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# Characterization of Cultural Traits by Means of Fractal Analysis

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## Abstract

Archaeology, as a science, dares to explain how extinct societies functioned. As in all sciences, knowledge is built through the classification of data. In this case, data appear as fragments of objects that human groups have left behind. Traditionally, archaeological classification systems use stylistic criteria to assign the belonging of fragments to a territory, to a moment in time, and to a culture. The underlying idea is that changes in the characteristics of objects respond to changes in cultural processes. Despite a long tradition in the analysis of archaeological material, there is still a significant subjective component in which the classification criteria should be. If the archaeologist uses one that is too broad, then fragments with very diverse characteristics can be included in the same group. Conversely, if the criterion is too narrow, fragments that are very similar to each other, but not identical, will not be considered of the same type. Conclusions that depend on the size of the tool used in the analysis do not seem to be very sound. Therefore, the limits of traditional archaeological analysis have been reached. New perspectives are required to move forward. In this chapter, it is proposed that social vestiges acquire fractal properties by the repeated iteration of culturally transmitted rules embedded in their production processes. Complex patterns emerge in a variety of cultural manifestations, but are all related to the way in which cultural practices of different groups occupy space: practices related to, for example, tool elaboration, symbolic representation or the choice of the geographic location where they settle. Fractal properties are the reflection of these cultural practices and the metrics that synthesizes the properties of each of the cultural manifestations is its fractal dimension. The fractal signature is built as a distinctive set of fractal dimensions of cultural traits of a social group. This is intended with the construction of the Xajay culture's "fractal signature." Xajay civilization flourished to the south of the northern border of Mesoamerica from around 350 AD until its collapse in 900 AD.

**Keywords:** cultural patterns, archaeology, fractal dimension, fractal signature, Xajay, characterization

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## 1. Introduction

Archaeology is the science of fragments. Archaeologists excavate to obtain information from fragments of cities, buildings, tools, pottery and a great diversity of objects that when put together form complex patterns of cultural manifestations. The objective of archaeological work is the classification of the fragments to assign them to a chronological period, to a certain usage, and to a specific social group. In this process, fragments acquire meaning.

Traditional classifications are built on discrimination by region, by types of objects, of material, of forms, etc. If the groupings are meaningful, the result will be the association of the different objects to a social group and to a culture. However, in the analysis of a territory that has been occupied by different cultures and which remains fit in more than one classification, traditional methods quickly reach their limits and a different approach is necessary. The hypothesis that guides our research is that a culture can be better characterized by means of the fractal analysis of the objects it produced and has left behind for the archaeologists to find and analyze. Supporting the hypothesis is the idea that the know-how that underlies the production of every cultural manifestation is not acquired individually, but is part of a complex social transmission system. Therefore, the fabrication of the distinctive paraphernalia of a group implies the reiteration of social practices that result in patterns that can be identified by fractal analysis.

## 2. The elusive Xajay

Our research uses artifacts from the Xajay culture to attempt its characterization by means of fractal analysis. The Xajay culture flourished from 350 to 900 AD in a semi-desert region close to the northern border of Mesoamerica in what is now central Mexico. The most important of their ceremonial centers is Pahñu, located 180 km to the northwest of modern Mexico City. In ancient times, the Mesoamerican border divided the nomadic (barbarian, hunter and forager) groups to the north from the sedentary (civilized and agricultural) population to the south.

The frontier condition of the Xajay, as well as other of their characteristics, has challenged a precise archaeological classification of this group. At times, they have been thought of as a development associated to the northern expansion of the great city of Teotihuacan, whereas at others they were considered as isolated groups that defended the entry to the central Mexican plateau from the northern barbarians.

Among the problems that arose while trying to classify the Xajay is that their architecture has decorative elements that are like those used at the moment in Monte Albán—located almost 500 km to the southeast—, but has other elements that are similar to those used by the Aztecs more than 700 years later [1]. Another classification predicament is that the Xajay ceremonial centers are contemporary to Teotihuacan and within its potential area of control; however, the influence of the big city on their small neighbors is almost insignificant. For example, the archaeological record of Teotihuacan-controlled sites always denotes the presence of artifacts

knapped from green obsidian. The access to the sources of this volcanic glass was monopolized by Teotihuacan for hundreds of years [2]. In contrast, the archaeological record of the Xajay sites only yields black obsidian obtained from local sources. A final inconsistent classification topic that is worth mentioning refers to the widespread stone engraving tradition in pre-Hispanic Central Mexico. Instead, the motifs of the Xajay petroglyphs are more like those of the Bajío region, located to the west of the Xajay sites [3].

With this conflicting evidence, every attempt to classify Xajay remains with respect to a previously known culture ended in ambiguity. As more archaeological material was obtained and analyzed, the prevailing confusion only increased. Also, classifications became so detailed that they became useless.

Here is where we believe that fractal analysis can make a difference. We used fractal analysis on some elements that had been previously used to attempt the characterization of the Xajay with two basic underlying questions: how similar among themselves are the remains; and how different are these remains with respect to those of other cultures?

### 3. The fractal nature of archaeological objects

The first thing that had to be done was to confirm the fractal nature of archaeological objects. If the challenge was to prove the usefulness of the fractal dimension for archaeological analysis, then it was important to justify why archaeological objects have fractal properties. It makes no sense to calculate the fractal dimension of something that is not fractal.

In mathematics, fractals are complex objects that show self-similarity at all scales resulting from the repeated iteration of a simple rule. In nature, again because of the reiteration of a simple rule, certain objects show self-similarity, but only on a limited number of scales due to the limitations imposed by matter. It is said that these are statistical fractals [4]. We propose that in the cultural domain, objects acquire statistical fractal properties due to the repetition of a simple (cultural) rule. This can explain the emergence of complex patterns in cultural manifestations, in domestic aspects such as pottery, textiles, and even hair-dos, but also in more social and aggregate phenomena such as urban and settlement patterns.

