_4, three dependent variables (*M*=3), nodal voltages V_1, V_2, V_3, and two nonlinear elements.

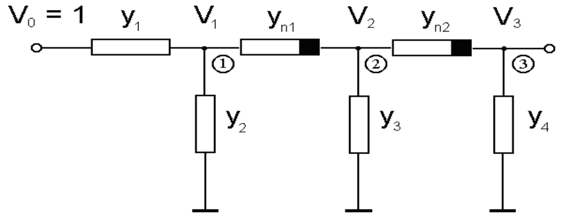


Fig. 1. Topology of three-node network

The nonlinear elements are defined as follows: $y_{n1} = a_{n1} + b_{n1}(V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2}(V_2 - V_3)^2$. The nonlinearity parameters are $b_{n1} = b_{n2} = 1$. The components of the vector *X* are defined by formulas $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4^2 = y_4$, $x_5 = V_1$, $x_6 = V_2$, $x_7 = V_3$. Defining the components x_1, x_2, x_3, x_4 by the above formulas automatically results in positive magnitudes of the conductances, which eliminates the issue of positive definiteness for each resistance and conductance and make it possible to carry out the optimization in the whole space of magnitudes of these variables without any limitations.

In this case we have a system of seven equations playing the role of the optimization algorithm, while the network model can be expressed by three nonlinear equations:

$$\frac{dx_i}{dt} = -\frac{\delta}{\delta x_i} F(X,U), \qquad i = 1,2,3,4 \tag{20}$$

$$\frac{dx_i}{dt} = -u_{i-4} \cdot \frac{\delta}{\delta x_i} F(X,U) + \frac{(1-u_{i-4})}{dt}\{-x_i(t-dt) + \eta_i(X)\}, \; i = 5,6,7$$

where $F(X,U) = C(X) + \sum_{j=1}^{3} u_j g_j^2(X)$,

$$
\begin{aligned}
&g_1(X) \equiv \left(x_1^2 + x_2^2 + a_{n1} + b_{n1}x_6^2\right)x_5 - \left(a_{n1} + b_{n1}x_6^2\right)x_6 - x_1^2 = 0 \\
&g_2(X) \equiv -\left(a_{n1} + b_{n1}x_6^2\right)x_5 + \left(x_3^2 + a_{n1} + b_{n1}x_6^2 + a_{n2} + b_{n2}x_7^2\right)x_6 - \left(a_{n2} + b_{n2}x_7^2\right)x_7 = 0 \qquad (21) \\
&g_3(X) \equiv -\left(a_{n2} + b_{n2}x_7^2\right)x_6 + \left(x_4^2 + a_{n2} + b_{n2}x_7^2\right)x_7 = 0
\end{aligned}
$$

The system (21) can be transformed into the following one:

$$
\left(1 - u_j\right)g_j\left(x_1, x_2, x_3, x_4, x_5, x_6, x_7\right) = 0 \qquad j = 1,2,3.
$$

The results of analysis of the full structural basis of design strategies, arising in this case at a fixed value of the controlling vector *U*, are given in Table 1.

N	Control functions vector U (u1, u2, u3)	Gradient method		Newton method		DFP method	
		Iterations number	Total design time (sec)	Iterations number	Total design time (sec)	Iterations number	Total design time (sec)
1	(0 0 0)	59	0.229	8	0.1331	12	0.0853
2	(0 0 1)	167	0.273	6	0.0637	17	0.0552
3	(0 1 0)	174	0.291	11	0.1193	15	0.0503
4	(0 1 1)	185	0.154	11	0.0758	22	0.0421
5	(1 0 0)	63	0.122	8	0.0986	22	0.0839
6	(1 0 1)	198	0.245	9	0.0905	19	0.0499
7	(1 1 0)	228	0.258	15	0.1382	18	0.0442
8	(1 1 1)	293	0.176	17	0.0853	33	0.0504

Table 1. Complete structural basis of design strategies for three- nodes passive network

Here we have 8 different strategies. The strategy corresponding to the control vector (0, 0, 0) is a traditional design strategy (TDS); the strategy with the control vector (1, 1, 1) is a modified traditional design strategy (MTDS) while the rest are some new intermediate strategies. Strategy 5 is optimal for the gradient method, strategy 2 is optimal for Newton's method and strategy 4 is optimal for the DFP method. Nevertheless, these strategies are not optimal in the whole. The optimal strategies for all methods have been found due to application of a special procedure – by variation of the control vector. The respective results are presented in Table 2.

N	Method	Optimal control functions vector U (u1, u2, u3)	Iterations number	Switching points	Total design time (sec)
1	Gradient method	(101); (000); (111)	81	3; 7	0.0636
2	Newton method	(111); (000); (011)	6	1; 2	0.0492
3	DFP method	(101); (011)	15	2	0.0301

Table 2. Characteristics of optimal design strategy for three optimization methods

The optimal strategy has two points of "switching" for gradient and Newton's methods and one "switching" point for DFP optimization methods. The gain in time for the optimal strategy, as compared with the traditional one, is 3.6 times for the gradient method, 2.7 for Newton's method and 2.83 times for the DFP method.

In the second case we analyze a network (Fig. 2) with five independent variables (*K*=5), conductances y_1, y_2, y_3, y_4, y_5, four dependent variables (*M*=4), nodal voltages V_1, V_2, V_3, V_4, and two nonlinear elements.

