Chapter 4

Relativistic Celestial Metrology: Dark Matter as an Inertial Gauge Effect

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Additional information is available at the end of the chapter

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Abstract

In canonical tetrad gravity, it is possible to identify the gauge variables, describing relativistic inertial effects, in Einstein general relativity. One of these is the York time, the trace of the extrinsic curvature of the instantaneous non-Euclidean 3-spaces (global Euclidean 3-spaces are forbidden by the equivalence principle). The extrinsic curvature depends both on gauge variables and on dynamical ones like the gravitational waves after linearization. The fixation of these gauge variables is done by relativistic metrology with its identification of time and space. Till now, the International Celestial Reference Frame ICRF uses Euclidean 3-spaces outside the Solar System. It is shown that York time and non-Euclidean 3-spaces may explain the main signatures of dark matter in ordinary space-time before using cosmology. Also dark energy may be connected to these inertial gauge effects, because both red-shift and luminosity distance depend on them.

Keywords: dark matter

1. Introduction

An extremely important, till now not explicitly clarified, point in Einstein general relativity (GR) (and in every generally covariant theory of gravity), whose gauge group is the group of diffeomorphisms of the Lorentzian 4-dimensional space-time,[1] is that the fixation of the gauge freedom is nothing else than *the establishment of conventions for relativistic metrology*, an operation performed from atomic physicists, NASA engineers and astronomers [2] with the introduction of a notion of clock synchronization and with a definition of the axes for the 4-coordinates in each point, that is, with the identification of a non-inertial frame of the space-time (global inertial frames are forbidden by the equivalence principle). See Ref. [3, 4] for a review of the existing conventions in the Solar System.

[1] See Ref. [1] for theoretical considerations concerning the nature of space and time in GR.

According with the International Astronomic Union IAU inside the Solar System, the choice of the 4-coordinates is *solved at the experimental level by the choice of a convention for the description of matter* based on special post-Newtonian (PN) solutions of linearized Einstein equations in a fixed given harmonic gauge [2–4]: (a) for satellites near the Earth (like the GPS ones) one uses NASA 4-coordinates compatible with the reference frames of the International Terrestrial Reference System ITRS2003[2] and of the Geocentric Celestial Reference System GCRS IAU2000; (b) for planets in the Solar System one uses the frame of the Barycentric Celestial Reference System BCRS-IAU2000.

These frames are compatible with the usual interpretation as *quasi-inertial frames* in Minkowski space-time and are metrology choices like the choice of a certain atomic clock as standard of time. However, already in the Solar System, the instantaneous 3-spaces are not Euclidean in the selected solutions, but the existing technology is not yet able to show it, being a property of order $O(1/c^2)$[3]

In astronomy, data like luminosity, light spectrum and angles are used to determine the positions of stars and galaxies and their temporal evolution in a 4-dimensional nearly Galilei space-time with the International Celestial Reference System ICRS [2, 3], a frame considered as a “quasi-inertial frame” and with all galactic dynamics described by PN gravity.

This is in accord with the smallness of the intrinsic 3-curvature of the 3-spaces as implied by the CMB data, a property included in the standard Friedmann-Lemaitre-Robertson-Walker (FLRW) Λ CDM cosmological model with its isotropy and homogeneity symmetries. However, to reconcile all the existing data with this 4-dimensional description, one must postulate the existence of *dark matter and dark energy* as the dominant components of the classical universe [6–8] after the recombination 3-surface (before it quantum mechanics is entering in the description and there is no acceptable description for the transition from quantum to classical astrophysics) already within galaxies before making the transition to cosmology and the replacement of ordinary space-time with the standard cosmological FLWR one, whose points describe a mean over a volume of 100 Mega-parsecs of the ordinary space-time. The attempts to avoid the appearance of “darkness” have led to many proposals of modifications of GR like MOND [9], $f(R)$ gravity [10–12] and the ones analyzed in Refs. [6–8].

After a description of ICRS and of the measurements in ordinary astrophysics (not cosmology) of quantities like luminosity distance, rotation curves of galaxies, gravitational lensing,…. implying “darkness,” we will study canonical ADM tetrad gravity and its gauge freedom after a suitable but arbitrary $3+1$ splitting of the space-time in a family of Einstein space-times able to include the extension of the models of particle physics to GR. We will identify which are the *gauge variables* to be fixed with astrophysical metrology and how the interpretation of “dark

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[2]A relativistic version of ITRS is not yet existing, so that one cannot yet connect the time of the atomic clocks in different laboratories to the clock on the Space Station with a suitable Lorentz transformations.

[3]See however the LATOR proposal [5] of measuring the deviation from 2π of the sum of the three angles of a triangle formed by the Space Station and two spacecrafts behind the Sun. When this non-Euclidean nature will be measured, one will have to redefine the standard of length measurements [2].

matter" and probably also of "dark energy" depends on the fixation of these gauge variables in a family of gauge-fixings different from the harmonic ones used in the IAU conventions. Therefore, our suggestion is that "*darkness*" *may be interpreted as a relativistic inertial effect* and that ICRS should be reformulated in a suitable relativistic way.

2. Astrophysical metrology

Reference data for positional astronomy, such as the data in astrometric star catalogs, are specified in the International Celestial Reference System ICRS [2, 3] with origin in the solar system barycenter and with kinematically non-rotating spatial axes fixed with respect to space according to the IAU conventions [2, 3]. It is based on the position of extragalactic radio sources that are distant enough to be considered stationary, in the limit of today's capabilities, and whose position is known with a precision of 0.001 arcsec, thanks to the Very Long Baseline Interferometry technique [13]. These sources are assumed to have no observable intrinsic angular momentum. The International Celestial Reference Frame ICRF is a realization of ICRS obtained by supposing that the origin is a quasi-inertial observer and that we have a quasi-inertial (essentially non-relativistic) reference frame with rectangular 3-coordinates in a nearly Galilean space-time whose 3-spaces are Euclidean.

However, a number of different categories of astronomical observations are explained in the usual Euclidean 3-space only in terms of so far undetermined dark matter and dark energy: rotational curves of galaxies [14–17], gravitational lensing [18–20], application of the virial theorem to galaxy clusters [21–23] and the acceleration of the expansion of the universe [24–29]. This already happens before the transition from the ordinary space-time to the cosmological one, the FLWR space-time which is not a Galilean space-time but has nearly internally flat 3-spaces and uses a theoretical cosmic time. What is still not explored is the possibility that in Einstein GR one can use non Euclidean 3-spaces with small internal 3-curvature, but with an extrinsic curvature (as 3-submanifolds of the space-time) depending on the gauge variables, namely on the metrology conventions.

In all the astronomical observations, the distance of the objects needs to be known. Measuring distances in astronomy is a difficult task, especially when dealing with extragalactic objects. Different methods must be applied at increasing distances, which need to be inter-calibrated appropriately. To get relevant quantities like distances and absolute luminosity of stars from the directly measured quantities, that is, apparent luminosity, angles and red-shift, it is important to know the geometry of the 3-spaces crossed by the propagating rays of light on null 4-geodesics of the space-time.