This definition of a cultural fractal fits perfectly well in archaeology. The simple rules that are repeated to form cultural fractals are the traditions that are transmitted from one generation to another and manifest themselves in the specific way in which things are done within a social group. For example, these traditions have to do with the sequence and strength of gestures that result in knapping a projectile point from an obsidian core, the steps involved in engraving blocks of tuff to obtain a certain symbol or glyph, or the way in which clay is knead and shaped to form a characteristic vessel, etc. All these social practices, repeated recursively, leave traces in the archaeological record and can be subject to fractal analysis. The fractal properties of cultural manifestations are the direct consequence of cultural practices. The metric that synthesizes the properties of each of the cultural manifestations is its fractal dimension.

In brief, it can be said that the fractal dimension measures the way in which an image fills the space that contains it. In a social domain, it is straightforward to think that two cultures will use space in distinct ways; their cultural manifestations will occupy space differently. Therefore, the fractal dimension of the objects they produce can be used to identify specific cultural traits.

Then, there is the problem of how to detect the fractal nature of the objects from the images that represent them, mostly because the fractal patterns are not always visible to the naked eye.

Box-counting is the most commonly used method for calculating fractal dimension from images. Essentially, this method calculates the fractal dimension as the relation between the number of squares (boxes) needed to cover an image and the size (scale) of the squares. If this relation is stable when the size of the squares changes, then there is scale invariance, and it can be said that the image fills the space that contains it in the same manner, with independence of scale. In other words, there is also some sort of self-similarity. We can be confident then that we are in the presence of an object with fractal properties, and that it makes sense to calculate its fractal dimension.

## 4. Choice of tools

The first issue we had to address was the choice of a tool to calculate the fractal dimension of the archaeological images. It is amazing that despite a relatively long history of fractal analysis in archaeology<sup>1</sup>, there is very little discussion of the results of research and even less of the methods employed. The literature we reviewed was of little help for deciding the software to use for the calculation of fractal dimension. We had access to five open-source or freely available programs<sup>2</sup> and to our surprise, each one gave a very different result for the fractal dimension of a same image using the same method (Box-counting).

Before going any further, it was necessary to clarify these differences and have certainty with respect to what was being calculated by each of the programs. In the end, we identified the main source of discrepancy in the way each program processed the images and converted them into binary files which are the input needed to use the box-counting method. We found other minor differences that derived from the way each program placed the grid of squares to cover the images, and from the way the size of the squares is increased in each iteration of the procedure<sup>3</sup>. By controlling the binary images that were used as input and by adjusting the parameters of grid placement and square size, we could obtain the same result for the fractal dimension of the same image calculated by each of the five programs.

<sup>1</sup> The earliest work that we are aware of is [12].

<sup>2</sup> We tested: (a) *FROG* (Fractal Researches on Geosciences) v1.0 developed by Jean-François Parrot of the Geography Institute at UNAM; (b) *Fractalyse* v2.4, developed at Université de Franche-Comté de Besançon; (c) *fractal3*, v3.4.7, developed by the National Agriculture Research Organization of Japan; (d) *HarFa*, v DEMO 5.5.30 developed at the Brno University of Technology, Czech Republic; and (e) *ImageJ* v1.46r developed by the US National Institutes of Health, and its plug-in *FracLac* v2.5, developed by A. Karperian at Charles Sturt University of Australia.

<sup>3</sup> For a complete description of the comparison of the five programs, see Ref. [20].

Having clarified the procedures of each of the programs, we decided to use *FracLac* for *ImageJ* because it allowed for a finer parametrization which gives better control over the process, it calculates not only the fractal dimension but other indicators, such as lacunarity that would result useful in our analysis, and finally because it was possible to batch process large sets of images.

A crucial lesson that can be drawn from this first step of the experiment is that software should not be taken as a black-box that produces a magical number: the fractal dimension. If there is not a discussion on methods, then the results can be very interesting, but will only be relevant for isolated cases. It is important to produce results that can be comparable to others so research can build on previous works.

## 5. Fractal dimensions of the Xajay culture

The first issue to define is what are the traits that can be measured and allow the characterization of a culture by means of fractal analysis. Archaeological evidence suggested that at least three distinctive cultural traits of the Xajay should be used: (a) the location in a specific landscape where the group built its ceremonial centers; (b) the stone engravings that can be found throughout what has been interpreted as their sacred territory; and (c) the projectile points included in offerings as part of their funerary practices.

### 5.1. Location

The study of location, landscape, and territory is common in archaeological research. They are topics that have also been approached by fractal analysis. A rigorous paper on the intervention, organization, and planning of space in pre-Hispanic times is that of Oleschko et al. [5] who proposed to use fractal geometry to identify the urban master-plan of the city of Teotihuacan. In their study, they analyzed satellite imagery and aerial photography of the archaeological zone. It is worth noting that an important part of their effort was devoted to verifying the fractal nature of what was represented in the images they analyzed. In particular, they proposed that the invariant fractal dimension that was obtained from images with different scales was evidence of self-similarity. The results of their calculations for the principal buildings of Teotihuacan as well as for the archaeological site as a whole gave a fractal dimension of 1.89 with minor variations. This outcome was the same in satellite imagery as well as in aerial photographs with scales of 1:30,000 and 1:5000. With this, the authors sustain their claim that the main buildings of Teotihuacan have fractal properties. Even more, they propose that, given the visual similarity of the plan of the Ciudadela complex with the mathematical fractal known as the Sierpinski Carpet—which coincidentally has a fractal dimension of 1.89—it was very possible that the urbanists and architects of Teotihuacan had in mind schemas and parameters like those that give place to the mathematical fractal.

