Chapter 16

Achievement Bests Framework, Cognitive Load Theory, and Equation Solving

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Additional information is available at the end of the chapter

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Abstract

The Framework of Achievement Bests provides an explanatory account into the process of optimization, which details how a person reaches from one level of best practice to that of a more optimal level. This framework, we contend, is significant in its explanatory account of personal growth, an internal state of flourishing, and the achievement of exceptionality. This chapter conceptualizes the applicability of the Framework of Achievement Bests to the context of instructional designs. We highlight the tenet of element interactivity, which is integral to the design of a particular mathematics instruction and its potential effectiveness. Element interactivity entails the interaction between elements within a learning material. Owing to the limited working memory capacity, an instruction that incurs high level of element interactivity would impose high cognitive load leading to reduced learning. Our conceptualization postulates the possible alignment between suboptimal and optimal instructional designs with realistic and optimal levels of best practice, respectively. This postulation (e.g., suboptimal instructional design → realistic level of best practice), which recognizes the importance of cognitive load imposition, is significant from a practical point of view. By focusing on instructional designs, it is possible to assist individuals to achieve optimal best practice in learning.

Keywords: achievement best frameworks, cognitive load theory, appropriate instructional design, element interactivity, equation solving

1. Introduction

Effective learning in school contexts is an important notion to consider. By all account, effective learning entails personal experience of deep, mastery learning, improvement in cognitive skills (e.g., problem solving), and the stimulation of interest and intellectual curiosity. The product of effective learning, in this sense, may include an improvement in academic

performance at the end of the school term. Ineffective learning, by contrast, may result in loss of interest, engagement in maladaptive outcomes, and superficial learning. This recognition places emphasis on a need for educators and researchers, alike, to focus on motivational initiatives, pedagogical strategies, and educational programs that could foster *engagement of* and *preference for* effective learning.

Our postulated positioning, based on previous research undertakings in the area of mathematics [1, 2], is that *appropriate instructional designs* may serve to facilitate and promote effective learning. Instructional designs, an important element of pedagogical practices in the teaching and learning processes, are central to the achievement of effective learning. We contend that, in this case, a particular instructional design is efficient when it imposes minimal *cognitive load on an individual's processing of information*. Instructional designs that impose high levels of cognitive load, by contrast, are ineffective and inefficient for implementation and practice. This line of research development, in general, has notable implications for us to consider, especially in relation to Teacher Education Pre-service preparation and training.

This chapter then, in accordance with the scope of the edited book, explores the importance of comparative instructional designs in the context of mathematics learning. Drawing from our previous work, we focus on the development of a conceptualization that emphasizes on the choosing of an appropriate instructional design for implementation. This conceptualization, in particular, focuses on the *achievement of optimal best in mathematics* [3, 4], taking into consideration *the negative impact of cognitive load imposition* [5, 6]. Furthermore, arising from this discussion, we consider methodological and theoretical issues for continuing research development into the area of instructional designs.

2. Achieving optimal functioning

Achieving optimal best in different subject matters is a central feat of human agency. This personal attribute emphasizes an internal state of determination and resilience to achieve optimal functioning. *Optimal functioning*, in contrast to the experience of stagnation, places emphasis on an individual's quest to fulfill his/her personal and psychological needs. Importantly, perhaps, the accomplishment of optimal best indicates the maximization in capability that an individual may demonstrate [4]. In the context of schooling, for example, optimal best for a Year 8 student may involve his/her understanding of mathematics equations involving special features (e.g., $10\%x = 20$, solve for x) (Appendix A). This level of exceptionality of mathematics learning, as mentioned, reflects the student's fullest potential for the stipulated time point.

Optimal best, in essence, coincides with the theoretical tenets of positive psychology [7, 8], which emphasize the *importance of human proactivity*, *personal fulfillment*, and the *aspiration to lead fruitful and meaningful lives*. Optimal best, consequently, indicates the development and manifestation of virtues, inner strengths, and resilience, and the achievement of exceptionality. These attributes and/or characteristics are positive, in nature. In recent years, researchers have advanced the study of optimal best, theoretically, methodologically, and empirically. Phan and colleagues [3, 4], for example, have developed the *Framework of Achievement Bests*,

detailing an underlying internal mechanism that could explain how an individual reaches a state of optimal functioning. This framework is significant as it contributes to existing work [7–9], and advances the inquiry into the tenets of optimal best.

2.1. The Framework of Achievement Bests

The Framework of Achievement Bests, developed by Phan and colleagues [3, 4, 10], explores the personal experience of optimal functioning. Optimal functioning, according to the authors, is defined as an internal state of experience and accomplishment that reflects maximization in capability (e.g., a Year 8 student's indication to learn and understand linear equations that involve multiple solution steps (e.g., $4/x = 11$, solve for x). "What is the best that I can accomplish?", in this instance, is a question that indicates an individual's self-awareness of his/her potential best practice.

