Chapter 5

Ricardo's Law of Comparative Advantage and the Law of Association: A Subjective Analysis

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Additional information is available at the end of the chapter

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Abstract

The law of association, which is a generalization of Ricardo's law of comparative advantage, is one of the most fundamental laws in economics, which explains the benefits of international trade in the macroscopic level and the division of labour in the microscopic one. However, the derivation of the law is traditionally based on aggregate production criterions rather than on the producers' subjective preferences. An economic law, which ignores subjective preferences cannot be regarded as a fundamental one. In this chapter, a subjective analysis of the law is presented, to the best of our knowledge, for the first time. It is shown that when subjective considerations are introduced the tendency to trade can be reduced. An algorithm is presented to illustrate the dynamics of the process, in which the information regarding the subjective preferences is transferred via the previous trading prices. Furthermore, the effect of specialization on the production frontiers, which is absent in most economics derivations of the law, is taken under consideration. It is shown that even if both producers are identical a non-trading state is unstable. It is therefore shown that counter to mainstream thinking, *comparative advantage is neither necessary nor is it a sufficient condition for trading*.

Keywords: Ricardo, law of comparative advantage, law of association, subjective preference ranking, division of labour, trading

1. Introduction

The law of association (LA), which is a generalisation of Ricardo's well-known law of comparative advantage (LCA) [1–4], can be regarded as one of the main corner stones of both micro- and macroeconomics. In the microscopic regime, it explains the motivation for basic trade, the division of labour, allocations of goods and production preferences. In the macroscopic realm not only does it shed light on international trading, but it is also a clear testimony for free trading and low tariffs. Despite its importance, this law is missing in most microeconomics and macroeconomics textbooks.

The LCA is usually known as Ricardo's law and we will also refer to it as such, albeit historical justice requires citing the fact that it should have been attributed to James Mill (for a discussion on this issue see Refs. [5, 6]).

Usually, the interested economists will find a discussion on the LCA in textbooks on international trading [7] or textbooks on general economics [8]. However, in these cases, the discussion usually focuses on international trading with a clear neglect of the law's implication on interpersonal exchange, namely, the LA.

A clear exception for this omission is the Austrian school of economics, which elaborates on the LA even in its most basic textbooks (see Refs. [9, 10]). They used this law, as was initially meant by Ricardo, to advocate for free trade and free interpersonal association, and as a tool to explain the process behind the division of labour between nations and individual.

However, there is some discrepancy in the Austrian adoption of the LCA. The Austrian school's approach is to avoid aggregate economic parameters, and yet, they use the LA to show that the aggregate production is raised due to trade. But does it mean that the conditions of both producers were improved as a consequence of trading? Despite Mises and Rothbard's attempt to claim that the LA does indicate mutual benefit, a complete Austrian analysis requires a subjective treatment, which is absent in their writings.

In all their writings, the Austrian economists stress that it is meaningless to discuss mutual utility. Only individual utility has a meaning. Moreover, they stress that utility has only ordinal meaning and not cardinal one [9, 10]. One can say that he prefers A over B, but it is meaningless to say by how much.

Therefore, even comparison between different individual's utility is, according to the Austrian school, meaningless. It should be stressed that this understanding was adopted by most modern schools of economics (on this subject see Ref. [11]).

The fact that two producers produce more of certain goods does not mean that they are better off. The problem can easily be emphasised by the following example: Suppose one individual is expert in making bread, and the other one is expert in making mud pies. Should one conclude from the LA that they both have to focus on the product they are best in producing? Surely not. The mud pie is useless for both. In this case, they both need to be focusing on producing bread. There is no point in wasting resources on producing mud pies.

Actually, it was Böhm-Bawerk, one of the prominent leaders of the Austrian school [12], who used the 'mud pie argument' to attack the classical economists in general and Karl Marx in particular for using the labour theory of value. The fact that a product 'costs' a certain amount of labour does not mean that it has some value.

The erroneous conclusions are a direct result of the absence of any subjective utility analysis in the derivation of the LA.

For a complete analysis, the subjective scale of preferences of the two producers/traders has to be incorporated in the analysis of the LA. The problem is that the traditional Austrian analysis is based on verbal arguments or very basic preference schedule tables. An extensive subjective Austrian treatment must incorporate two-dimensional preference matrices along the Ricardian's argument.

Another discrepancy arises in the literature in connection to specialisation. It was stated very clearly by many economists and can easily be grasped by the layman that specialisation increases the productivity of every one of the merchants prior to trading. Nevertheless, this effect is also neglected in the analysis or, at best, analysed separately from the benefits of trading.

See, for example, two contemporary economists (pages 4 and 48 in Ref. [7] and even Ref. [13]), which both recognise that specialisation increases the productivity of each one of the producers (not only the aggregate productivity), but they fail to incorporate this point in the LA analysis.

It is the object of this chapter to fix these two problems and to analyse the LA with subjective preferences and with the effect of specialisation.

2. The traditional Ricardo's law of comparative advantage

Let there be two individuals (1 and 2), both of them can produce two consumption commodities: A and B.

Let the maximum number of units of good A and of good B that the first individual (hereinafter we will adopt the title 'producer') produces are A_1 and B_1, respectively. Similarly, the maximum number of units of the same goods (A and B) that the second producer produces are A_2 and B_2, respectively. Therefore, the first producer is constrained by the equation

$$\frac{a_1}{A_1} + \frac{b_1}{B_1} \leq 1 \tag{1}$$

where a_1 and b_1 are the number of units the first individual produces, and similarly

$$\frac{a_2}{A_2} + \frac{b_2}{B_2} \leq 1 \tag{2}$$

where a_2 and b_2 are the number of units the second individual produces.