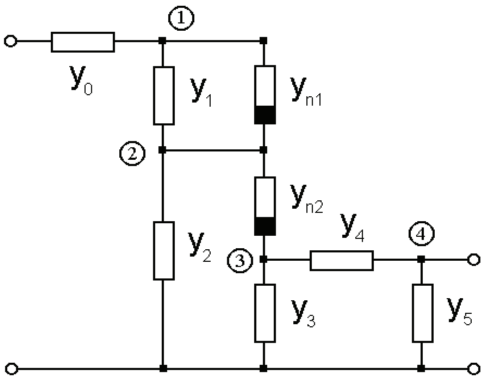


Fig. 2. Topology of four-node network

The nonlinear elements are defined by formulas $y_{n1} = a_{n1} + b_{n1} \cdot (V_1 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_2 - V_3)^2$. System (16) includes nine equations, and system's model includes four equations. The results of analysis of the full set of strategies are presented in Table 3.

N	Control functions vector U (u1,u2,u3,u4)	Gradient method		Newton method		DFP method	
		Iterations number	Total design time (sec)	Iterations number	Total design time (sec)	Iterations number	Total design time (sec)
1	(0 0 0 0)	114	0.819	10	0.366	15	0.187
2	(0 0 0 1)	87	0.388	8	0.251	26	0.207
3	(0 0 1 0)	51	0.269	10	0.361	11	0.103
4	(0 0 1 1)	77	0.176	8	0.171	29	0.129
5	(0 1 0 0)	100	0.526	7	0.251	12	0.112
6	(0 1 0 1)	217	0.486	9	0.189	14	0.061
7	(0 1 1 0)	166	0.338	14	0.273	19	0.076
8	(0 1 1 1)	402	0.596	11	0.182	34	0.104
9	(1 0 0 0)	111	0.586	5	0.181	11	0.103
10	(1 0 0 1)	73	0.166	7	0.149	14	0.062
11	(1 0 1 0)	115	0.256	8	0.167	16	0.104
12	(1 0 1 1)	135	0.225	10	0.184	20	0.068
13	(1 1 0 0)	519	1.055	13	0.253	14	0.056
14	(1 1 0 1)	595	0.882	10	0.166	18	0.055
15	(1 1 1 0)	159	0.169	15	0.185	20	0.046
16	(1 1 1 1)	330	0.276	24	0.234	51	0.101

Table 3. Complete set of design strategies for four- nodes passive network

There are 16 different design strategies for this case. Of interest is the fact that among all the diversity of the design strategies, for the gradient method we have thirteen strategies, for Newton's method – 15, and for DFP – 14 strategies, whose processor time is less than that of traditional strategy 1 with the vector of controlling functions such as (0, 0, 0, 0). Among the whole stock of the strategies, the optimal one is strategy 10 for the gradient method, which provides a gain in time by a factor of 4.93 – compared to the traditional strategy, strategy 10 for Newton's method with a gain by 2.46, and strategy 14 – for the DFP method, with a gain by 4.06 times.

The optimal or, to be more exact, quasi-optimal strategies have been found by using a special procedure, whose application results are given in Table 4.

N	Method	Optimal control functions vector U (u1, u2, u3, u4)	Iterations number	Switching points	Total design time (sec)
1	Gradient method	(1011); (0000); (1110)	53	3; 4	0.0644
2	Newton method	(1010); (1001)	6	3	0.1265
3	DFP method	(0111); (1110)	17	2	0.0405

Table 4. Characteristics of quasi-optimal design strategy for three optimization methods

Here we have two switching point for the gradient method and one – for Newton's method and DFP method. These quasi-optimal strategies provide a gain in time by a factor of 12.7 for the gradient method, 2.89 for Newton's method, and 4.6 for DFP method.

In the next case we analyzed an electronic network of an amplifier in two transistors (Fig. 3). The design process was performed for dc conditions. The transistors are represented by the steady-state Ebers-Moll model embedded in the SPICE system (Massobrio & Antognetti, 1993). The objective function was defined as the sum of squared differences between present and current values of voltages across transistor's junctions.

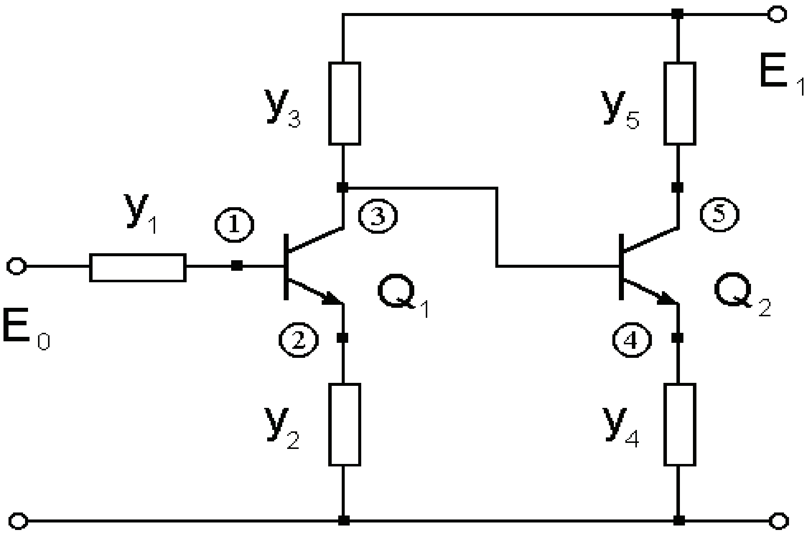


Fig. 3. Circuit topology for two-stage transistor amplifier

For this example we establish five independent variables y_1, y_2, y_3, y_4, y_5 (*K*=5), and five dependent variables V_1, V_2, V_3, V_4, V_5 (*M*=5). In this case the optimization algorithm is based on a system of ten equations, and the network model is defined by five nonlinear equations. The full set of design strategies includes 32 different strategies. Table 5 contains the results of analysis of the TDS, and some other strategies among those requiring less time for implementation than the traditional one. Strategy 17 is optimal for gradient method and strategy 13 is optimal for DFP method. It makes possible to obtain a gain in time, as compared to the traditional one, by 19.7 times for the gradient method, and by 26.2 times for the DFP method. However, as in the previous examples, these strategies are not "optimal-in-whole". The results of quasi-optimal strategies are listed in Table 6.