The Framework of Achievement Bests draws comparison with Fraillon's [27] theorization of *optimization*, which is a psychological process that focuses on an individual's optimal best from some point of self-reference. Phan and colleagues' [3, 4, 10] conceptualization of achievement bests depicts two major levels of best practice: (i) *realistic level of best practice* (i.e., denoted as RL), which entails what an individual is realistically capable of accomplishing, at present (e.g., what can I actually do, at present, in Algebra?; how much do I know…..?), and (ii) *optimal level of best practice* (i.e., denoted as OL), which is defined as an individual's accurate indication of projected accomplishment that is exceptional, in nature (i.e., as of today, what is the best that I can do for this topical theme, realistically?). Reaching an optimal level of best practice from a realistic level of best practice reflects, in this case, a state of flourishing or optimal experience. **Figure 1** illustrates the Framework of Achievement Bests, in its totality [4, 11].

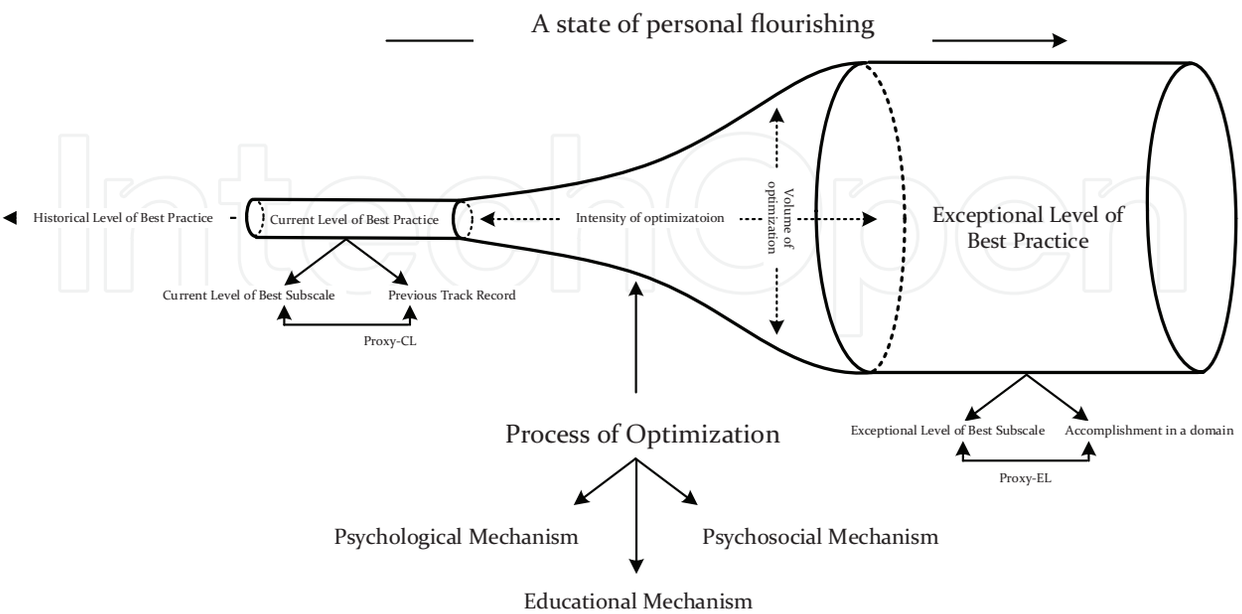

Figure 1. The Framework of Achievement Bests. Source: Adapted from Ref. [1].

The Framework of Achievement Bests is unique for its attempts to explain the "achievement" of optimal functioning. That is, from **Figure 1**, an individual's point of reference is his/her realistic level of best practice (note: as time progresses, an individual's realistic level of best practice becomes his/her historical level of best practice, which is defined as *previous record of accomplishment* in a subject matter). This level of best practice, consequently, serves as a source and/or a reference point by which an individual would use to formulate his/her optimal level of best practice. The *zone of optimization*, which refers to the "difference" or "range" between the realistic level and the optimal level of best practice (i.e., OL-RL), in essence, delves into the process of optimization. In other words, as Phan and colleagues explain, optimization is a psychological process that serves to optimize an individual's internal state of functioning to "progress" from one level to that of another level.

The psychological process of optimization, recently updated in terms of theorization [10], varies in terms of *intensity* and *scope* (or volume). The intensity of optimization emphasizes *the extent and amount of resources* (e.g., appropriate instructional design) needed to optimize an individual's state of functioning. The scope (or volume) of optimization, by contrast, focuses on the *amount of effort* and *time* (e.g., the extent to which a student is motivated to invest effort and time) needed to optimize an individual's state of functioning. Optimizing a small zone of optimization in mathematics learning (e.g., knowing how to solve one-step equations such as $x + 3 = 5$, to knowing how to solve one-step equations such as $x + 4 = -7$), for instance, may require only a small amount of effort and time, and/or the amount of resource (e.g., limited scaffolding from a teacher). Both equations (i.e., $x + 3 = 5$ and $x + 4 = -7$) share identical problem structure except that the latter equation has a negative number (i.e., −7), and thus may pose a difficulty for students [12]. By contrast, it will require more resources (e.g., effective instructional design), effort, and time to optimize a large zone of optimization in mathematics learning (e.g., knowing how to solve one-step equations such as $y + 3 = 7$ (**Figure 2(a)**, to knowing how to solve one-step equations such as $\frac{4}{a} = 2$ (**Figure 2(b)**). This is because the one-step equation such as $\frac{4}{a} = 2$ has more solution steps than the one-step equation such as $y + 3 = 7$, irrespective of the methods (i.e., balance or inverse). The differential efficiency between the balance and inverse methods will be discussed later. Similarly, in physical education,