These constrain equations are usually termed: the production possibility frontier (PPF).

Now if $A_1 > A_2$, but $B_1 < B_2$, then it is clear that the first producer has an absolute advantage over the second one in producing units of good A and vice versa in producing units of good B. In this case, it is clear that trading will be beneficial to both producers.

The novelty of the LA is the notion that even in the case where $A_1 > A_2$ and $B_1 > B_2$, where clearly the second producer has no absolute advantage in the production of either commodities, they still can benefit from exchange.

$$\text{If } \frac{A_2}{B_2} < \frac{A_1}{B_1} \tag{3}$$

then, the first producer has a comparative advantage in producing A, while the second producer has a comparative advantage in producing B. It is easy to see that in this case, there is a common interest for the exchange. Suppose that ΔA units of A are exchanged for ΔB units of B, i.e. the first producer sells ΔA units of A for ΔB units of B. Clearly, the first producer would agree to this exchange provided the price, i.e. the ratio

$p \equiv \Delta A/\Delta B$ obeys the inequality

$$p = \frac{\Delta A}{\Delta B} < \frac{A_1}{B_1} \tag{4}$$

Otherwise, this producer can produce the commodity instead of buying it. Similarly, the second producer would agree to this exchange provided the price is larger than

$$p = \frac{\Delta A}{\Delta B} > \frac{A_2}{B_2} \tag{5}$$

for exactly the same reason.

Therefore, if inequality (3) holds, then there is a price regime in which they will both benefit from the exchange. This is the traditional LA.

3. Production maximization analysis

Clearly, aggregate production analysis cannot justify the subjective behaviour of the producers, however, the improvement in the producers' status can be quantified by the excess production with respect to the producer's PPF. The reason for that is the producer is indifferent to its position on the PPF. Thus, any improvement in its status is achieved by advancing in the perpendicular direction to the production frontier.

If after trading the first producer has a_1 units of A and b_1 units of B and the second producer has a_2 units of A and b_2 units of B, then the distances between their current status and their PPF (which quantifies their production improvement) are

$$\Delta D_1 = \frac{a_1/A_1 + b_1/B_1 - 1}{\sqrt{A_1^{-2} + B_1^{-2}}} \text{ and } \Delta D_2 = \frac{a_2/A_2 + b_2/B_2 - 1}{\sqrt{A_2^{-2} + B_2^{-2}}}, \tag{6}$$

respectively. After trading ΔA units of A for ΔB units of B

$$\Delta D_1 = \frac{-\Delta A/A_1 + \Delta B/B_1}{\sqrt{A_1^{-2} + B_1^{-2}}} \text{ and } \Delta D_2 = \frac{\Delta A/A_2 - \Delta B/B_2}{\sqrt{A_2^{-2} + B_2^{-2}}}. \tag{7}$$

One can therefore evaluate the price p^*, for which both producers have the same gain, i.e. $\Delta D_1(p^*) = \Delta D_2(p^*)$,

$$p^* = \frac{B_2\sqrt{A_1^2 + B_1^2} + B_1\sqrt{A_2^2 + B_2^2}}{A_2\sqrt{A_1^2 + B_1^2} + A_1\sqrt{A_2^2 + B_2^2}} = \frac{\cos\theta_1 + \cos\theta_2}{\sin\theta_1 + \sin\theta_2}, \tag{8}$$

$$\text{where } \tan\theta_1 \equiv A_1/B_1 \text{ and } \tan\theta_2 \equiv A_2/B_2. \tag{9}$$

At this price, the production gain of both producers is equal to

$$\Delta D_1(p^*) = \Delta D_2(p^*) = \Delta A\frac{\sin(|\theta_2 - \theta_1|)}{\sin\theta_2 + \sin\theta_1} = \Delta B\frac{\sin(|\theta_2 - \theta_1|)}{\cos\theta_2 + \cos\theta_1} \tag{10}$$

4. Subjective analysis

However, clearly something is missing in these production analysis. It is clear that production in itself is not the economic goal. Hence, in what sense, the producer condition is better after the exchange than before it?

One must assume that while the individuals have a comparative advantage in the production of one of the goods, they want or need both of them, and in the process of analysing the best option to act (producing or a combination of producing and trading), the individual chooses the option, which yields the best combination of goods. But what is the best combination? An evaluation method is required.Historically, the tool for situation evaluation was the utility function. However, as was realised by the Austrian school of economics and was later accepted among most economists [11], the situation preference ranking cannot have a cardinal meaning (as the utility function suggest) but only ordinal one.

The problem is, that creating a list, i.e. a table, of preferences, when there are multiple parameters or many degrees of freedom, is doable, but cumbersome and complicates the economic analysis. This may be the reason, that Rothbard, which used several times lists of preferences, used them only in relatively simple cases. In the problem under discussion, the actors are both producers and traders. Their decisions are based on two stages.

In what follows, we introduce the actors' ranking matrices: $R_1(a_1,b_1)$ and $R_2(a_2,b_2)$. That is, every state of the first producer is described by two parameter: the number of units in his/her possession of commodity A (a_1) and of commodity B (b_1). Similarly, the states of the second producer are described by the equivalent parameter a_2 and b_2, respectively. Therefore, instead of presenting the scenarios as a single list, which includes all options, we present them with two two-dimensional matrices $R_1(a_1,b_1)$ and $R_2(a_2,b_2)$.

Clearly, since A and B are goods, the utility increases with the number of units, i.e.

$$R_n(a_n + 1, b_n) > R_n(a_n, b_n) \text{ and } R_n(a_n, b_n + 1) > R_n(a_n, b_n) \tag{11}$$

for $n = 1,2$.

