Chapter 10

Fractal Geometry and Porosity

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Abstract

A fractal is an object or a structure that is self-similar in all length scales. Fractal geometry is an excellent mathematical tool used in the study of irregular geometric objects. The concept of the fractal dimension, *D*, as a measure of complexity is defined. The concept of fractal geometry is closely linked to scale invariance, and it provides a framework for the analysis of natural phenomena in various scientific and engineering domains. The relevance of the power law scaling relationships is discussed. Fractal characteristics of porous media and the characteristic method of the porous media are also discussed. Different methods of analysis on the permeability of porous media are discussed in this chapter.

Keywords: fractal geometry, fractal structure, fractal dimension, porous media, permeability

1. Introduction

Fractal is one of the subjects, which recently attracted attention in natural science and social science. A fractal is defined as a geometric object whose fractal dimension is larger than its topological dimension. Many fractals also have a property of self-similarity; within the fractal lies another copy of the same fractal, smaller but complete. Mandelbrot [1] referred to fractals as structures consisting of parts that, in some sense, are similar to integers; fractals are of a fine (non-integer) dimension (*D*) that is always smaller than the topological dimension. In the past 40 decades, fractal theory has significantly contributed to the characterization of the distribution of physical or other quantities with a geometric support. In science and engineering, fractal geometry provides a wide range and powerful theoretical framework that is used to describe complex systems, which have been successfully applied to the quantitative description of microstructures such as surface roughness and amorphous metal structure [2].

Typically, microstructure elements can be explained using the Euclidean dimension (d). With respect to point defects (e.g., vacancies and interstitial atoms), $d = 0$; with respect to linear defects (dislocations), $d = 1$; with respect to planar defects (twins), $d = 2$; and with respect to three-dimensional (3-D) formations, $d = 3$. Nonetheless, the Euclidean dimension cannot be used to illustrate structural elements differing from standard ones (e.g., points or straight lines). Thus, a well-known grain boundary, being the most significant element of the microstructure, is curvilinear, and this form can be described by the fractal dimension (D) correlating to $1 \leq D \leq 2$. Surface defects may also be illustrated using the fine dimension that will commensurate to the range $2 \leq D \leq 3$ [3]. Therefore, fractal theory introduces a new quantitative parameter-fractal dimension for illustrating structures, which, because of its universal nature, is appropriate for illustrating structures in systems types. With a system such as a deformed solid, the fractal concept provides the possibility of quantitatively illustrating the elements of the initial microstructure (e.g., phases, grain, boundaries, etc.) and the structures formed during deformation [3]. Fractal theory thus provides a new and effective method for characterizing complex structure of the engineering materials. The theory of fractals is considered a basis for quantitative description by means of the fractal dimension of various structures.

An extremely disordered morphology, such as surface roughness and porous media having the self-similarity property, is scrutinized by fractal geometry. This implies that the morphology stays similar in magnification over a broad range. Another significant attribute of natural fracture is that their formation needs the supply of a large amount of energy externally [3]. If microstructure formation is preferentially caused by a phenomena taking place outside of thermodynamic equilibrium, they are also characterized by fractal property. This implies that the description of highly disordered microstructures on the basis of conventional approach is not sufficient [3]. Thus, most of the objects that occur in nature are disordered and irregular, and they do not follow the Euclidean illustration because of the scale-dependent measures of length, area, and volume [4, 5]. Examples of such objects are the surfaces of mountains, coastlines, microstructure of metals, and so on. These objects are termed fractals and are illustrated by a non-integral dimension known as fractal dimension [1]. The fractal property is a physical property expressed at the super-molecular level, at a microscopic scale, and at a macroscopic scale.

The phenomenon of fractal is ubiquitous in a wide array of materials, such as the fracture of nanoparticle composites [6–8], the growth of crystal [9–12], the quasicrystal structure [13], the fracture of martensite morphology [14, 15], the porous materials [16–19], and the deposited film [20–25]. These materials are of uncommon class of disordered materials and usually show complex microstructures. Fractal theory has been widely used in many fields of modern science since it was presented by Mandelbrot [1] in 1982. It has been used in studying permeability of porous media [17, 26–28], dual-porosity medium [29, 30], evaluating dislocation structure [31], simulating the failure of concrete [32–35], analyzing fracture surfaces or network [36, 37], and thermal conductivity performance [38]. Fractal has also been used to establish the morphology of highly irregular objects imbedded onto two- and three-dimensional spaces and is defined as two- and three-dimensional fractal dimensions [39].