N	Control functions vector U (u1,u2,u3,u4,u5)	Gradient method		DFP method	
		Iterations number	Total design time (sec)	Iterations number	Total design time (sec)
1	(0 0 0 0 0)	979	20.466	274	7.361
2	(0 0 1 0 0)	889	15.396	196	6.381
3	(0 0 1 0 1)	704	9.571	57	1.474
4	(0 0 1 1 0)	989	13.455	251	5.818
5	(0 0 1 1 1)	512	5.405	154	2.986
6	(0 1 1 0 0)	859	11.861	218	5.631
7	(0 1 1 0 1)	420	4.503	125	2.522
8	(0 1 1 1 0)	751	8.011	129	2.591
9	(0 1 1 1 1)	528	4.228	90	1.371
10	(1 0 1 0 0)	780	10.745	199	2.956
11	(1 0 1 0 1)	249	1.734	62	0.462
12	(1 0 1 1 0)	1253	13.297	135	1.545
13	(1 0 1 1 1)	386	2.161	30	0.281
14	(1 1 1 0 0)	1683	17.702	105	1.187
15	(1 1 1 0 1)	263	1.471	56	1.769
16	(1 1 1 1 0)	1191	9.459	77	0.656
17	(1 1 1 1 1)	637	1.039	65	0.296

Table 5. Some design strategies for two-stage transistor amplifier

N	Method	Optimal control functions vector U (u1, u2, u3, u4, u5)	Iterations number	Switching points	Total design time (sec)	Computer time gain
1	Gradient method	(11101); (11111)	220	11	0.403	50.8
2	DFP method	(10111); (11111)	37	11	0.157	46.9

Table 6. Characteristics of quasi-optimal design strategy for two-stage transistor amplifier

The optimal strategy in both cases has a single switching point. The gain in time, as compared to the traditional strategy, in this case is 50.8 times for the gradient method and 46.9 times for DFP method.
The results of analyzed passive circuits for M from 1 to 5 are presented in Fig. 4 for three different optimization procedures.

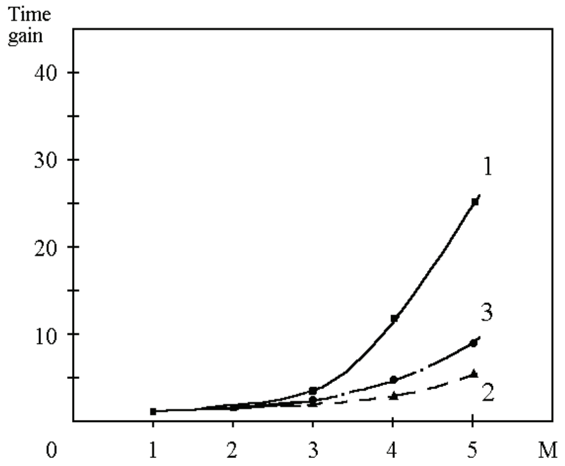


Fig. 4. Computer time gain for passive networks

This is the computer time gain of the optimum design strategy with respect to the traditional design strategy as the function of the dependent parameters' number M. The traditional design approach is not time-optimal and the time gain increases very fast with the M increasing.
The results of the active networks optimization for different number of transistor cells N_{TR} are shown in Fig. 5.

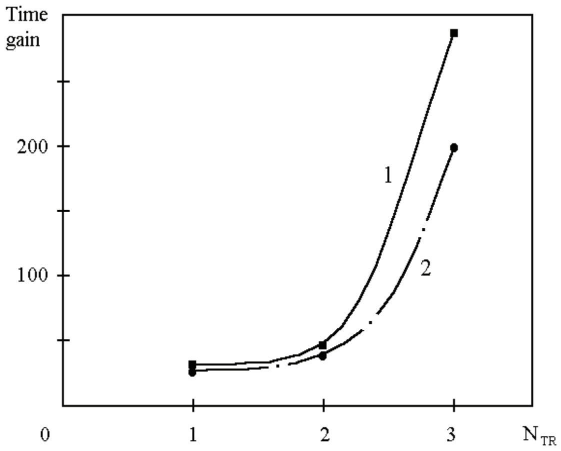


Fig. 5. Computer time gain for active networks for different number of transistor cells

This result confirms the rule that the total computer time gain of the time-optimal design strategy increases when the complexity of the network increases. The obtained results permit us to make the following current conclusions:

1) In practice, the traditional design strategy is never optimal in terms of time.

2) The new design strategies form a basis, which can be used for generation of a design strategy optimal by the running time criterion.

3) The potential gain in time, attained with the aid of optimal strategy, grows as the size and complexity of the electronic network increase.

5. Acceleration effect

We can obtain the possible gain in the computation time for quasi-optimal strategy. However, this expected gain can become reality only if we manage to generate the algorithm making it possible to determine the optimal trajectory of the design process. Thus the problem of investigation of major intrinsic properties and restrictions of the optimal trajectory of design is the principal task to be resolved in creation of the optimal algorithm.