The most important methods rely on the absolute intrinsic luminosity L of a *standard candle* compared to the apparent brightness F as measured on Earth. In terms of these quantities, one defines the *luminosity distance* [6, 18–20] of a luminous object $d_L = \sqrt{L/4\pi F}$, which is the proper distance of an object at rest with respect to the observer in a Euclidean stationary universe. In an expanding universe, the luminosity distance is dependent on the red-shift z of the light arriving on the Earth from the object and to the comoving distance r_1. If a_o is the scale factor, H_o is the Hubble constant, $\dot{H}_o$ its time derivative, and if one keeps only the first-order terms in the expansion, one has

$$d_L = a_o\, r_1\, (1+z) = \frac{z}{H_o}\, [1 + (1 + \frac{\dot{H}_o}{2\, H_o^2})\, z], \tag{1}$$

Also used is the *angular diameter distance* $d_A = D/\theta$, where θ is the angular diameter of the source as measured by the observer and D is the diameter of well-known close galaxies. Also the angular diameter distance depends on the red-shift: $d_A = \frac{a_o\, r_1}{1+z} = \frac{z}{H_o}\, \left[1 - (1 - \frac{\dot{H}_o}{2\, H_o^2})\, z\right]$.

In both luminosity distance and angular diameter distance, the terms which depart from Euclidean geometry enter only at higher orders, which depend on the rate of expansion of the universe and on the curvature parameter. For the galaxies with the most reliable rotation curves that are within a range of a few tens of Mega-parsec, they can be neglected, and we can consider the 3-space to be Euclidean. Higher order terms need instead to be considered when the objects have a distance of hundreds of Mega-parsec or more.

For larger z, one has to take into account a model of cosmology: In a FLWR metric, one has $F = L/[4\,\pi\,(a_o\, r_1)^2\,(1+z)^2]$ with $a_o = 1/(1+z)$ and with r_1 depending also on z.

Assuming that all supernovae (SN) Ia have the same intrinsic luminosity, it was found [26–29] that the SN1a's at $z \le 0.5$ are about 10 per cent fainter than expected, and this has been interpreted as evidence of an accelerated expansion of the universe and dark energy has been invoked to take care of the accelerated expansion.

3. Einstein general relativity

We shall use the formulation of Einstein GR in a 4-dimensional Lorentzian space-time (the one used in classical astrophysics, not in cosmology, after the recombination surface for the propagation of light) with the Lagrangian description implied by the ADM action principle [30, 31], because it allows to make the transition to the canonical formalism and to use Dirac theory of constraints [32], in particular to use the Shanmugadhasan canonical transformation [33, 34] to find canonical bases adapted to the constraints (see Ref. [35] for reviews). Light and visible stars and galaxies constitute the matter.

We will restrict ourselves to *globally hyperbolic, topologically trivial* and *asymptotically Minkowskian space-times*, in the absence of Killing symmetries (see Ref. [36] for their inclusion as Dirac constraints) and with the asymptotic SPI symmetries at spatial infinity of Ref. [37] restricted to the asymptotic ADM Poincaré group [38] by eliminating the super-translations with suitable boundary conditions on the 4-metric. This framework is defined in Refs. [39–43], where the matter consists of electrically charged positive-energy scalar point particles plus the electromagnetic field. In the limit of vanishing Newton constant ($G = 0$), the asymptotic Poincare' group becomes the Poincare' group of particle physics, where elementary particles are always considered as irreducible representations of this group.

While in the family of spatially compact without spatial boundary space-times[4], considered in loop quantum gravity [44, 45], the Dirac Hamiltonian is a combination of constraints because the canonical Hamiltonian vanishes, in our space-times there is not a frozen picture, because the canonical Hamiltonian is the weak ADM energy $\hat{E}_{ADM}$[5] plus a combination of constraints. In the absence of matter, Christodoulou-Klainermann space-times [46] are compatible with this description.

In the ADM Lagrangian, the basic variable is the 4-metric ${}^4g_{\mu\nu}(x)$ of the space-time (x^μ are local 4-coordinates with an arbitrary origin): it determines the dynamical chrono-geometrical structure of space-time by means of the line element $ds^2 = {}^4g_{\mu\nu}(x)\, dx^\mu\, dx^\nu$, and it teaches to massless particles which are the allowed trajectories in each point.

However, to include the coupling of gravity to the spin of fermions, we must use ADM *tetrad gravity*: the 10 components of the 4-metric appearing in the ADM Lagrangian are decomposed on a set of cotetrads [31] $E^{(\alpha)}_\mu(x)$, ${}^4g_{\mu\nu}(x) = E^{(\alpha)}_\mu(x)\, \eta_{(\alpha)(\beta)}\, E^{(\beta)}_\nu(x)$.[6] This leads to an interpretation of gravity based on a congruence of time-like observers endowed with orthonormal tetrads $E^\mu_{(\alpha)}(x)$ (i.e., the inverse of the cotetrads $E^\mu_{(\alpha)}(x)\, E^{(\beta)}_\mu(x) = \delta^{(\beta)}_{(\alpha)}$): in each point of space-time, the time-like axis is the unit 4-velocity of a time-like observer, whereas the spatial axes are a (gauge) convention for the three gyroscopes of the observer.

A. Metrology as the Fixation of the Gauge Freedom of General Relativity

While the ADM action for metric gravity is invariant under space-time diffeomorphisms, the decomposition of the 4-metric on the cotetrads gives an ADM action [30] invariant not only under the space-time diffeomorphisms but also on a local O(3,1) Lorentz group describing the freedom in the orientation and transport of the gyroscopes along the time-like world lines of observers. Let us remark that the same gauge freedoms are present in all the generally covariant formulations of GR proposed as modifications of Einstein GR.

In electromagnetism and in Yang-Mills theories, the Lagrangian description in terms of potentials implies the presence of a gauge group acting on an internal space and implying the gauge nature of certain scalar and longitudinal components of the potentials: the gauge fixings imply the description of physics in terms of electric and magnetic fields or of their non-abelian analogues. Instead, in the metric formulation of GR, the gauge freedom is connected with the freedom in the choice of the metrology conventions, described in the previous section, for the definitions of clocks (i.e., time) and 3-space in each point of the space-time. As we shall see a metrology convention implies the fixation of 8 of the 10 components of the 4-metric, so that the remaining two components describe the physical degrees of freedom of the gravitational field

[4]Therefore, it is not possible to define a Poincare' group and to find a connection with particle physics.

[5]It is a volume integral over 3-space of a coordinate-dependent energy density. It is weakly equal to the *strong* ADM energy, which is a flux through a 2-surface at spatial infinity.