In the limit where the units of the goods are arbitrarily small, the continuum limit can be used, in which case, Eq. (11) can be written as $\frac{\partial R_n(a_n, b_n)}{\partial a_n} > 0$ and $\frac{\partial R_n(a_n, b_n)}{\partial b_n} > 0$.

The law of diminishing marginal utility (LDMU) is traditionally formulated by demanding a concave shape for the utility function. However, in the absence of a utility function, it is meaningless to apply this criterion on the preference ranking matrix. A better approach is to notice that the decline in the marginal utility of a certain good is actually manifested by the relative increase in the ranking of *other goods*. Therefore, the LDMU can be stated mathematically as

$$R(a, b + 1) - R(a, b) \leq R(a + 1, b + 1) - R(a + 1, b)$$

Or, equivalently, in a more symmetric form

$$R(a, b + 1) + R(a + 1, b) \leq R(a + 1, b + 1) + R(a, b) \tag{12}$$

Similarly, Eq. (12) can be rewritten in the continuum limit as $\frac{\partial^2 R(a, b)}{\partial a \partial b} \geq 0$.

Any individual would prefer to increase the value of its current ranking $R_n(a_n,b_n)$ (for $n = 1,2$) by changing his/her state parameters a_n and b_n (by producing and trading goods).

That is, if $R_n(a_n, b_n) > R_n(\overline{a}_n, \overline{b}_n)$, then the nth producer would prefer the state (a_n,b_n) over the state $(\overline{a}_n, \overline{b}_n)$. Clearly, if both $a_n > \overline{a}_n$ and $b_n > \overline{b}_n$, then $R_n(a_n, b_n) > R_n(\overline{a}_n, \overline{b}_n)$; however, in many cases, the ranking is improved even when $a_n > \overline{a}_n$ but $b_n < \overline{b}_n$ or when $a_n < \overline{a}_n$ and $b_n > \overline{b}_n$, which depends on the subjective ranking of both individuals.

Prior to trading the producer needs to produce the goods. The decision on the amount to produce depends on the ranking matrix under the relevant constrictions (1) and (2).

In **Figure 1**, such a two-dimensional ranking matrix is illustrated. For simplicity, we assume that the two producers have the same ranking, i.e. $R_1(a,b) = R_2(a,b)$ for any (a,b); however, this is not a restrictive assumption. Despite the fact that in this example, the two producers have the same preference ranking, their production's decision is different due to their different

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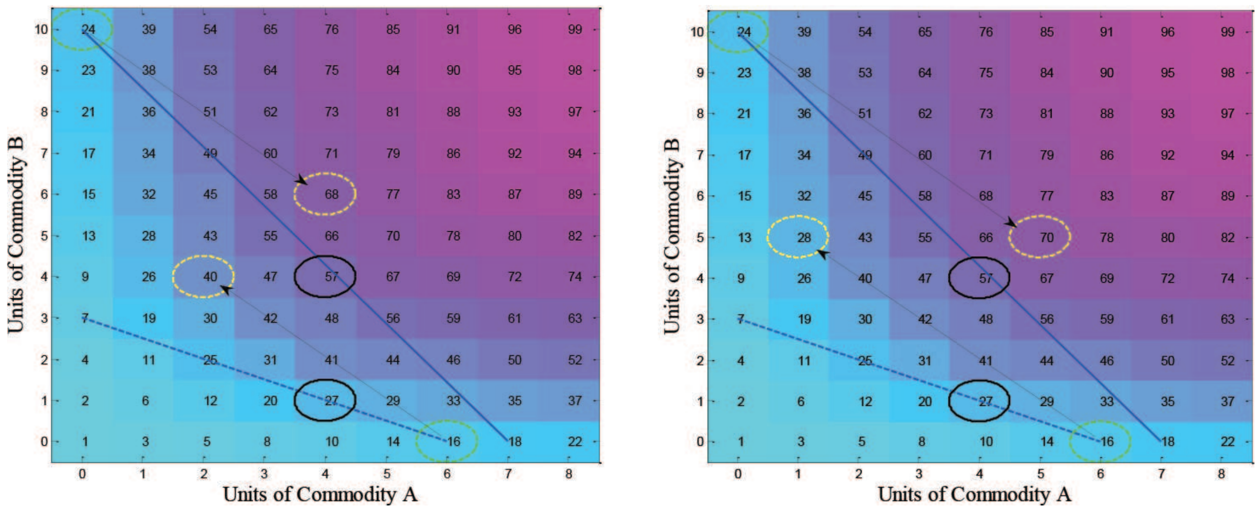

Figure 1. Two possible scenarios of trading. In the scenario on the left, the two producers temporally worsen their ranking, which decreases from 27 and 57 (without trading) to 16 and 24, respectively, to increase it to 40 and 68, respectively, after trading. On the right, the number of units, which exchange hands, was increased. As a result, the ranking of the first producer decreases to 28 but the ranking of the second producer increases to 70.

production abilities (different PPFs). Trading will take place provided there are Δa and Δb, which can be either positive or negative, so that

$$R_1(a_1 - \Delta a, b_1 + \Delta b) > R_1(a_1, b_1) \tag{13}$$

$$R_2(a_2 + \Delta a, b_2 - \Delta b) > R_2(a_2, b_2) \tag{14}$$

In the example presented in **Figure 1**, $A_2 = 7$, $B_2 = 10$, $A_1 = 6$ and $B_1 = 3$. Therefore, without trading, the best ranking that the production constrains allow for the first producer is $R_1(a_1 = 4, b_1 = 1) = 27$ and for the second producer is $R_2(a_2 = 4, b_2 = 4) = 57$. Since $0.7 = A_2/B_2 < A_1/B_1 = 2$, there is an advantage for the first producer to produce extra units of A and to sell them to the second producer for units of B.