Based on the concepts suggested, it is possible to analyze new effects (Zemliak, 2002) arising in the process of network design by the control theory methods. The inquiries started from a simplest nonlinear circuit with a single node and two parameters (*N*=2), which has no practical applications but serves a good illustration for comprehension of the process in the network design based on the new methodology. After that an *N*-dimensional problem will be considered. All the examples included demonstrate some phenomenon, which may be called the effect of acceleration of design process. The latter arises because of different behavior of design trajectories having different control functions. The circuit diagram of the nonlinear electronic network with a single node is shown in Fig. 1.

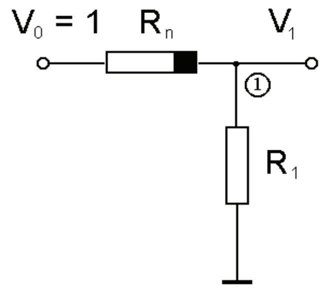


Fig. 6. Simplest nonlinear electronic network

Assume that the element R_n is described by a nonlinear function having the form $R_n = r_{10} + b_n \cdot V_1^2$. For this example we defined only two parameters: resistance R_1 as independent parameter (*K*=1), and nodal voltage V_1 as dependent parameter (*M*=1). In this example, as well as in the subsequent ones, we also assume that all the resistances are positive-valued. For automatically meeting the latter requirement the following definition of the vector *X* can be used: $X = (x_1, x_2)$ where $x_1^2 \equiv R_1$ and $x_2 \equiv V_1$.

The structural basis of various design strategies, defined for the control vector *U* in our case consists of two strategies – at u_1=0 and u_1=1. The first one is the TDS while the second is the

MTDS. The design trajectories for the initial point $X^0 = (1,1)$, are depicted in Fig. 7a: the solid line – for TDS, and the dashed one – for MTDS. The number of iterations and the processor time in the first case are equal to 44 and 0.092_{10}-3 s and in the second case – 78 and 0.149_{10}-3 s. As can be seen, the traditional strategy is preferable. An insignificant reduction of the designing time (5%) can be obtained if in the process of motion the control function u_1 changes form 0 to 1 at step 18. A different result is observed in the event of selecting a negative value of initial point for the variable x_2, for instance, -1, i.e., $X^0 = (1,-1)$. The trajectories corresponding to this situation are depicted in Fig. 7b.

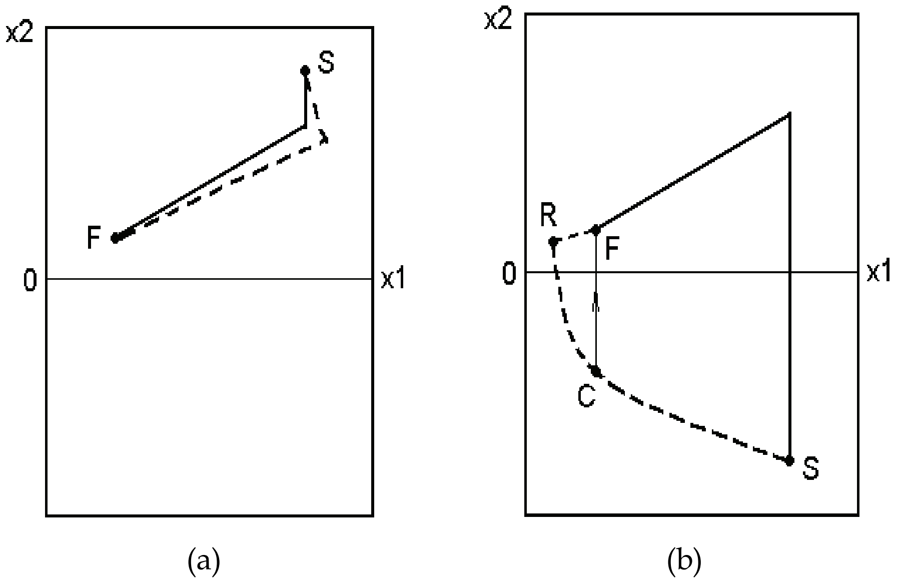


Fig. 7. Trajectories for TDS (solid) and for MTDS (dash) for: (a) X_{in} =(1,1), (b) X_{in} =(1,-1)

The trajectory corresponding to TDS remains almost unchanged. In the first case we have a jump downward from the initial point to the line corresponding to the matched solution. In the second case we have the jump to the same line, but upward. Since the jump occurs instantaneously, the time in both cases is the same. A somewhat different situation is observed for MTDS trajectory. At the negative initial value of the variable x_2 (x_2 = -1) the first part of the trajectory lies in the unfeasible (negative in terms of the variable x_2) half-space, while the second part – in the positive one. It is pertinent to note that the motion of the current point over the first part of the trajectory from the point *S* to point *R* occurs rapidly enough, and then slows down. The total time in this event is larger than at the positive initial approximation. We emphasize that the trajectories of both different strategies approach the final point *F* of the design process from opposite sides. There opens a possibility for accelerating the process by changing the control function u_1 from 1 to 0 in the *C*, which represents the projection of the final point *F* on the trajectory corresponding to MTDS. In this case the optimal strategy has two parts. The first one, described by the curve *SC*, corresponds to u_1=1, and MTDS lies in the physically unreal space. In the point *C* the value of the control function u_1 takes the zero value, and we make a jump to the final point *F* or close to it, which depends on the step of integration and on prescribed accuracy. The second part of the trajectory, starting in the point *C* and corresponding to u_1=0, and TDS either degenerates into a jump, so that we have a single step in addition to the first part of the trajectory, or into several (few in number) additional steps corresponding to TDS. The number of iterations for our example, corresponding to u_1=1, is equal to 9, plus a single step (a jump) corresponding to u_1=0. The time for this optimal trajectory is 0.0194$_{10}$-3 s, which means acceleration of the process by a factor of 4.7. The effect of acceleration can be also observed for more complex examples. However, in this case the trajectories lie in the *N*-dimensional space, and we must analyze various projections of *N*-dimensional curves. Some different passive and active nonlinear networks were investigated to observe an additional acceleration effect. The potential computer time gain of the optimum design strategy with an additional acceleration as the function of the transistor cell number N_{TR} is presented in Fig. 8 for two different optimization procedures (gradient and DFP methods).