[6](α) are flat indices and $\eta_{(\alpha)(\beta)}$ is the flat 4-metric of Minkowski space-time. The signature of the 4-metrics is $\epsilon = \pm$ so that $\eta_{(\alpha)(\beta)} = \epsilon\,(1; -1, -1, -1)$. $\epsilon = 1$ is the convention of particle physics, whereas $\epsilon = -1$ is the convention usually used in GR

(the gravitational waves (GW) of its linearization in the case of weak fields). In tetrad gravity, we have 16 fields, but the extra 6 fields are fixed by metrology conventions on the orientation of three gyroscopes and on their transport along time-like world lines in each point of the space-time.

In special relativity, the metrology conventions amount to the choice of a standard atomic clock and of the instantaneous Euclidean 3-spaces of a *global inertial frame*, whose extension to *global non-inertial frames* was done in Ref. [47] with an application to relativistic atomic physics described in Ref. [48].

In GR, due to the equivalence principle forbidding the existence of global inertial frames, one has to use the cited theory of global non-inertial frames in the form of the so-called *3+1 point of view*[7]: one gives the world line of a time-like observer and a nice foliation of the space-time whose leaves are the instantaneous 3-spaces. Instead of standard local 4-coordinates x^μ centered in a point of the observer world line, one uses *4-scalar observer-dependent radar 4-coordinates*[8] $\sigma^A = (\tau;\, \sigma^r)$, where τ is an arbitrary increasing function of the observer proper time and σ^r is curvilinear 3-coordinates on the 3-spaces Σ_τ (diffeomorphic to R^3) with the observer as origin.

The inverse transformation $\sigma^A \mapsto x^\mu = z^\mu(\tau,\, \sigma^r)$ defines the embeddings of the 3-spaces Σ_τ into the space-time and the induced 4-metric is $g_{AB}[z(\tau,\sigma^r)] = z^\mu_A(\tau,\sigma^r)\, z^\nu_B(\tau,\sigma^r)\, g_{\mu\nu}(z(\tau,\sigma^r))$, where $z^\mu_A = \partial z^\mu / \partial\, \sigma^A$, while the cotetrads take the form $E^{(\alpha)}_A(\tau,\sigma^r) = z^\mu_A(\tau,\sigma^r)\, E^{(\alpha)}_\mu(z(\tau,\sigma^r))$. As shown explicitly in Ref. [51], the use of the 4-scalar radar 4-coordinates implies that *the ten components* ${}^4g_{AB}(\tau,\sigma^r)$ *and the sixteen components* $E^{(\alpha)}_A(\tau,\sigma^r)$ *are 4-scalars of the space-time*. Also, all the components of *radar tensors* (i.e., tensors expressed in radar 4-coordinates) are 4-scalars of the space-time.

While the 4-vectors $z^\mu_r(\tau,\sigma^u)$ are tangent to Σ_τ, so that in each point of the 3-space, the unit normal $l^\mu(\tau,\sigma^u)$ is proportional to $\epsilon^\mu{}_{\alpha\beta\gamma}\, z^\alpha_1(\tau,\sigma^r)\, z^\beta_2(\tau,\sigma^r)\, z^\gamma_3(\tau,\sigma^u)$, we have $z^\mu_\tau(\tau,\sigma^r) = N(\tau,\sigma^r)\, l^\mu(\tau,\sigma^r) + N^r(\tau,\sigma^r)\, z^\mu_r(\tau,\sigma^r)$, where $N(\tau,\sigma^r) = \epsilon\, z^\mu_\tau(\tau,\sigma^r)\, l_\mu(\tau,\sigma^r)$ and $N_r(\tau,\sigma^r) = -\epsilon g_{\tau r}(\tau,\sigma^r)$ are the lapse and shift functions of canonical GR.

In the chosen family of space-times, the foliation needed for the 3+1 splitting is nice and admissible if the lapse function satisfies $N(\tau,\sigma^r) > 0$ in every point of Σ_τ,[9] if $\epsilon\, {}^4g_{\tau\tau}(\tau,\sigma^r) > 0$[10] and if the positive-definite 3-metric ${}^3g_{rs}(\tau,\sigma^u) = -\epsilon\, {}^4g_{rs}(\tau,\sigma^u)$ has three positive eigenvalues. These are the Møller conditions [52, 53].

Moreover, all the 3-spaces Σ_τ must tend to the same space-like hyperplane at spatial infinity. Due to the imposed absence of super-translations [39, 40], the non-Euclidean 3-spaces are orthogonal to the conserved ADM 4-momentum at spatial infinity; therefore, each 3-space is a

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[7]Instead the usually used 1+3 point of view using the world line of a time-like observer leads only to local coordinate systems like the Riemann and Fermi ones valid only in a neighborhood of a time-like world line, because locally the 3-spaces are identified with the tangent spaces orthogonal to the observer 4-velocity so that they intersect each other.

[8]They were introduced by Bondi in Ref. [49, 50].

[9]Therefore, the 3-spaces never intersect, avoiding the coordinate singularity of Fermi coordinates.

[10]This property avoids the coordinate singularity of the rotating disk.

non-inertial rest frame [39–41] of the 3-universe with vanishing ADM 3-momentum, and there are asymptotic inertial observers with spatial axes identified by means of the fixed stars of star catalogues. In each 3-space Σ_τ, there are cotriads ${}^3e_{(a)r}(\tau,\sigma^r) = \sum_b R_{(a)(b)}(\alpha_{(c)}(\tau,\sigma^r))\,{}^3\bar{e}_{(a)r}(\tau,\sigma^r)$ defined modulo rotations ($R_{(a)(b)}$ are rotation matrices and $\alpha_{(a)}(\tau,\sigma^r)$ are angles).

In Refs. [39–43], there is a parametrization of tetrads, cotetrads and 4-metric in the framework of the 3+1 splitting of space-time. The basic configuration variables, that is, the cotetrads, are connected to cotetrads adapted to the 3+1 splitting of space-time (so that the adapted time-like tetrad is the unit normal to the 3-space Σ_τ) by standard Wigner boosts $L^{(\alpha)}{}_{(\beta)}$ for time-like vectors depending upon boost parameters $\varphi_{(a)}(\tau,\sigma^r)$: ${}^4E^{(\alpha)}_A = L^{(\alpha)}{}_{(\beta)}(\varphi_{(a)})\,{}^4E^{o}_A(\beta)$. The adapted tetrads and cotetrads have the expression[11],[12]