(a) Balance method

Line 1	$y + 5 = 13$	(-5) on both sides
Line 2	$-5 \quad -5$	
Line 3	$y = 8$	

Inverse method

Line 1	$y + 5 = 13$	(+ 5 becomes – 5)
Line 2	$y = 13 - 5$	
Line 3	$y = 8$	

(b) Balance method

Line 1	$\frac{4}{a} = 2$	$(\times a)$ on both sides
Line 2	$\times a \quad \times a$	
Line 3	$4 = 2a$	$(\div 2)$ on both sides
Line 4	$\div 2 \quad \div 2$	
Line 5	$2 = a$	

Inverse method

Line 1	$\frac{4}{a} = 2$	$(\div a$ becomes $\times a)$
Line 2	$4 = 2 \times a$	$(\times 2$ becomes $\div 2)$
Line 3	$4 \div 2 = a$	
Line 4	$2 = a$	

Figure 2. (a) One-step equation involving one operational and two relational lines. (b) One-step equation involving two operational lines and three relational lines.

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optimizing an individual's functioning in a physical activity (e.g., running from 500 m to 5 km) may require much more effort, time, and resources. On this basis, the "quantitative" difference between the realistic level of best practice and the optimal level of best practice, which in this case reflects its complex nature, determines the intensity and scope of optimization.

In essence, the psychological process of optimization encompasses the utilization of resources, and the expenditure of time and effort in order to optimize an individual's internal state of functioning. Optimization, based on existing theorizations, involves more than just personal scaffolding from a capable authority figure [13]. According to Phan and colleagues' representation, there are three types of "mechanisms" [4] that operate to optimize a person's state of functioning. The sequencing of this psychological process is as follows:

i. The initiation and execution of *optimizing agents* (i.e., *psychological mechanisms*, *educational practices*, and/or *psychosocial mechanisms*) that operate to influence the internal personal processes for learning and performance. There are three types of optimizing agents, namely (i) *psychological mechanisms*, such as a person's self-efficacy beliefs for learning [14, 15], hope [16, 17], and motivation, in general [18], (ii) *educational practices*, such as instructional efficiency and appropriate pedagogical approach that enable better comprehension and understanding of the instructional materials [1, 4], and (iii) *psychosocial factors*, such as the impact of the home social environment that may shape a student's state of functioning [19].

ii. Upon the positive influences of optimizing agents, internal personal processes of *persistence* [20–22], *effort expenditure* [20, 23, 24], and *effective functioning* [25–27] are activated. This activation, in turn, plays a central role in motivating an organism to reach an optimal level of functioning.

Research development emphasizing the operational nature of optimization is in its early stage of evolution. In terms of empirical research, for example, a few researchers have used quantitative methodological designs to study the explanatory functioning of the three mentioned mechanisms [11, 28]. Phan and colleagues have used the Optimal Outcome Questionnaire (OOQ) [29] to explore the importance of "profiling of best practice" [4, 11], and the predictive and explanatory effects of the different types of optimizing mechanisms [28]. Aside from empirical research, it is also possible to explore optimal best practice and the operational nature of optimization from the perspective of conceptualization, using authentic contexts. In this section of the book chapter, we provide an in-depth discussion of a conceptualization of optimal best in the area of mathematics learning.

3. The importance of algebra

Mathematics educators acknowledge the prominent role of algebra in mathematics learning and curriculum development [30, 31]. They regard algebraic skills as a "gatekeeper" that facilitates the engagement of higher-order mathematical thinking skills [32]. Algebraic skills are useful not only for solving real-life problems (e.g., "If your father wants to increase your

weekly allowance of $20 by 5%, what is your new allowance?" [33] but also enable effective learning in comparable subject areas, such as Physics and Chemistry (e.g., "A solution contains 1.1 g of sodium nitrate, $NaNO_3$ in 250 ml of solution, what is the molarity of this solution?") [34–36].

Regarding the use of algebra to solve real-life problems, successful problem solvers differ from unsuccessful problem solvers in their use of *schematic knowledge* to set up an equation that could then generate a solution [36, 37]. However, unless the problem solvers possess *adequate equation-solving skills*, they are unlikely to obtain the solution even if they have succeeded in setting up an equation pertaining to the schematic knowledge. Since equation solving is an integral component of the algebra problem-solving process [38], it is timely that we propose *appropriate instructions* that could facilitate optimal learning experiences of equation solving. On this basis, reflecting the Framework of Achievement Bests, we contend that appropriate instructional designs could serve to optimize students' understanding of equation solving.