In **Figure 1**, two such options are presented. In both cases the producers decided to specialise in a single product, the one which they have a comparative advantage with. By specialising they knowingly decreases *temporarily* their preference ranking. The preference ranking of the first producer reduces temporarily from the maximum 27 to 16, and the ranking of the second producer reduces from the maximum value 57 to 24. After trading, there is a substantial increase in the preference ranking. In the left scenario, the first producer's preference ranking increases to 40 and that of the second one increases to 68. It is shown that if the second producer wishes to increase its preference ranking even further to 70, it must be on the account of a substantial reduction in the preference ranking of the first producer (to 28), albeit it is still higher than the pre-trading maximum ranking (27). As was emphasised in Mises and Rothbard writings [9, 10], the final state depends on the bargaining merits of the two producers (now merchants). However, while Mises and Rothbards emphasised that the price is a matter of bargaining, they ignored the fact that the *amount of exchange good* is also a matter of bargaining *even for the same price*. In the two scenarios, which are presented in **Figure 1**, the exchange price is the same, however, there is a difference in the number of units, which took place in the exchange. In both cases the price is 1 (1 units of A for 1 units of B), however, while the first producer prefers exchanging 5 units, the second producer prefers exchanging 6 units. So, the bargaining is not on the price but on the number of units, and the problem is that when the first one gains the second one losses and vice versa. This contradiction of wants did not occur in Mises and Rothbard writings because they *did not take the preference ranking into account*.

In **Figure 2**, we see two additional behaviours, which were neglected or ignored in previous writings. In the left panel, we can see a scenario in which non-specialisation yields better outcomes to both participants. The preference ranking of the first producer is improved (from 28 in the right panel of **Figure 1** to 30), and even the second producer gains, for this production scheme (1 unit of A and 8 units of B), do not consume all his temporal resources (since 8/10 + 1/7 < 1).

In the right panel of **Figure 2**, all the possible outcomes after trading are plotted by light circles. As can be easily seen, not all options, which were predicted by the LA are allowed and again the discrepancy is the fact that the traditional LA ignores the subjective preference scaling. For example, consider the scenario, in which they both specialise, i.e., the first producer produces 6 units of A and the second producer produces 10 units of B, and then 2 units of A are exchanged for 2 units of B. On the face of it, if the LA is considered, this is a legitimate transaction. However, while the first producer gains from that exchange (his preference ranking increases from the previous maximum of 27 to 41), the second producer clearly loses (from 57 to 51), and therefore, he/she will have no motivation to participate in such a transaction. He/she may, however, decide to produce 2 units of A and 7 of units of B and then trade 2 (or even better 3) units of A to 2 (or 3) units of B. In which case, they both gain (the first producer's preference ranking increases to 41 or 42 and the preference ranking of the second one increases to 66 or 67).

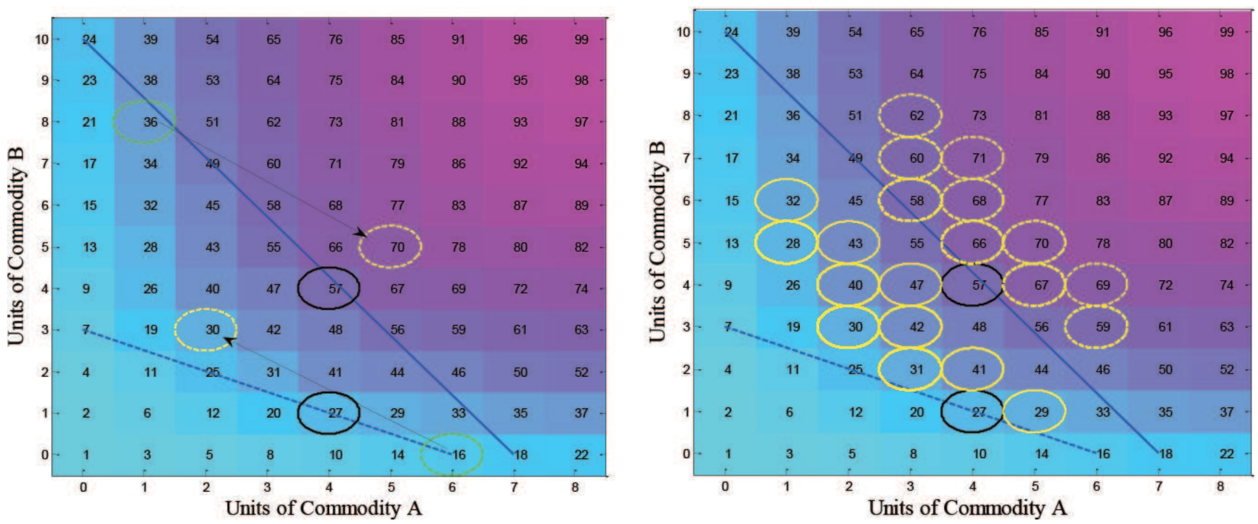

Figure 2. On the left panel, the final scenario is better than the right scenario in **Figure 1** despite the fact that the second producer produces less. On the right panel all the final trading scenarios are plotted. Due to the ranking matrix not every transaction is possible.