$$
\begin{aligned}
&{}^4E^A_{(\alpha)} \overset{def}{=} {}^4\overset{\circ}{E}{}^A_{(\beta)}\, L^{(\beta)}{}_{(\alpha)}(\varphi_{(a)}) \overset{def}{=} \overset{\circ}{\bar{E}}{}^A_{(o)}\, L^{(o)}{}_{(\alpha)}(\varphi_{(c)}) + \\
&\qquad + \sum_{ab} {}^4\overset{\circ}{\bar{E}}{}^A_{(b)}\, R^T_{(b)(a)}(\alpha_{(c)})\, L^{(a)}{}_{(\alpha)}(\varphi_{(c)}),\\
&{}^4g_{AB} = E^{(\alpha)}_A\, {}^4\eta_{(\alpha)(\beta)}\, E^{(\beta)}_B = \\
&\qquad = {}^4\overset{\circ}{E}{}^{(\alpha)}_A\, {}^4\eta_{(\alpha)(\beta)}\, {}^4\overset{\circ}{E}{}^{(\beta)}_B = {}^4\overset{\circ}{\bar{E}}{}^{(\alpha)}_A\, {}^4\eta_{(\alpha)(\beta)}\, {}^4\overset{\circ}{\bar{E}}{}^{(\beta)}_B,\\
&{}^4\overset{\circ}{\bar{E}}{}^A_{(o)} = {}^4\overset{\circ}{E}{}^A_{(o)} = \frac{1}{1+n}\,(1;\, -\sum_a \bar{n}_{(a)}\, {}^3\bar{e}^r_{(a)}) = l^A, \qquad {}^4\overset{\circ}{\bar{E}}{}^A_{(a)} = (0; {}^3\bar{e}^r_{(a)}),\\
&{}^4\overset{\circ}{\bar{E}}{}^{(o)}_A = {}^4\overset{\circ}{E}{}^{(o)}_A = (1+n)\,(1;\, \vec{0}) = \epsilon l_A, \qquad {}^4\overset{\circ}{\bar{E}}{}^{(a)}_A = (\bar{n}_{(a)}; {}^3\bar{e}_{(a)r}),\\
&{}^4\overset{\circ}{E}{}^{(a)}_A = \sum_b R_{(a)(b)}\,(\bar{n}_{(b)}; {}^3\bar{e}_{(b)r}),\\
&{}^4g_{\tau\tau} = \epsilon\,[(1+n)^2 - \sum_a \bar{n}^2_{(a)}], \qquad {}^4g_{\tau r} = -\epsilon\, n_r = -\epsilon \sum_a \bar{n}_{(a)}\, {}^3\bar{e}_{(a)r},\\
&{}^4g_{rs} = -\epsilon\, {}^3g_{rs} = -\epsilon \sum_a {}^3\bar{e}_{(a)r}\, {}^3\bar{e}_{(a)s}, \qquad \sqrt{-g} = \sqrt{|{}^4g|} = \frac{\sqrt{{}^3g}}{\sqrt{\epsilon\, {}^4g^{\tau\tau}}} = \sqrt{\gamma}\,(1+n).
\end{aligned}
\tag{2}
$$

From Eq. (5.5) of the third paper in Ref. [43], we assume the following (direction-independent, so to kill super-translations) boundary conditions at spatial infinity ($r = \sqrt{\sum_r (\sigma^r)^2}$; $\epsilon > 0$; $M = const.$): $n(\tau,\sigma^r) \rightarrow_{r\rightarrow\infty} O(r^{-(2+\epsilon)})$, $\pi_n(\tau,\sigma^r) \rightarrow_{r\rightarrow\infty} O(r^{-3})$, $n_{(a)}(\tau,\sigma^r) \rightarrow_{r\rightarrow\infty} O(r^{-\epsilon})$, $\pi_{n_{(a)}}(\tau,\sigma^r) \rightarrow_{r\rightarrow\infty} O(r^{-3})$, $\varphi_{(a)}(\tau,\sigma^r) \rightarrow_{r\rightarrow\infty} O(r^{-(1+\epsilon)})$, $\pi_{\varphi_{(a)}}(\tau,\sigma^r) \rightarrow_{r\rightarrow\infty} O(r^{-2})$, ${}^3e_{(a)r}(\tau,\sigma^r) \rightarrow_{r\rightarrow\infty} \left(1+\frac{M}{2r}\right)\delta_{ar} + O(r^{-3/2})$, ${}^3\pi^r_{(a)}(\tau,\sigma^r) \rightarrow_{r\rightarrow\infty} O(r^{-5/2})$.

[11] $N(\tau,\sigma^r) = 1 + n(\tau,\sigma^r)$ and $n_{(a)}(\tau,\sigma^r) = (N^r\,{}^3e^r_{(a)})(\tau,\sigma^r) = \sum_b R_{(a)(b)}(\alpha_{(c)}(\tau,\sigma^r))\,\bar{n}_{(b)}(\tau,\sigma^r)$ are the lapse and shift functions respectively.

[12] ${}^4\overset{\circ}{E}{}^A_{(\beta)}$ and ${}^4\overset{\circ}{\bar{E}}{}^A_{(o)}$ are tetrads adapted to the 3+1 splitting.

As shown in Refs. [[39–43], due to the existence of the asymptotic ADM Poincare' group, the isolated system *gravitational field plus matter*, namely the 3-universe, has the mass given by the ADM weak energy and the spin by the ADM angular momentum. Therefore, at each time, the 3-universe can be described as a decoupled non-covariant non-observable external pseudo-particle (the center of mass of the 3-universe) carrying a pole (the mass)-dipole (the spin) structure. Since the ADM 3-momentum vanishes due to the rest-frame condition, the conjugate non-observable internal center of mass of the 3-universe may be eliminated from the observable variables by imposing the vanishing of the ADM Lorentz boosts.

As a conclusion to fix the gauge in GR with a metrology convention, so to visualize the associated gauge-dependent inertial effects, we need to separate the gauge variables from the dynamical ones, the so-called Dirac observables (DO), and only the Hamiltonian formalism has the tools to face this problem. The usual criticism that this can be done only in a non-covariant coordinate-dependent way is avoided due to the use of the radar coordinates implying the existence of 4-scalar tensors.

B. Canonical ADM Tetrad Gravity and Its Gauge Variables

The parametrization of cotetrads given in the previous subsection for ADM tetrad gravity implies [40] that the ADM action may be considered function of the 16 configurational variables $\varphi_{(a)}$, $1+n$, $n_{(a)}$, ${}^3e_{(a)r}$. At the Hamiltonian level, there is a phase space spanned by these 16 configuration variables and their conjugated 16 momenta, and there are *14 first class constraints*. Ten of them are primary constraints (the vanishing of the 7 momenta of boosts, lapse and shift variables plus three constraints describing the gauge freedom in the rotation on the flat indices (a) of the cotriads), whereas four are secondary ones (the super-Hamiltonian and super-momentum constraints). Therefore, there are 14 gauge variables describing *inertial effects* and 2 canonical pairs of physical degrees of freedom describing the *tidal effects* of the gravitational field (namely GW in the weak field limit).