Other examples are [6, 7] that propose a relation between the intra-site settlement pattern with kinship and between the regional distribution of sites with warfare. The argument focuses on the rank-size distribution observed in archaeological materials and in the settlement patterns to sustain their fractal properties; however, they also mention that fractal dimension



can be calculated from images. It is interesting that the authors warn explicitly that neither everything is fractal nor all dynamics are nonlinear, but that many archaeological patterns are fractal and should be described and analyzed properly.

Landscape has also been a topic in the analysis of the Xajay. The location where this group built all their ceremonial centers is peculiar. The eruption of a huge caldera about 4 million years ago formed vast tuff plains that have since suffered erosion and given place to plateaus or mesas that end abruptly on their northern side. It is precisely on the edge of these mesas that the Xajay decided to place their ceremonial centers. Because of this characteristic, the Xajay were once named the Mesas Culture [8].

We built on the assumption that if the location of the sites is peculiar, then it should have a specific fractal dimension and of course, it should be different from that of other locations. We used topography as represented by contour lines as a proxy for location to measure its fractal dimension. Contour lines are abstractions that represent points in a map with the same altitude. The more rugged the terrain, contour lines will be closer to each other and have more twists and bends: they will fill the map space in a more complete way. Consequently, it should be expected that the fractal dimension of a topographical map of a mountainous landscape should be greater than that of a flat location.

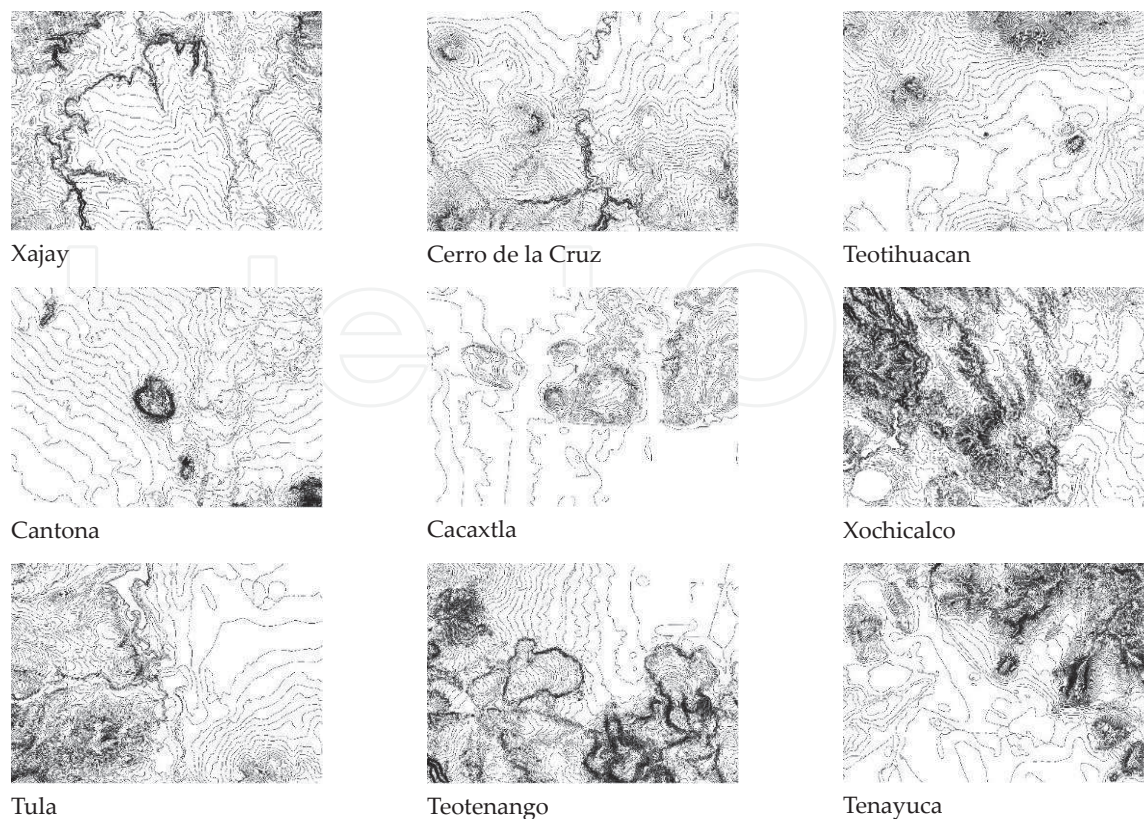
To prove this hypothesis, topographical maps were obtained for the location of the Xajay ceremonial centers and eight other archaeological sites in Mexico: Teotihuacan, Cantona, Cacaxtla, Xochicalco, Teotenango, Tula, Tenayuca, and Cerro de la Cruz. However, we were not only interested in quantifying the ruggedness of the terrain but also in sustaining that the precise location of the archaeological sites, at least for the Xajay, had a cultural meaning. Therefore, we expected that the fractal dimension of the topography of the location of the Xajay ceremonial centers would be like that of Cerro de la Cruz with whom, it is known [9], they share cultural traits—among others the placement of sites on edges of cliffs.

Contour lines were extracted from the digital elevation models provided online by the Mexican Institute of Geography (INEGI). The resolution of the digital elevation model files is 15 m per pixel and differences in altitude represented by each contour line is 10 m. The generated maps, covering approximately  $10 \times 15$  km in a scale of 1:50,000 were stored in TIFF files of  $3500 \times 2480$  pixels and are shown in **Table 1**.