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2. Fractal structure

Fractal structure is a structure that is characterized with self-similarity, that is, it is composed of such fragments whose structural motif is repeated if the scale changes. Fractal structure outlined the degree of occupancy of a structure in a space (dimension), which is not an integer value. Therefore, n-dimensional fractal occupies an intermediate position that lies between the n-dimensional and $(n + 1)$-dimensional objects. Recursive functions are used to construct a fractal object. An important characteristic of fractal structure is the scale independence [40]. Thus, fractal structures do not have a single length scale, and fractal processes (e.g., time series) cannot be characterized by a single time scale [41]. Fractal structures are associated to rough or fragmented geometric structures [42]. The complexity of a fractal structure is described by its fractal dimension; this is greater than the topological dimension. It is much easier to obtain fractal dimension from datasets by using fractal analysis, for example, digital images, obtained from the investigation of natural phenomena, and from theoretical models. Different techniques to perform fractal analysis include box-counting, lacunarity analysis, multifractal analysis, and mass methods. An interesting application of fractal analysis is the description of fractured surfaces [43]. Mandelbrot et al. [42] have shown that fractured surfaces are fractal. Zhang [44] reported a quadratic polynomial relationship between the rock burst tendency and fractal dimension of fracture surface. A fractal dimension threshold of $\hat{d}_f$ was found, and there was a positive correlativity between the rock burst tendency index and the fractal dimension when $d_f \le \hat{d}_f$, an inverse correlativity when $d_f \le \hat{d}_f$. In the investigation of fractured surfaces, Liang and Wu investigated the relationship between the fracture surface fractal dimension and the impact strength of polypropylene nanocomposites. A strong correlation was observed, and it indicated that the fracture surface of the composites was fractal, and the relationship between the impact strength and fractal dimension of the composites obeyed roughly exponential function [7]. Lung et al. have also demonstrated that there is a relation between the roughness and the fractal dimension of the surface [45].

3. Fractal analysis

Fractal analysis is defined as a contemporary method of applying non-traditional mathematics to patterns that defy understanding with traditional Euclidean concepts. It means assessing the fractal description of data, and it is a common technique to study a variety of problems. It consists of different methods assigned to a fractal dimension and other fractal characteristics to a dataset. It, in essence, measures complexity using fractal dimension. In fractal analysis, other different parameters can also be assessed [43], for instance, lacunarity and succolarity, and can be used to classify and segment images [46]. Whatever type of fractal analysis has to be done, it always rests on some kind of fractal dimension. In fractal analysis, complexity is a change in detail with change in scale. The simplest form of fractal dimension is described using the relation in Eq. (1).

$$N = \frac{1}{S^D} \tag{1}$$

where N is the number of self-similar "pieces," S is the linear scaling factor (sizes) of the pieces to the whole, D is the dimension that characterizes the (invariant) relationship between size and number. Rearranging the elements in Eq. (1), one can solve for D.

$$D = -\left(\frac{\log N}{\log S}\right) \tag{2}$$

D is an algebraic equation, that is, Eq. (1) can give a dimension, which is the concept of geometry, not algebra. Let's say, one-dimensional line is cut into pieces, each of which is a fraction (S) of the original line, like making $S = 1/4$. For example, one-dimensional line can be cut into pieces such that each one-fourth will be the size of the original line, then N will be equal to four little lines. Then one can say that Eq. (1) gives the line a fractal dimension $D = 1$, because $N = 1/\left(\frac{1}{4}\right)^1$. If a two-dimensional square area is cut into pieces, the side of which is one-fourth the size of the original square, then $N = 16$ little squares. Eq. (1) will then tell that the area of the square has a fractal dimension $D = 2$, because $N = 1/\left(\frac{1}{4}\right)^2$. If a three-dimensional cube volume is cut into pieces, such that the side of which is one-fourth the size of the original cube, then N will be equal to 64 little cubes. Eq. (1) then tells that the volume of the cube has a fractal dimension of $D = 3$, because $N = 1/\left(\frac{1}{4}\right)^3$. No matter the value of S, N will still be found as $1/S^D$pieces when one-, two-, and three-dimensional objects have been cut into pieces. Thus, Eq. (1) gives the correct fractal dimension for one-dimensional line, two-dimensional area, and three-dimensional volume [47].