Referring to our previous mention, a conceptualization that involves a focus on instructional designs could in effect provide evidence that attests to the explanatory power of the Framework of Achievement Bests. In this analysis, we contend that appropriate instructional designs could serve as educational mechanisms to optimize students' learning experiences in mathematics. Appropriateness of an instructional design is determined, in part, from its inverse association with the *negative impact of cognitive load imposition* [5, 6]. In our recent studies, for example, we proposed a theoretical position, which posits that optimal instructions that impose low cognitive load may generate positive emotions, resulting in an increase in motivation to learn equation solving. By contrast, however, suboptimal instructions are more likely to result in high cognitive load imposition, which may then generate negative emotions and a decline in motivation to learn equation solving.

4. Cognitive load theory: a theoretical overview

Human cognitive architecture, compromising of both working and long-term memory, is central to the importance of *cognitive load theory* [5]. The working memory is severely limited in its capacity to process unfamiliar information [39]; however, this limitation disappears when familiar information is retrieved from the long-term memory for processing. By contrast, long-term memory has an unlimited capacity, which enables it to store a large amount of information for an infinite period. Cognitive load theory, proposed by Sweller [5, 40], is an instructional theory that attempts to explain why a specific instruction will or will not work.

Three types of cognitive load affect the design of a specific instruction:

i. *Extraneous cognitive load*, which is imposed by an inappropriate instruction. We can change the design of inappropriate instruction to reduce extraneous cognitive load.

ii. *Intrinsic cognitive load*, which is imposed by the inherent complexity of a learning unit (or a material). We can change either the design of the instruction [33, 41] or the knowledge base of a learner to reduce intrinsic cognitive load.

iii. *Germane cognitive load*, which entails an investment of cognitive resources to assist in the learning of relevant aspects of the instructional material. We can change the design of an instruction in order to increase germane cognitive load. For example, one way to improve problem-solving skills is to provide learners with variability practice, involving the identification of a category of problems that share a similar problem structure but have different contexts [42].

Recent research development on cognitive load theory has highlighted an important concept, known as *element interactivity*, that exists across the three types of cognitive load [43]. Element interactivity, in this case, emphasizes the *interaction that exists between elements within a learning material*. An element refers to anything that requires learning (e.g., a number, a symbol, a concept, a procedure, etc.) [44]. Under this conceptualization of cognitive load theory, the level of element interactivity determines the extent to which a particular type of cognitive load would exert its influence on the design of an instruction. Why is this the case? There are three possible reasons as to why this is the case: (i) the level of element interactivity determines the intrinsic nature of the material and, thus, the intrinsic cognitive load, (ii) the level of element interactivity determines, in part, the appropriateness and/or inappropriateness of an instruction and its extraneous cognitive load, and (iii) the level of element interactivity determines the beneficial design feature of an instruction and, thus, its corresponding germane cognitive load. Because cognitive resources, in the case of germane cognitive load, facilitate in the learning of relevant aspects of instructional material, germane cognitive load is not an independent source of cognitive load; rather, it is incorporated in the intrinsic cognitive load.

5. Element interactivity, learning, and understanding

Learning material reflects low-element interactivity knowledge if we can learn each element independently of another element [1]. In mathematical numeracy, a student can learn to recognize a number (e.g., 5) independently of another number (e.g., 9). Learning individual numbers therefore constitutes low-element interactivity knowledge, as each number is independent and may be learned in isolation. Moreover, because a student can learn to recognize individual numbers sequentially (e.g., "5" and then "6"), minimal working memory resources are involved when a student learns to recognize each number. Manipulation of multiple interactive elements simultaneously, by contrast, reflects high-element interactivity knowledge. In the case of learning how to solve a simple one-step equation, such as $x - 7 = 13$, a student would need to understand the role of the variable x, and the quantitative relationship between the elements, by which the left-hand side of the equation is equaled to the right-hand side. Manipulating multiple interactive elements simultaneously in order to solve an equation would impose a heavy cognitive load.

Because there is limited association between individual numbers, a student can learn to recognize a vast number of numerals individually, via memorization (e.g., rote learning). However, since multiple elements within a linear equation interact, a student must learn these elements simultaneously rather than individually. On this basis, learning to solve an equation may pose a challenge for a student because it requires him/her to understand the relation between

the multiple interactive elements. In essence, understanding applies only to high-element interactivity material, but not to low-element interactivity material. Mathematics learning normally involves students learning multiple mathematical concepts simultaneously, which consequently imposes high-element interactivity and high cognitive load [45]. Thus, for optimal learning experience, it is important that we design appropriate instructions that could minimize the burden of the working memory, which in turn would help students learn mathematical concepts.

6. Balance method, inverse method, and element interactivity

Based on the Framework of Achievement Bests [4], it is plausible to postulate that appropriate instructional designs and pedagogical practices could serve to optimize students' learning experiences in mathematics. This postulation reflects, in part, our previous research undertakings that involved secondary school students in Australia and Malaysia. We contend that pedagogical practices (i.e., instructional designs) used by teachers are comparative, resulting in perceived differences in terms of effectiveness. Cognitive load imposition [5, 43], as explained, may assist and/or determine the effectiveness of a particular pedagogical approach. In mathematics learning, the two popular methods that facilitate the acquisition of equation-solving skills are the *balance* and *inverse* methods (**Figure 2(a)**). The balance method is popular among Western countries [46], whereas some Asian countries (e.g., Singapore, Korea, and Japan) have introduced and preferred the inverse method in primary mathematics curriculum [47].