Because different images can have very similar fractal dimensions, lacunarity was also calculated to discriminate more effectively between images. Lacunarity was proposed by Mandelbrot [10] as a complementary measure to fractal dimension. If the fractal dimension measures the way an image fills the space that contains it, lacunarity measures the holes or the lumpiness of the image, the way in which it does not fill space.

In **Figure 1**, the fractal dimensions and lacunarities of the topographical maps of the nine archaeological sites are plotted.

Even if the limited number of points in the sample do not allow for statistically significant inferences, it is relevant that the points representing the values of maps of the Xajay ceremonial centers and of Cerro de la Cruz are proximate to each other, which corresponds to what



**Table 1.** Topographical maps of archaeological sites.

was expected because of the shared cultural traits. In this case, topography has a cultural meaning. The other sites have topographies whose combination of fractal dimension and lacunarity are quite separate from the Xajay; hence, we can conclude that they do not share with them their characteristic cultural placement on the edge of cliffs.

These results confirm that the particularity of the location of the Xajay ceremonial centers is more than something perceptual: it has been measured. The placement of the sites on the edges of cliffs is now associated to specific values of fractal dimension and lacunarity of their topographies. In this way, it can be used to distinguish objectively between preferences of ancient peoples for the building of their sites.

## 5.2. Petroglyphs

The existence of petroglyphs in and around the ceremonial centers is another trait that has been mentioned as characteristic of the Xajay culture [8]. There are almost 200 petroglyphs registered in the five Xajay ceremonial centers<sup>4</sup>, being Zidada the one where they have been found more abundantly. A selection of 23 images of petroglyphs from the survey records of Zidada was prepared for measuring their fractal dimension. The petroglyph motif was

<sup>4</sup> Apart from Pahñu, the main ceremonial center, the Xajay built Zidada, Taxangú, Cerrito, and Zethé, all of them on the northern edge of cliffs. The maximum distance between the sites (Zidada to the west and Zethé to the east) is less than 8 km.



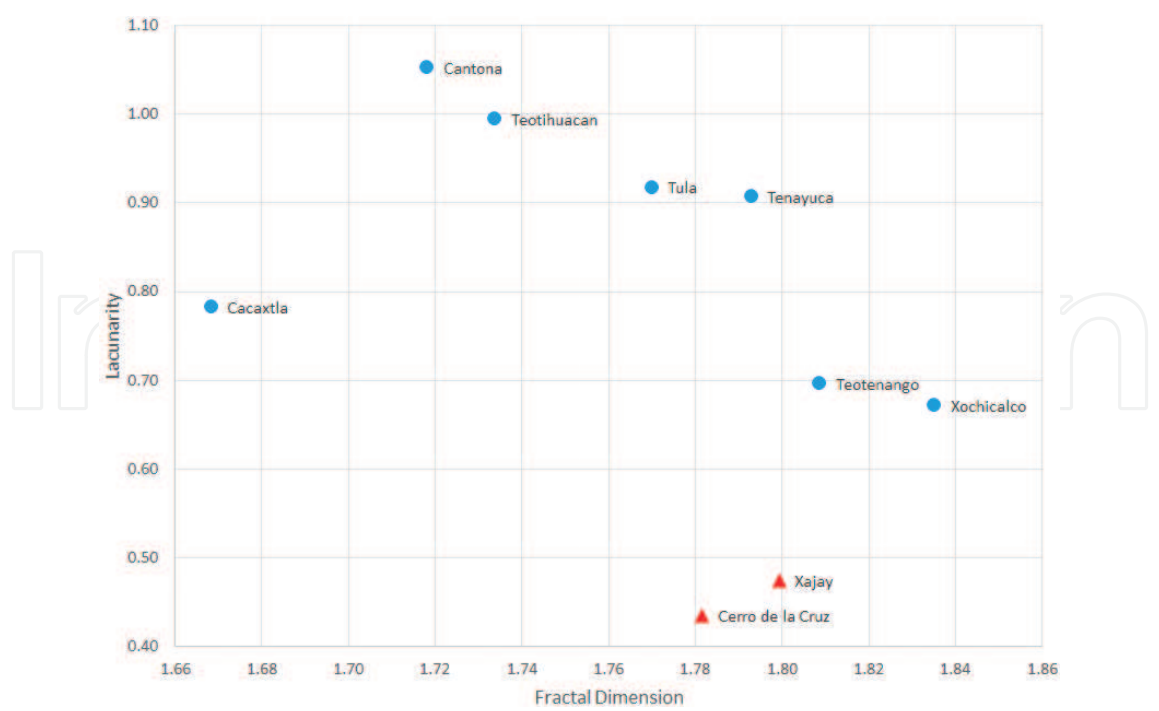


Figure 1. Fractal dimension and lacunarity of the location of archaeological sites.

retraced as a black line on the digitized original survey photographs. Then, the background photograph was erased and the file was converted to a binary format. Table 2 illustrates this process.

The mean fractal dimension of the sample of 23 images is 1.1234 with a standard deviation of 0.0688, denoting a relatively compact grouping. This can be indicative that petroglyphs in and around the Xajay ceremonial centers are specific of this culture. The values of the fractal dimension of the Zidada petroglyphs were compared to those of three other pre-Hispanic petroglyphs from the Xajay sites of Pahñu and Taxangú and to that of a modern stone carving found near Zidada. Assuming normality in the distribution, it can be sustained that there is more than 95% probability that the pre-Hispanic petroglyphs belong to the same set as the ones from Zidada. With the same probability, we can conclude that the modern petroglyph has a motif that does not belong to the Xajay culture. Another comparison was made


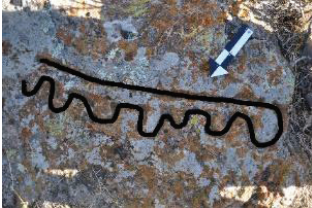

Original survey photograph	Retraced motif	Binary image
		
Zidada 11		

Table 2. Example of the extraction of petroglyph motif.

with two petroglyphs that functioned as astronomical markers from the site of Xihuingo [11], contemporary to Pahñu, but with a strong influence from Teotihuacan. Not surprisingly, the fractal dimension of these petroglyphs fell outside of the 95% confidence interval for the Xajay motifs.