The basis of canonical variables for this formulation of tetrad gravity, naturally adapted to 7 of the 14 first-class constraints, is (only the momenta ${}^3\pi^r_{(a)}$ conjugated to the cotriads are not vanishing)

$$\begin{array}{cccc} \varphi_{(a)} & n & \overline{n}_{(a)} & {}^3e_{(a)r} \\ \pi_{\varphi_{(a)}} \approx 0 & \pi_n \approx 0 & \pi_{n_{(a)}} \approx 0 & |{}^3\pi^r_{(a)} \end{array} \tag{3}$$

In Ref. [42], a York canonical basis, adapted to 10 first-class constraints (not to the super-Hamiltonian and super-momentum ones, whose solution is unknown), was identified by means of a Shanmugadhasan canonical transformation [33, 34]; this allows *for the first time to get the explicit identification of the inertial and tidal variables*. It implements the York map of Ref. [54] and diagonalizes the York-Lichnerowicz approach [55]. Its final form is[13]

[13]G is Newton constant. The set of numerical parameters $\gamma_{\bar{a}a}$ satisfies $\sum_u \gamma_{\bar{a}u} = 0$, $\sum_u \gamma_{\bar{a}u}\gamma_{\bar{b}u} = \delta_{\bar{a}\bar{b}}$, $\sum \bar{a}\,\gamma_{\bar{a}u}\gamma_{\bar{a}v} = \delta_{uv} - \frac{1}{3}$. Each solution of these equations defines a different York canonical basis.

$$\begin{array}{cccccccc} \varphi_{(a)} & \alpha_{(a)} & n & \overline{n}_{(a)} & \theta^r & \tilde{\phi} & & R_{\overline{a}} \\ \pi_{\varphi_{(a)}} \approx 0 & \pi^{(\alpha)}_{(a)} \approx 0 & \pi_n \approx 0 & \pi_{\overline{n}_{(a)}} \approx 0 & \pi^{(\theta)}_r & \pi\tilde{\phi} = \frac{c^3}{12\pi G}\, {}^3K & & \Pi_{\overline{a}} \end{array} \tag{4}$$

$$\begin{aligned} {}^3e_{(a)r} &= \sum_b R_{(a)(b)}(\alpha_{(c)})\, V_{rb}(\theta^i)\, \tilde{\phi}1/3\, e^{\sum_{\overline{a}}^{1,2} \gamma_{\overline{a}a} R_{\overline{a}}}, \\ {}^4g_{\tau\tau} &= \epsilon[(1+n)^2 - \sum_a \overline{n}^2_{(a)}], \\ {}^4g_{\tau r} &= -\epsilon\, \overline{n}_{(a)}\, V_{ra}(\theta^i)\, \tilde{\phi}1/3\, e^{\sum_{\overline{a}}^{1,2} \gamma_{\overline{a}a} R_{\overline{a}}}, \\ {}^4g_{rs} &= -\epsilon\, {}^3g_{rs} = -\epsilon\, \tilde{\phi}2/3 \sum_a V_{ra}(\theta^i)\, V_{sa}(\theta^i)\, e^{\sum_{\overline{a}}^{1,2} \gamma_{\overline{a}a} R_{\overline{a}}}, \\ \tilde{\phi} &= \sqrt{det\, {}^3g_{rs}}, \end{aligned} \tag{5}$$

In this York canonical basis, the *inertial effects* are described by the arbitrary gauge variables[14] $\alpha_{(a)}$, $\varphi_{(a)}$, $1+n$, $\overline{n}_{(a)}$, θ^i, ${}^3K$, whereas the *tidal effects*, that is, the physical degrees of freedom of the gravitational field (the two polarizations of GW in the linearized theory), by the two canonical pairs $R_{\overline{a}}$ and $\Pi_{\overline{a}}$, $\overline{a} = 1, 2$ ($R\overline{a}$ are eigenvalues of the 3-metric with determinant one).

The momenta $\pi^{(\theta)}_r(\tau, \sigma^r)$ and the 3-volume element $\tilde{\phi}(\tau, \sigma^r) = \sqrt{det\, {}^3g_{rs}(\tau, \sigma^u)}$ have to be found as solutions of the super-momentum ($\mathcal{H}_{(a)}(\tau, \sigma^r) \approx 0$) and super-Hamiltonian (i.e., the Lichnerowicz equation [55] $\mathcal{H}(\tau, \sigma^r) \approx 0$) constraints, respectively.

Instead, the DO's (gauge invariant under the Hamiltonian gauge transformations generated by all the first class constraints; see Ref. [51]) of the gravitational field are not known[15]; they would be the two pairs of 4-scalar tidal variables in a Shanmugadhasan canonical basis adapted to all the 14 first class constraints.

The extra O(3,1) gauge freedom of the tetrads[16] is described by the gauge variables $\alpha_{(a)}(\tau, \sigma^r)$, $\varphi_{(a)}(\tau, \sigma^r)$. In the *Schwinger time gauges,* one imposes the gauge fixings $\varphi_{(a)}(\tau, \sigma^r) \approx 0$, $\alpha_{(a)}(\tau, \sigma^r) \approx 0$ so that the time-like tetrad coincides with the unit normal to the 3-space and the space-like ones became tangent to it (namely the tetrads become adapted to the 3+1 splitting).

[14] $\alpha_{(a)}$, $\varphi_{(a)}$, θ^i and ${}^3K$ are the primary gauge variables, whereas n and $\overline{n}_{(a)}$ are the secondary ones, which are determined as a consequence of the gauge fixing of the primary ones.

[15] $R_{\overline{a}}$, $\Pi_{\overline{a}}$ are not gauge invariant under the Hamiltonian gauge transformations generated by the super-Hamiltonian and super-momentum constraints.

[16] The gauge freedom for each observer to choose three gyroscopes as spatial axes and to choose the law for their transport along the time-like world line.

The gauge angles $\theta^i(\tau,\sigma^r)$[17] describe the freedom in the choice of the axes for the 3-coordinates σ^r on each 3-space: their fixation implies the determination of the shift gauge variables $\overline{n}_{(a)}$, namely the appearances of gravitomagnetism in the chosen 3-coordinate system [55]. The *3-orthogonal gauges* are defined by the gauge fixings $\theta^i(\tau,\sigma^r)\approx 0$: in them, the 3-metric ${}^3g_{rs}(\tau,\sigma^u) = -\epsilon\,{}^4g_{rs}(\tau,\sigma^u) = \delta_{rs}\,\tilde{\phi}2/3\,e^2\sum\overline{a}^{1,2}\;\gamma_{\bar{a}r}\,R_{\bar{a}}$ is diagonal.

Only one momentum is a gauge variable (a reflection of the Lorentz signature): the *York time* [56, 57], that is, the trace ${}^3K(\tau,\sigma^r)$ of the *extrinsic curvature* of the non-Euclidean 3-spaces as 3-submanifolds of space-time.[18] This inertial effect describes the GR version of the special-relativistic gauge freedom in clock synchronization [47, 48] when one has to describe physics in non-inertial frames. Its fixation determines the lapse function.