In this section of the book chapter, we discuss the characteristics of the balance and inverse methods for effective learning in mathematics. *Differentiation* between the two methods involves clarity and explanation of the solution procedure of one-step equations, which may involve understanding of the difference between *relational* and *operational lines* [2]. A relational line indicates the relationship between the elements on the left side of the equation, which is equaled to the right side of the equation (e.g., Lines 1 and 3 in **Figure 2(a)**). By contrast, an operational line refers to the application of a mathematical operation that changes the state of the equation, and yet at the same time preserves its equality (e.g., Line 2 in **Figure 2(a)**).

6.1. Balance method

In accordance with **Figure 2(a)**, Line 1 is a relational line and it involves six elements, consisting of y, 5, 13, and three concepts. These three concepts are as follows: (i) y represents an unknown number, (ii) the "=" sign describes a quantitative relation between elements, with the left side of the equation equals to the right side, and (iii) to find y, the learner needs to perform the same operation on both sides in order to balance the equation. A learner is required to coordinate the interaction between the six elements simultaneously. By contrast, Line 2 is an operational line that involves three elements and consists of a number (i.e., −5) and two concepts. The two concepts require the learner to cancel +5 with −5 on the left side of the equation, and to perform 13 − 5 on the right side of the equation in order to maintain the equality of the equation. Interaction between elements occurs on both sides of the equation when the

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learner performs +5 with −5 on the left side of the equation as well as 13 − 5 on the right side of the equation. Lastly, Line 3 is regarded as a relational line; it consists of three elements such as *y*, 8, and one concept. The concept requires the learner to be able to process Lines 1 and 2 successfully so that he or she would then know that *y* equals to 8 is the solution.

6.2. Inverse method

The inverse method differs from the balance method for Line 2, but not for Line 1 and/or Line 3. Line 2 is an operational line and it involves four elements, consisting of *y*, 13, −5, and one concept. This concept requires the use of an inverse operation: move +5 from the left side of Line 1 to become −5 on the right side of Line 2 in order to balance the equation. Element interactivity occurs on one right side of the equation, where −5 interacts with 13. Overall, then, the inverse method incurs only half of the interactive elements as the balance method for the operational line (i.e., Line 2). Consequently, the inverse method imposes lower element interactivity and therefore lower cognitive load than the balance method.

6.3. Differential element interactivity between the balance and inverse methods

For both the balance and inverse methods of mathematics learning, understanding can only occur when learners simultaneously assimilate multiple interactive elements that arise within each line, and across the three lines of the solution procedure. For each relational line, the level of element interactivity arises from the interaction of elements within and between the left side and right side of the equation. Because the level of element interactivity is caused by the intrinsic nature of the equation, there is no differential element interactivity between the balance and inverse methods. By contrast, differential element interactivity between the balance and inverse methods favors the inverse method for the operational line. Interaction between elements occurs on both sides of the equation for the balance method, but only on one side of the equation for the inverse method. In other words, the balance method incurs twice as many interactive elements as the inverse method for each operational line. Nevertheless, for a simple one-step equation (e.g., $y + 3 = 7$) that consists of one operational line and two relational lines (**Figure 2(a)**), the total cognitive load required to process the level of element interactivity would expect to be low for both the balance and inverse methods. Indeed, research has shown that the inverse method is not better than the balance method for one-step equations that consist of one operational and two relational lines in the solution procedure [1, 48, 49].

The inverse method, as shown, is comparable with the balance method for simple one-step equations that involve one operational line and two relational lines (e.g., $y + 3 = 7$). The inverse method, however, is more advantageous when complex one-step equations consisting of two operational lines and three relational lines are involved (e.g., **Figure 2(b)** [48]. Compared with simple one-step equations, the level of element interactivity of the complex one-step equations for both the balance and inverse methods has increased because of an increase in both operational lines (2 vs. 1) and relational lines (3 vs. 2). Nonetheless, the ratio of the interactive elements between the balance and inverse methods remains the same (i.e., 2:1), irrespective of the number of operational line. Having said this, the total number of interactive elements for two operational lines is twice the total number of interactive elements for one operational line,

irrespective of whether it is the balance method or inverse method. Consequently, as revealed by prior studies, differential element interactivity between the balance and inverse methods favors the inverse method for complex one-step equations that involve two operational lines and three relational lines [1, 48, 49].

7. Special features

Aside from operational and relational lines, the presence of *special features* that involve complex elements also increases the complexity of one-step equations (see Appendix A). Operating with negative numbers is an integral component of middle-school mathematics curriculum. Having said this, operating with negative numbers continually poses challenges for school-age students [50, 51]. For example, in relation to multiplication, many students struggle with problems that have two negative numbers in algebraic expression problems (e.g., $-4(5x-2)$ [50]. Furthermore, aside from negative numbers, students also commit errors when operating with fractions [52]. Finally, to compound this difficulty, many students also fail to engage in mathematical reasoning that emphasizes the connection between fraction, percentage, and decimal [53].