3.1. Concepts of the fractal dimension

The ratio that gives statistical index of intricacy and compares how detailed a shape (fractal pattern) changes with the scale at which it is measured is called fractal dimension. It is sometimes identified by a measure of the space filling volume of a pattern that states how a fractal scale is different from the space it is rooted in; a fractal dimension is not always an integer [48–50]. There are several different concepts of the fractal dimension of a geometrical configuration [5].

There are several ways of measuring length-related fractal dimensions. Mandelbrot [51] first proposed the concept of a fractal dimension to describe structures, which look the same at all length scales. His concept takes into account the measuring of the perimeter of an object with several lengths of rulers (spans or calipers) (using a trace method). For a fractal object, the plot of the log of the perimeter against the log of the ruler lengths will give a straight line with a negative slope S. This plot will then result to $D = 1 - S$ [52]. Although this is mainly mathematical concept, many examples in nature that can be closely approximated to fractal objects are available for only over a particular range of scale. The likes of these objects are generally named self-similar in order to indicate their scale invariant structure. The common attribute of such objects is that their length (for a curve object, otherwise it could be the area or volume)

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mainly rests on the length scale used for measuring it, and the fractal dimension provides the exact nature of this reliance [53]. Fractal dimensions (D) are numbers used to quantify these properties [5]. In fractal geometry [1], the fractal dimension, D, is given as:

$$D = \lim_{r \to 0} \frac{\log(N_r)}{\log(1/r)} \tag{3}$$

This is a statistical quantity that shows how a fractal totally fills the space when viewed at finer scales.

The second concept was proven by Pentland on the basis that the image of a fractal object is also a fractal [54], which has made scientific investigations on the methods of estimating the fractal dimensions of images. Many researchers have put great efforts into this field of fractal geometry, and many methods for estimating fractal dimensions of certain objects have been proposed since the establishment of fractal geometry theory. Typical methods of this concept involve the use of spectral analysis and box-counting. Usually, spectral analysis method applies fast Fourier transformation (FFT) to image in order to obtain the coefficients and mean spectral energy density. The fractal dimension can be evaluated by analyzing the power law reliance of spectral energy density and the square size [55]. The box-counting method is the widely used method for calculating fractal dimensions in the natural sciences; this is called box-counting dimension. It is a method based on the concept of "covering" the border, it is also known as the grid method. Sets of square boxes (i.e., grids) are used here in order to cover the border. Each set is represented by a box size. The number of boxes essential to cover the border is considered a function of the box size. **Figure 1** is an example of the log of the number of covering boxes of each size times the length of a box edge plotted against the log of the length of a box edge. Furthermore, a straight line with slope S which is equal to the dimension D will be obtained [52]. The slope is defined as the amount of change along the Y-axis, divided by the amount of change along the X-axis. Any result with a steeper slope shows that the object is more "fractal," which means it gains in complexity as the box size reduces. Any result with a lower-valued slope shows that the object is closer to a straight line, less "fractal," and that the amount of detail does not grow as quickly with an increase in magnification. Again, the 3-D space containing a specific object, partitioned into boxes of a certain size and how many boxes could fill up the object, is also accounted for. With the use of ratio r in Eq. (1), in order to decide the box size, the box-counting method will account for the total number of boxes (i.e., Nr of Eq. (1)) that are needed to form the object. The fractal dimension D of Eq. (1) can then be estimated from the least square linear fit of $\log(Nr)$ versus $\log(1/r)$ by counting Nr for different scaling ratio r [56].

Several traditional box-counting methods have been used for the calculation of the fractal dimensions of images, this includes differential box-counting (DBC) method [57], Chen et al.'s approach [58], the reticular cell counting method [59], Feng's method [60], and so on. DBC method has been proved to have better performance than other methods [61]. Many analyses have been done in order to improve the DBC method [62–64].