The Dirac Hamiltonian is $H_D = \frac{1}{c}\,\hat{E}_{ADM} + \int d^3\sigma\,[n\,\mathcal{H} - \overline{n}_{(a)}\,\mathcal{H}_{(a)}](\tau,\sigma^u) + \int d^3\sigma\,[\lambda_n\,\pi_n + \lambda_{\overline{n}_{(a)}}\,\pi_{\overline{n}_{(a)}} + \lambda_{\varphi_{(a)}}\,\pi_{\varphi_{(a)}} + \lambda_{\alpha(a)}\,\pi^{(\alpha)}_{(a)}](\tau,\sigma^u)$, where the weak ADM energy is an explicit function of all the variables, and the λ's are arbitrary Dirac multipliers (to be determined as a consequence of the gauge fixings).

In the family of Schwinger time gauges, the fixation of the primary gauge variables ${}^3K(\tau,\sigma^r)$, $\theta^i(\tau,\sigma^r)$ implies elliptic equations on the instantaneous 3-space Σ_τ for the determination of the lapse and shift functions (the secondary gauge variables) and then of their Dirac multipliers λ's. Instead in the usually used harmonic gauges, one imposes the primary gauge fixing $\chi^A(\tau,\sigma^r) = \partial_\tau\left((1+n(\tau,\sigma^r))\,{}^3e(\tau,\sigma^r)\,{}^4g^{\tau A}(\tau,\sigma^r)\right)\approx 0$, whose stability in time, that is, $\partial_\tau\,\chi^A(\tau,\sigma^r)\;\approx 0$, implies hyperbolic equations for the lapse and shift functions, namely the necessity of Cauchy conditions in the past for these metrology gauge variables.

This parametrization of canonical tetrad gravity clarifies the meaning of the metrology conventions.

The fixation of the York time determines the sequence of instantaneous non-Euclidean 3-spaces Σ_τ of the 3+1 splitting of space-time centered on an observer either on the Earth or on the Space Station[19]: all the clocks on each 3-space are synchronized with the atomic clock (τ is its proper time) of the observer at the intersection of the 3-space with the observer world line. This time metrology convention implies also the determination of the lapse function, which describes how the unit of time of the atomic clock changes when one goes from a 3-space to an infinitesimally near successive one. The metrology conventions on the choice of the three space coordinates σ^r also imply the determination of the shift functions, which say in which point of the infinitesimally near next 3-space there are the same 3-coordinates of the chosen point on the original 3-space.

[17]They identify the direction cosines of the tangents to the three coordinate lines in each point of the 3-space Σ_τ.

[18]It is absent in the Galilean space-time of Newtonian gravity with its absolute notions of time and Euclidean 3-space.

[19]The detailed structure of these non-Euclidean 3-spaces depends on the extrinsic curvature 3-tensor ${}^3K_{rs}$, which depends not only from all the gauge variables but also on the tidal variables, so that it is determined by the chosen solution of Einstein equations.

C. Einstein Hamilton Equations of Tetrad Gravity and their Linearization

In the York canonical basis, the Hamilton equations generated by the Dirac Hamiltonian $H_D = \hat{E}_{ADM} + (constraints)$ are divided into four groups after the fixation of the O(3,1) gauge variables with the Schwinger time gauges:

A. Four contracted Bianchi identities, namely the evolution equations for $\tilde{\phi}$ and $\pi_i^{(\theta)}$ (they say that given a solution of the constraints on a Cauchy surface, it remains a solution also at later times).

B. Four evolution equation for the four basic primary gauge variables θ^i and ${}^3K$: these equations determine the lapse and the shift functions once four gauge fixings for the basic gauge variables are added.

C. four evolution equations for the tidal variables $R\bar{a}$, $\Pi\bar{a}$;

D. the Hamilton equations for matter, when present.

The Hamilton equations become completely deterministic after a fixation of the gauge freedom. In the York canonical basis, it is convenient to use a family of *non-harmonic 3-orthogonal Schwinger time gauges* $\alpha_{(a)}(\tau,\sigma^r)\approx 0$, $\varphi_{(a)}(\tau,\sigma^r)\approx 0$, $\theta^i(\tau,\sigma^r)\approx 0$, ${}^3K(\tau,\sigma^r)\approx F(\tau,\sigma^r)$ parametrized by the numerical values $F(\tau,\sigma^r)$ of the York time ${}^3K(\tau,\sigma^r)$ and having the 3-metric in the 3-spaces diagonal and well determined lapse and shift functions. In these gauges, given a solution of the super-momentum and super-Hamiltonian constraints, one can find a solution of Einstein's equations in radar 4-coordinates adapted to a time-like observer giving the Cauchy data on an initial 3-space only for the tidal variables. This happens in the associated 3+1 splitting of space-time with dynamically selected instantaneous 3-spaces in accord with Ref. [1]. Then, one can pass to adapted world 4-coordinates ($x^\mu = z^\mu(\tau,\sigma^r) = x_o^\mu + \epsilon_A^\mu\ \sigma^A$) and can describe the solution in every 4-coordinate system by means of 4-diffeomorphisms.

In Ref. [43], this class of asymptotically Minkowskian space-times without super-translations is used to study the coupling of N charged scalar point particles (with the inertial and gravitational masses equal as required by the equivalence principle) plus the electromagnetic field to ADM tetrad gravity. The use of Grassmann-valued electric charges and the signs of the energy of the particles allows to regularize the self-energies. The theory can be reformulated in terms of transverse electromagnetic fields by using the non-covariant radiation gauge; this allows to extract the generalization of the Coulomb interaction among the particles in the Riemannian instantaneous 3-spaces of global non-inertial frames.

From the Hamilton equations in the York canonical basis [43], followed by a Hamiltonian Post-Minkowskian (HPM) linearization (disregarding terms of order $O(G^2)$ in the Newton constant and using an ultra-violet cutoff for matter) with the asymptotic flat Minkowski 4-metric at spatial infinity as background, it has been possible to develop a theory of GW's with asymptotic background propagating in the non-Euclidean 3-spaces Σ_τ of a family of *non-harmonic 3-orthogonal Schwinger time gauges* $\alpha_{(a)}(\tau,\sigma^r)\approx 0$, $\varphi_{(a)}(\tau,\sigma^r)\approx 0$, $\theta^i(\tau,\sigma^r)\approx 0$, ${}^3K(\tau,\sigma^r)\approx F(\tau,\sigma^r)$ parametrized by the numerical values $F(\tau,\sigma^r)$ of the York time ${}^3K(\tau,\sigma^r)$ and having the 3-metric in the 3-spaces diagonal and well-determined lapse and shift functions.

Since the celestial reference frame ICRS has diagonal 3-metric, our 3-orthogonal Schwinger time gauges are a good choice for celestial metrology.