On this basis, when the number of operational lines and relational lines is kept constant in one-step equations, operations with special features (see Appendix A) pose an additional challenge for students. For example, the equation $2x = 6$ shares a similar structural feature with that of the equation $10\%x = 20$ and, consequently, both have the same level of element interactivity. However, $10\%x = 20$ poses a greater challenge than $2x = 6$, owing to the fact that the latter equation has a percentage (i.e., 10%). The percentage (i.e., 10%) is regarded as a complex element because it comprises not only a number (i.e., 10) but also a percentage sign (i.e., %).

In summary, from the discussion so far, what can we say about the two pedagogical approaches: inverse versus balance? We argue that the inverse method, preferred by many Asian countries, is more effective than the balance method for two major attributes: (i) the number of operational lines and relational lines that exist and (ii) the presence of special features in the equations. Indeed, our previous research undertakings have provided evidence that the inverse method is better than the balance method for complex one-step equations that involve two operational lines and three relational lines. The inverse method, though, is comparable to the balance method for simple one-step equations that involve one operational line and two relational lines [48]. Furthermore, as our research showed, the presence of special features favored the inverse method when the number of operational and relational lines is kept constant [54].

8. Mathematical equivalence

Mathematics education researchers have regarded conceptual and procedural knowledge as essential components of mathematics proficiency [55, 56]. According to Rittle-Johnson, Siegler [57],

conceptual knowledge refers to the principle that governs a domain, and *procedural knowledge* refers to a sequence of actions to obtain a solution. The extent to which students have acquired procedural knowledge of one-step equations is reflected in their ability to solve one-step equations [1]. However, the acquisition of conceptual knowledge for one-step equations is concerned with students' understanding of the mathematical equivalence (i.e., "=" sign concept) with respect to both relational and operational lines [1]. Apparently, the relational understanding of the equal sign ("=") is critical to a student's success in solving equations [58].

We recently explored the issue of equal sign with reference to the two comparative pedagogical approaches, balance versus inverse. Using a two-group pretest-posttest experimental design, we found that the inverse group had no advantage over the balance group with regard to students' understanding of the equal sign for the relational line [1]. For example, presented with an equation such as $x + 6 = 11$, students could justify that the "=" sign indicated "balance, equal, etc." There are two ways of presenting the "=" sign concept with respect to the operational line: (i) balance method: $x + 3 = 5$, $x + 3 - 3 = 5 - 3$ and (ii) inverse method: $x + 3 = 5$, $x = 5 - 3$. When students were asked to judge whether a pair of equations was equivalent (e.g., balance method: $x + 3 = 5$, $x + 3 - 3 = 5 - 3$), both the balance and inverse groups performed better when the pair of equations was presented using the inverse method [1]. This evidence suggests that, in general, the differential element interactivity favors the inverse method for the operational line.

9. Achieving optimal best for one-step equations

Experience of optimal best in mathematics learning, according to Phan et al. [4], may involve demonstration of competence for not only the simple percentage problems but also percentage problems that are more complex. A realistic level of best practice, by contrast, reflects the demonstration of competence for simple percentage problems only. This conceptualization of achievement bests is significant and highlights variations in personal functioning in different subject domains of academia. A realistic level of best practice serves as a point of self-reference for determination and/or aspiration of an optimal level of best practice. A student's determination of his/her level of optimal best, in part, depends on what he/she is capable of, at present. In the context of mathematics learning, we postulate pedagogical practices (e.g., an appropriate instructional design), involving the impact of element interactivity and cognitive load imposition, that could associate with differing levels of best practice. For example, in relation to our discussion so far, we conceptualize that an optimal instructional design devised to assist in the achievement of optimal best in complex percentage problems would impose a lower level of element interactivity. Suboptimal instructional designs devised for a realistic level of best practice in simple percentage problems, by contrast, would impose a higher level of element interactivity. Furthermore, as noted, the level of element interactivity is directly proportionate to the degree of cognitive imposition [5, 6].

As discussed, the balance method imposes twice as many interactive elements as the inverse method for each operational. In regard to the acquisition of procedural knowledge in equation

solving, prior studies have revealed the superiority of the inverse method over the balance method for solving complex one-step equations, but not for simple one-step equations [48]. This testament has credence, given that the complex one-step equations have more operational lines (2 vs. 1) and relational lines (2 vs. 3) than the simple one-step equations. Moreover, the superiority of the inverse method over the balance method also extends to one-step equations involving special features (e.g., 12%x = 28).

In relation to the acquisition of conceptual knowledge in equation solving, we also found that the inverse method is better than the balance method when the "=" sign concept is applied to the operational line, but not to the relational line [1]. Thus, the inverse method is better than the balance method in facilitating the acquisition of both procedural and conceptual knowledge of one-step equations. This evidence provides empirical support for our proposition, regarding the *alignment between optimal instructions* (i.e., the inverse method) and *the demonstration of competence not only for simple one-step equations* (i.e., a realistic level of best practice) *but also for complex one-step equations* (i.e., an optimal level of best practice). At the same time, we propose an analogous *alignment between suboptimal instructions* (i.e., the balance method) and *the demonstration of competence for simple one-step equations* (i.e., realistic level of best practice).