A third concept was developed by Flook [65], and the method of this concept is called the dilation method. Dilation, in this case, means a widening and smoothing of the border. This

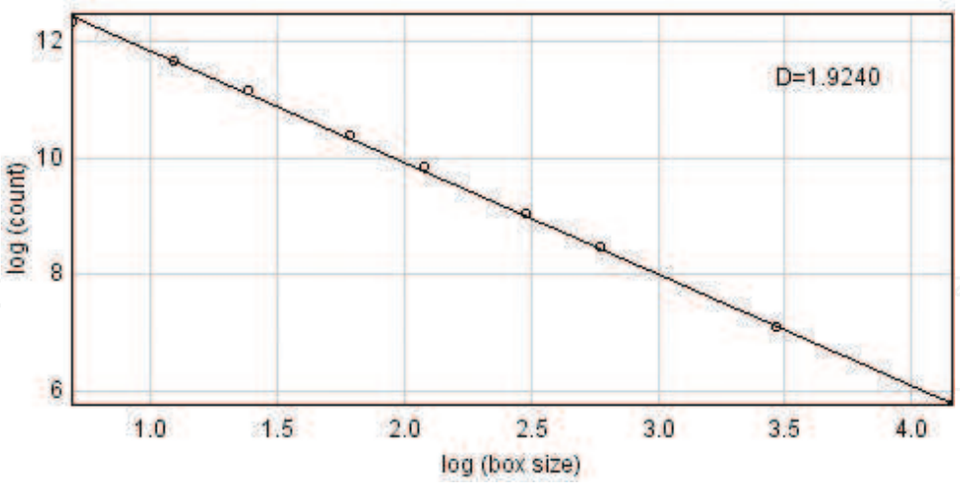

Figure 1. Example of fractal dimension of a material.

can be accomplished by convolution operation with a binary disk, that is, all the non-zero components of all the convolution kernels have a (Boolean) unitary value. The result is a thickened, but grey-scale border. All non-zero pixels are thresholded to a Boolean one when returning this border to Boolean one values. The speed at which the total surface area of the border raises as a function of the diameter of the convolution kernel relies on the dimension *D*. The log of each resulting area, divided by the kernel diameter, is plotted against the log of the kernel diameter [52]. A straight line results with a negative slope *S*, and *D* can be further estimated.

3.2. Power law scaling relationship

A functional relationship between two quantities is known as power law. This relationship takes place when a relative change in one quantity results in a proportional change in the other quantity and independent of the initial size of those quantities; thus, one quantity varies as power law to another. The characteristic of fractals is known as power law scaling. Therefore, a relationship, which yields a straight line on log-log coordinates, can often identify an object or phenomena as fractal. Although not all power law relationships are due to fractals, an observer needs to consider the existence of such relationship in order to know if the system is self-similar [66]. Self-similarities indicate the existence of scaling relationship which implies the type of a relationship called "power law." Thus, self-similarities give rise to the power law scaling. The power law scalings are shown as a straight line when the logarithm of the measurement is plotted against the logarithm of the scale at which it is measured. Fractal dimension is based on self-similarities; thus, power law scaling can be used to determine the fractal dimension. For a set to achieve the complexity and irregularity of a fractal, the number of self-similar pieces must be related to their size by power law [47]. The power law scaling describes how the property $L(r)$ of the system depends on the scale r at which it is measured using the relation in Eq. (4).

$$L(r) = Ar^{\alpha} \tag{4}$$

The fractal dimension describes how the number of pieces of a system depends on the scale r, using the relation in Eq. (5).

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$$N(r) = Br^{-D} \tag{5}$$

where B is a constant. The similarity between Eqs. (4) and (5) means that one can determine the fractal dimension D from the scaling exponent α if one knows how the measure property $L(r)$ depends on the number of pieces $N(r)$. For example, for each little square of sides of an object with two-dimensional area, the surface area is proportional to r^2. Thus, one can determine the fractal dimension of the exterior of such an object by showing that the scaling relationship of the surface area depends on the scale r. For example, to determine the fractal dimension D from the scaling exponent is to derive the function of the dimension $f(D)$, such that the property measured is proportional to $r^{f(D)}$ [66]. If the experimentally determined scaling of the measured property is proportional to r^{α}, then the power of the scale r can be equated to the relation in Eq. (6):

$$f(D) = \alpha \tag{6}$$

Then, one can solve for D.

4. Fractal characteristics of porous media

Porous media include many manmade as well as natural materials. All solid substances are in fact porous either to some degree or at some length scale [67]. A porous medium is a randomly multi-connected medium with channels randomly obstructed. The quantity that measures how "holed" the medium is due to the presence of these channels, and it is called the porosity of the medium. A pore network description can represent the porous medium as an ensemble of pores and throats of different geometries and sizes that can take values from appropriate distributions [68]. Therefore, fractal theory gives a favorable layer of structures of different models that will address the complexity of the disordered, heterogeneous, and hierarchical porous media like soil, materials with fracture networks. Theoretically, Yu et al. [69] provided an overview of the physical properties of ideal fractal porous media and explained how natural heterogeneous materials can exhibit both mass- and pore-fractal scaling. Cihan et al. [70] reported new analytical models for predicting the saturated hydraulic conductivity based on the Menger sponge mass fractal. They tested their model predictions against lattice Boltzmann simulations of flow performed in different configurations of the Menger sponge.