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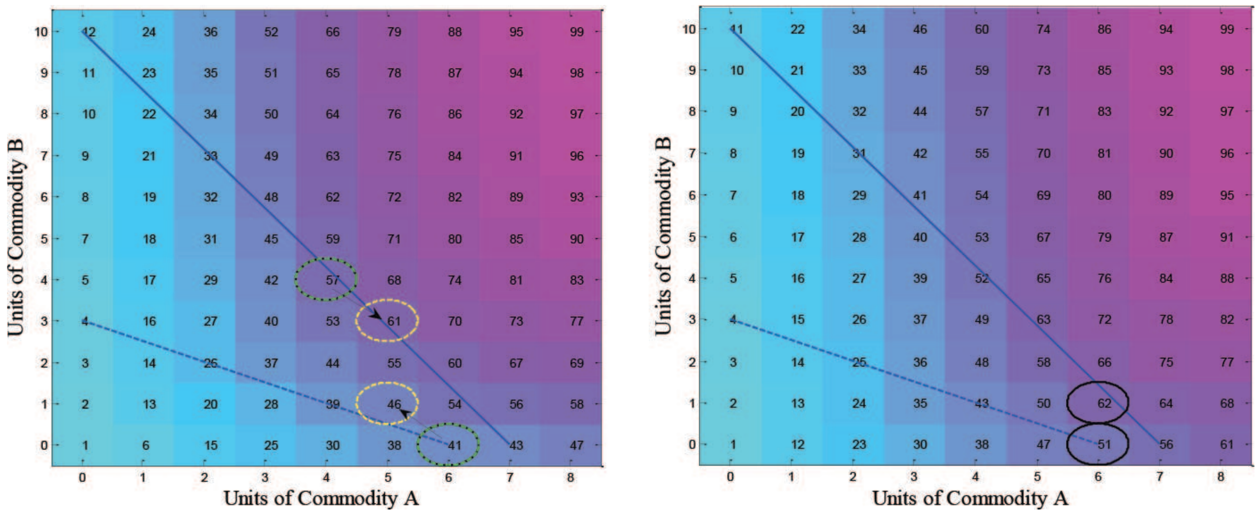

Figure 3. Two asymmetric scenarios. Despite the fact that there is a clear comparative advantage of the two producers, and in principle there are numerous trading options, in practice, due to the asymmetry between the utility of the two commodities (commodity A is more desirable than commodity B); then, the trading options are extremely limited. In the left panel there is only one option for trading, and in the right panel there is none.

In the previous ranking, the matrix was approximately symmetric with respect to the diagonal (for example, the ranking of 7 units of A and 2 of B is 50, while the ranking of 2 units of A and 7 of B is similar, i.e. 49). In **Figure 3**, this pseudo symmetry is broken. In the two scenarios, which are presented in **Figure 3**, the two producers clearly prefer units of A over units of B, and therefore trading is substantially depressed. In the left scenario, there is only one option for trading, and in any case, there is no clear motivation to the second producer to produce more units of B.

In the right scenario of trading is suppressed completely. The first producer cannot make anything which can motivate the second producer to trade with. In this latter scenario, the suppression of trading reduces also the motivation for the division of labour. Both producers behave like separate entities.

5. Dynamics

The main dilemma, which the producers must resolve, is that by specialisation they have to take a risk. When they specialise, they produce too many products which they do not need, and therefore, they temporarily reduce their preference ranking. The source of the problem is that they do not know the preference ranking of the *other* producers. It seems contradictory to base specialisation on subjective analysis, since if the producer is familiar only with his own preference scale, then how can he judge, what would the other producer want to buy from him?

Clearly, in a single trading event, this dilemma has no solution; however, in successive trading events (multiple iterations), the dilemma is solved, since *objective* information is transferred via the *price* of the previous trade.

The producers/traders follow the following *algorithm*. The algorithm consists of three stages: the initial condition, the entrepreneurial stage, and the bargaining stage. The last two stages (B and C) are repeated iteratively between one trading event and the next.

A. Initial state

$n = 0$ (iteration number)

Initial production values

$a_1^{(0)} = a_1^*, b_1^{(0)} = b_1^*, a_2^{(0)} = a_2^*, b_2^{(0)} = b_2^*$ (the asterisks stand for the best values prior to trading)

Initial price value

$\Delta A^{(0)} = \Delta A^*, \Delta B^{(0)} = \Delta B^*$ (the initial price is guessed by the producers)

B. Entrepreneurial stage

$n \leftarrow n + 1$ (increment the iteration index)

Each producer checks different production working points, i.e. they check the effect of different increment/decrement Δa_1 and Δa_2 (under the constrain of fixed prices, i.e. given $\Delta A^{(n-1)}$ and $\Delta B^{(n-1)}$).

Mathematically, it means vary Δa_1 and if

$$R_1\left(a_1^{(n-1)} + \Delta a_1 - \Delta A^{(n-1)}, b_1\left[a_1^{(n-1)} + \Delta a_1\right] + \Delta B^{(n-1)}\right) > R_1\left(a_1^{(n-1)}, b_1\left[a_1^{(n-1)}\right]\right)$$

then

$$a_1^{(n)} = a_1^{(n-1)} + \Delta a_1$$

Similarly, vary Δa_2 and if

$$R_2\left(a_2^{(n-1)} - \Delta a_2 + \Delta A^{(n-1)}, b_2\left[a_2^{(n-1)} + \Delta a_2\right] - \Delta B^{(n-1)}\right) > R_2\left(a_2^{(n-1)}, b_2\left[a_2^{(n-1)}\right]\right)$$

Then,

$$a_2^{(n)} = a_2^{(n-1)} + \Delta a_2.$$

where $b_1[x]$ and $b_2[x]$ represents the production frontiers of the two producers, i.e.

$$b_1[x] \leq floor[B_1(1 - x/A_1)]$$

$$b_2[x] \leq floor[B_2(1 - x/A_2)]$$

where *floor*[] is the floor rounding function (rounds the argument to the nearest integers towards minus infinity).