An important question, certainly, entails the constructive application of the Framework of Achievement Bests in the context of academic learning. The Framework of Achievement Bests may provide grounding to assist educators in their teaching practices. This application may take into consideration the impact of cognitive load theory [5, 43], and its subsequent influence on the development of appropriate instructional designs. For example, the use of the inverse method is likely to assist middle-school students to achieve an optimal level of best practice to solve complex one-step equations. Consequently, competence in solving complex one-step equations may enable middle-school students to apply such skills to solve real-life problems. Consider a problem that reflects a real-life situation, for example: "Sally wants to invite her friends to her birthday party. She has 15 lollies and she wants to give three lollies for each friend. How many friends should Sally invite for her birthday?" A student, in this case, could use algebra to "set up" the equation – for instance: 15/x = 3, solve for x. Because this equation involves two operational lines and three relational lines, it is obvious then that the balance method would reflect high cognitive load imposition, and subsequently hinder students' learning. The inverse method, by contrast, would associate with low cognitive load imposition, enabling students to solve such equation.

10. Cognitive load and motivation

Worked example is one of the popular instructional designs that has extensively been researched [59]. The merit of worked example depends largely on its design. For example, we could use illustrations (e.g., a diagram) to represent the problem situation of the specific problem, which in turn would increase germane cognitive load and hence improve students' problem-solving skills [33, 41]. Aside from worked example, other pedagogical strategies to increase germane cognitive load include the *incorporation of self-explanation* [60], and *high contextual*

inference problem contexts [61]. Investing germane cognitive load to assist learning is in accordance with *deliberate practice*, whereby engagement in practice activities serves to assist learners to develop *expertise* in the domain [62]. Having said this, difficulties may arise, as van Gog, Ericsson [62] argued, whereby learners' lack of motivation may deter their willingness to invest germane cognitive load, and/or to engage in deliberate practice activities with a view to improve learning.

A review of the empirical literature indicates that, to date, research development into the relationship between germane cognitive load and students' motivational beliefs is inconclusive. The *Goal-based Scenarios* (GBS) technique used in multimedia instructional designs, for example, is advantageous by motivating learners to study the instructional material, which then leads to improved understanding of the material, in total [63]. Nevertheless, despite this pedagogical initiative, there is little, if any, association between students' motivational beliefs and their perceived increase in germane cognitive load. In another study, however, Rey and Buchwald [64] found that the *probability of success*, a subdimension of motivation, was partially associated with the investment of cognitive load in learning. On this basis, evidence pertaining to the relationship between motivational beliefs and investment of germane cognitive load is inconclusive and requires further research development. For example, in a recent development, we proposed a theoretical model that conceptualized the relationships between optimal and suboptimal instructional designs (e.g., varying levels of element interactivity), and levels of best practice (varying levels of motivation) in the domain of percentage problems. Our theorization, as shown in **Figure 3**, is holistic and seeks to illuminate the combined effects of cognitive (e.g., cognitive load imposition), affective (e.g., a heightened state of anxiety), and motivational (e.g., personal self-efficacy beliefs) dimensions of effective learning to facilitate optimal best in the percentage problems.

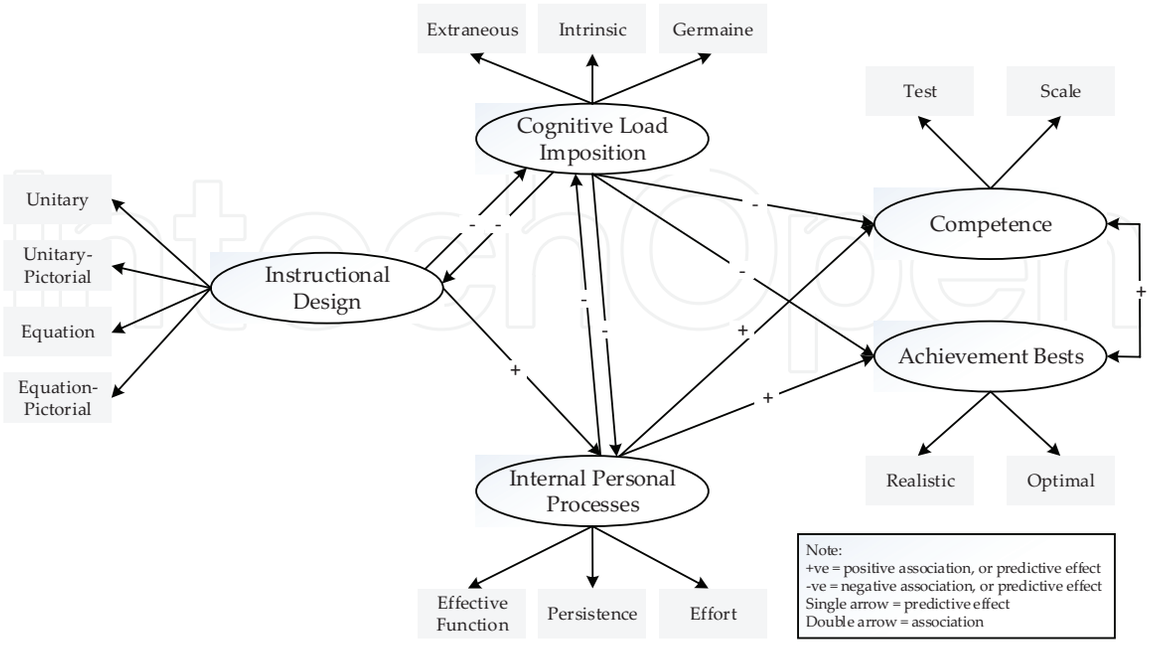

Figure 3. Proposed relationships between instructional designs, cognitive load, motivational processes, and achievement bests. Source: Adapted from Ref. [4].