Fractal models have been used to describe the solid volume, the pore volume, or the interface between the two phases of porous media. In the past three decades, fractal models of pore space were developed and used in the petroleum physics with application in hydraulic system and in engineering communities with the application electrical conductivity [71, 72]. Turcotte [73] proposed a fractal fragmentation model, which identified a physical basis for the existence of fractal soils in the scale invariance of the fragmentation of soil particles. Hence, his model elucidated a mechanism in which scale-independent fragmentation processes could form fractal distribution of particles, giving theoretical legitimacy to the study of fractal models on porous media. Fragmentation can be viewed as the chief mechanism of physical weathering [67].

4.1. Characteristic method of porous media

Hunt [67] stated that model characteristics are defined so that the porosity and water retention functions are identical to those of the discrete and explicit fractal model of Rieu and Sposito [74] (called hereafter the RS model). They began with a description of virtual pore size fractions in a porous medium that permits a facile foundation that conceptualized the fractal of solid matrix and pore space. These concepts resulted in equations used in solving the porosity and bulk density of both the size fractions and the porous medium in terms of a characteristic fractal dimension, D.

A porous medium with a porosity from a broad range of pore sizes was considered, the porosity decreases in mean (or median) diameter from $p_0 - p_{m-1} (m \geq 1)$. A bulk element of the porous medium has the volume V_0, massive enough to contain all sizes of pore; it has porosity ϕ and the dry bulk density σ_0. They divided the pore-volume distribution of V_0 mathematically into m virtual pore size fractions, with the ith virtual size fraction defined by:

$$P_i \equiv V_i - V_{i+1} \ (i = 0, \ldots, m-1) \tag{7}$$

where P_i represents the volume of pores entirely made of size p_i contained in V_i which is the ith partial volume of the porous medium, which itself has all pores of size $\leq p_i$. The partial volume V_{i+1} is therefore incorporated in V_i and the partial volume V_{m-1} is then incorporated in the smallest pore-size fraction P_{m-1}, together with the residual solid volume symbolized by V_m. They stated that the solid material whose volume is V_m will not be chemically or mineralogically homogeneous. Its mass density, symbolized by σ_m, represents an average "primary particle" density. Eq. (7) gives the bulk volume of the porous medium which can mathematically be represented as the summation of m increments of the basic pore-size fraction $P_0 - P_{m-1}$ added to a residual solid volume V_m:

$$V_0 = \sum_{i=1}^{m-1} P_i + V_m \tag{8}$$

The porosity of the medium can then be given as [74]:

$$\phi \equiv \frac{(V_0 - N^m V_m)}{V_0} \tag{9}$$

$$= 1 - (1 - \Gamma)^m \tag{10}$$

Going by the fractal dimension of Eq. (3), proposed by Mandelbrot [1], the fractal dimension is related closely to the pore coefficient, Γ [74].

$$\Gamma = 1 - Nr^3 \tag{11}$$

which, with Eq. (3), results to:

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$$\Gamma = 1 - r^{3-D} \ (\Gamma < 1,\ r < 1) \tag{12}$$

It was shown from Eq. (12) that in a fractal porous medium where pore sizes are scaled by the same ratio $r < 1$, the fractal dimension increases with the decrease in the magnitude of the pore coefficient [74]. Thus, the relation between the porosity and the fractal dimension from Eqs. (10) and (12) gives:

$$\phi = 1 - (r^{3-D})^m \tag{13}$$

For a given value of the exponent m, the porosity of a fractal porous medium decreases as the fractal dimension increases. Further, Eq. (13) shows that the fractal dimension of a porous medium must be < 3.