*Note that in this stage, every producer's decisions depend only on his/her own preference ranking matrix.

C. Bargaining stage

During the bargaining stage, the price values (ΔA and ΔB) vary until *both* conditions

$$R_1\left(a_1^{(n)} - \Delta A, b_1\left[a_1^{(n)}\right] + \Delta B\right) > R_1\left(a_1^{(n-1)}, b_1\left[a_1^{(n-1)}\right]\right)$$

and

$R_2\left(a_2^{(n)} + \Delta A, b_2\left[a_2^{(n)}\right] - \Delta B\right) > R_2\left(a_2^{(n-1)}, b_2\left[a_2^{(n-1)}\right]\right)$ apply.

In which case

$$\Delta A^{(n)} \leftarrow \Delta A$$

$$\Delta B^{(n)} \leftarrow \Delta B$$

D. Update the parameters and go back to the entrepreneurial stage (B)

It should be stressed that the iterations are essential for successful trading. Without iterations no knowledge can be transmitted between the producers, and the entrepreneurial act would be futile. A similar iterative process can formalise the Mengerian [14] and Misesian [15] origin of money.

6. The effect of specialisation

Plato [16] attributed the division of labour to the diversity in people's merits, i.e., the baker specialises in making bread, while the carpenter specialises in making tables because the baker has a talent for making bread and the carpenter has the talent for making tables. Smith [17] emphasised that the division of labour does not rely on diversity in the population inborn talents. The division of labour itself is beneficial and creates wealth to the community.

The classical LA does not take this effect into consideration. But the comparative advantage itself is a function of specialisation.

If when specialising the first producer can produce A_1 units of commodity A or B_1 units of commodity B, it does not mean that he/she can produce $A_1/2$ units of A and $B_1/2$ units of B. In fact, the joint production must be *lower* than that.

Therefore, formulation of the production possibility frontier by a straight line is a very loose constrain. In fact, the real constrain curve is more convex; an example of which is illustrated in **Figure 4**.

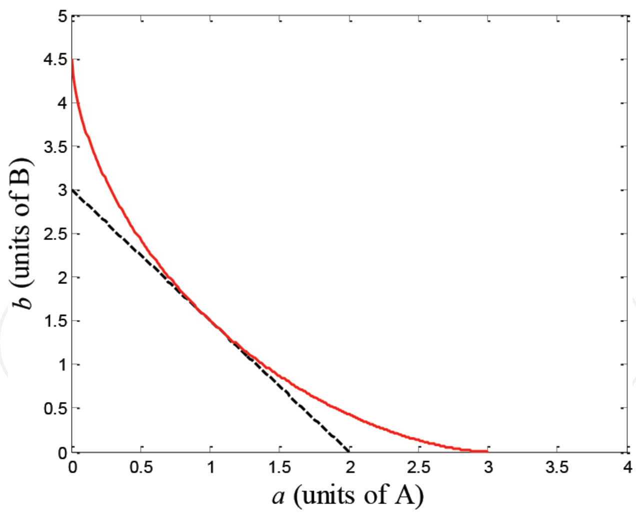

Figure 4. The effect of specialisation on the production frontier. The dashed line corresponds to the case before specialisation (in this case $A = 2$ and $B = 3$), while the solid curve stands for the convex production frontier in the presence of specialisation with the specialisation production gain factor of $F = 1.5$ (for which case $\alpha = 0.6309$).

We therefore present here, to the best of our knowledge, for the first time a mathematical presentation of a more realistic formulation of the production frontiers. Instead of Eqs. (1) and (2), the PPF can be written

$$\left(\frac{a_1}{A_1}\right)^{\alpha_1} + \left(\frac{b_1}{B_1}\right)^{\beta_1} \leq 1 \text{ and } \left(\frac{a_2}{A_2}\right)^{\alpha_2} + \left(\frac{b_2}{B_2}\right)^{\beta_2} \leq 1 \tag{15}$$

where α_1, β_1, α_2 and β_2 are constants smaller than 1.

In this case, the generic dynamics are essentially similar to the previous section except for the change in the production frontiers, namely

$$b_1[x] \leq floor\left\{B_1\left[1 - \left(\frac{x}{A_1}\right)^{\alpha_1}\right]^{1/\beta_1}\right\} \text{ and } b_2[x] \leq floor\left\{B_2\left[1 - \left(\frac{x}{A_2}\right)^{\alpha_2}\right]^{1/\beta_2}\right\}. \tag{16}$$

It can easily be shown that if $\alpha = \beta$, then the production is increased by a factor (see **Figure 4**)

$$F = 2^{1/\alpha - 1}. \tag{17}$$

In other words, if without specialisation the production frontier is bounded by $a/A + b/B = 1$, and specialisation increases its production by a factor of F, then the new production frontier under specialisation is

$$\left(\frac{a}{AF}\right)^{\alpha(F)} + \left(\frac{b}{BF}\right)^{\alpha(F)} = 1 \text{ when } \alpha(F) = \frac{1}{1 + \log_2 F}. \tag{18}$$

In **Figure 4**, the effect of specialisation on the production frontier is illustrated for $F = 1.5$, which corresponds to $\alpha = 0.6309$.

In **Figure 5**, for example, $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0.5$, which according to Eq. (17) corresponds to a production gain factor of F = 2. In general, the better the specialisation or the longer the internship required, the smaller is the related exponent, for example, if the second producer is an expert in producing A, then α_1 is small.