These results support and give a numerical expression to the idea that the Xajay elaborated a specific type of petroglyphs in association with their ceremonial sites. Xajay petroglyphs are similar and this is one of their characteristic cultural traits.

### 5.3. Projectile points

Orton and Grace, two British archaeologists, made team with two computer specialists and conducted a pioneer research on the fractal properties of archaeological lithic material [12]. The authors confirmed their intuition that the fractal dimension would change depending on the different erosive processes that affected flint flakes, but could not meet their objective of determining the use that had been given to the artifacts by the differences in the fractal dimension of the images of microware types. Nonetheless, they set the ground for further research based on fractal analysis in archaeology.

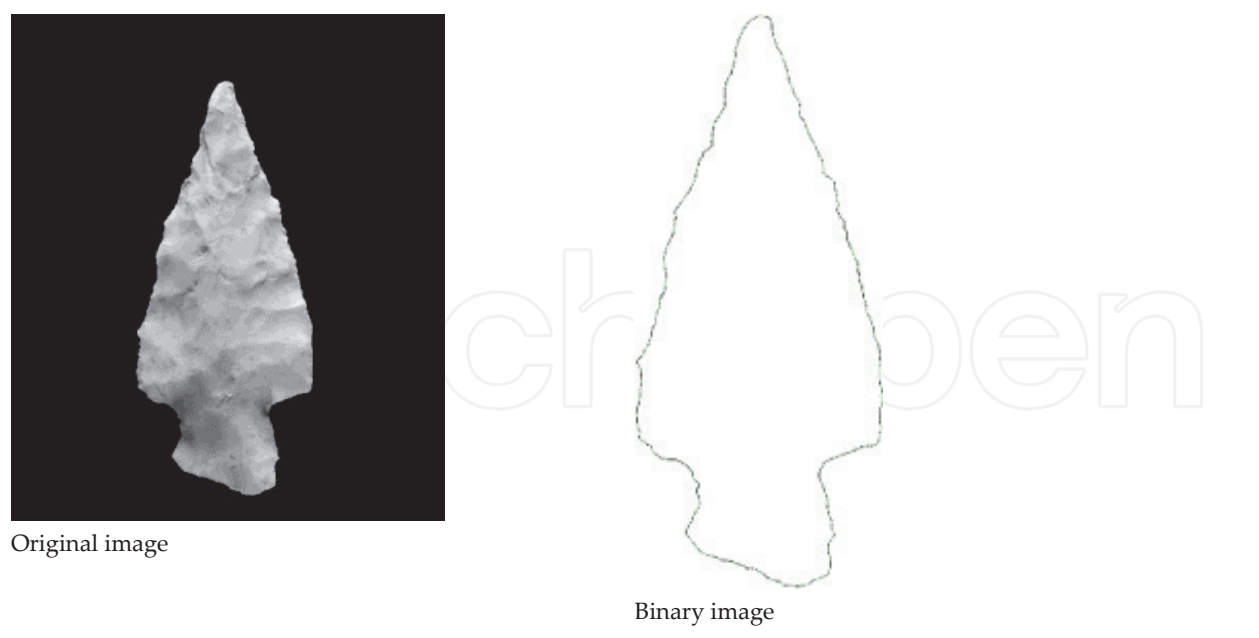
During excavations at the Xajay site of Pahñu, a significant quantity of archaeological material was recovered. Of special interest was the material of an offering placed in a cache under the northwest corner of the main pyramid around 500 AD. Among the findings, archaeologist recovered a great diversity of projectile points, of different forms and made of different materials. These points were used to explore if, despite the diversity, some specific Xajay trait could be revealed through the fractal analysis of their images.

As with the petroglyphs, the images of the points had to be prepared for their processing. The silhouette or edge of the images of the dorsal side of 54 projectile points were extracted from the original digitized photographs and saved in a binary format. **Table 3** illustrates the process.

Fractal dimension and lacunarity were calculated for the 54 images of projectile points. In both calculations, the dispersion of values is relatively small, which can indicate a clustering of the points in terms of these characteristics.

Plotting fractal dimension against lacunarity of the images reveals what could be two groups within the Xajay projectile points. A first group formed by the points whose image has a fractal dimension less than 1.09, and the second group has fractal dimensions greater than this value.

To find an explanation for this segmentation, the data were compared to the information contained in the lithic database of the excavation project. The grouping of the data could be explained by differences in the material out of which the points were knapped. The points with a fractal dimension less than 1.09 were made from obsidian with two exceptions. The only point made from basalt belongs to this group, and there is one made from flint that requires a more detailed analysis to determine why it has such a low fractal dimension compared to the other flint points. The second group of points with a fractal dimension greater