Moreover, integration over the continuous pore size distribution between qr and r, where q is the ratio of radii of successive pore classes in fractal soil, r is the pore radius, $q < 1$ is an arbitrary factor, yields the contribution to the porosity from each size class obtained by RS model. The distribution of pore sizes is defined by the following probability density function [75]:

$$W(r) = \frac{3 - D_p}{r_m^{3-D_p}} r^{-1-D_p} r_0 \le r \le r_m \tag{14}$$

where r_0 and r_m refer explicitly to the minimum and maximum pore radii, respectively. The power law distribution of pore sizes is bounded by a maximum radius, r_m, and truncated at the minimum radius, r_0. Eq. (14), as written, is compatible with a volume, r^3, for a pore of radius r and D_p describes the pore space. The result for the total porosity derived from Eq. (14) is given in [75] as:

$$\phi = \frac{3 - D_p}{r_m^{3-D_p}} \int_{r_0}^{r_m} r^3 r^{-1-D_p} dr = 1 - \left(\frac{r_0}{r_m}\right)^{3-D_p} \tag{15}$$

Eq. (15) is exactly as in RS (Eq. (13)). If a particular geometry for the pore shape is envisioned, it is possible to change the normalization factor to maintain the result for the porosity and also maintain the correspondence to RS [67].

4.2. Fractal analysis on the permeability of porous media

The fluid flow through porous media is governed by geometrical properties, such as porosity, properties of the flowing fluid, the connection and the tortuosity of the pore space.

The transport phenomena in porous media, that include single-phase and multiphase fluid flow through porous media, electrical and acoustical transport in porous media, and heat transfer in porous media, are focused on common interests and have emerged as a separate field of study [76–79]. A matrix of a porous medium combined with fractured networks is

called the dual-porosity medium. In the dual-porosity media, fracture and matrix are generally considered as different media, each with its own property. Thus, gas flow through these dual-porosity media could consist of two physically distinguished migration processes: one is associated to the movement of gas through the larger-scale fractures, that is, a permeability flow, which can be described by Darcy law, the other is related to the movement of gas inside matrix blocks, that is, diffusion processes, which may be involved in several different mechanisms, subject to the pore size [30, 74, 80].

In reality, surfaces of capillaries are rough and have great impact on fluid flow behavior and permeability of a porous medium. Analytically, permeability expression is a function of the relative roughness, the tortuosity fractal dimensions, capillaries sizes, and surface roughness, together with the microstructural parameters (such as the characteristic length, the maximum and minimum pore diameters, and the fractal dimensions) [19].

4.3. Methods of fractal analysis on the permeability of porous media

Fractal, multifractal, Gaussian, and log-normal models have been initiated, perhaps in all scale range. The validation of unchanging theoretical framework used in calculating transport properties, at least at some scales, has the capacity of eliminating much confusion regarding the appropriate theoretical approaches used and the appropriate model to choose [67]. Investigation on gas flow through a dual-porosity medium, for example, a flow domain made up of matrix blocks (with low permeability) implanted in a network of fractures, is not common. Physical and computer modeling are commonly used for permeability of porous media. Different methods of analysis on the permeability of porous media will be discussed in this section.

Zheng and Yu [30] studied the permeability of a gas with the use of matrix porous media embedded with randomly distributed fractal-like tree networks. The scientific expression for gas permeability in dual-porosity media was obtained based on the pore size of matrix and the mother channel diameter of embedded fractal-like tree networks having fractal distribution. It was discovered that gas permeability was a function of structural parameters, which includes the fractal dimensions for pore area and tortuous capillaries, porosity and the maximum diameter of matrix, the length ratio, the diameter ratio, the branching levels, and angle of the embedded networks used for dual-porosity media. The model that was initiated does not contain any empirical constant. The model predictions were validated with the available experimental data and simulating results, a fair agreement among them was found. An analysis of the influences of geometrical parameters on the gas permeability in the media was done.

Khlaifat et al. [80] experimentally studied a single-phase gas flow through fractured porous media of tight sand formation of Travis Peak Formation under different operation conditions. Their study enhanced gas recovery from low permeability reservoirs by the creation of a single fracture perpendicular to the flow direction. The porous medium sample that was taken into account was a slot-pore-type tight sand from the Travis Peak Formation with permeability in the microdarcy range and a porosity of 7%. A number of single-phase experiments that include water and gas were performed at different pressure drops conditions ranging from 100 to 600 psig and at overburden pressures of 2000, 3000, and 4000 psig, respectively. It was shown from

their results that the sample used was very sensitive to overburden pressure. Again, it was shown from the experimental data that the presence of a fracture in a low permeability porous media is the main factor responsible for reinforcing the gas recovery from tight gas reservoirs. The presence of a fracture reinforces the gas flow, due to the increase in overall permeability and the creation of different flow patterns, which locally shifted the two-phase flow away from capillary force domination region. Furthermore, the fracture aperture played a significant role in enhancing flow due to both reconfigurations of connecting pores and joining of the non-connecting pores to the flow network.