In this scenario, prior to trading, the first producer chooses to produce only six units of A and the second producer only two units of A and two of B. However, the knowledge of possible future trading persuade the second producer to abandon the production of A and to specialise only in B. After trading their preference, ranking is improved considerably from 16 to 40 for the first producer and from 25 to 68 for the second one.

The convexity of the specialisation curve increases dramatically the trading possibilities. As the right panel of **Figure 5** illustrates (compare it to **Figure 2**), there is another important conclusion from this analysis. The specialisation itself creates the incentive for exchange. In the classical LA, there should be a diversity in the producers' abilities. But this analysis demonstrates that even if the producers are initially *identical in any respect*, i.e. they have the same preference schedule (same ranking) and the same production constrains, the convexity of the production frontier creates the incentive for exchange.

For example, suppose the two producers have the same production capabilities, i.e. their production constrains are:

$$\sqrt{\frac{a}{7}} + \sqrt{\frac{b}{10}} \leq 1 \tag{19}$$

where a and b stand for both producers, i.e. for either (a_1,b_1) and (a_2,b_2).

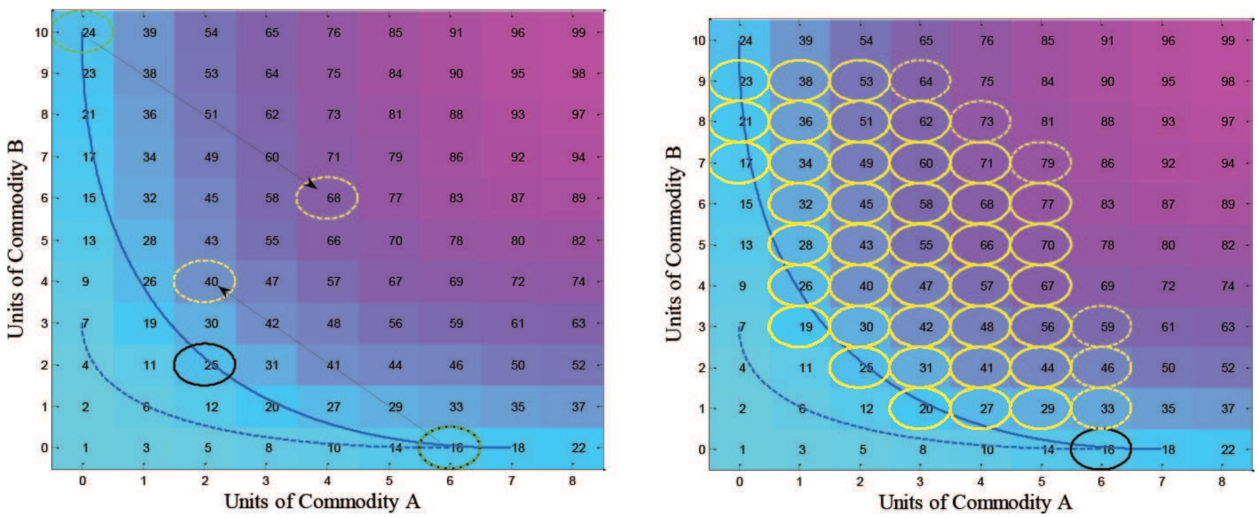

Figure 5. The effect of specialisation on the constrain curves and on the possibilities for trading. Due to specialisation, the production constrains curve become convex. On the left panel such a scenario is illustrated. The dotted and the solid curves represent the production boundaries of the first and second producers, which in the absence of trading the maximum achievable preference ranking is 16 and 25, respectively. With the possibility of trading the motivation for specialisation increases. Specialisation temporarily reduces the ranking of the second producer to 24, and after trading their ranking increases to 40 and 68, respectively. In the right panel all the possible trading scenarios are presented by light circles.

In which case without trading, they both prefer to produce two units of both A and B (gaining a ranking 25); however, this is not a stable situation, because the first entrepreneur, which will decide to produce more than 5 (6–10) units of B, which should not be too complicated a task, because his/her production limit is 10 units of B, can trade 2 (or more) units of B for a single units of A and to improve his/her preference ranking at least to 26. A similar argument applies to an entrepreneur, who decides to manufacture more than 5 units of A. He/she can trade 2 (or more) units of A for a single unit of B and to improve his/her ranking to (at least) 27. Moreover, the process cannot stop here, while one entrepreneur decides to specialise in one commodity, the motivation for the second one to specialise in the other commodity increases. The dynamic process, which is described in Section 5, can stop only at full production (note that leisure cannot be regarded as a commodity in this simple model) when one produces 7 units of A and the other produces 10 units of B. The eventual state of the two producers depends on their bargaining skills and cannot be determined *a priori*.

7. Instabilities

When applying the entrepreneurial-trading algorithm (Section 5) on the specialisation case, it can be shown that stable state in the absence of trading, i.e. the state the producer choose without the option of trading, becomes unstable in the case of trading. We will show that despite the fact that the two producers are identical ($\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$ and $R_1 = R_2$), the non-trading status is unstable, the logic of which was explained in the previous section. In this section, it will be shown mathematically.

For simplicity we choose $\alpha = \beta$ (Eq. (18)) for *both* producers, but it can easily be generalised to $\alpha \neq \beta$. Moreover, to simplify the analysis we use dimensionless variables, i.e. Eq. (18) can be rewritten as

$$\xi^\alpha + \eta^\alpha = 1, \tag{20}$$

where $\xi \equiv a/AF$ and $\eta \equiv b/BF$.