**Table 3.** Example of the extraction of the silhouette of a point.

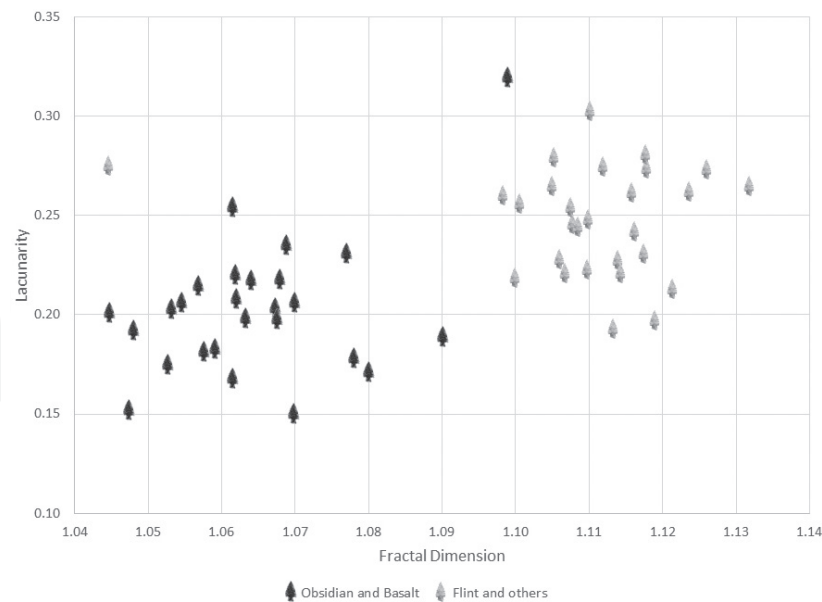
than 1.09 comprises 20 points made from flint, 5 from chalcedony, 1 from calcite, 1 from rhyolite, and 1 from obsidian. **Figure 2** shows the segmentation of the points with respect to their knapping material.

The mean fractal dimensions of these two groups of points resulted different with a confidence interval of more than 99%. Also, and this is not clearly visible to the naked eye, the mean lacunarity of the two groups is likewise different. It follows that, as a complementary measure, lacunarity is also useful to distinguish between these two groups<sup>5</sup>.

This result could lead us to think that fractal dimensions and lacunarity are simply distinguishing some physical attributes of the raw material, and not a cultural practice, which, in the last instance, is the relevant criterion to characterize a human group. However, if the physical properties are explaining the clustering of fractal dimensions and lacunarities, then as many groups as different raw materials would be discernible, and this is not the case.

To further explore if the segmentation of fractal dimensions was due to the physical properties of the raw material, the Xajay collection was compared to the images of other points obtained from the Internet. The criteria for selecting the points were that they should not be very different in form than the Xajay so the differences could not be explained by this reason. For contrasting, four very distinct Clovis points were also chosen, which in principle should have a very different fractal dimension than the Xajay points. Most of the images were taken from the online catalogue of the American Southwest Virtual Museum [13]. The comparative set of images was composed of 15 points from Arizona, of which 2 were obsidian, 4 Clovis

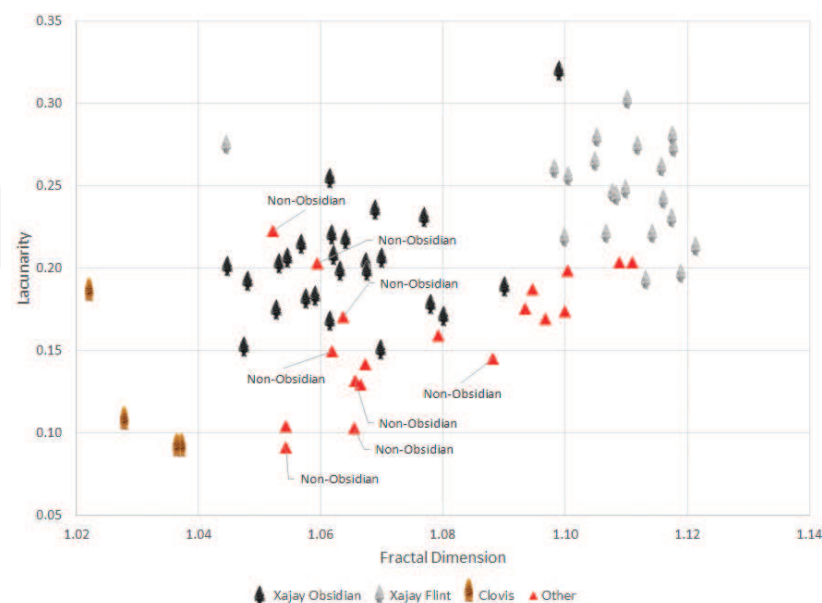
<sup>5</sup> The value of Student's *t*-statistic for the independent sample equality of means test for fractal dimensions was 11.093, with 52 degrees of freedom. The two-tailed confidence interval is greater than 99%. For lacunarities, Student's *t*-statistic was 5.320 with the same degrees of freedom and confidence interval as for fractal dimensions.



**Figure 2.** Fractal dimension and lacunarity of Xajay projectile points.

points from several locations in the United States, 1 point from Kansas, and 1 obsidian point from an unknown provenance in the southwest of the United States. An obsidian point from Teotihuacan and a flint point from Belize were also included in this set. **Figure 3** plots the fractal dimension and lacunarity of the ensemble of points.

The first thing that is worth noting is that the four Clovis points clearly form a distinct group with the lowest fractal dimensions of all. The number of observations is very small, and statistically significant inferences cannot be made, but once again, the potential of fractal



**Figure 3.** Fractal dimension and lacunarity of projectile points.



dimensions to distinguish among groups of objects is manifest. The Clovis points are visibly not made from obsidian but probably from a sedimentary material. The fact that their fractal dimensions are smaller than those calculated for the Xajay points in general and much smaller than those of the points made from sedimentary material gives some weight to the argument that the fractal dimension is not measuring the physical attributes of the artifacts.

As for the set of points from the American Southwest and the other isolated points, the fractal dimensions of those knapped from obsidian are all in the range of the Xajay points made from that material. Also, fractal dimensions of the points made from other material are distributed in the whole range of values of the Xajay points, independently of the raw material used in their manufacture. Once again, there is an indication that fractal dimensions are measuring something more than the simple attributes of raw materials.