11. Cognitive load imposition, internal personal processes, and achievement bests

An important focus of inquiry for development entails the potential associations between cognitive load impositions, internal personal processes of learning, and levels of best practice. This development, reflected in our recent conceptualization [4], indicates a concerted effort to integrate three major strands of research, namely cognitive processes, motivational beliefs and affective dimensions, and achievement bests. We urge researchers and educators to consider this theoretical model for research development. This empirical validation is worth noting and may indicate significance regarding the impact of an integration of different strands of inquiries. For example, in relation to affective responses, Ashcraft and Kirk [65] found that heightened anxiety levels negatively influenced the working memory capacity to process different types of mathematic-learning tasks. It is plausible to assume that a proportion of the work memory resources is used to "counter" the heightened state of anxiety, and on this basis, very little is left for processing of information. Similar evidence has been reported in a simulation training study in the area of Medical Education [66]. In this study by Fraser et al. [66], the authors found that negative affective responses (e.g., anxiety) increased extraneous cognitive load imposition, which then led to a decrease in the working memory capacity for learning. However, from the results, the relationship between cognitive load imposition, positive emotions, and learning outcomes was less predictable.

Our theorization, as shown in **Figure 3**, has a number of proposed associations for consideration. Central to our conceptual model is the recognition and inclusion of the two major theories: cognitive load imposition [5, 43] and achievement bests [4, 11]. Importantly, of course, a focus of inquiry may involve the use of both theories to inform the development of appropriate pedagogical practices (e.g., an instructional design) to promote effective learning experiences. For example, suboptimal instructional designs (e.g., the balance method), which directly associate with negative cognitive load imposition, could have adverse effects on motivational beliefs and achievement of optimal best (e.g., the achievement of realistic best practice only, which, in this case, may involve simple one-step equations).

In relation to what we have discussed so far, it is evident that in the context of mathematics learning, comparative instructional designs may have differing effects on students' understanding. Future research undertakings may pursue this inquiry, delving into the relationships between comparative instructional designs (e.g., balance vs. inverse) and levels of best practice. This postulation, emphasizing two contrasting associations (i.e., balance method ↔ simple one-step equations vs. inverse method ↔ complex one-step equation, where ↔ = closely aligned association), is of value for testament, especially when we consider its potentials to influence motivational beliefs and affective responses. Our argument, overall, based on previous research development, is that *the inverse method is superior to the balance method for effective learning*. This conviction, we contend, draws to the fact that the inverse method (i) imposes low cognitive load imposition, enabling a learner to understand both simple and complex one-step equations, (ii) elicits positive

affective responses (e.g., happiness), consequently as a result of a learner's ability to understand and to demonstrate the mastery of complex one-step equations, and (iii) reflects, correspondingly, a high score on the Optimal Best Subscale of the Optimal Outcomes Questionnaire (e.g., I feel positive when I am asked to solve complex one-step equations).

We recently developed, as mentioned, the Optimal Outcomes Questionnaire [29], which has two subscales, the Realistic Best Subscale (i.e., consists of eight items) and the Optimal Best Subscale (i.e., consists of eight items). Aside from focusing on the importance of "profiling" of best practice [11, 28], we contend that this questionnaire could measure and assess students' motivational levels and affective responses as a result of their exposures to different instructional designs. This recognition, which we recommend for further research advancement, indicates the importance of *diagnostic assessment* of motivational levels and achievement bests that arise from varying levels of cognitive load imposition.

Appendix

Solution procedure for one-step equations that have special features in the test items

Equation type	Balance method
Negative numbers	$a-2 = 3$ $+2 \quad +2$ $a = -1$
A decimal number	$\frac{x}{0.5} = 5$ $\times 0.5 \quad \times 0.5$ $x = 2.5$
A percentage	$10\,\%\, x = 20$ $\div 10\,\% \quad \div 10\%$ $x = 200$
A fraction or a decimal as a solution	$3m = 2$ $\div 3 \quad \div 3$ $m = 2/3$
Pronumeral on the right side	$1 = 2p$ $\div 2 \quad \div 2$ $0.5 = p$
**Negative pronumeral*	$6 - q = 10$ $-6 \quad -6$ $-q = 4$ $\div(-1) \quad \div(-1)$ $q = -4$
**Pronumeral as a denominator*	$\frac{4}{a} = 2$ $\times a \quad \times a$ $4 = 2a$ $\div 2 \quad \div 2$ $2 = a$

Note: The solution procedure of those equations marked by * has two operational lines (e.g., −6 on both sides, and ÷(−1) on both sides) and thus impose higher element interactivity than other equations that have one operational line (e.g., +2 on both sides).

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