The open problem is that the GCRS and BCRS conventions in the Solar System are using the special harmonic gauge of IAU [2, 3], in which the lapse function satisfies a hyperbolic equation like the tidal variables and needs initial data in the past, differently from what happens in the 3-orthogonal Schwinger time gauges. See Subsection 3.3 of the third paper in Ref. [43] for the comparison of the IAU harmonic gauge for BCRS with the 3-orthogonal gauges and Subsection 3.3 of the second paper in Ref. [43] for the equations identifying the 4-coordinate transformation from the 3-orthogonal gauges to the harmonic ones after the linearization, which have to be solved to get the reformulation of IAU conventions in our gauges.

4. Dark matter as a relativistic inertial effect

The linearized HPM Hamilton equations for point particles of mass m_i, $i = 1, .., N$[20], whose world lines $x^\mu_i(\tau) = z^\mu(\tau, \eta^r_i(\tau))$ are identified by radar 3-coordinates $\eta^r_i(\tau)$ due to the 3+1 splitting, and for the electromagnetic field coupled to tetrad gravity have been written explicitly in Refs. [43]: among the forces acting on matter, there are both the inertial potentials and the GW's.

In the third paper of Ref. [43], electro-magnetism is eliminated and there is a detailed studied of the HPM equations of motion of the particles. Then, the PN expansion of these regularized HPM equations of motion for the particles was studied, and it was shown that the particle 3-coordinates $\eta^r_i(\tau = ct) = \tilde{\eta}^r_i(t)$ (coinciding with the Newtonian coordinates of the world lines at this level of approximation) satisfy the equation of motion

$$\begin{aligned}&\frac{d}{dt}\left[m_i\left(1+\frac{1}{c}\,\frac{d}{dt}\,{}^3\tilde{\mathcal{K}}_{(1)}(t,\vec{\tilde{\eta}}_i\,(t))\right)\frac{d\,\tilde{\eta}^r_i(t)}{dt}\right]\overset{\circ}{=}\\&-G\,\frac{\partial}{\partial\,\tilde{\eta}^r_i}\sum_{j\neq i}\eta_j\,\frac{m_i\,m_j}{|\vec{\tilde{\eta}}_i\,(t)-\vec{\tilde{\eta}}_j\,(t)|}+\mathcal{O}(G^2).\end{aligned} \tag{6}$$

where at the lowest order, there is the standard Newton gravitational force

$$\vec{F}_{i(Newton)}(t) = -m_i\,G\,\frac{\partial}{\partial\,\tilde{\eta}^r_i}\sum_{j\neq i}\frac{m_j}{|\vec{\tilde{\eta}}_i\,(t)-\vec{\tilde{\eta}}_j\,(t)|} = -m_i\,\frac{\partial\,\Phi(t,\vec{\tilde{\eta}}_i(t))}{\partial\,\tilde{\eta}^r_i}. \tag{7}$$

Since Eqs. (4) imply

$$\epsilon^4 g_{\tau\tau}(\tau = ct, \sigma^r) - 1 = 2\,n(\tau = ct, \sigma^r) + O(G^2) = 2\,\frac{\Phi(t,\sigma^r)}{c^2} - \frac{2}{c}\,\frac{\partial}{\partial\,t}\,{}^3\tilde{\mathcal{K}}_{(1)}(\tau = ct, \sigma^r) + O(G^2), \tag{8}$$

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[20] m_i is both the inertial and the gravitational mass, since they coincide in Einstein GR due to the equivalence principle.

there is a 0.5 PN *inertial effect* (hidden in the lapse function) not existing in the Newton theory where the Euclidean 3-space is an absolute notion like the Newtonian time. It does not depend on the York time ${}^3K_{(1)}$ but on the *non-local York time* (Δ is the Laplacian associated to the asymptotic Minkowski 4-metric)

$$ {}^3\tilde{\mathcal{K}}_{(1)}(\tau,\sigma^r) = \left(\frac{1}{\Delta}\,{}^3K_{(1)}\right)(\tau,\sigma^r). \tag{9} $$

If we put ${}^3K_{(1)} = 0$, the standard results about binaries are reproduced.

The term in the non-local York time can be *interpreted* as the introduction of an *effective (time-, velocity- and position-dependent) inertial mass term* for the kinetic energy of each particle:

$$ m_i \mapsto m_i \left(1 + \frac{1}{c}\,\frac{d}{dt}\,{}^3\tilde{\mathcal{K}}_{(1)}(t,\vec{\tilde{\eta}}_i(t))\right) \tag{10} $$

in each instantaneous 3-space. Since, in the Newton potential, there are the gravitational masses m_i of the particles, the effect is due to a modification of the effective inertial mass in each non-Euclidean 3-space depending on its shape as a 3-submanifold of space-time. Therefore, we find *it is the equality of the inertial and gravitational masses of Newtonian gravity to be violated* in a gauge-dependent way in Einstein GR!

In the two-body case, one gets that for Keplerian circular orbits of radius r the modulus of the relative 3-velocity can be written in the form $\sqrt{\frac{G\,(m+\Delta\,m(r))}{r}}$ with $\Delta\,m(r)$ function only of ${}^3\tilde{\mathcal{K}}_{(1)}$.

The data on the *rotation curves of spiral galaxies* [14–17] imply that the relative 3-velocity goes to constant for large r instead of vanishing like in Kepler theory. As shown in Subsection 6.4 of the third paper in Ref. [43], this result can be simulated by fitting $\Delta\,m(r)$ (i.e., the non-local York time) to the experimental data with $\Delta\,m(r)$ interpreted as a *dark matter halo* around the galaxy.

Therefore, this dark matter can be explained as a *relativistic inertial gauge effect* consequence of the non-trivial shape of the non-Euclidean 3-space as a 3-submanifold of space-time. There is the concrete possibility to explain the rotation curves of galaxies [14–17] as a *relativistic inertial effect inside Einstein GR* (choice of a non-local York time compatible with observations) without modifications: (a) of Newton gravity like in MOND [9]; (b) of GR like in $f(R)$ theories [10–12]; (c) of particle physics with the introduction of WIMPS [58].

A similar interpretation (see Subsections 6.2 and 6.3 of the third paper in Ref. [43]) can be given for the other two main signatures of the existence of dark matter in the observed masses of galaxies and clusters of galaxies, namely *the mass determination with weak and strong gravitational lensing*[21] [18–20] and *the mass determination with the virial theorem* [21–23].

[21]In the case of gravitational lensing Einstein's deflection angle, $\alpha = 4\,G\,M/c^2\,\xi$ (ξ is the impact parameter of the ray of light deflected at the position of the mass M) has $M = M_{baryon} + M_{DM}$ with the dark matter term given by $G\,M_{DM} = -2\,c^2 \mid \vec{\sigma} \mid \partial_\tau\,{}^3\tilde{\mathcal{K}}_{(1)}(\tau,\sigma^u)$.