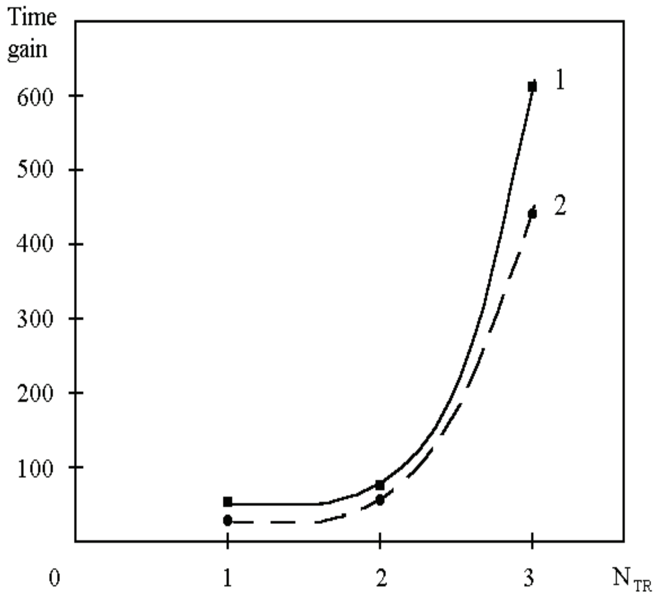


Fig. 8. Computer time gain for active networks with acceleration effect for different number of transistor cells. 1-Gradient method, 2-DFP method

Fig. 8 shows that the time gain for three-stage transistor amplifier increase till 500-600 due to the acceleration effect.

The analysis of acceleration effect shows that this effect appears with special initial conditions. For instance for the network in Fig. 6 this effect exists when the initial point has a negative value of the coordinate x_2, and the value of this coordinate itself, corresponding to the position under the special line - separatrix, are the sufficient conditions for obtaining the acceleration effect. A more detailed analysis shows that in reality these conditions are not necessary. Fig. 9 shows the phase portrait of design process, corresponding to MTDS for the network in Fig. 6, but for all possible values of the coordinate x_2.

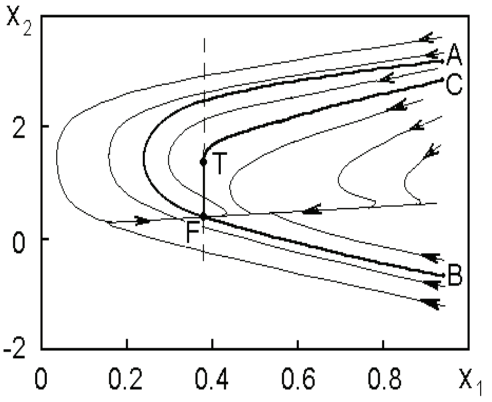
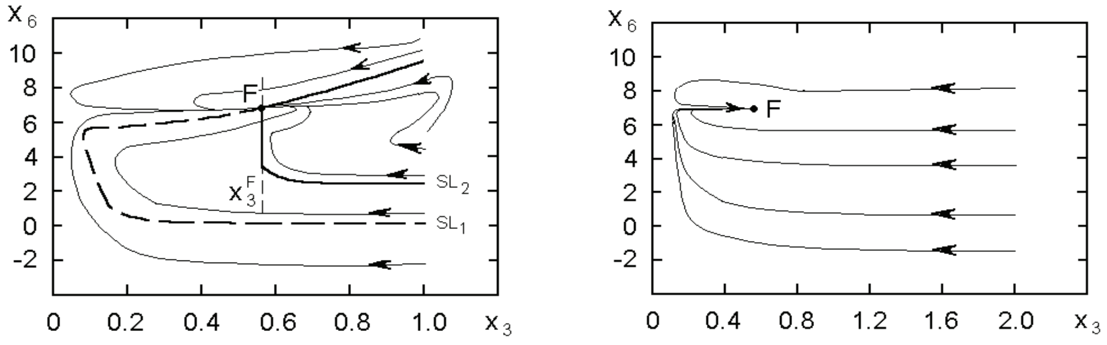


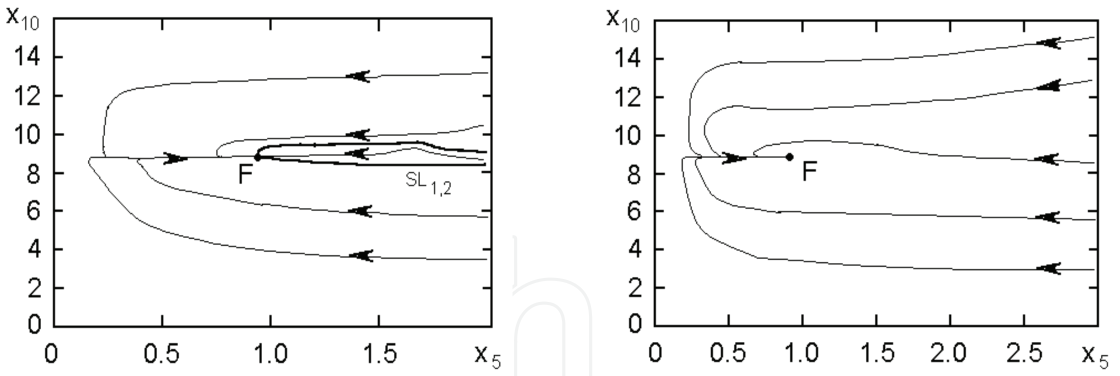
Fig. 9. Design trajectories for MTDS and separatrix for one-node network

The portrait includes two types of separatrix. The first one is separatrix *AFB* separating the trajectories approaching the final point from the left and from the right. The second type separatrix *CTFB* divides the phase space in two other subspaces. The points of the subspace surrounded by this separatrix do not give trajectories permitting to realize the acceleration effect. Conversely, the points lying outside the separatrix correspond to trajectories able to produce the acceleration effect. The jump to the final point can be performed from below as well as from above the separatrix *CTFB* along the extension of the line *TF*. These geometric conditions are necessary and sufficient for existence of the acceleration effect.