Even if the differences in fractal dimensions were related to the raw material, it could be argued that it is due to the precise gestures involved in working with them, and this is cultural. Artifacts are manufactured from each raw material through a carefully thought out sequence of interrelated actions. The knowledge behind this sequence is transmitted within each human group.

## 6. The fractal signature

Up to this point of the research, we have obtained a handful of fractal dimensions, each one of them saying something with respect to a trait of the Xajay culture. The next step was to integrate them into a more synthetic measure that could represent the ensemble of the Xajay culture. A characteristic required of this synthetic measure was that it distinguishes the Xajay from any culture with whom it was compared. The fractal signature is a measure that complies with this condition.

Given the ambiguity with which the concepts of fractal dimension and fractal signature are used in literature, their distinction is worth a brief discussion. The lack of precision in the use of these terms is not exclusive of fractal archaeological research. At times, the distinction is solved by choosing one, or the other, or simply by making them interchangeable. See, for example, Refs. [14, 15].

It is amazing that the most precise definitions of fractal signature occur in the extremes of the observation spectrum. At a macro level, in astronomy, near galaxies are distinguished by the fractal dimension of their images at different resolutions [16]. With a method, similar to box-counting, the area of a galaxy in an image is calculated by counting the non-black pixels. The authors observed that the area of the galaxies change when the resolution of the image changes. For them, the fractal signature is the relation between the area of a galaxy calculated from different images and the resolution of the images.

In the opposite extreme of the spectrum, there are several medical science papers that use the fractal signature for the analysis of the microscopic structure of bone tissue to detect anomalies. The definition used in some of these papers is like the one used in the study of galaxies.

The fractal signature is the variability of fractal dimension calculated from images with a different resolution: “a fractal signature [...] is simply the estimated fractal dimension as a function of scale” [17].

The problem faced by astronomers and physicians is that the fractal dimension of objects represented in the images they were using (galaxies or microscopic structure of tissue) varied as a function of the resolution of the image. In other words, the fractal dimension of an object changed depending on the “magnification” with which the image had been captured. There was thus a need for a method permitting their comparison, independently of the capture conditions.

The above-mentioned works are based on the study of Peleg et al. [18], who used mathematical morphology procedures to solve a general problem arising from the characterization of textures. The sharpness of digital images changes with their resolution, which in principle conflicts with the properties of fractal objects that should be self-similar independently of scale. The fractal signature was defined to measure the degree of detail or sharpness that is lost when the resolution of the image is reduced. “The magnitude of the fractal signature  $S(\varepsilon)$  relates to the amount of detail that is lost when the size of the measuring yardstick passes  $\varepsilon$ .” [18] Note that  $\varepsilon$  refers to the measurement scale or to the resolution of the image. Another way of understanding this would be that the fractal signature measures the change in the fractal dimension of objects calculated from images that represent different “levels of observation.”

The usefulness of fractal signatures in research that propose to measure the detail of “texture” of tissues or galaxies from microscopic or telescopic observations is clear. In social sciences, and particularly in the research that we present, the analysis of “texture” of cultural remains is not only done through successive magnifications, but the different “levels of observation” imply seeing the archaeological context from diverse domains, opening the field of vision toward object of varied nature.

We propose that the concept of fractal signature in archaeology can be recovered if it is considered as the measure of change in fractal dimension of the different aspects of material culture. In this case, fractal dimension would not change only because of the resolution of images of a single object, but because the observations would include distinct aspects of culture. The fractal signature would not be a single number, but a set of numbers each one measuring the fractal dimension of a particularity of the archaeological context. Under the hypothesis that archaeological objects have fractal properties due to the repeated iteration of cultural rules, it is reasonable to assume that those that govern the fabrication of pottery are different than those applied to architecture and urbanism, and also different than those present in sculpture, stone engraving or any other activity that leaves traces in the archaeological context. Therefore, each aspect or element of a social group has a specific fractal dimension, and the ensemble of these dimensions is what constitutes their fractal signature.

The obvious question that arises is how many aspects of a culture should be observed to have an adequate characterization. In other words, how many elements should be measured to have an effective fractal signature. Fractal dimension can be used to measure

as many aspects of a culture as is required to build a fractal signature that characterizes adequately a human group. In principle, the inclusion of additional traits would be never ending; the number of dimensions of the cultural space would increase incessantly. Nonetheless, the same result should be expected: each group will have a unique location in that space. The way this is understood is related to the way in which groups, societies or cultures are looked at. Each one has endless components, their difference is not in the similarity or lack of the components, but in the way in which these interact and combine. In other words, what is important is to understand how the components form a dynamic system with transformation rules of its own. Considering this, we propose that the fractal signature is the representation of the systemic dynamics of the group in the  $n$ -dimensional space of culture.

In what follows we will present some ideas of how a prototype of the Xajay fractal signature could begin to be assembled from the set of fractal dimensions of the attributes that were measured. To facilitate presentation, the fractal signature will consider only the mean fractal dimensions of the topography of the placement of the sites, of the petroglyph motifs and of the silhouettes of the projectile points, as shown in **Table 4**.

Likewise, it can be represented in a vector notation in the three-dimensional cultural space defined by topography  $\times$  petroglyphs  $\times$  points.

$$\overrightarrow{\text{XajayFS}} = [1.7973, 1.1243, 1.0882] \tag{1}$$

This combination of fractal dimensions occupies a unique position in this space and is therefore proposed that the fractal signature expressed in this manner is a measurement of the specificity of the Xajay.

The usefulness of the fractal signature to make distinctions between groups would be given by the fact that the measurements of the attributes of the distinct groups would occupy different positions in this space.