A well-testing technique for Devonian shale gas reservoirs characterized by a low storage and high flow-capacity natural fractures fed by a high storage, low flow-capacity rock matrix was developed by Kucuk and Sawye [81] by using analytical methods and numerical simulator. They developed analytical solutions in order to analyze the basic fractured reservoir measurable factors that influence well productivity. These measurable factors are fracture system porosity and permeability, matrix porosity and permeability, and matrix size. They found that the traditional way of testing the well does not usually work for fractured Devonian shale gas reservoirs. Most of the time, the two parallel straight lines with a vertical separation are not shown in the semi-log plot of the drawdown and build-up data. They further found that the inter-porosity flow parameter is not the only parameter, which characterizes the nature of semi-log straight line.

A permeability model assumed to be comprised of a bundle of tortuous capillaries whose size distribution and roughness of surfaces follow the fractal scaling laws has been derived for porous media [19]. The proposed model includes the effects of the fractal dimensions for size distributions of capillaries, for tortuosity of tortuous capillaries, and for roughness of surfaces on the permeability. The proposed model is given by Eq. (16):

$$K_R = \frac{\pi L_0^{1-D_T} D_f \lambda_{\max}^{3+D_T}}{128A(3+D_T-D_f)}(1-\varepsilon)^4 \tag{16}$$

where K_R denotes the permeability for flow in porous media with roughened surfaces. Eq. (16) indicates that the permeability is a function of the relative roughness ε, the fractal dimensions D_T (the fractal dimension for tortuosity of tortuous capillaries) and D_f (the fractal dimension for pore space), as well as the structural parameters A (cross-sectional area), L_0 (the representative length or straight line along the flow direction of a capillary), and $\lambda_{\max}$ (maximum capillary diameter). Eq. (16) also shows that the higher the relative roughness, the lower the permeability value; this can be explained by saying that the flow resistance is increased with the increase in roughness. This is consistent with a physical situation [19].

The proposed Eq. (16) was found to be a function of the relative roughness ε, the fractal dimension D_T for tortuosity of tortuous capillaries, and structural parameters A, L_0, and $\lambda_{\max}$. The ratio of the permeability for rough capillaries to that for smooth capillaries follows the quadruplicate power law of $(1-\varepsilon)$ given by Eq. (17). That is, Eq. (17) indicated that the decrease of permeability for porous media with roughened surfaces in capillaries follows the quadruplicate power law of $(1-\varepsilon)$. The authors concluded that the permeability of porous media with roughened capillaries will be drastically decreased with the increase in relative

roughness, and the proposed model can reveal more mechanisms that affect the flow resistance in porous media than conventional models [19]. K in Eq. (17) is the permeability of porous media with smooth capillaries.

$$\eta = K_R/K = (1-\varepsilon)^4 \tag{17}$$

Zinovik and Poulikakos [82] evaluated the relationships between porosity and permeability for a set of fractal models with porosity approaching unity and a finite permeability. Prefractal tube bundles generated by finite iterations of the corresponding geometric fractals can be used as a model porous medium where permeability-porosity relationships are derived analytically as explicit algebraic equations. Their investigation showed that the tube bundles generated by finite iterations of the corresponding geometric fractals can be used to model porous media where the permeability-porosity relationships are derived analytically. It was further shown that the model of prefractal tube bundles can be used to obtain fitting curves of the permeability of high porosity metal foams and to provide insight on permeability-porosity correlations of the capillary model of porous media.

All the methods discussed here have shown that the permeability of a porous media is strongly affected by its local geometry and connectivity, the matrix size of the material, and the pores available for flow. All the methods gave concept and knowledge of fractal geometry in relation to the characterization of the porous structure with respect to the permeability of the porous media.

5. Conclusion

Fractal is considered a self-similar system. It has been confirmed that the fractal technique is a powerful technique that has been successfully used in the characterization of the geometric and structural properties of fractal surfaces and pore structures of porous materials. It gives an understanding on how the geometry affects the physical and chemical properties of systems since their complex patterns are better described in terms of fractal geometry if the self-similarity is satisfied. It also builds a bridge between micro-morphology and macro performance. This chapter shows that the structural and functional characters of porous materials depend on the pore structure, which can be described effectively by the fractal theory.

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