The examples given below refer to design of a transistorized amplifier in one and two transistors presented in Fig. 3. The design of the single- and two-stage amplifiers has been performed separately. In the case of the single-stage amplifier, there are three independent variables y_1, y_2, y_3 (*K*=3), and three dependent ones V_1, V_2, V_3 (*M*=3). The vector *X* includes six components: $x_1^2 = y_1$, $x_2^2 = y_2$, $x_3^2 = y_3$, $x_4 = V_1$, $x_5 = V_2$, $x_6 = V_3$. For the two-stage amplifier, the vector *X* contains 10 components. Fig. 10 shows two-dimensional projections of phase portraits of the design process for MTDS in both cases: *a, b* – for the single stage, and *c, d* – for the two-stage amplifier. Fig. 10a,b illustrate behavior of the trajectories on the plane $x_3 - x_6$ at initial values x_i^0 =1,0, i =1,2,3 (case *a*) and at initial values x_i^0 =2,0 (case *b*). We can see a pronounced difference in behavior of phase trajectories for passive and active networks.



(a) (b)



(c) (d)

Fig. 10. Family of curves that correspond to MTDS and separatrix for: (a), (b) one-stage; (c), (d) two-stage amplifier

Projections of the separatrix *SL₁* and separatrix *SL₂* are clearly expressed in the case of x_i^0 =1,0, which is indicative of the presence or absence of the trajectories offer the possibility of the "jump" into the final point of the design process. Of interest is the fact that an increase in network complexity results in expansion of the domain of existence of acceleration effect, which can be seen in Fig 10*c* for the two-stage amplifier. Here we analyze the behavior of projections of trajectories on the plane $x_5 - x_{10}$ at x_i^0 =2,0, *i* =1,2,…,5. The zone confined by the separatrix, where the acceleration effect is absent, becomes narrower for the two-stage amplifier. An increase in initial values of originally independent variables x_i^0 up to 3,0, *i* =1,2,…,5 for the two-stage amplifier (Fig. 10*d*) results in disappearance of separatrix projections – as in the case of the single-stage network. Based on analysis of the above examples we come to the conclusion that complication of electronic network structure and an increase in initial values of originally independent variables expands the domain of existence of the acceleration effect of design process.

The optimal choice of the initial point of the design process permits to realize the acceleration effect with a larger probability. Analysis of trajectories for different design strategies shows that the separatrix concept is useful for comprehension and determination of necessary and sufficient conditions of existence of the design acceleration effect. The separatrix divides the whole phase space of design into a domain where we can achieve the acceleration effect, and a domain in which this effect does not exist. The first domain may be used for constructing the optimal design trajectory. Selection of the initial point of design process outside the domain encircled by separatrix constitutes the necessary and sufficient conditions for existence of the acceleration effect. In the general case, a separatrix is a hyper surface having an intricate structure. However, the real situation is simplified in the most important case, corresponding to active nonlinear networks, because of narrowing the area inside the separatrix, or its complete disappearance – at the initial values of the originally independent variables large enough. It means that the acceleration effect can be realized almost in any case for the networks of large complexity.

6. Stability analysis

Basic concepts of a new methodology in analogue networks optimization in terms of the control theory were stated in previous sections. It was shown that the new approach potentially allows to significantly decreasing the processor time used to design the circuit. This quality appears due to a new possibility of controlling the design process by redistributing computational burden between the circuit's analysis and the procedure of parametric optimization. It may be considered to be a proven fact that traditional design strategy (TDS) including the circuit's analysis at every step of its design is not optimal with respect to time. More over the benefit in time used to design the circuit for some optimal or more precisely quasi-optimal strategy compared to TDS increases with increasing size and complexity of the designed circuit. This optimal strategy and corresponding design's trajectory were obtained using special search procedure and serve only as a proof existing strategies which are much more optimal than TDS. However, it is clear that the problem lies in the ability to move along an optimal trajectory of the circuit's design process from the very beginning of designing the circuit. Only in this case it is possible to obtain the mentioned potentially tremendous advantage in time, which corresponds to the optimal design strategy. During the building the optimal strategy and its corresponding trajectory at the present moment it is necessary to analyze their most significant characteristics. The study of the optimal trajectory's qualitative characteristics and their differences from those of the other trajectories appears to be the only possible way to solve the problem.

The discovery of an effect expecting additional acceleration of the design process and exploration of conditions determining this effect's existence lead to increased time advantage and serve as an initial point of quasi-optimal design strategy building. The analysis of this effect allowed to state three most significant moments: 1) to obtain the acceleration effect the initial point of the design process should be chosen outside the domain limited with a special hypersurface (separatrix), 2) the acceleration effect appears during a transition from a trajectory corresponding to a modified traditional design strategy (MTDS) to the trajectory which corresponds to TDS and from any trajectory similar to MTDS to any trajectory similar to the trajectory of TDS, 3) the most significant element of the acceleration effect is an exact position of the switch point corresponding to a transition from one strategy to another.