Now, suppose that prior to trading the highest preference ranking is reached at $\xi_0 \equiv a_0/AF$ and $\eta_0 \equiv b_0/BF$. Any decision to deviate from this optimal point will worsen their status and reduce their ranking. However, the deviations are not symmetric, i.e. if one producer decides to produce more of A, i.e. $\xi_0 + \delta$ and the other decides to produce less, i.e. $\xi_0 - \delta$ then, for an arbitrary small perturbation δ their production of B corresponds to

$$\eta_{before}(\xi_0 \pm \delta) \cong (1 - \xi_0^\alpha)^{1/\alpha} \mp \delta \xi_0^{\alpha-1}(1 - \xi_0^\alpha)^{1/\alpha - 1} + \frac{1}{2}\delta^2(1 - \alpha)\xi_0^{\alpha-2}(1 - \xi_0^\alpha)^{1/\alpha - 2} \tag{21}$$

The subscript stands for '*before* trading'.

Since (ξ_0, η_0) is the point with the highest ranking, then at this point the gradient of the ranking matrix is perpendicular to the slope of the production frontier, i.e. any advancement

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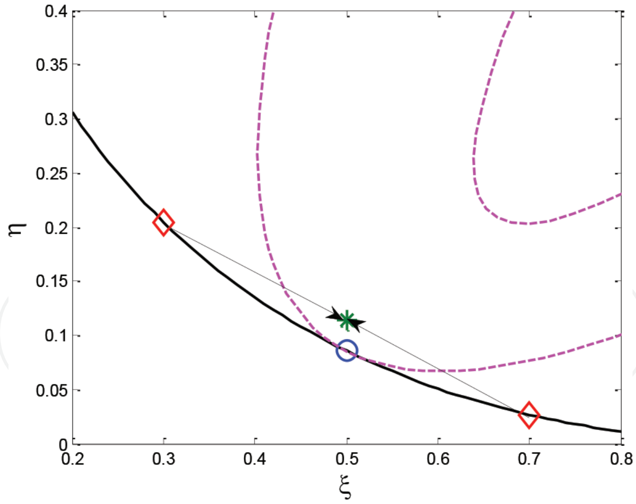

Figure 6. Illustration of the instability created by specialisation. The circle represents the pre-trading stable point ($\xi_0 = 0.5, \eta_0 = 0.0858$), the diamonds represents the perturbed (Eq. (21) with δ= 0.2) pre-trading condition. The arrows describe the trading process (from the diamonds to the asterisk, which represents Eq. (23)). The dashed curves represents indifference curves, while the solid curve stands for the production frontier.

in the normal direction (perpendicular to the production frontier) will necessarily improve the producers' ranking.

The trading occurs in the linear regime, i.e. δ units of ξ can be traded for $\delta\xi_0^{\alpha-1}(1-\xi_0^{\alpha})^{1/\alpha-1}$ units of η, i.e. for the price

$$\Delta B/\Delta A = B\Delta\eta/A\Delta\xi = (B/A)\xi_0^{\alpha-1}(1-\xi_0^{\alpha})^{1/\alpha-1}. \tag{22}$$

Therefore, after trading their status exceeds the production frontier (see **Figure 6**)

$$\eta_{after}(\xi_0 \pm \delta) \cong (1-\xi_0^{\alpha})^{1/\alpha} + \frac{1}{2}\delta^2(1-\alpha)\xi_0^{\alpha-2}(1-\xi_0^{\alpha})^{1/\alpha-2} > \eta(\xi_0) \tag{23}$$

whose distance from the production frontier is approximately

$$\Delta D \cong \frac{1}{2}\delta^2 \frac{(1-\alpha)\xi_0^{\alpha-2}(1-\xi_0^{\alpha})^{1/\alpha-2}}{\sqrt{\xi_0^{2\alpha-2}(1-\xi_0^{\alpha})^{2/\alpha-2}-1}}. \tag{24}$$

Thus, we see that if trading is an option, then the stable maximum ranking point (ξ_0, η_0) becomes unstable, since any deviation from this point will necessarily improve the producers' ranking.

8. Summary and conclusions

The law of association is well known as one of the most fundamental laws in economics. It is traditionally believed that advantage, either absolute of comparative, is a sufficient condition for trading. It is shown in this chapter that one of the sources of this belief is that no subjective analysis is used in the derivation of the LA. This is a major flaw in the law's derivation, since it is well known that the utility is a subjective property and any fundamental law should be based on subjective grounds.

We first presented the traditional law of association (Section 2). In the absence of subjective analysis, we used the distance from the production frontier to quantify the improvement in the status of both producers as a consequence of specialisation and trading (Section 3). Using this tool, we derive the price, in which the gain of both producers is the same.

Then, we present an analysis, which is based on subjective preference ranking. It is shown that comparative advantage is an insufficient condition for trading (Section 4).

In Section 5, we present the dynamic of the process, which is based on subjective analysis. The object of this section is to answer one of the main dilemmas in specialisation—the lack of information regarding the *other* producers' preference ranking. An algorithm, which solves this dilemma, is presented, where the information is carried via the objective price level of the previous trading.

In Section 6, we investigate the effect of specialisation, which is also absent in the traditional analysis of the LA. It is shown that the specialisation bend the production frontier to a convex curve (a novel mathematical presentation for this bending is suggested). As a consequence, the motivation for trading increases, and therefore, there is no need for any advantage (absolute or comparative) to encourage trading. The producers can have identical production frontier and identical preference ranking and yet they would prefer to trade.

In Section 7, we show that specialisation breaks the stability of the pre-trading status, and creates trading opportunities even when the producers are identical.

Thus, when subjective considerations are introduced to the analysis, advantage between producers is neither necessary nor sufficient a condition for trading.

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