Therefore, there is the possibility of describing part (or maybe all) dark matter as a *relativistic inertial effect*.

The quoted three main experimental signatures of dark matter are well-defined functional of the time and space derivatives of the non-local York time[22] the inertial gauge variable describing the general relativistic remnant of the gauge freedom in clock synchronization.

Since the time evolution of the signatures of dark matter is not known, at best from the data, we can extract information only on a *mean value in time* of the time- and space derivatives of the non-local York time. Since from Eq. (7), we see that $-\partial_\tau\, {}^3\tilde{\mathcal{K}}_{(1)}(\tau, \sigma^r)$ is a modification of the Newton potential, we can assume that in Einstein GR the gauge variable non-local York time can be equated to the time-independent potentials $V(\sigma^r)$ used either in phenomenology or in modified theories of GR to describe dark matter in either galaxies or cluster of galaxies. Then, we can make the ansatz ${}^3\tilde{\mathcal{K}}_{(1)}(\tau, \sigma^r) = -\tau\, V(\sigma^r)$ and find the local York time ${}^3K_{(1)}(\tau, \sigma^r) = \Delta\, {}^3\tilde{\mathcal{K}}_{(1)}(\tau, \sigma^r)$ connected with the dark matter of the chosen either galaxy or cluster of galaxies.

Since there is no indication of dark matter in the voids existing among the clusters of galaxies, we can get an idea on the form of the local York time in the 3-space Σ_τ (i.e., the whole 3-universe) by summing its value for all the known galaxies and clusters of galaxies. This would produce an indication of which could be a metrology convention on the inertial gauge variable describing the general relativistic gauge freedom in clock synchronization in the Einstein space-time outside the Solar System. One expects that, with this metrology convention, the resulting 3-spaces (each one with all the clocks synchronized) are nearly Euclidean except where there is need of introducing dark matter.

In Ref. [59], there is a first attempt to fit some data of dark matter by using a Yukawa-like ansatz on the non-local York time of a galaxy. In each galaxy, the Yukawa-like potential of $f(R)$ theories [10–12] is put equal to a contribution to the extra potential depending on the non-local York time present in the lapse function appearing in Eq. (8); in this way, the good fits of the rotation curves of galaxies obtainable with $f(R)$ theories can be reproduced inside Einstein's GR as an inertial gauge effect.

5. Metrology against darkness

In conclusion, a suitable metrology convention on the inertial gauge variable York time could reduce or maybe eliminate the necessity of introducing dark matter in the classical universe and in its extension to classical cosmology after the recombination surface.

A needed natural proposal is now to define a *Post-Minkowskian ICRS* with non-Euclidean 3-spaces, whose intrinsic 3-curvature (due essentially to GW and matter) is small, in such a way

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[22] $\partial_\tau\, {}^3\tilde{\mathcal{K}}_{(1)}(\tau,\sigma^r)$ in the gravitational lensing case, $\frac{d}{dt}\, {}^3\tilde{\mathcal{K}}_{(1)}(c\,t, \vec{\tilde{\eta}}_i(t)) = (\frac{\partial}{\partial t} + \dot{\vec{\tilde{\eta}}}_i(t) \cdot \frac{\partial}{\partial \vec{\tilde{\eta}}_i})\, {}^3\tilde{\mathcal{K}}_{(1)}(c\,t, \tilde{\eta}(\vec{t}))$ in the rotation curve case and $\frac{d^2}{dt^2}\, {}^3\tilde{\mathcal{K}}_{(1)}(c\,t, \vec{\tilde{\eta}}_i\,(t))$ in the virial theorem case.

that the York time be (at least partially) fitted to the observational data implying the presence of dark matter. As a consequence, BCRS would be its quasi-Minkowskian approximation for the Solar System.[23] Let us remark that the 3-spaces can be quasi-Euclidean (i.e. with a small internal 3-curvature tensor), as required by CMB data in the astrophysical context, even when their shape as 3-submanifolds of space-time is not trivial and is described by a not-small York time.

In this way, one would get a solution to the gauge problem for the PM space-times of GR: one chooses a reference system of 4-coordinates in a 3-orthogonal gauge selected by the observational conventions for matter. A PM definition of ICRS will be also useful for the ESA-GAIA mission [60] (cartography of the Milky Way) and for the possible anomalies (different from the already explained Pioneer one) inside the Solar System [5].

Regarding *dark energy* in cosmology [24–29], we can remark that in the FLRW cosmological solution, the Killing symmetries connected with homogeneity and isotropy imply (τ is the cosmic time, $a(\tau)$ the scale factor) ${}^3K(\tau) = -\frac{\dot{a}(\tau)}{a(\tau)} = -H$, namely the York time is no more a gauge variable but coincides with the Hubble constant. However, in cosmological perturbation theory, we have ${}^3K = -H + {}^3K_{(1)}$ at the first order with ${}^3K_{(1)}$ being again an inertial gauge variable.

Let us also remark that in Szekeres space-times [61–63], that is, in inhomogeneous space-times without Killing symmetries, the York time remains an inertial gauge variable.

As said in Section2, the red-shift and luminosity distance of SNIa is a signal of *dark energy*. In Section3 of the third paper in Ref. [43], there is the evaluation of the dependence on the non-local York time of the PM time-like geodesics, whereas in Section4 of that paper, there is evaluated the dependence on it of the PM null geodesics, of the PM red-shift, of the PM geodesics deviation equation, of the PM luminosity distance and of the Hubble old red-shift distance relation (becoming the Hubble law if cosmology in introduced in the description). Like in the case of dark matter, one has a dependence on the second derivatives ∂_τ^2, $\partial_\tau\, \partial_r$ and $\partial_r\, \partial_s$ of the non-local York time now concentrated along the either time- or null geodesics. Therefore, also, this indication of dark energy is metrology dependent!

Let us also remark that in the back-reaction approach [64–69], in which to take into account the inhomogeneity of the observed universe when trying to get a cosmological description of it, one considers spatial mean values on large scales, dark energy in cosmology is a byproduct of the nonlinearities of GR. In this approach, one gets that the spatial average of the 4-scalar gauge variable York time gives the effective Hubble constant of this approach.

Finally, as shown in Eq. (10) of the last paper in Ref. [35], it can be shown that the York time is responsible for the negative terms in the kinetic energy term in the ADM energy, whose existence was known but whose explicit form could be given only in the York canonical basis. It is therefore possible that the connected Landau-Lifschitz energy-momentum pseudo-tensor [70] of GR could be reformulated as the energy-momentum tensor of a viscous pseudo-fluid,

[23]To test this possibility, one has to study the transition from harmonic gauges to 3-orthogonal ones in linearized Einstein GR.

which could have a negative pressure for certain choices of the York time like the dark energy fluid in FLWR cosmology.

In conclusion, the York time has a central position in all the cases where darkness is required to fit the data!

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