To obtain an optimal sequence of switching points during the design process it is necessary to select a special criterion, which depends on the internal properties of the design strategy. The problem of searching for the optimal with respect to time design strategy deals with a more general problem of convergence and stability of each trajectory. On the basis of experiment, the design time for each strategy determines by properties of convergence and stability of corresponding trajectory. One of the common approaches of analysis of dynamic systems stability is based on the direct Lyapunov method (Barbashin, 1967; Rouche et al., 1977). We consider that the time design algorithm is a dynamic controlled process. In this case, the main control aim is determined as minimization problem of transient time of this process. As result, the analysis of stability and characteristics of transient process (process of designing is one of these) for each trajectory are possible on the basis of the direct Lyapunov method. Let's introduce Lyapunov function of process of designing. It will be used for analysis of properties and structure of optimal algorithm and for searching of optimal switch point positions of control vector particularly.

There is a certain freedom of Lyapunov function choice as the latter has more than one form. Let's denote the Lyapunov function of process of designing (1)–(5) in form:

$$V\left(X\right)=\sum_{i}\left(x_{i}-a_{i}\right)^{2} \tag{22}$$

where a_i is a stationary value of coordinate x_i. The set of all coefficients a_i is the main result of process of designing as the minimum of target function $C(X)$ is achieved at these values of coefficients, i.e. the aim of designing is succeeded. It is clear, that these coefficients are accurately known only at the end of designing. The other variables $y_i = x_i - a_i$ could be determined instead of x_i variables. In this case equation (5) takes the form:

$$V(Y)=\sum_{i} y_i^2 \tag{23}$$

Taking into account the new variables y_i, the process of designing (1)–(5) remains the same form. However, equation (23) satisfies all conditions of Lyapunov function definition. Indeed, this function is piecewise continuous function having piecewise continuous first partial derivatives. In addition, three main properties of function (23) V(Y)>0, 2) V(0)=0, and 3) $V(Y)\to\infty$ for $\|Y\| \to \infty$) are presented. In this case we obtain the possibility to analyze the stability of equilibrium position (point *Y*=0) by Lyapunov theorem. On other hand, the stability of point $a=(a_1,a_2,...,a_N)$ analyzes on basis of (22). It is clear, both of these problems are identical. The point $a=(a_1,a_2,...,a_N)$ can be defined only at the end of the process of designing that is inconvenience of equation (22). As result, we could analyze the stability of various designing strategies by the equation (22) if the problem's solution (i.e. point *a*) was determined already in another way. Moreover, the possibility to control the stability of process during optimization procedure is of interest. In this case we have to determine another form of Lyapunov function which would be irrespective of final stationary point *a*. Let's define Lyapunov function in the form:

$$V(X,U)=\left[F(X,U)\right]^r \tag{24}$$

$$V\left(X,U\right)=\sum_{i}\left(\frac{\partial F\left(X,U\right)}{\partial x_{i}}\right)^{2} \tag{25}$$

where $F(X,U)$ is a generalize target function of process of designing and $r>0$. Under additional conditions both of these equations determine Lyapunov function having properties similar to (23) in the sufficiently great neighborhood of stationary point. Meanwhile the dependence on control vector *U* appears too. Indeed, we can see that the value of (24) is equal zero in the stationary point if the target function of this process $C(X)$ in the same point is equal zero as well. The equation (24) is positive defined function in all points distinct from a stationary point as the function $C(X)$ is nonnegative. The function $V(X,U)$ increases without bound when we are going away from the stationary point.

The equation (25) also determines Lyapunov function if $\partial F / \partial x_i$ = 0 in the stationary point $a = (a_1, a_2, \ldots, a_N)$ and *V(a,U)*=0. On the other hand, *V(X,U)*>0 for all X. In conclusion, Lyapunov function determined by (25) is a function of vector *U* i.e. all coordinates x_i depend on *U*. The third property of Lyapunov function is proven wrong as the behavior of function *V(X,U)* when $\|X\| \to \infty$ is unknown. However, as known a posteriori, the function *V(X,U)* is the increasing function in the sufficiently great neighborhood of a stationary point. According to Lyapunov method, the information about trajectory stability is connected with time derivative of Lyapunov function. Direct calculation of time derivative of Lyapunov function $\overset{\bullet}{V}$ lets estimate the dynamic system stability. The process of designing and a corresponding trajectory is stable if this derivative is negative. On the other part, the direct Lyapunov method gives sufficient but not necessary stability conditions. This implies that the process can lose stability or can remain stable in the case of positive derivative. The appearance of positive values of derivative $\overset{\bullet}{V}$ on set of positive measure only states the instability displaying in a few growth of Lyapunov function instead of decreasing latter. If such process exists far from the stationary point, then the process of designing is divergent function and we cannot obtain the solution of this trajectory. In this case the initial point of process of designing or strategy should be changed. If the positive derivative $\overset{\bullet}{V}$ appears at the end of process of designing (i.e. not far from the stationary point), then we could say that the process of designing significantly decelerates. This designing strategy is going round in a circle and cannot provide required accuracy. As result, the engineering time substantially grows. This effect is well known in practical designing. If we obtain the unacceptable accuracy, the strategy of designing or initial point have to be changed. The detailed behavioral analysis of Lyapunov function and its derivative for different strategies of designing makes it possible to choose the perspective strategies. This analysis also allows determining on qualitative level the relationship between design time and Lyapunov function and its derivative being the main factors of the process of designing.