To illustrate this, fractal dimensions of the three domains that form the prototypical cultural space were obtained for Teotihuacan and Cantona, enabling the comparison with the Xajay. This is presented in **Table 5**.

A graphical representation of these fractal signatures in the three-dimensional cultural space is shown in **Figure 4**.

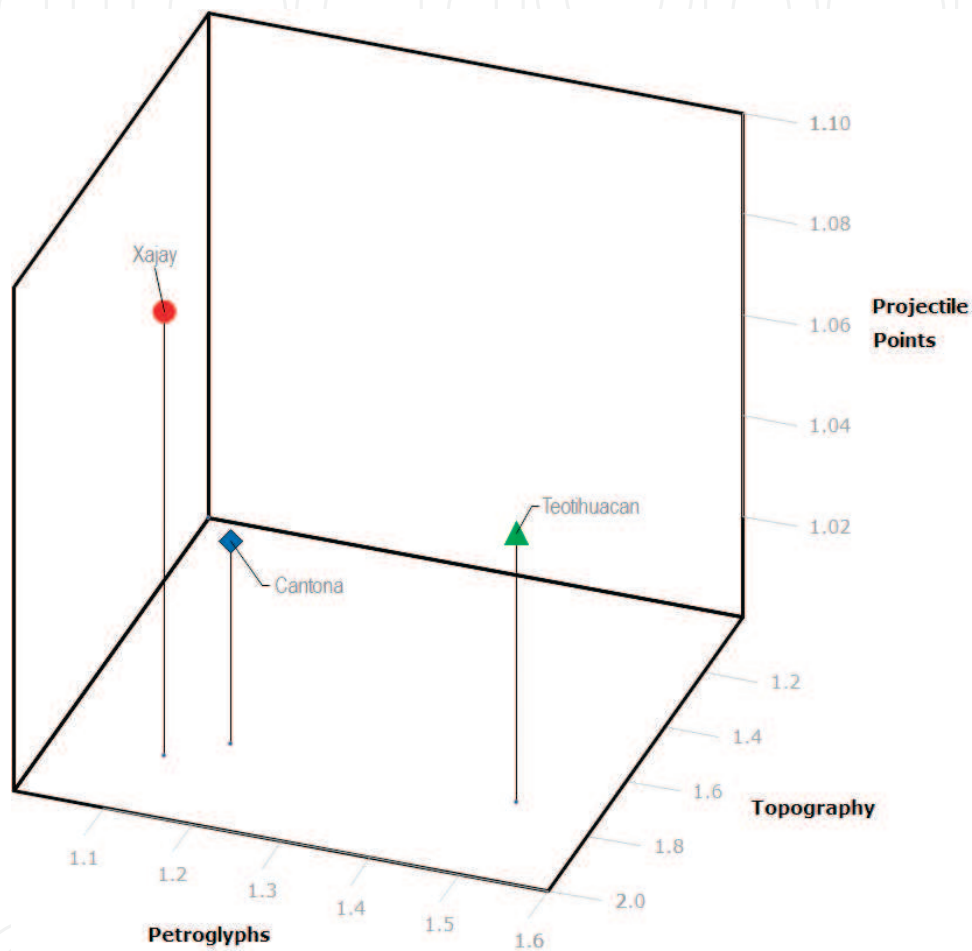
The graph clearly shows how each cultural group occupies a unique position in the space determined by the fractal dimensions of their attributes.

Topography	Petroglyphs	Projectile points
1.7973	1.1243	1.0882

**Table 4.** Prototype of the Xajay fractal signature.

	Topography	Petroglyphs	Projectile points
Xajay	1.7973	1.1243	1.0882
Teotihuacan	1.7339	1.5067	1.0535
Cantona	1.7181	1.1819	1.0402

**Table 5.** Three attribute fractal signature of the Xajay, Teotihuacan, and Cantona.



**Figure 4.** Fractal signature of the Xajay, Teotihuacan, and Cantona.

## 7. Conclusions

The driving purpose of our research was to assign quantitative measures to the classifications made by archaeologists in their day-to-day work. Classification of objects and cultural traits are made from the knowledge and experience acquired through research and are always biased in some way and have a certain dose of ambiguity. It is not our intention to demerit archaeologists. Their work has achieved the non-easy task of giving form to the history of social transformations. Moreover, when societies under study do not leave written records, research becomes even harder and the only thing archaeologists can get a grasp on to reconstruct history from are fragments of



objects. With these fragments, the functioning of societies and the relations between its elements have to be recreated. To reduce subjectiveness in archaeological work, we have proposed a model to characterize different cultures by means of their fractal dimensions. The bet is that, independently of how many elements (fractal dimensions) are considered, there will never be two groups that occupy the same location in the  $n$ -dimensional space of culture, given that each point in this space (each fractal signature) is the representation of one and only one social group.

Characterization of social groups by means of fractal analysis opens the door to a great variety of future research. For example, the configuration of the territory occupied in pre-Hispanic times can be represented by a cloud of points in cultural space, each one associated to a social group. The points of this cloud could get closer or move away from each other, giving place in certain moments to clusters of cultural significance. One of these clusters is probably Mesoamerica.

Another line of research, maybe much more ambitious, is related to the discovery of the “generators” of the fractal properties of cultural objects, namely the simple rules whose repetition produces fractal patterns. These are the lines developed in Ron Eglash’s classic work on African fractals [19]. In this chapter, we have shown the potential of the fractal signature to characterize the Xajay from the fractal properties of different aspects of their culture, but there was no attempt to discover the cultural rules that produced them. If such rules could be determined, if their repetition could be identified as processes, an important step would be taken in the understanding of the interactions of the different components of the Xajay material culture.

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