Two-stage transistor amplifier, depicted in Fig.3, is used for stability analysis of different strategies of design. The direct calculation of time derivative $\overset{\bullet}{V}$ of Lyapunov function, determined by (29) for r=0.5, shown that the derivative is negative in the initial point of design for all trajectories, i.e., all possible design's strategies and its trajectories are stable at the beginning if integration step of system (1) is enough small. In the same time, when the current point of trajectory reaches some ε-neighbourhood of the stationary point

$(a_1, a_2, ..., a_N)$, the derivative of Lyapunov function comes positive and the current design's strategy loses stability. This implies that this strategy dose not ensure the convergence of trajectories to the stationary point $(a_1, a_2, ..., a_N)$ starting from some value of ε-neighbourhood, i.e. achievement of minimum of target function *F(X,U)* and so function *C(X)* with accuracy to ε dose not guarantee. In fact, each trajectory has eigen ε-neighbourhood determining maximum available accuracy for this one and the convergence problem arises inside this area. The process of designing significantly decelerates for current strategy before approaching of the critical value of ε-neighbourhood. Alias, the derivative $\dot{V}$ remains negative but has enough small absolute value. The results for two-stage amplifier are presented in Table 7. The design realized on the basis of strategies coming into structural basis 2^M and determined by control vector *U*. The appearance of positive values of derivative $\dot{V}$ on set of positive measure determines the termination of process of designing. Process optimization realized on the basis of equation (18) and gradient method with a variable optimal step t_s hereupon this step t_s could be both small and large. As result, we have non-smooth behavior of derivative from one step to other.

N	Control vector	Iterations	Computer	Critical value of
	U(u1,u2,u3,u4,u5)	number	time (sec)	ε -neighborhood
1	(0 0 0 0 0)	3177	7.25	2.78E-08
2	(0 0 0 0 1)	3074	8.02	3.36E-07
3	(0 0 0 1 1)	11438	26.36	8.18E-07
4	(0 0 1 0 1)	799	1.16	9.38E-09
5	(0 0 1 1 0)	1798	2.61	1.61E-08
6	(0 1 0 1 1)	43431	76.89	3.16E-05
7	(0 1 1 0 0)	1378	2.25	1.67E-08
8	(0 1 1 0 1)	571	0.72	6.83E-09
9	(0 1 1 1 0)	1542	2.03	2.05E-08
10	(1 0 0 1 1)	11839	21.37	1.68E-05
11	(1 0 1 0 0)	2097	3.57	5.47E-07
12	(1 0 1 1 0)	6026	8.31	4.94E-07
13	(1 1 1 0 0)	6602	8.84	7.41E-07
14	(1 1 1 0 1)	935	0.71	1.33E-08
15	(1 1 1 1 0)	2340	2.31	1.62E-07
16	(1 1 1 1 1)	1502	0.38	1.09E-08

Table 7. Critical value of the ε-neighborhood for design strategies for two-stage amplifier

For smoothing of derivative $\dot{V}$ the value averaging on the interval 30 steps was used. The analysis of results of Table 7 has shown some important laws. First of all, the strong correlation between processor time and critical value of ε -neighbourhood, after which the value of derivative $\dot{V}$ stays positive, is presented. As a rule, the fewer available value of ε -neighbourhood corresponds to the fewer processor time. We could order all strategies in Table 7. from the smallest processor time (0.38 sec, No. 16) to the longest one (76.89 sec, No. 6).

On the other hand, the strategies in ascending order of critical value of ε -neighbourhood are presented in Table 8.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Number of strategies regulated by the computer tim	16	14	8	4	9	7	15	5	11	1	2	12	13	10	3	6
Number of strategies regulated by the ε -neighborhood	8	4	16	14	5	15	7	9	1	2	12	11	13	3	10	6

Table 8. Strategies' ordering by processor time and by critical value of ε -neighborhood

The No. of each strategy in Table 8 determined by two different ways of order is slightly different. Two strategies (13 and 6) have the same number. Seven strategies have the difference in one place, four ones – in two places, and three strategies – in three places. The average difference is equal to 1.5. Taking into account that the critical values of ε -neighbourhood are obtained approximately by the averaging during integration of system (1) we can see that the correspondence of processor time with critical value of ε -neighbourhood is enough acceptable. Contrariwise, the parameters of ε -neighbourhood are obtained on the basis of Lyapunov function and its derivative analysis. Therefore, we could say that the strong correlation between processor time and properties of Lyapunov function is presented.

From the analysis above the assumption is induced: Lyapunov function of process of designing and its derivative are enough informative source to select more perspective design strategies.

7. Conclusion

The traditional approach for the analogue network optimization is not time-optimal. The problem of the minimal-time design algorithm construction can be solved adequately on the basis of the control theory. The network optimization process in this case is formulated as a controllable dynamic system. Analysis of the different examples gives the possibility to conclude that the potential computer time gain of the time-optimal design strategy increases when the size and complexity of the system increase. The Lyapunov function of the optimization process and its time derivative include the sufficient information to select more perspective strategies. The above-described approach gives the possibility to search the time-optimal algorithm as the approximate solution of the typical problem of the optimal control theory.

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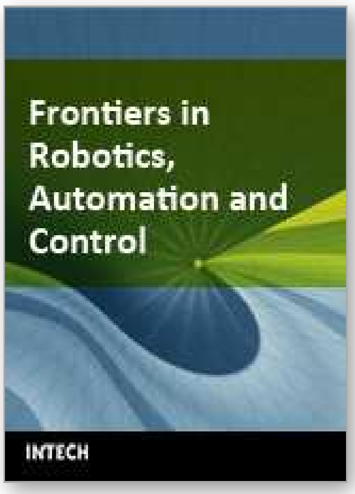
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